

I – Systems, Lesson 4, Modeling Systems of Linear Inequalities (r. 2018)

SYSTEMS

Modeling Systems of Linear Inequalities

CC Standard	NG Standard
A-CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. <i>For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.</i>	AI-A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. e.g., Represent inequalities describing nutritional and cost constraints on combinations of different foods.

LEARNING OBJECTIVES

Students will be able to:

- 1) Create a system of linear inequalities from a real-world context.

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson <ul style="list-style-type: none">- activate students' prior knowledge- vocabulary- learning objective(s)- big ideas: direct instruction- modeling	guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work <ul style="list-style-type: none">- developing essential skills- Regents exam questions- formative assessment assignment (exit slip, explain the math, or journal entry)

VOCABULARY

see key words below

BIG IDEAS

Modeling systems of linear inequalities is similar to modeling systems of linear equations, except that an inequality sign is used instead of an equal sign.

Key English Words and Their Mathematical Translations

These English Words	Usually Mean	Examples: <i>English becomes math</i>
is, are	equals	<i>the sum of 5 and x is 20 becomes $5 + x = 20$</i>
more than, greater than	inequality >	<i>x is greater than y becomes $x > y$ x is more than 5 becomes $x > 5$ 5 is more than x becomes $5 > x$</i>
greater than or equal to, a minimum of, at least	inequality ≥	<i>x is greater than or equal to y becomes the minimum of x is 5 becomes x is at least 20 becomes</i>
less than	inequality <	<i>x is less than y becomes x is less than 5 becomes 5 is less than x becomes</i>
less than or equal to, a maximum of, not more than	Inequality ≤	<i>X is less than or equal to y becomes The maximum of x is 5 becomes X is not more than becomes</i>

General Approach

The general approach is as follows:

1. Read and understand the entire problem.
2. Underline key words, focusing on variables, operations, and equalities or inequalities.
3. Convert the key words to mathematical notation (consider meaningful variable names other than x and y).
4. Write two or more inequalities with the same variables.
5. Check the final system of linear inequalities for reasonableness.

<i>Example</i>	<i>Inequalities</i>
<p>A high school drama club is putting on their annual theater production. There is a maximum of 800 tickets for the show. The costs of the tickets are \$6 before the day of the show and \$9 on the day of the show. To meet the expenses of the show, the club must sell at least \$5,000 worth of tickets.</p> <p>a) Write a system of inequalities that represent this situation. b) The club sells 440 tickets before the day of the show. Is it possible to sell enough additional tickets on the day of the show to at least meet the expenses of the show? Justify your answer.</p>	<p><i>Inequality #1.</i></p> $b + d \leq 800$ <p><i>Inequality #2.</i></p> $6b + 9d \geq 5000$
<p>Variables: Let b represent the number of tickets sold <i>before</i> the day of the show. Let d represent the number of tickets sold <i>the day</i> of the show.</p>	
<p>Solution Strategy: Substitute 440 for b in both inequalities.</p> <p><i>Inequality #1.</i></p> $b + d \leq 800$ $440 + d \leq 800$ $d \leq 360$	
<p>They can sell no more than 360 tickets on the day of the show.</p>	

Inequality #2.

$$6b + 9d \geq 5000$$

$$6(440) + 9d \geq 5000$$

$$2640 + 9d \geq 5000$$

$$9d \geq 5000 - 2640$$

$$9d \geq 2360$$

$$d \geq 262.\overline{22}$$

They need to sell at least 263 tickets on the day of the show.

Yes, it is possible to sell enough additional tickets on the day of the show to meet expenses.

NOTE: Systems of inequalities often have an infinite number of solutions. Graphs are useful to represent the solution sets for such systems. Graphing systems of inequalities is covered in [Systems](#), Lesson 5, [Graphing Systems of Linear Inequalities](#).

DEVELOPING ESSENTIAL SKILLS

Model each context below with a system of inequalities. Define the variables. *Do not solve.*

Contexts	Systems of Inequalities
Nazmun has at least \$5,000 in a savings account at the bank. Her savings account balance is more than 5 times greater than her checking account balance.	Let S represent Nazmun's savings account balance. Let C represent Nazmun's checking account balance. Write: $\begin{cases} c \geq 5000 \\ s \geq 5c \end{cases}$
The senior spirit committee is selling food to raise money for the prom. They need to raise at least \$500. A deluxe meal with dessert costs \$10. A sandwich meal with potato chips costs \$5. They have enough food to sell at most 100 meals.	Let d represent the number of <i>deluxe</i> meals. Let s represent the number of <i>sandwich</i> meals. Write: $\begin{cases} 10d + 5s \geq 500 \\ d + s \leq 100 \end{cases}$
Dr. Steve is going to Sal's Diner to buy sandwiches. A small sandwich costs \$3.50 and larger hoagie costs \$5.00. He needs to buy at least 20 sandwiches, and he can spend no more than \$88.	Let s represent the number of <i>sandwiches</i> . Let h represent the number of <i>hoagies</i> . Write: $\begin{cases} 3.5s + 5h \leq 88 \\ s + h \geq 20 \end{cases}$
The girls soccer team is doing a fundraiser for new soccer uniforms. They need to raise at least \$2,000. A local merchant has promised to donate up to 150 plain and deluxe t-shirts to help the team with their fundraiser. Plain t-shirts sell for \$8 each and fancy t-shirts sell for \$12 each.	Let p represent the number of <i>plain t-shirts</i> . Let d represent the number of <i>deluxe-shirts</i> . Write: $\begin{cases} 8p + 12d \geq 2000 \\ p + d \leq 150 \end{cases}$

<p>Tenzin is working math problems to prepare for the high stakes math exam required for graduation. He wants to work at least 200 math problems before the exam. He estimates that it will take 10 minutes to work a multiple choice problem and 15 minutes to work an open-end problem. He can spend at most 1300 minutes working math problems before the exam. Write a system of inequalities to help Tenzin decide how many multiple choice problems and how many open-end problems he should work before the exam.</p>	<p>Let x represent the number of multiple choice problems. Let y represent the number of open-end problems.</p> <p>Write: $\begin{cases} 10x + 15y \leq 1300 \\ x + y \geq 200 \end{cases}$</p>
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REGENTS EXAM QUESTIONS (through June 2018)

A.CED.A.3: Modeling Systems of Linear Inequalities

- 269) A high school drama club is putting on their annual theater production. There is a maximum of 800 tickets for the show. The costs of the tickets are \$6 before the day of the show and \$9 on the day of the show. To meet the expenses of the show, the club must sell at least \$5,000 worth of tickets.
- a) Write a system of inequalities that represent this situation.
b) The club sells 440 tickets before the day of the show. Is it possible to sell enough additional tickets on the day of the show to at least meet the expenses of the show? Justify your answer.

- 270) A drama club is selling tickets to the spring musical. The auditorium holds 200 people. Tickets cost \$12 at the door and \$8.50 if purchased in advance. The drama club has a goal of selling at least \$1000 worth of tickets to Saturday’s show.

Write a system of inequalities that can be used to model this scenario.

If 50 tickets are sold in advance, what is the minimum number of tickets that must be sold at the door so that the club meets its goal? Justify your answer.

- 271) The drama club is running a lemonade stand to raise money for its new production. A local grocery store donated cans of lemonade and bottles of water. Cans of lemonade sell for \$2 each and bottles of water sell for \$1.50 each. The club needs to raise at least \$500 to cover the cost of renting costumes. The students can accept a maximum of 360 cans and bottles. Write a system of inequalities that can be used to represent this situation. The club sells 144 cans of lemonade. What is the *least* number of bottles of water that must be sold to cover the cost of renting costumes? Justify your answer.

- 272) Jordan works for a landscape company during his summer vacation. He is paid \$12 per hour for mowing lawns and \$14 per hour for planting gardens. He can work a maximum of 40 hours per week, and would like to earn at least \$250 this week. If m represents the number of hours mowing lawns and g represents the number of hours planting gardens, which system of inequalities could be used to represent the given conditions?

- | | |
|----------------------|----------------------|
| 1) $m + g \leq 40$ | 3) $m + g \leq 40$ |
| $12m + 14g \geq 250$ | $12m + 14g \leq 250$ |

$$2) \quad m + g \geq 40$$

$$12m + 14g \leq 250$$

$$4) \quad m + g \geq 40$$

$$12m + 14g \geq 250$$

SOLUTIONS

269) ANS:

a) $Eq.1 \quad p + d \leq 800$
 $Eq.2 \quad \$6p + \$9d \geq 5000$

b) Yes, it is possible. They will need to sell 263 or more tickets on the day of the show. They have 360 tickets left.

Strategy: Write a system of equations, then use it to answer part b.

STEP 1.

Let p represent the number of tickets sold before the day of the show.

Let d represent the number of tickets sold on the day of the show.

Write: $Eq.1 \quad p + d \leq 800$
 $Eq.2 \quad \$6p + \$9d \geq \$5000$

STEP 2. Substitute 440 for p in both equations and solve.

$Eq.1 \quad p + d \leq 800$ $440 + d \leq 800$ $d \leq 800 - 440$ $d \leq 360$	$Eq.2 \quad \$6p + \$9d \geq \$5000$ $\$6(440) + \$9d \geq \$5000$ $\$2640 + \$9d \geq \$5000$ $\$9d \geq \$5000 - \$2640$ $\$9d \geq \2360 $d \geq \frac{\$2360}{\$9}$ $d \geq 262.2$
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DIMS? Does It Make Sense? Yes. They could cover their costs by selling 263 tickets and make almost \$9000 over costs if they sell 360 tickets on the day of the show.

PTS: 2 NAT: A.CED.A.3 TOP: Modeling Systems of Linear Inequalities

270) ANS:

Answer: 48 Tickets

PART 1: Write a system of inequalities.

Let D represent the number of tickets sold at the door.

Let A represent the number of tickets sold in advance.

$$12D + 8.50A \geq 1000$$

$$D + A \leq 200$$

PART 2: Solve for 50 tickets sold in advance.

$$12D + 8.50A \geq 1000$$

$$12D + 8.50(50) \geq 1000$$

$$12D + 425 \geq 1000$$

$$12D \geq 575$$

$$D \geq \frac{575}{12}$$

$$D \geq 47.916$$

The drama club needs to sell at least 48 tickets at the door to meet its goal of making \$1000.

PTS: 4 NAT: A.REI.A.2

271) ANS:

STEP 1. Write a system of inequalities.

Let L represent a can of lemondade.

Let W represent a bottle of water.

Write:

Equation 1
$2L + 1.5W \geq 500$
Equation 2
$L + W \leq 360$

STEP 2. Use Equation 1 to determine the least amount of W required when L=144.

$$2L + 1.5W \geq 500$$

$$2(144) + 1.5W \geq 500$$

$$288 + 1.5W \geq 500$$

$$1.5W \geq 212$$

$$W \geq \frac{212}{1.5}$$

$$W \geq 141.\overline{33}$$

You cannot sell $\overline{.33}$ bottles of water, so the drama club needs to sell at least

142 bottles of water

PTS: 4 NAT: A.CED.A.3 TOP: Modeling Systems of Linear Inequalities

272) ANS: 1

Strategy: Translate the words into two inequalities.

Let m represent the number of hours mowing.

Let g represent the number of hours gardening.

He is paid \$12 per hour for mowing lawns and \$14 per hour for planting gardens. He can work a maximum of 40 hours per week, and would like to earn at least \$250 this week.

Inequality #1 Hours per week.

$$m + g \leq 40$$

This inequality says:

the number of hours mowing (m) and the number of hours gardening (g) must be less than or equal to 40 hours.

Inequality #2 Money earned

$$12m + 14g \geq 250$$

This inequality says:

the money earned mowing ($12m$) and the money earned gardening ($14g$)
must be greater than or equal to \$250.