The University of the State of New York

328TH HIGH SCHOOL EXAMINATION

TRIGONOMETRY

Wednesday, August 22, 1956 — 12 m. to 3 p.m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (a) names of schools where you have studied, (b) number of weeks and recitations a week in trigonometry previous to entering summer high school, (c) number of recitations in this subject attended in summer high school of 1956.

The minimum time requirement is four or five recitations a week for half a school year. The summer school session will be considered the equivalent of one semester's work during the regular session (four or five recitations a week for half a school year).

For those pupils who have met the time requirement the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1956 is required.

Part II

Answer three questions from this part. Show all work unless otherwise directed.

21 *A* Derive the law of sines. [Consider only the case in which the triangle is acute.] [6]

B Indicate the correct completion for *each* of the following by writing the letter *a, b* or *c* after the proper number on your answer paper: [2, 2]

(1) Using the data $A = 125°$, $b = 18$ and $a = 15$, it is possible to construct (a) two triangles (b) only one triangle (c) no triangle

(2) In triangle ABC, $A = 20°$ and $B = 40°$. The value of the ratio $a : b$ to the *nearest hundredth* is (a) .53 (b) .52 (c) .50

22 Prove the identities:

a $\cos^2 A \tan A = \dfrac{2 \sin A}{\sec A + \cos A + \sin^2 A \sec A}$ [5]

b $\dfrac{\cos 2A - \cos A}{2 \cos A + 1} = \cos A - 1$ [5]

23 Find, to the *nearest degree,* all values of x between 0° and 180° which satisfy the equation: $2 \sec^2 x - \tan x - 5 = 0$. [10]

24 *a* On the same set of axes *sketch* the graphs of $y = \frac{1}{2} \sin x$ and $y = 2 \cos x$ as x varies from 0 to 2π radians. [4, 4]

b From the graphs made in answer to part *a*, determine the number of values of x between 0 and 2π radians which satisfy the equation $\frac{1}{2} \sin x = 2 \cos x$. [2]

[1] [OVER]

25 List the numbers 1–5 on your answer paper, and tell whether *each* statement is always true, sometimes true or never true by writing the word *always, sometimes* or *never* opposite the appropriate number on your answer paper. [10]

In each of the following, A and B are positive acute angles:

(1) $\tan \dfrac{A}{2} \cot \dfrac{A}{2} = 1$

(2) $\sec^2 \dfrac{A}{3} - \tan^2 \dfrac{A}{3} = 1$

(3) $\sin B + \cos B = 1$

(4) $\sec A \csc A = 1$

(5) $\sin(A + B) = 1$ when $\cos A = m$ and $\sin B = m$

Part III

Answer two questions from this part. Show all work.

26 In triangle ABC, $a = 384$, $b = 288$ and $c = 562$. Find C to the *nearest ten minutes*. [10]

27 A surveyor runs a level base line along the bank of a stream from point A to point B, a distance of 250 feet. From A and B he sights C, a point on the opposite shore. Angle $BAC = 28° 40'$ and angle $ABC = 43° 30'$. Find, to the *nearest foot*, the width of the river at C. [4, 6]

28 In triangle ABC, $a = 344$, $b = 708$ and $A = 16° 20'$. Find B to the *nearest ten minutes*. [10]

29 An airplane flies 220 miles from airfield A to airfield B on a course N 57° 30' E. From B it flies a course S 42° 50' E to C, a distance of 380 miles. If the plane returns to A directly from C, what course must it follow, to the *nearest ten minutes?* [5, 4, 1]

[2]

Fill in the following lines:

Name of pupil..Name of school..

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 Express sin 215° as a function of a positive acute angle. 1................

2 Through how many radians does the minute hand of a clock rotate in 40 minutes? 2................

3 An angle of 2.5 radians at the center of a circle intercepts an arc whose length is 10. Find the radius of the circle. 3................

4 Find the positive value of $\cot(\cos^{-1}\frac{1}{2})$. 4................

5 Angle A is in the first quadrant. Express sin A in terms of tan A. 5................

6 If tan $x = 2$ and tan $y = 5$, find $\tan(x + y)$. 6................

7 In triangle ABC, sin $A = .3$, sin $B = .8$ and $b = 12$. Find a. 7................

8 In triangle ABC, $a = 4$, $b = 7$ and $c = 8$. Find the cosine of the largest angle of the triangle. 8................

9 If x is a positive acute angle and sin $x = \dfrac{3}{\sqrt{10}}$, find sin $2x$. 9................

10 Find log cos 35° 27′. 10................

11 Find the number whose logarithm is $9.2846 - 10$. 11................

12 Find tan 63° 38′. 12................

13 In isosceles triangle ABC, vertex angle C equals 106° and AB equals 20. Find, to the *nearest tenth,* the altitude to AB. 13................

14 In triangle ABC, $a = 10$, $b = 4$ and $C = 31° 20′$. Find the area of the triangle to the *nearest integer.* 14................

15 In triangle ABC, $a = 24$, $b = 8$ and $C = 96°$. Find tan $\frac{1}{2}(A - B)$ to the *nearest hundredth.* 15................

[3] [OVER]

Directions (16–20): Indicate the correct completion for *each* of the following by writing on the line at the right the letter *a, b* or *c*.

16 The minimum value of $\frac{1}{2}\sin 4x$ is　(*a*) —2　(*b*) —1　(*c*) —$\frac{1}{2}$　16. . . . .

17 The expression $\dfrac{\sin 33° + \sin 3°}{\cos 33° + \cos 3°}$ equals　(*a*) tan 36°

(*b*) tan 18°　　(*c*) cot 15°　　　　　　　　　　　　17. . . . .

18 Cos (—320°) equals (*a*) cos 40°　(*b*) cos 140°　(*c*) cos 220°　18. . . . .

19 Cos 3*A* cos *A* + sin 3*A* sin *A* equals　　(*a*) cos 4*A*　　(*b*) sin 2*A*
(*c*) cos 2*A*　　　　　　　　　　　　　　　　　　　　19. . . . .

20 As angle *x* increases from 180° to 270°, the value of sin 2*x*
(*a*) increases　　(*b*) increases and then decreases　　(*c*) decreases and then
increases　　　　　　　　　　　　　　　　　　　　　20. . . . .

[4]

FOR TEACHERS ONLY

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INSTRUCTIONS FOR RATING
TRIGONOMETRY

Wednesday, August 22, 1956 — 12 m. to 3 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow $2\frac{1}{2}$ credits for each correct answer; allow no partial credit. Do not allow credit if the answers to questions 10 and 12 are not expressed to four decimal places. For questions 16–20, allow credit if the pupil has written the correct answer instead of the letter *a, b* or *c*.

(1) $-\sin 35°$ *or* $-\cos 55°$

(2) $\dfrac{4}{3}\pi$

(3) 4

(4) $\dfrac{1}{\sqrt{3}}$ *or* $\dfrac{\sqrt{3}}{3}$

(5) $\dfrac{\tan A}{\sqrt{\tan^2 A + 1}}$

(6) $-\frac{7}{9}$

(7) 4.5

(8) $\frac{1}{56}$

(9) $\frac{3}{5}$

(10) $9.9110 - 10$ *or* $\bar{1}.9110$

(11) 0.1926

(12) 2.0175

(13) 7.5

(14) 10

(15) 0.45

(16) *c*

(17) *b*

(18) *a*

(19) *c*

(20) *b*