H – Quadratics, Lesson 3, Modeling Quadratics (r. 2018)

QUADRATICS
Modeling Quadratics

Common Core Standard	**Next Generation Standard**
A-CED.1 Create equations and inequalities in one variable ~~and use them to solve problems~~. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* PARCC: Tasks are limited to linear, quadratic, or exponential equations with integer exponents.	**AI-A.CED.1** Create equations and inequalities in one variable to **represent a real-world context**. (Shared standard with Algebra II) Notes: • **This is strictly the development of the model (equation/inequality).** • **Limit equations to linear, quadratic, and exponentials of the form** $f(x) = a(b)^x$ **where** $a > 0$ **and** $b > 0$ ($b \neq 1$). • **Work with geometric sequences may involve an exponential equation/formula of the form** $a_n = ar^{n-1}$, **where a is the first term and r is the common ratio.** • **Inequalities are limited to linear inequalities.** • Algebra I tasks do not involve compound inequalities.

NOTE: This lesson is related to Expressions and Equations, Lesson 4, Modeling Linear Equations

LEARNING OBJECTIVES

Students will be able to:

model quadratic equations that reflect real-world contexts, including:

a. product of consecutive integer contexts,
b. product of ages contexts, and
c. squared number contexts.

Overview of Lesson

Teacher Centered Introduction	**Student Centered Activities**
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

consecutive integers   consecutive odd integers   consecutive even integers

BIG IDEAS

General Approach

The general approach to modelling quadratics is:

1. Read and understand the entire problem.
2. Underline key words, focusing on variables, operations, and equalities or inequalities.
3. Convert the key words to mathematical notation (consider meaningful variable names other than x and y).
4. Write the final expression or equation.
5. Check the final expression or equation for reasonableness.

Product of Consecutive Integer Problems: The key to solving *product* of consecutive integer problems is also defining the variables.

Typical Problem in Context	Mathematical Translation	Hints and Strategies
Find three consecutive positive even integers such that the product of the second and third integers is twenty more than ten times the first integer.	6, 8, 10. Three consecutive even integers are *x*, $x+2$ and $x+4$. $(x+2)(x+4) = 10x+20$ $x^2+6x+8 = 10x+20$ $x^2-4x-12=0$ $(x-6)(x+2)=0$ $x=6$	For consecutive integer problems, define your variables as x, x + 1, etc. For consecutive *even or odd* integer problems, define your variables as x, x +2, etc.

Product of Ages Problems: The key to solving *product* of consecutive integer problems is also defining the variables.

Typical Problem in Context	Mathematical Translation	Hints and Strategies
Brian is 3 years older than Doug. The product of their ages is 40. How old is Doug?	Let d represent Doug's age. Let d+3 represent Brian's age. Let $d(d+3)=40$ represent the product of their ages. Solve for *d*. $d(d+3)=40$ $d^2+3d=40$ $d^2+3d-40=0$ $(d+8)(d-5)=0$ $d=\{-8,5\}$ Reject -8 because age cannot be negative. Doug is 5 years old.	Define your variables carefully.

Squared Number Problems:

Typical Problem in Context	Mathematical Translation	Hints and Strategies

When 36 is subtracted from the square of a number, the result is five times the number. What is the positive solution?	9 Let the square of a number be represented by x^2 Let five times the number be represented by $5x$ Write: $x^2 - 36 = 5x$ $x^2 - 5x - 36 = 0$ $(x-9)(x+4) = 0$ $x = \{-4, 9\}$ The problem says to select the positive solution.	Underline key words.

DEVELOPING ESSENTIAL SKILLS

1) Find three consecutive positive even integers such that the product of the second and third integers is twenty more than ten times the first integer. [Only an algebraic solution can receive full credit.]

2) When 36 is subtracted from the square of a number, the result is five times the number. What is the positive solution?
   1) 9
   2) 6
   3) 3
   4) 4

3) Noj is 5 years older than Jacob. The product of their ages is 84. How old is Noj?

4) The square of a positive number is 24 more than 5 times the number. What is the value of the number?

5) Find three consecutive odd integers such that the product of the first and the second exceeds the third by 8.

6) Three brothers have ages that are consecutive even integers. The product of the first and third boys' ages is 20 more than twice the second boy's age. Find the age of *each* of the three boys.

7) Tamara has two sisters. One of the sisters is 7 years older than Tamara. The other sister is 3 years younger than Tamara. The product of Tamara's sisters' ages is 24. How old is Tamara?

Answers

1) 6, 8, 10.
   Let x represent the first integer.
   Let x+2 represent the second integer.
   Let x+4 represent the third integer
   Write

$$(x+2)(x+4)=10x+20$$
$$x^2+6x+8=10x+20$$
$$x^2-4x-12=0$$
$$(x-6)(x+2)=0$$
$$x=\{-2,6\}$$

Reject the negative integer solution for x because the problem calls for a positive integer solution.

2) Let x^2 represent the square of a number.
Let 5x represent five times the number.
Write:

$$x^2-36=5x$$
$$x^2-5x-36=0$$
$$(x-9)(x+4)=0$$
$$x=\{-4,9\}$$

The positive solution is 9.

3) Let N represent Noj's age.
Let N-5 represent Jacob's age.
Write:

$$N(N-5)=84$$
$$N^2-5N-84=0$$
$$(N-12)(N+7)=0$$
$$N=\{-7,12\}$$

Reject the negative solution because age cannot be negative.
Noj is 12 years old.

4) Let x^2 represent the square of a number.
Let 5x represent 5 times the number.
Write:

$$x^2=24+5x$$
$$x^2-5x-24=0$$
$$(x-8)(x+3)=0$$
$$x=\{-3,8\}$$

Reject the negative solution
The number is 8.

5) Let x represent the first odd integer.
Let x+2 represent the second consecutive odd integer.
Let x+4 represent the third consecutive odd integer.
Write:

$$x(x+2)-(x+4)=8$$
$$x^2+2x-x-4=8$$
$$x^2+x-12=0$$
$$(x+4)(x-3)=0$$
$$x=\{-4,3\}$$

Reject the even integer solution.
The three consecutive odd integers are 3, 5, and 7

6) Let x represent the age of the first brother.
Let x+2 represent the age of the second brother.
Let x+4 represent the age of the third brother.
Write:

$$x(x+4)=20+2(x+2)$$
$$x^2+4x=20+2x+4$$
$$x^2-2x=24$$
$$x^2-2x-24=0$$
$$(x+6)(x-4)=0$$
$$x=\{-6,4\}$$

Reject the negative solution because age cannot be negative.
The ages of the three brothers are 4, 6, and 8.

7) Let x represent Tamara's age.
Let x+7 represent the age of Tamara's older sister.
Let x-3 represent the age of Tamara's younger sister.
Write:

$$(x+7)(x-3)=24$$
$$x^2+7x-3x-21=24$$
$$x^2+4x-21=24$$
$$x^2+4x-45=0$$
$$(x+9)(x-5)=0$$
$$x=\{-9,5\}$$

Reject the negative solution.
Tamara's age is 5.

REGENTS EXAM QUESTIONS (through June 2018)

A.CED.A.1: Modeling Quadratics

208) Sam and Jeremy have ages that are consecutive odd integers. The product of their ages is 783. Which equation could be used to find Jeremy's age, *j*, if he is the younger man?

1) $j^2 + 2 = 783$

2) $j^2 - 2 = 783$

3) $j^2 + 2j = 783$

4) $j^2 - 2j = 783$

209) Abigail's and Gina's ages are consecutive integers. Abigail is younger than Gina and Gina's age is represented by *x*. If the difference of the square of Gina's age and eight times Abigail's age is 17, which equation could be used to find Gina's age?

1) $(x+1)^2 - 8x = 17$

2) $(x-1)^2 - 8x = 17$

3) $x^2 - 8(x+1) = 17$

4) $x^2 - 8(x-1) = 17$

SOLUTIONS

208) ANS: 3

Strategy: Deconstruct the problem to find the information needed to write the equation.

Let *j* represent Jeremy's age. The last sentence says *j* represents Jeremy's age.

Let $\left(j+2\right)$ represent Sam's age. The problem tells us that Sam and Jeremy have ages that are **consecutive odd integers**. The consecutive odd integers that could be ages are $\{1, 3, 5, 7, 9, \ldots\ldots\}$ and each odd integer is 2 more that the odd integer before it. Thus, if Jeremy is 2 years younger than Sam, as the problem says, then Sam's age can be represented as $\left(j+2\right)$.

The second sentence says, "The product of their ages is 783." Product is the result of multiplication, so we can write $j\left(j+2\right) = 783$. Since this is not an answer choice, we must manipulate the equation:

$$j\left(j+2\right) = 783$$

$$j^2 + 2j = 783$$

Our equation is now identical to answer choice *c*, which is the correct answer.

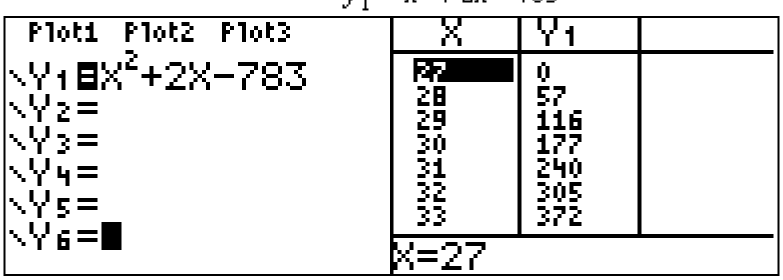
DIMS? Does It Make Sense? Yes. Jeremy is 27 and Sam is 29. The product of their ages is $27 \times 29 = 783$.
In order to input this into a graphing calculator, the equation must be transformed as follows:

$$j^2 + 2j = 783$$

$$j^2 + 2j - 783 = 0$$

$$0 = j^2 + 2j - 783$$

$$y_1 = x^2 + 2x - 783$$



PTS: 2 NAT: A.CED.A.1 TOP: Modeling Quadratics

209) ANS: 4

If Gina’s age is *x*, then Abigail’s age is $x-1$.
The square of Gina’s age is represented by x^2.
Eight times Abigail's age can be represented as $8(x-1)$.
The difference of the square of Gina's age and eight times Abigail's age is 17 can be represented as $x^2-8(x-1)=17$.
Check by solving for x, as follows:

$$\begin{aligned} x^2-8(x-1)&=17 \\ x^2-8x+8&=17 \\ x^2-8x-9&=0 \\ (x-9)(x+1)&=0 \\ x&=9 \end{aligned}$$

Gina is 9 years old and Abigail is 8 years old.

PTS: 2 NAT: A.CED.A.1 TOP: Modeling Quadratics