The University of the State of New York
The State Education Department

EXAMINATION IN EXPERIMENTAL TWELFTH YEAR MATHEMATICS

June 1962

Name of pupil..... Name of school.....

Part I

Answer twenty-five of the thirty questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answer on the line at the right.

Questions 1-9: Write the number preceding the expression that best completes the statement or answers the question.

1 If d represents the statement "f is a differentiable function" and c represents the statement "f is a continuous function", which symbolic statement claims that "f is a differentiable function only if it is continuous"?

(1) $c \longrightarrow d$
(2) $d \longrightarrow \sim c$
(3) $c \longleftrightarrow d$
(4) $\sim c \longrightarrow \sim d$
(5) $\sim c \longrightarrow d$ 1.....

2 Which of the following is a tautology?

(1) $[(p \wedge q) \vee p] \longleftrightarrow (p \vee q)$

(2) $(\sim p \vee q) \longleftrightarrow (p \longrightarrow q)$

(3) $[\sim(p \longrightarrow q)] \longleftrightarrow (\sim p \longrightarrow \sim q)$

(4) $[(p \wedge q) \vee \sim p] \longleftrightarrow q$

(5) $[(p \longrightarrow q) \vee (q \longrightarrow p)] \longleftrightarrow (p \longleftrightarrow q)$ 2.....

3 The negation of $\forall_x \exists_y (x = y + 2)$ is equivalent to

(1) $\forall_x \forall_y (x \neq y + 2)$

(2) $\forall_y \exists_x (y = x + 2)$

(3) $\exists_x \exists_y (x \neq y + 2)$

(4) $\exists_x \forall_y (x \neq y + 2)$

(5) $\exists_y \forall_x (x = y + 2)$ 3.....

4 The converse of $\sim p \longrightarrow q$ is equivalent to

(1) $p \longrightarrow \sim q$
(2) $p \longrightarrow q$
(3) $\sim q \longrightarrow p$
(4) $q \longrightarrow p$
(5) $\sim p \longrightarrow \sim q$ 4.....

5 Given that $\sim p \longrightarrow q$ and $\sim r \longrightarrow \sim q$ are accepted statements. Select the statement that is <u>not</u> a valid conclusion.

(1) $\sim p \longrightarrow r$
(2) $\sim r \longrightarrow p$
(3) $\sim p \wedge r$
(4) $p \vee r$
(5) $\sim (\sim p \wedge \sim r)$ 5.....

6 In the Venn diagram, A, B and C are the interiors of the circles lying within the rectangle I. The shaded area is represented by

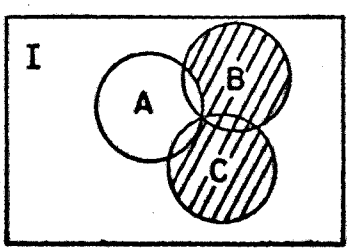
(1) A'

(2) $A \cup (B \cap C)$

(3) $A' \cap (B \cup C)$

(4) $(B \cap C)'$

(5) $A \cap (B \cup C)'$



6.....

7 Which statement is implied by the condition that $A \cap B'$ is the empty set?

(1) $A \cup B'$ is the empty set.
(2) $A' \cap B$ is the empty set.
(3) $A \supset B$.
(4) $A \subset B$.
(5) $A = B$. 7.....

8 Which of the following is a rational integral function of x?

(1) $x + \frac{3}{x}$

(2) $2x^3 + x^{\frac{2}{3}} - 7$

(3) $\sqrt{x^2 - 9}$

(4) $x^{-4} + 6x^{-2} + 9$

(5) $\frac{\pi x^2}{2} - 3x\sqrt{3}$ 8.....

9 To establish that a set $G = \{x, y, z, \ldots\}$ and an operation indicated by * form a group, certain properties must be noted. Select the condition that is <u>not</u> necessary.

(1) $\exists_e \forall_x (e * x = x)$

(2) $\forall_x \exists_{x'} (x * x' = e)$

(3) $\forall_x \forall_y (x * y = y * x)$

(4) $\forall_x \forall_y \forall_z [(x * y) * z = x * (y * z)]$

(5) $\forall_x \forall_y \exists_z (z = x * y)$ 9.....

10 List the elements in the set $A \cap B$ if $A = \{1, 3, 5\}$ and $B = \{x \mid (0 < x < 10) \wedge (x \text{ is prime})\}$. 10.....

11 If $F(x) = x^2 + 2$ and $G(x) = 1 - \frac{1}{x}$, write the expression $G[F(x)]$ in terms of x. 11.....

12 What real value of x can not be included in the domain of f if f is complex and given by the rule $f(x) = \frac{3}{\sqrt{5 - x}}$? 12.....

13 If $f(x) = \frac{x^2 - x}{x^2 - 1}$ for $x \neq 1$ and if f(x) is to be continuous at x = 1, how must f(1) be defined? 13.....

14 Find the value of k if (x + 1) is a factor of $x^8 + kx^3 - 2x + 1$. 14.....

15 Write $[\sqrt{2}\ (\cos 45^\circ + i \sin 45^\circ)]^3$ in the form a + bi. 15.....

16 The n nth roots of unity form a group under multiplication. For n = 4, find the inverse element of -i. 16.....

17 Using the ordered pair (x,y) as an equivalent expression for $\frac{x}{y}$, write as an ordered pair the sum of (3,8) and (2,3). 17.....

18 Find the value of x if $\log(x + 1) + \log(x - 3) = \log 5$. 18.....

19 The membership of a club numbers twelve persons, including three officers. How many committees of four persons may be selected if at least one committee member must be an officer? 19.....

20 Find the probability of rolling a "seven" or an "eleven" on a single roll of a pair of dice. 20.....

21 Evaluate: $\sum_{n=1}^{5} (2n - 1)^2$. 21.....

22 Find the smallest positive integer which satisfies the congruence $2x + 4 \equiv 0 \bmod 11$. 22.....

23 If f is a nonconstant function defined so that $V_x V_y[f(x + y) = f(x) \cdot f(y)]$, determine the value of f(0). 23.....

24 Evaluate: $\lim_{h \to 0} \frac{(4 - x - h)^2 - (4 - x)^2}{h}$ 24.....

25 Find the average rate of change of y with respect to x of the function $y = x^3 - 2x - 5$ over the interval from x = 1 to x = 3. 25.....

26 A tangent to the curve $y = x^2 - 6x + 8$ has a slope of 4. Find the coordinates of the point at which the tangent touches the curve. 26.....

27 A particle moves along a line in such a way that its distance S in feet from the starting point at the end of t seconds is given by the equation $S = 3t^2 - 5t$. Find the velocity in feet per second at the end of 3 seconds. 27.....

28 Find the length of the line segment joining the points of intersection of the graphs of $x^2 - y^2 = 9$ and x = 5. 28.....

29 Find the coordinates of the midpoint of the line segment joining A(4,3,-6) and B(6,-1,0). 29.....

30 The surface area of a certain sphere is 100 square units. What is the surface area of a sphere having a radius measuring twice the radius of the given sphere? 30.....

Part II

Answer five questions from this part.

31 Find to the nearest tenth the real root of the equation $2x^3 - 3x - 3 = 0$. [10]

32 Prove by mathematical induction that, if n is a positive integer, then 3 is a factor of $n^3 + 2n$. [10]

33 a Graph $\{(x,y) \mid (2y - x - 4)(2y + x + 4) = 0\}$. [4]

b Graph $\{(x,y) \mid (2y - x - 4)(2y + x + 4) > 0\}$. [6]

34 A function f is defined for all real numbers by the equation $f(x) = Ax^2 + Bx + C$ where A, B and C are certain constants.

a Write an equation which expresses the fact that the graph of f passes through the point (-1,-15). [2]

b Now express the fact that the graph of f has a relative maximum at the point (2,3). [You will need two equations.] [4]

c Using the results of parts a and b, determine the values of A, B and C, given that the graph of f passes through the point (-1,-15) and has a relative maximum at the point (2,3). [4]

35 An open can has the form of a right circular cylinder and contains 16π cubic inches. Find the radius of the base and the height of the can that will require the least amount of material. [Leave answers in radical form.] [10]

36 a Sketch the portion of the surface $\{(x,y,z) \mid 8x + 20y + 15z = 120\}$ that lies in the first octant, labeling the coordinates of the intercepts. [4]

b Find the volume of the pyramid formed by this surface and the coordinate planes. [3]

c If a wire model of this pyramid is to be made, find to the nearest tenth the total length of wire needed. [3]

37 A container holds six cards, on each of which is painted one of the digits 0,1,2,3,4,5; no digit appears on more than one card. A card is drawn at random and the digit x appearing on it is recorded. From the remaining five cards, another is drawn at random and the digit y appearing on it is recorded.

a List the elements of the sample space OR graph the sample space for this experiment. [4]

b Find the probabilities for the following events: [2,2,2]

$E_1 = \{(x,y) \mid x = 5 \vee y = 5\}$

$E_2 = \{(x,y) \mid x + y = 5\}$

$E_3 = \{(x,y) \mid x + y < 5\}$

38 A function f is defined for positive real numbers by the expression: $f(x) = x^2 - 2x + 2 \ (x > 0)$

a What is the domain of f? [1]

b Sketch the graph of f. [4]

c Describe the range of f. [2]

d Sketch the graph of f^{-1}. [2]

e Is f^{-1} a function? [1]

FOR TEACHERS ONLY

INSTRUCTIONS FOR RATING

EXAMINATION IN EXPERIMENTAL TWELFTH YEAR MATHEMATICS

June 1962

Part I

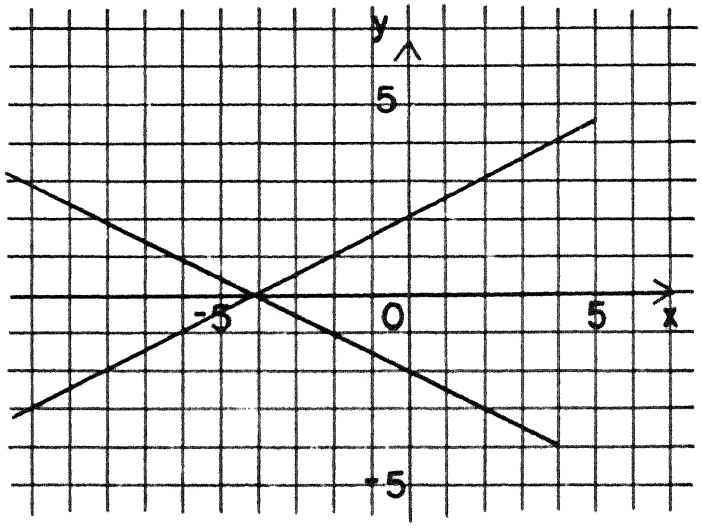
Allow 50 credits, 2 credits for each of 25 of the following:

(1) 4
(2) 2
(3) 4
(4) 1
(5) 3
(6) 3
(7) 4
(8) 5
(9) 3
(10) $\{3,5\}$
(11) $1 - \frac{1}{x^2+2}$
(12) 5
(13) $\frac{1}{2}$
(14) 4
(15) $-2+2i$
(16) i
(17) (25,24)
(18) 4
(19) 369
(20) $\frac{2}{9}$
(21) 165
(22) 9
(23) 1
(24) $-8+2x$
(25) 11
(26) (5,3)
(27) 13
(28) 8
(29) (5,1,-3)
(30) 400

Part II

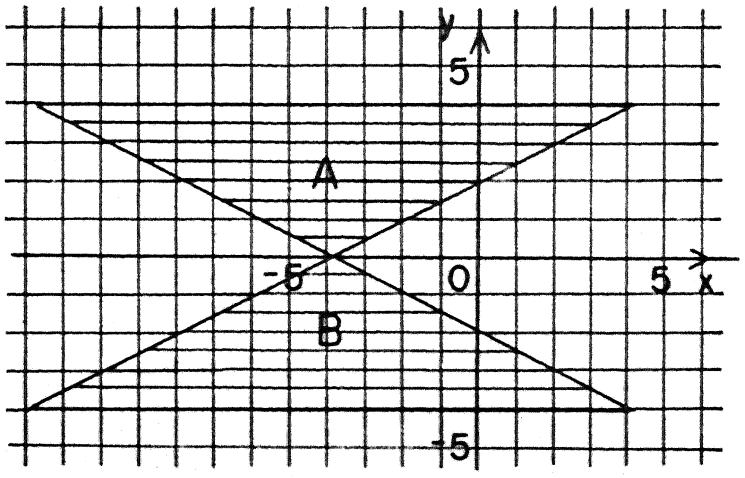
(31) 1.6 [10]

(33) a



[4]

b



[6]

Shaded area, regions A and B, not including the boundary lines

34 a $A - B + C = -15$ [2]

b $4A + B = 0$
$4A + 2B + C = 3$ [4]

c $A = -2,\ B = 8,\ C = -5$ [4]

35 $r = \sqrt[3]{16},\ h = \sqrt[3]{16}$ [10]

36 a The coordinates of the intercepts are (15,0,0), (0,6,0), (0,0,8). [4]

b 120 [3]

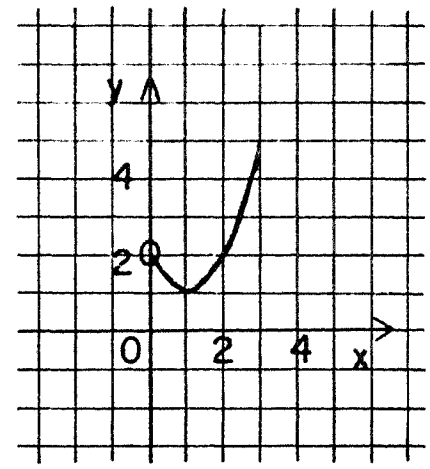
c 72.2 [3]

37 a There are 30 elements. [4]

b $\frac{1}{3}, \frac{1}{5}, \frac{2}{5}$ [2,2,2]

38 a Domain: {Positive real numbers} [1]

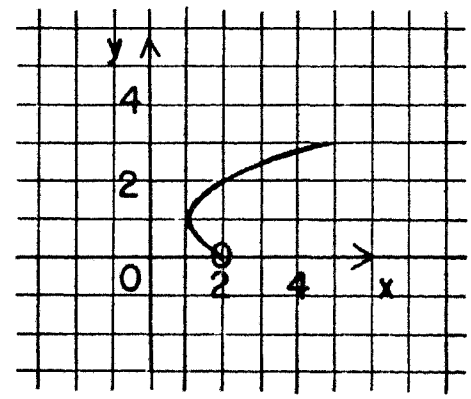
b



[4]

c Range: $\{x \mid x \geq 1\}$ [2]

d



[2]

e No [1]