**I – Systems, Lesson 6, Quadratic-Linear Systems (r. 2018)**

# SYSTEMS
# Quadratic-Linear Systems

| **Common Core Standard** | **Next Generation Standards** |
|---|---|
| **No Current Standard in New York** | **AI-A.REI.7a** Solve a system, with rational solutions, consisting of a linear equation and a quadratic equation (parabolas only) in two variables both algebraically and graphically.<br>(Shared standard with Algebra II) |
| **A-REI.D.11** Explain why the x-coordinates of the points where the graphs of the equations $y=f(x)$ and $y=g(x)$ intersect are the solutions of the equation $f(x)=g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, ~~or find successive approximations~~. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential~~, and logarithmic functions~~.<br>~~PARCC: Tasks that assess conceptual understanding of the indicated concept may involve any of the function types mentioned in the standard except exponential and logarithmic functions. Finding the solutions approximately is limited to cases where f(x) and g(x) are polynomial functions.~~ | **AI-A.REI.11** Given the equations $y = f(x)$ and $y = g(x)$:<br>i) **recognize** that each x-coordinate of the intersection(s) is the solution to the equation $f(x) = g(x)$;<br>ii)find the solutions approximately using technology to graph the functions or make tables of values; and<br>iii) **interpret the solution in context**.<br>(Shared standard with Algebra II)<br>**Notes: Algebra I tasks are limited to cases where *f(x)* and *g(x)* are linear, polynomial, absolute value, and exponential functions of the form** $f(x) = a(b)^x$ **where** $a > 0$ **and** $b > 0$ **(**$b \neq 1$**).**<br>**Students should be taught to find the solutions approximately by using technology to graph the functions *and* by making tables of values. When solving any problem, students can choose either strategy.** |

## LEARNING OBJECTIVES

Students will be able to:

1) solve quadratic-linear systems of equations algebraically or by graphing.

**Overview of Lesson**

| **Teacher Centered Introduction** | **Student Centered Activities** |
|---|---|
| **Overview of Lesson**<br>**- activate students' prior knowledge**<br>**- vocabulary**<br>**- learning objective(s)**<br>**- big ideas: direct instruction**<br>**- modeling** | **guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work**<br>**- developing essential skills**<br>**- Regents exam questions**<br>**- formative assessment assignment (exit slip, explain the math, or journal entry)** |

## VOCABULARY

linear equation quadratic equation solution

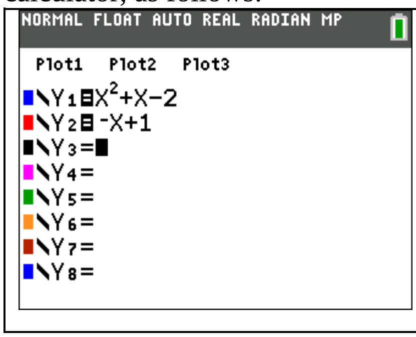
# BIG IDEAS

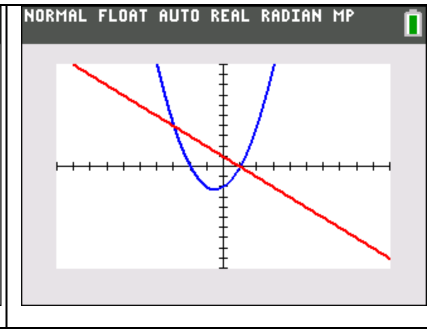
Quadratic-linear systems are solved in the same ways that systems of linear equations and/or systems of linear inequalities are solved, either algebraically or by graphing.

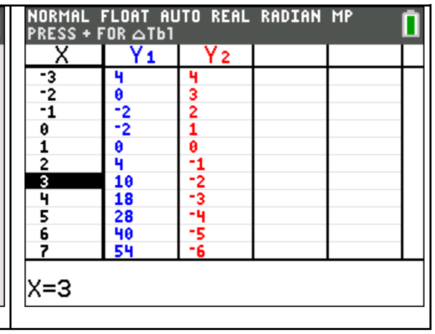
A **solution of a system** of equations makes each equation in the system true. Solutions can be found using three different views of a function. Quadratic linear systems will have:

- no solution (the graphs do not intersect),
- one solution (the graphs intersect at one point)
- two solutions (the graphs intersect at two points).

Example: If $y_1 = x^2 + x + 2$ and $y_2 = -x + 1$, then the solution may be found using a graphing calculator, as follows:




The solutions to this quadratric-linear system are (-3,4) and (1,0).

NOTE: The calculate intersection function of some graphing calculators can be used to identify solutions.

How to Solve a Quadratic Linear System Algebraically

| Step 1 | Step 2 | Step 3 | Step 4 | Step 4 |
|---|---|---|---|---|
| Isolate the same variable in both equations. | Set the opposite expressions equal to one another. | Solve for the first variable. NOTE: Strategies other than factoring can be used. | Input the solutions from Step 3 into an equation and solve for the second variable. | Write the solutions as ordered pairs. |
| $y = x^2 + 6x + 3$<br>$y = 3x + 7$ | $x^2 + 6x + 3 = 3x + 7$ | $x^2 + 6x + 3 = 3x + 7$<br>$x^2 + 3x - 4 = 0$<br>$(x+4)(x-1) = 0$<br>$x = \{-4, 1\}$ | $y = 3x + 7$<br>$y = 3(-4) + 7$<br>$y = -5$<br><br>$y = 3x + 7$<br>$y = 3(1) + 7$<br>$y = 10$ | Two solutions:<br><br>(-4,-5)<br>and<br>(1,10) |

# DEVELOPING ESSENTIAL SKILLS

Solve the following quadratic-linear systems of equations algebraically and by graphing.

| 1. | 2. | 3. | 4. | 5. |
|---|---|---|---|---|
| $y=x^2-4x+6$<br>$y=x+2$ | $y=x^2-9x+18$<br>$y=x+2$ | $y=x^2-10x+14$<br>$y=7x-16$ | $y=x^2+5x+4$<br>$y=x+4$ | $y=x^2+8x+16$<br>$y=x+6$ |

Answers

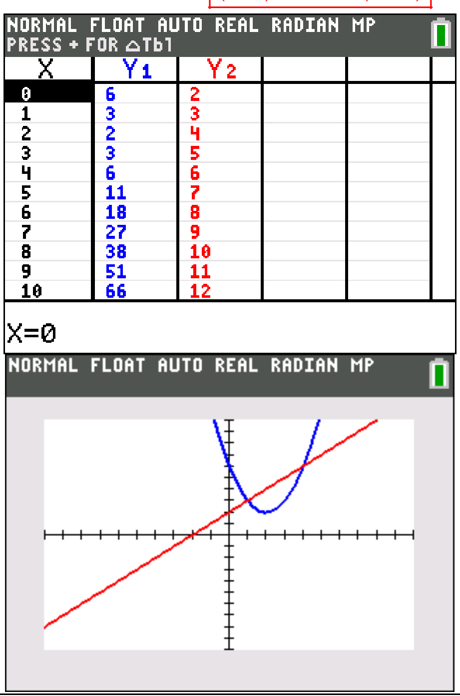
**1.**

$$y=x^2-4x+6$$
$$y=x+2$$

$$x^2-4x+6=x+2$$
$$x^2-5x+4=0$$
$$(x-4)(x-1)=0$$
$$x=\{1,4\}$$

$$y=x+2$$
$$y=(1)+2=3$$
$$y=(4)+2=6$$

$$\boxed{(1,3)\ \ and\ \ (4,6)}$$




| X | Y1 | Y2 |
|---|---|---|
| 0 | 6 | 2 |
| 1 | 3 | 3 |
| 2 | 2 | 4 |
| 3 | 3 | 5 |
| 4 | 6 | 6 |
| 5 | 11 | 7 |
| 6 | 18 | 8 |
| 7 | 27 | 9 |
| 8 | 38 | 10 |
| 9 | 51 | 11 |
| 10 | 66 | 12 |

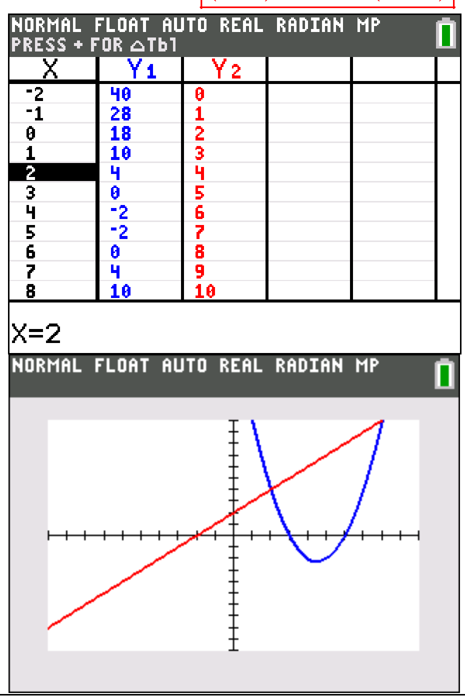
**2.**

$$y=x^2-9x+18$$
$$y=x+2$$

$$x^2-9x+18=x+2$$
$$x^2-10x+16=0$$
$$(x-8)(x-2)=0$$
$$x=\{2,8\}$$

$$y=x+2$$
$$y=(2)+2=4$$
$$y=(8)+2=10$$

$$\boxed{(2,4)\ \ and\ \ (8,10)}$$




| X | Y1 | Y2 |
|---|---|---|
| -2 | 40 | 0 |
| -1 | 28 | 1 |
| 0 | 18 | 2 |
| 1 | 10 | 3 |
| 2 | 4 | 4 |
| 3 | 0 | 5 |
| 4 | -2 | 6 |
| 5 | -2 | 7 |
| 6 | 0 | 8 |
| 7 | 4 | 9 |
| 8 | 10 | 10 |

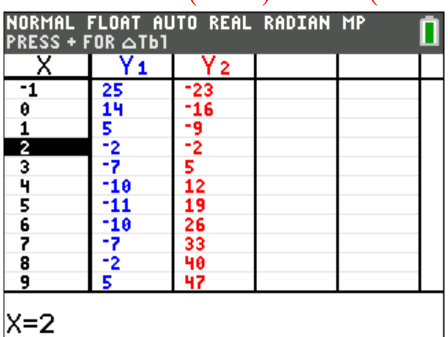
3.

$$y = x^2 - 10x + 14$$
$$y = 7x - 16$$

$$x^2 - 10x + 14 = 7x - 16$$
$$x^2 - 17x + 30 = 0$$
$$(x-15)(x-2) = 0$$
$$x = \{2, 15\}$$

$$y = 7x - 16$$
$$y = 7(2) - 16 = -2$$
$$y = 7(15) - 16 = 89$$

$$(2,-2) \quad and \quad (15,89)$$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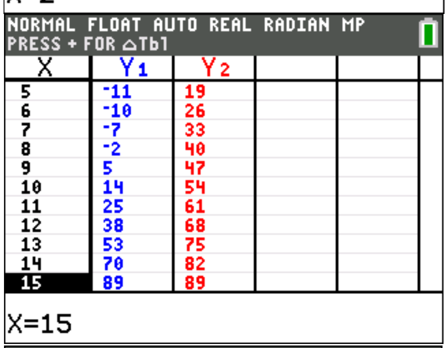
| X | Y1 | Y2 |
|---|---|---|
| -1 | 25 | -23 |
| 0 | 14 | -16 |
| 1 | 5 | -9 |
| 2 | -2 | -2 |
| 3 | -7 | 5 |
| 4 | -10 | 12 |
| 5 | -11 | 19 |
| 6 | -10 | 26 |
| 7 | -7 | 33 |
| 8 | -2 | 40 |
| 9 | 5 | 47 |

X=2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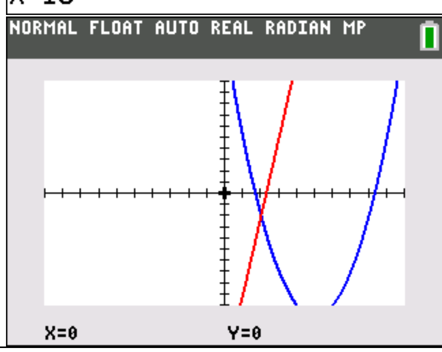
| X | Y1 | Y2 |
|---|---|---|
| 5 | -11 | 19 |
| 6 | -10 | 26 |
| 7 | -7 | 33 |
| 8 | -2 | 40 |
| 9 | 5 | 47 |
| 10 | 14 | 54 |
| 11 | 25 | 61 |
| 12 | 38 | 68 |
| 13 | 53 | 75 |
| 14 | 70 | 82 |
| 15 | 89 | 89 |

X=15



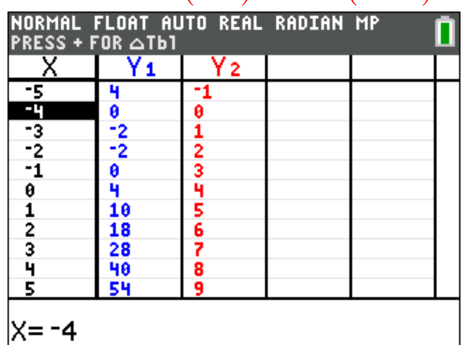

4.

$$y = x^2 + 5x + 4$$
$$y = x + 4$$

$$x^2 + 5x + 4 = x + 4$$
$$x^2 + 4x = x + 4$$
$$x(x+4)$$
$$x = \{0, -4\}$$

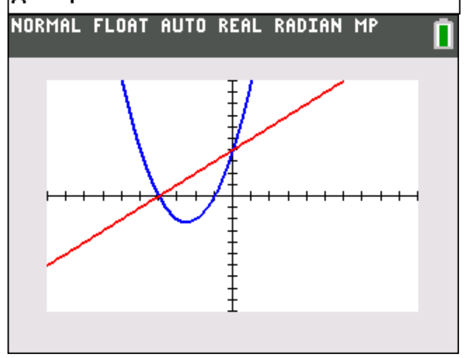
$$y = x + 4$$
$$y = (0) + 4 = 4$$
$$y = (-4) + 4 = 0$$

$$(0,4) \quad and \quad (-4,0)$$


NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR △Tbl

| X | Y1 | Y2 |
|---|---|---|
| -5 | 4 | -1 |
| -4 | 0 | 0 |
| -3 | -2 | 1 |
| -2 | -2 | 2 |
| -1 | 0 | 3 |
| 0 | 4 | 4 |
| 1 | 10 | 5 |
| 2 | 18 | 6 |
| 3 | 28 | 7 |
| 4 | 40 | 8 |
| 5 | 54 | 9 |

X= -4



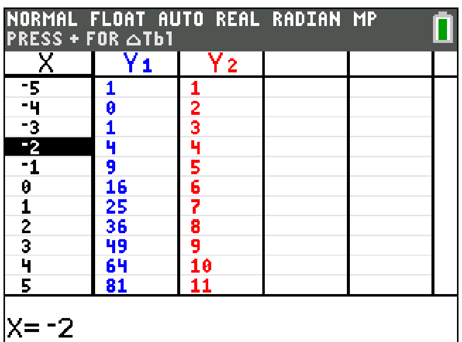
5.

$$y = x^2 + 8x + 16$$
$$y = x + 6$$

$$x^2 + 8x + 16 = x + 6$$
$$x^2 + 7x + 10 = 0$$
$$(x+5)(x+2) = 0$$
$$x = \{-5, -2\}$$

$$y = x + 6$$
$$y = (-5) + 6 = 1$$
$$y = (-2) + 6 = 4$$

$$\boxed{(-5,1) \quad and \quad (-2,4)}$$



NORMAL FLOAT AUTO REAL RADIAN MP
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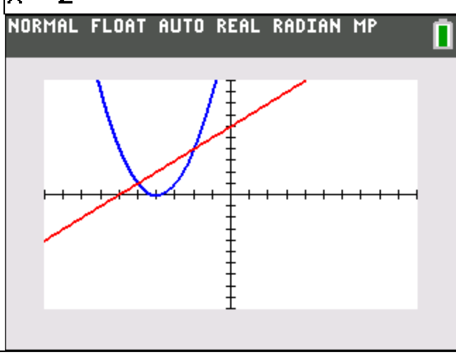
| X | Y1 | Y2 | | |
|---|---|---|---|---|
| -5 | 1 | 1 | | |
| -4 | 0 | 2 | | |
| -3 | 1 | 3 | | |
| -2 | 4 | 4 | | |
| -1 | 9 | 5 | | |
| 0 | 16 | 6 | | |
| 1 | 25 | 7 | | |
| 2 | 36 | 8 | | |
| 3 | 49 | 9 | | |
| 4 | 64 | 10 | | |
| 5 | 81 | 11 | | |

X= -2

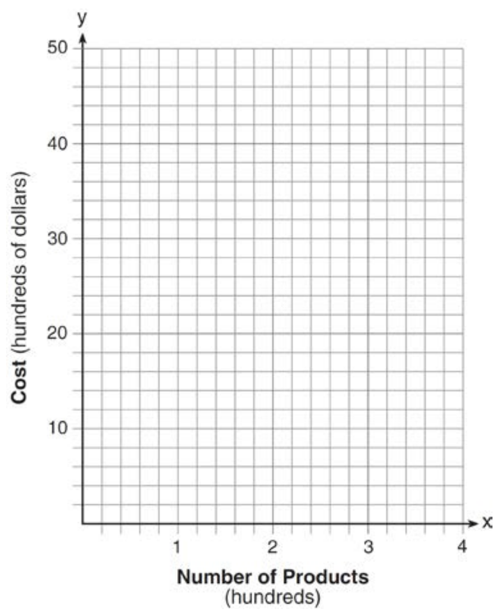


# A.REI.C.7, A.REI.D.11: SOLVE QUADRATIC-LINEAR SYSTEMS

287) The graphs of $y = x^2 - 3$ and $y = 3x - 4$ intersect at approximately

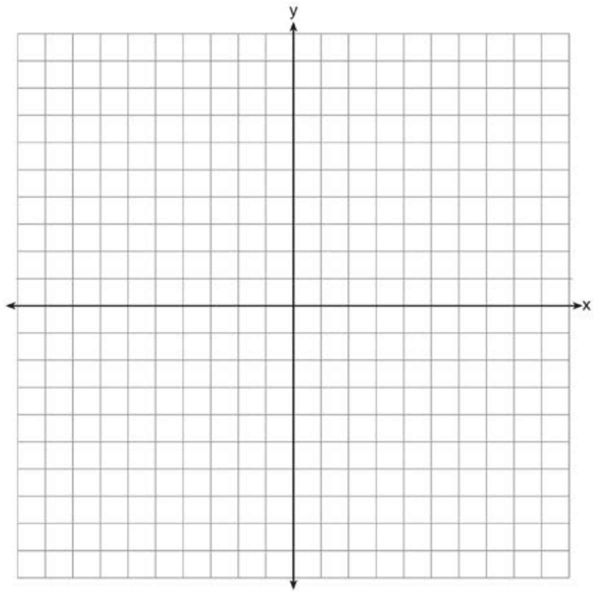
1) $(0.38, -2.85)$, only
2) $(2.62, 3.85)$, only
3) $(0.38, -2.85)$ and $(2.62, 3.85)$
4) $(0.38, -2.85)$ and $(3.85, 2.62)$

288) A company is considering building a manufacturing plant. They determine the weekly production cost at site *A* to be $A(x) = 3x^2$ while the production cost at site *B* is $B(x) = 8x + 3$, where *x* represents the number of products, *in hundreds*, and $A(x)$ and $B(x)$ are the production costs, *in hundreds of dollars*. Graph the production cost functions on the set of axes below and label them site *A* and site *B*.




State the positive value(s) of *x* for which the production costs at the two sites are equal. Explain how you determined your answer. If the company plans on manufacturing 200 products per week, which site should they use? Justify your answer.

289) Let $f(x) = -2x^2$ and $g(x) = 2x - 4$. On the set of axes below, draw the graphs of $y = f(x)$ and $y = g(x)$.



Using this graph, determine and state *all* values of $x$ for which $f(x) = g(x)$.

290) John and Sarah are each saving money for a car. The total amount of money John will save is given by the function $f(x) = 60 + 5x$. The total amount of money Sarah will save is given by the function $g(x) = x^2 + 46$. After how many weeks, *x*, will they have the same amount of money saved? Explain how you arrived at your answer.

291) If $f(x) = x^2 - 2x - 8$ and $g(x) = \frac{1}{4}x - 1$, for which value of *x* is $f(x) = g(x)$?

1) $-1.75$ and $-1.438$
2) $-1.75$ and 4
3) $-1.438$ and 0
4) 4 and 0

292) If $f(x) = x^2$ and $g(x) = x$, determine the value(s) of *x* that satisfy the equation $f(x) = g(x)$.

293) Given: $g(x) = 2x^2 + 3x + 10$

$k(x) = 2x + 16$

Solve the equation $g(x) = 2k(x)$ algebraically for *x*, to the *nearest tenth*. Explain why you chose the method you used to solve this quadratic equation.

## **SOLUTIONS**

287) ANS: 3

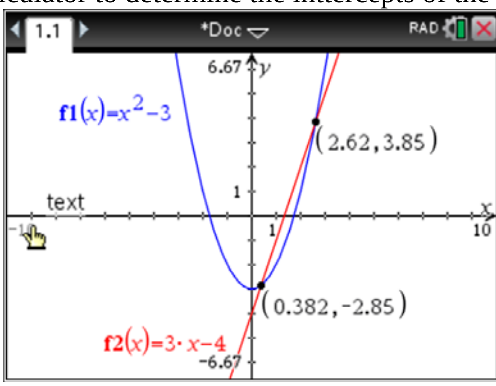
Strategy #1. Solve $y = x^2 - 3$ and $y = 3x - 4$ as a system of equations.

$y = x^2 - 3$ and $y = 3x - 4$

$$x^2 - 3 = 3x - 4$$
$$x^2 - 3x = -1$$
$$\left(x - \frac{3}{2}\right)^2 = -1 + \left(\frac{-3}{2}\right)^2$$
$$\left(x - \frac{3}{2}\right)^2 = -1 + \left(\frac{9}{4}\right)$$
$$\left(x - \frac{3}{2}\right)^2 = \frac{5}{4}$$
$$x - \frac{3}{2} = \pm\frac{\sqrt{5}}{2}$$
$$x = \frac{3 \pm \sqrt{5}}{2}$$

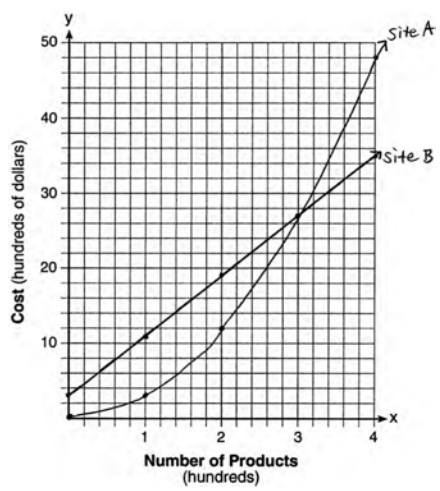
The values of x that satisfy the system are:
$x = \dfrac{3 + \sqrt{5}}{2} \approx 2.62$ and $x = \dfrac{3 - \sqrt{5}}{2} \approx .38$

Strategy #2. Use a graphing calculator to determine the intercepts of the graphs of the two equations.




PTS: 2 NAT: A.REI.C.7 TOP: Quadratic-Linear Systems
KEY: algebraically

288) ANS:

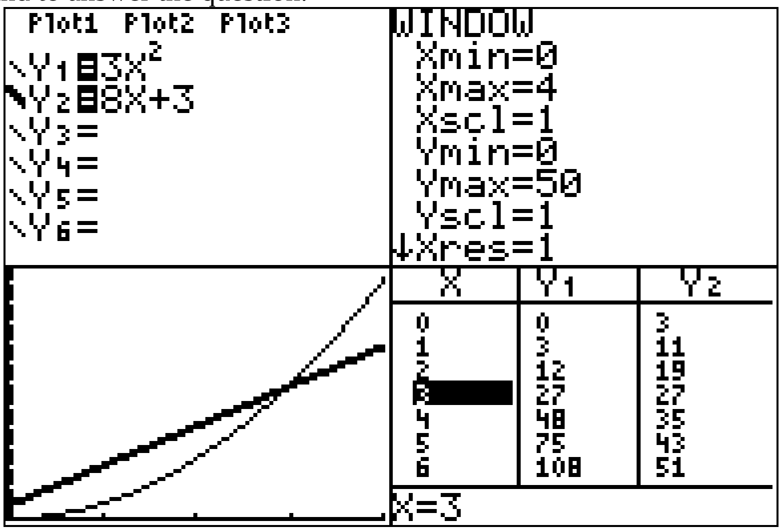



a)

b) The graphs of the production costs are equal when $x = 3$.

c) The company should use Site $A$, because the costs of Site $A$ are lower when $x = 2$.

Strategy: Input both functions into a graphing calculator and use the table and graph views to construct the graph on paper and to answer the question.



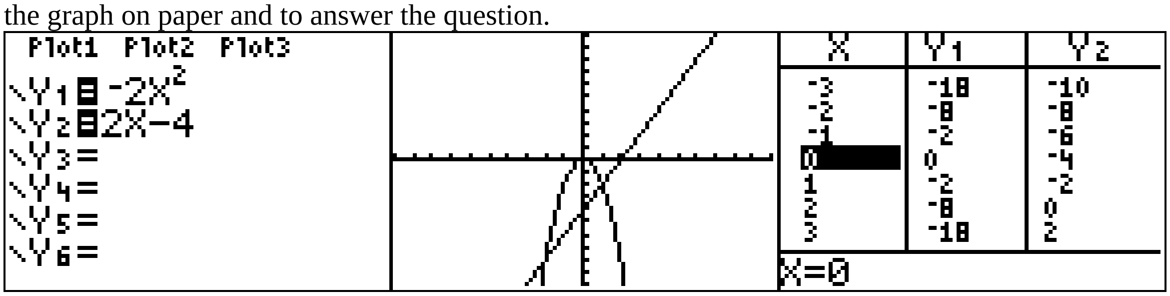

PTS: 6 NAT: A.REI.D.11 TOP: Quadratic-Linear Systems

289) ANS:

a) 

b) $f(x) = g(x)$ when $x = -2$ and $x = 1$.

Strategy: Input both functions into a graphing calculator and use the table and graph views to construct the graph on paper and to answer the question.




PTS: 4 NAT: A.REI.D.11 TOP: Quadratic-Linear Systems

290) ANS:

John and Sarah will have the same amount of money saved at 7 weeks. I set the expressions representing their savings equal to each other and solved for the positive value of *x* by factoring.

Strategy: Set the expressions representing their savings equal to one another and solve for *x*.

$$f(x) = 60 + 5x \text{ and } g(x) = x^2 + 46$$

$$\text{Let } f(x) = g(x)$$

$$x^2 + 46 = 60 + 5x$$

$$x^2 - 5x - 14 = 0$$

$$(x - 7)(x + 2) = 0$$

$$x = 7$$

DIMS? Does It Make Sense? Yes. After 7 weeks, John and Sarah will each have $95.00.

| John's Savings | Sarah's Savings |
|---|---|
| $f(x) = 60 + 5x$<br>$f(7) = 60 + 5(7)$<br>$f(7) = 60 + 35$<br>$f(7) = 95$ | $g(x) = x^2 + 46$<br>$g(7) = (7)^2 + 46$<br>$g(7) = 49 + 46$<br>$g(7) = 95$ |

PTS: 2 NAT: A.REI.D.11 TOP: Quadratic-Linear Systems

291) ANS: 2

Strategy: Set both expressions equal to one another and solve for *x*.

$$f(x) = x^2 - 2x - 8 \quad and \quad g(x) = \frac{1}{4}x - 1$$

$$Let\ f(x) = g(x)$$

$$x^2 - 2x - 8 = \frac{1}{4}x - 1$$

$$4x^2 - 8x - 32 = x - 4$$

$$4x^2 - 9x - 28 = 0$$

$$(4x + 7)(x - 4) = 0$$

$$x = -\frac{7}{4} \quad and \quad x = 4$$

$$f(-1.75) = g(-1.75)$$

$$and$$

$$f(4) = g(4)$$

PTS: 2 NAT: A.REI.D.11 TOP: Quadratic-Linear Systems

292) ANS:

$x = \{0, 1\}$

Given: $f(x) = x^2$ and $g(x) = x$, find $f(x) = g(x)$ as follows:

$$f(x) = g(x)$$

$$x^2 = x$$

$$x^2 - x = 0$$

$$x(x - 1) = 0$$

Therefore: $x = 0$ and $(x - 1) = 0$

$$x = 1$$

PTS: 2 NAT: A.REI.D.11 TOP: Quadratic-Linear Systems

KEY: AI

293) ANS:

$x \approx \{-3.1, 3.6\}$

$$g(x) = 2k(x)$$

$$2x^2 + 3x + 10 = 2(2x + 16)$$

$$2x^2 + 3x + 10 = 4x + 32$$

$$2x^2 - x - 22 = 0$$

The quadratic formula can be used to solve this quadratic in standard form, where $a = 2$, $b = -1$, and $c = -22$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-22)}}{2(2)}$$

$$x = \frac{1 \pm \sqrt{177}}{4}$$

$$x = \frac{1 + \sqrt{177}}{4} = 3.576033 \approx 3.6$$

$$x = \frac{1 - \sqrt{177}}{4} = -3.076033 \approx -3.1$$

The quadratic formula was chosen because it works with any quadratic equation.

PTS: 4 NAT: A.REI.D.11 TOP: Quadratic-Linear Systems
KEY: AI