POLYNOMIALS
Operations with Polynomials

Common Core Standard	Next Generation Standard
A-APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply poly-nomials.	**AI-A.APR.1** Add, subtract, and multiply polynomials and recognize that the result of the operation is also a pol-ynomial. This forms a system analogous to the integers. **Note: This standard is a fluency recommendation for Algebra I. Fluency in adding, subtracting and multi-plying polynomials supports students throughout their work in algebra, as well as in their symbolic work with functions.**

LEARNING OBJECTIVES

Students will be able to:

1) add, subtact, and multiply polynomials.

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson - activate students' prior knowledge - vocabulary - learning objective(s) - big ideas: direct instruction - modeling	guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work - developing essential skills - Regents exam questions - formative assessment assignment (exit slip, explain the math, or journal entry)

VOCABULARY

- **Polynomial:**   A monomial or the sum of two or more monomials whose exponents are positive.
  Example: $5a^2 + ba - 3$
- **Monomial:**   A polynomial with one term; it is a number, a variable, or the product of a number (the coefficient) and one or more variables
  Examples: $-\dfrac{1}{4}$, x^2, $4a^2b$, -1.2, $m^2n^3p^4$
- **Binomial:**   An algebraic expression consisting of two terms
  Example $(5a + 6)$
- **Trinomial:**   A polynomial with exactly three terms.
  Example $(a^2 + 2a - 3)$
- **Like Terms:**   Like terms must have **exactly the same base and the same exponent**.   Their coefficients may be different.   Real numbers are like terms.
  Example: Given the expression

$$1x^2 + 2y + 3x^2 + 4x + 5x^3 + 6y^2 + 7y + 8x^3 + 9y^2,$$
the following are like terms:
$$1x^2 \text{ and } 3x^2$$
$$2y \text{ and } 7y$$
4x has no other like terms in the expression
$$5x^3 \text{ and } 8x^3$$
$$6y^2 \text{ and } 9y^2$$
Like terms in the same expression can be combined by adding their coefficients.
$$1x^2 \text{ and } 3x^2 = 4x^2$$
$$2y \text{ and } 7y = 9y$$
4x has no other like terms in the expression = 4x
$$5x^3 \text{ and } 8x^3 = 13x^3$$
$$6y^2 \text{ and } 9y^2 = 15y^2$$
$$1x^2 + 2y + 3x^2 + 4x + 5x^3 + 6y^2 + 7y + 8x^3 + 9y^2 = 4x^2 + 9y + 4x + 13x^3 + 15y^2$$

BIG IDEAS

Adding and Subtracting Polynomials

To add or subtract polynomials, arrange the polynomials one above the other with like terms in the same columns.   Then, add or subtract the coefficients of the like terms in each column and write a new expression.

Addition Example	**Subtraction** Example
Add: $\left(3r^4 - 9r^3 - 8\right) + \left(4r^4 + 8r^3 - 8\right)$	Subtract: $\left(3r^4 - 9r^3 - 8\right) - \left(4r^4 + 8r^3 - 8\right)$
$\begin{array}{ccc} 3r^4 & -9r^3 & -8 \\ \underline{4r^4} & \underline{+8r^3} & \underline{-8} \\ 7r^4 & -r^3 & -16 \end{array}$	$\begin{array}{ccc} 3r^4 & -9r^3 & -8 \\ \underline{-\left(4r^4\right)} & \underline{-\left(+8r^3\right)} & \underline{-(-8)} \\ -1r^4 & -17\ r^3 & +0 \end{array}$

Multiplying Polynomials

To multiply two polynomials, multiply each term in the first polynomial by each term in the second polynomial, then combine like terms.

Example:

Multiply: $\left(-8r^2 - 9r + 7\right)\left(-5r + 1\right)$

STEP 1: Multiply the first term in the first polynomial by each term in the second polynomial, as follows:

$$-8r^2\left(-5r+1\right)$$
$$-8r^2\left(-5r\right) + -8r^2\left(1\right)$$
$$\boxed{40r^3 - 8r^2}$$

STEP 2. Multiply the next term in the first polynomial by each term in the second polynomial, as follows:

$$-9r\left(-5r+1\right)$$
$$-9r\left(-5r\right)+-9r\left(1\right)$$
$$\boxed{45r^2\text{-}9r}$$

STEP 3. Multiply the next term in the first polynomial by each term in the second polynomial, as follows:
$$7\left(-5r+1\right)$$
$$7\left(-5r\right)+7\left(1\right)$$
$$\boxed{\text{-}35r+7}$$

STEP 4. Combine like terms from each step.
$$40r^3 \text{ -}8r^2 +45r^2 \text{ -}9r \text{ -}35r +7$$
$$\boxed{40r^3 \quad +37r^2 \quad \quad \text{-}44r \quad +7}$$

DEVELOPING ESSENTIAL SKILLS

1. When $3g^2 - 4g + 2$ is subtracted from $7g^2 + 5g - 1$, the difference is
    a. $-4g^2 - 9g + 3$
    b. $4g^2 + g + 1$
    c. $4g^2 + 9g - 3$
    d. $10g^2 + g + 1$

2. When $4x^2 + 7x - 5$ is subtracted from $9x^2 - 2x + 3$, the result is
    a. $5x^2 + 5x - 2$
    b. $5x^2 - 9x + 8$
    c. $-5x^2 + 5x - 2$
    d. $-5x^2 + 9x - 8$

3. The sum of $4x^3 + 6x^2 + 2x - 3$ and $3x^3 + 3x^2 - 5x - 5$ is
    a. $7x^3 + 3x^2 - 3x - 8$
    b. $7x^3 + 3x^2 + 7x + 2$
    c. $7x^3 + 9x^2 - 3x - 8$
    d. $7x^6 + 9x^4 - 3x^2 - 8$

4. What is the result when $2x^2 + 3xy - 6$ is subtracted from $x^2 - 7xy + 2$?
    a. $-x^2 - 10xy + 8$
    b. $x^2 + 10xy - 8$
    c. $-x^2 - 4xy - 4$
    d. $x^2 - 4xy - 4$

5. When $5x + 4y$ is subtracted from $5x - 4y$, the difference is
    a. 0
    b. $10x$
    c. $8y$
    d. $-8y$

6. What is the sum of $-3x^2 - 7x + 9$ and $-5x^2 + 6x - 4$?
    a. $-8x^2 - x + 5$
    b. $-8x^4 - x + 5$
    c. $-8x^2 - 13x + 13$
    d. $-8x^4 - 13x^2 + 13$

7. When $8x^2 + 3x + 2$ is subtracted from $9x^2 - 3x - 4$, the result is
    a. $x^2 - 2$
    b. $17x^2 - 2$
    c. $-x^2 + 6x + 6$
    d. $x^2 - 6x - 6$

8. The sum of $3x^2 + 5x - 6$ and $-x^2 + 3x + 9$ is
    a. $2x^2 + 8x - 15$
    b. $2x^2 + 8x + 3$
    c. $2x^4 + 8x^2 + 3$
    d. $4x^2 + 2x - 15$

9. When $2x^2 - 3x + 2$ is subtracted from $4x^2 - 5x + 2$, the result is
    a. $2x^2 - 2x$
    c. $-2x^2 - 8x + 4$

b. $-2x^2 + 2x$
d. $2x^2 - 8x + 4$

10. The sum of $8n^2 - 3n + 10$ and $-3n^2 - 6n - 7$ is
   a. $5n^2 - 9n + 3$
   c. $-11n^2 - 9n - 17$
   b. $5n^2 - 3n - 17$
   d. $-11n^2 - 3n + 3$

11. What is the result when $4x^2 - 17x + 36$ is subtracted from $2x^2 - 5x + 25$?
   a. $6x^2 - 22x + 61$
   c. $-2x^2 - 22x + 61$
   b. $2x^2 - 12x + 11$
   d. $-2x^2 + 12x - 11$

12. When $6x^2 - 4x + 3$ is subtracted from $3x^2 - 2x + 3$, the result is
   a. $3x^2 - 2x$
   c. $3x^2 - 6x + 6$
   b. $-3x^2 + 2x$
   d. $-3x^2 - 6x + 6$

13. What is the product of $(c + 8)$ and $(c - 5)$?
   a. $c^2 + 3c - 40$
   c. $c^2 + 13c - 40$
   b. $c^2 - 3c - 40$
   d. $c^2 - 40$

14. The expression $\left(a^2 + b^2\right)^2$ is equivalent to
   a. $a^4 + b^4$
   c. $a^4 + 2a^2b^2 + b^4$
   b. $a^4 + a^2b^2 + b^4$
   d. $a^4 + 4a^2b^2 + b^4$

15. The expression $(x - 6)^2$ is equivalent to
   a. $x^2 - 36$
   c. $x^2 - 12x + 36$
   b. $x^2 + 36$
   d. $x^2 + 12x + 36$

16. The length of a rectangle is represented by $x^2 + 3x + 2$, and the width is represented by $4x$. Express the perimeter of the rectangle as a trinomial. Express the area of the rectangle as a trinomial.

17. What is the product of $(3x + 2)$ and $(x - 7)$?
   a. $3x^2 - 14$
   c. $3x^2 - 19x - 14$
   b. $3x^2 - 5x - 14$
   d. $3x^2 - 23x - 14$

18. What is the product of $-3x^2y$ and $(5xy^2 + xy)$?
   a. $-15x^3y^3 - 3x^3y^2$
   c. $-15x^2y^2 - 3x^2y$
   b. $-15x^3y^3 - 3x^3y$
   d. $-15x^3y^3 + xy$

Answers

1. ANS: C
2. ANS: B
3. ANS: C
4. ANS: A
5. ANS: D
6. ANS: A
7. ANS: D
8. ANS: B
9. ANS: A
10. ANS: A
11. ANS: D
12. ANS: B
13. ANS: A
14. ANS: C
15. ANS: C

16. ANS:
$$P = 2(x^2 + 3x + 2) + 2(4x) = 2x^2 + 6x + 4 + 8x = 2x^2 + 14x + 4$$
$$A = 4x(x^2 + 3x + 2) = 4x^3 + 12x^2 + 8x$$

17. ANS: C
18. ANS: A

A.APR.A.1: Operations with Polynomials

330) If $A = 3x^2 + 5x - 6$ and $B = -2x^2 - 6x + 7$, then $A - B$ equals
   1) $-5x^2 - 11x + 13$ 3) $-5x^2 - x + 1$
   2) $5x^2 + 11x - 13$ 4) $5x^2 - x + 1$

331) Express the product of $2x^2 + 7x - 10$ and $x + 5$ in standard form.

332) Fred is given a rectangular piece of paper. If the length of Fred's piece of paper is represented by $2x - 6$ and the width is represented by $3x - 5$, then the paper has a total area represented by
   1) $5x - 11$ 3) $10x - 22$
   2) $6x^2 - 28x + 30$ 4) $6x^2 - 6x - 11$

333) Subtract $5x^2 + 2x - 11$ from $3x^2 + 8x - 7$. Express the result as a trinomial.

334) If the difference $(3x^2 - 2x + 5) - (x^2 + 3x - 2)$ is multiplied by $\frac{1}{2}x^2$, what is the result, written in standard form?

335) Which trinomial is equivalent to $3(x - 2)^2 - 2(x - 1)$?
   1) $3x^2 - 2x - 10$ 3) $3x^2 - 14x + 10$
   2) $3x^2 - 2x - 14$ 4) $3x^2 - 14x + 14$

336) When $(2x - 3)^2$ is subtracted from $5x^2$, the result is
   1) $x^2 - 12x - 9$ 3) $x^2 + 12x - 9$
   2) $x^2 - 12x + 9$ 4) $x^2 + 12x + 9$

337) The expression $3(x^2 - 1) - (x^2 - 7x + 10)$ is equivalent to
   1) $2x^2 - 7x + 7$ 3) $2x^2 - 7x + 9$
   2) $2x^2 + 7x - 13$ 4) $2x^2 + 7x - 11$

338) What is the product of $2x + 3$ and $4x^2 - 5x + 6$?
   1) $8x^3 - 2x^2 + 3x + 18$ 3) $8x^3 + 2x^2 - 3x + 18$
   2) $8x^3 - 2x^2 - 3x + 18$ 4) $8x^3 + 2x^2 + 3x + 18$

339) Which expression is equivalent to $2(3g - 4) - (8g + 3)$?
   1) $-2g - 1$ 3) $-2g - 7$
   2) $-2g - 5$ 4) $-2g - 11$

340) Express in simplest form: $(3x^2 + 4x - 8) - (-2x^2 + 4x + 2)$

341) Write the expression $5x + 4x^2(2x + 7) - 6x^2 - 9x$ as a polynomial in standard form.

342) Which polynomial is twice the sum of $4x^2 - x + 1$ and $-6x^2 + x - 4$?
   1) $-2x^2 - 3$ 3) $-4x^2 - 6$
   2) $-4x^2 - 3$ 4) $-2x^2 + x - 5$

<u>SOLUTIONS</u>

330) ANS: 2
   Strategy: To subtract, change the signs of the subtrahend and add.

Given: $3x^2 + 5x - 6$ $-\left(-2x^2 - 6x + 7\right)$	Change the signs and add: $3x^2 + 5x - 6$ $+2x^2 + 6x - 7$ $5x^2 + 11x - 13$

   PTS: 2   NAT: A.APR.A.1   TOP: Addition and Subtraction of Polynomials
   KEY: subtraction

331) ANS:
   $2x^3 + 17x^2 + 25x - 50$

   Strategy: Use the distribution property to multiply polynomials, then simplify.

   STEP 1.   Use the distributive property
   $$(2x^2 + 7x - 10)(x + 5)$$
   $$2x^3 + 10x^2 + 7x^2 + 35x - 10x - 50$$
   $$2x^3 + 17x^2 + 25x - 50$$

   STEP 2.   Simplify by combining like terms.
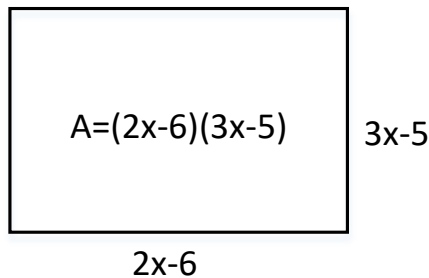 $$2x^3 + 10x^2 + 7x^2 + 35x - 10x - 50$$
   $$2x^3 + 17x^2 + 25x - 50$$

   PTS: 2   NAT: A.APR.A.1   TOP: Multiplication of Polynomials

332) ANS: 2
   Strategy: Draw a picture and use the area formula for a rectange: $A = lw$.

   A=(2x-6)(3x-5)   3x-5

   2x-6

   $$A = (2x - 6)(3x - 5)$$
   $$A = 6x^2 - 10x - 18x + 30$$
   $$A = 6x^2 - 28x + 30$$

   PTS: 2   NAT: A.APR.A.1   TOP: Multiplication of Polynomials

333) ANS:

Strategy: To subtract, change the signs of the subtrahend and add.

Given: $3x^2 + 8x - 7$ $-\left(5x^2 + 2x - 11\right)$	Change the signs and add: $3x^2 + 8x - 7$ $\underline{-5x^2 - 2x + 11}$ $- 2x^2 + 6x + 4$

PTS: 2   NAT: A.APR.A.1   TOP: Addition and Subtraction of Polynomials
KEY: subtraction

334) ANS:

$$x^4 - \frac{5}{2}x^3 + \frac{7}{2}x^2$$

Strategy.   First, find the difference between $(3x^2 - 2x + 5) - (x^2 + 3x - 2)$, the use the distributive property to multiply the difference by $\frac{1}{2}x^2$.   Simplify as necessary.

STEP 1.   Find the difference between $(3x^2 - 2x + 5) - (x^2 + 3x - 2)$.   To subtract polynomials, change the signs of the subtrahend and add.

Given: $(3x^2 - 2x + 5)$ $\underline{-(x^2 + 3x - 2)}$	Change the signs and add: $3x^2 - 2x + 5$ $\underline{-x^2 - 3x + 2}$ $2x^2 - 5x + 7$

STEP 2.   Multiply $2x^2 - 5x + 7$ by $\frac{1}{2}x^2$.

$$\frac{1}{2}x^2\left(2x^2 - 5x + 7\right)$$

$$x^4 - \frac{5}{2}x^3 + \frac{7}{2}x^2$$

PTS: 2   NAT: A.APR.A.1   TOP: Operations with Polynomials
KEY: multiplication

335) ANS: 4
Strategy: Expand and simplify the expression $3(x - 2)^2 - 2(x - 1)$

STEP 1   Expand the expression.

$$3(x^2 - 4x + 4) - 2(x - 1)$$

$$3x^2 - 12x + 12 - 2x + 2$$

STEP 2:   Simplify the expanded expression by combining like terms.

$$3x^2 - 12x + 12 - 2x + 2$$

$$3x^2 - 14x + 14$$

PTS: 2   NAT: A.APR.A.1   TOP: Operations with Polynomials
KEY: mixed

336) ANS: 3
Strategy: Expand the binomial, then subtract it from $5x^2$.

$$5x^2 - (2x - 3)^2$$

$$5x^2 - (2x - 3)(2x - 3)$$

$$5x^2 - (4x^2 - 6x - 6x + 9)$$

$$5x^2 - (4x^2 - 12x + 9)$$

$$5x^2 - 4x^2 + 12x - 9$$

$$x^2 + 12x - 9$$

PTS: 2   NAT: A.APR.A.1   TOP: Operations with Polynomials
KEY: multiplication

337) ANS: 2

$$3(x^2 - 1) - (x^2 - 7x + 10)$$

$$3x^2 - 3 - x^2 + 7x - 10$$

$$2x^2 + 7x - 13$$

PTS: 2   NAT: A.APR.A.1   TOP: Operations with Polynomials
KEY: subtraction

338) ANS: 3
Strategy: Use the distributive property

$$(2x + 3)\left(4x^2 - 5x + 6\right)$$

$$8x^3 - 10x^2 + 12x + 12x^2 - 15x + 18$$

$$8x^3 \left(-10x^2 + 12x^2\right)(+12x - 15x) + 18$$

$$8x^3 + 2x^2 - 3x + 18$$

PTS: 2   NAT: A.APR.A.1

339) ANS: 4

Given	$2(3g - 4) - (8g + 3)$
Distributive Property	$6g - 8 - 8g - 3$
Combine Like Terms	-2g-11

PTS: 2   NAT: A.APR.A.1   TOP: Operations with Polynomials
KEY: subtraction

340) ANS:
$$5x^2 - 10$$

$$3x^2 + 4x - 8$$
$$\underline{-(-2x^2 + 4x + 2)}$$
$$5x^2 \qquad - 10$$

PTS: 2   NAT: A.APR.A.1   TOP: Operations with Polynomials
KEY: subtraction

341) ANS:

$$5x + 4x^2(2x + 7) - 6x^2 - 9x$$
$$5x + 8x^3 + 28x^2 - 6x^2 - 9x$$
$$8x^3 + 28x^2 - 6x^2 - 9x + 5x$$
$$8x^3 + 22x^2 - 4x \text{ Answer}$$

PTS: 2   NAT: A.APR.A.1   TOP: Operations with Polynomials
KEY: multiplication

342) ANS: 3
STEP 1. Solve for the sum of $4x^2 - x + 1$ and $-6x^2 + x - 4$.
$$4x^2 - x + 1$$
$$\underline{-6x^2 + x - 4}$$
$$-2x^2 \qquad - 3$$
STEP 2. Solve for twice the sum of $-2x^2 - 3$.
$$2(-2x^2 - 3) = -4x^2 - 6$$

PTS: 2   NAT: A.APR.A.1   TOP: Operations with Polynomials
KEY: addition