K – Polynomials, Lesson 4, Factoring the Difference of Perfect Squares (r. 2018)

POLYNOMIALS
Factoring the Difference of Perfect Squares

Common Core Standard	Next Generation Standard
A-SSE.2 Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$, *thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$. PARCC: Tasks limited to numerical and polynomial expressions in one variable. Recognize $53^2 - 47^2$ as a difference of squares and see an opportunity to rewrite it in the easier-to -evaluate form (53+47)(53-47). See an opportunity to rewrite $a^2 + 9a + 14$ as (a+7)(a+2). NYSED: Does not include factoring by grouping and factoring the sum and difference of cubes.	**AI-A.SSE.2 Recognize** and use the structure of an expression to identify ways to rewrite it. (Shared standard with Algebra II) **e.g.,** $\mathbf{x^3 - x^2 - x = x(x^2 - x - 1)}$ $\mathbf{53^2 - 47^2 = (53 + 47)(53 - 47)}$ $\mathbf{16x^2 - 36 = (4x)^2 - (6)^2 = (4x + 6)(4x - 6) = 4(2x + 3)(2x - 3)}$ **or** $\mathbf{16x^2 - 36 = 4(4x^2 - 9) = 4(2x + 3)(2x - 3)}$ $\mathbf{-2x^2 + 8x + 10 = -2(x^2 - 4x - 5) = -2(x - 5)(x + 1)}$ $\mathbf{x^4 + 6x^2 - 7 = (x^2 + 7)(x^2 - 1) = (x^2 + 7)(x + 1)(x - 1)}$ Note: Algebra I expressions are limited to numerical and polynomial expressions in one variable. **Use factoring techniques such as factoring out a greatest common factor, factoring the difference of two perfect squares, factoring trinomials of the form ax2+bx+c with a lead coefficient of 1, or a combination of methods to factor completely.** Factoring will not involve factoring by grouping and factoring the sum and difference of cubes.

LEARNING OBJECTIVES

Students will be able to:

1) factor the difference of perfect squares.

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

Completely factor
Perfect square binomial
Square of a number
Square root of a number

BIG IDEA

General Rule

$$\left(a^2-b^2\right)=\left(a+b\right)\left(a-b\right)$$

Examples

$$x^2-4=\left(x+2\right)\left(x-2\right)$$

$$x^4-9=\left(x^2+3\right)\left(x^2-3\right)$$

DEVELOPING ESSENTIAL SKILLS

1. The expression x^2-16 is equivalent to
   a. $(x+2)(x-8)$ c. $(x+4)(x-4)$
   b. $(x-2)(x+8)$ d. $(x+8)(x-8)$
2. Factored, the expression $16x^2-25y^2$ is equivalent to
   a. $(4x-5y)(4x+5y)$ c. $(8x-5y)(8x+5y)$
   b. $(4x-5y)(4x-5y)$ d. $(8x-5y)(8x-5y)$
3. The expression $9x^2-100$ is equivalent to
   a. $(9x-10)(x+10)$ c. $(3x-100)(3x-1)$
   b. $(3x-10)(3x+10)$ d. $(9x-100)(x+1)$
4. Factor completely: $4x^3-36x$
5. Which expression is equivalent to $9x^2-16$?
   a. $(3x+4)(3x-4)$ c. $(3x+8)(3x-8)$
   b. $(3x-4)(3x-4)$ d. $(3x-8)(3x-8)$
6. If Ann correctly factors an expression that is the difference of two perfect squares, her factors could be
   a. $(2x+y)(x-2y)$ c. $(x-4)(x-4)$
   b. $(2x+3y)(2x-3y)$ d. $(2y-5)(y-5)$
7. Which expression is equivalent to $121-x^2$?
   a. $(x-11)(x-11)$ c. $(11-x)(11+x)$
   b. $(x+11)(x-11)$ d. $(11-x)(11-x)$
8. When a^3-4a is factored completely, the result is
   a. $(a-2)(a+2)$ c. $a^2(a-4)$
   b. $a(a-2)(a+2)$ d. $a(a-2)^2$
9. The expression x^2-36y^2 is equivalent to
   a. $(x-6y)(x-6y)$ c. $(x+6y)(x-6y)$
   b. $(x-18y)(x-18y)$ d. $(x+18y)(x-18y)$
10. Which expression represents $36x^2-100y^6$ factored completely?
    a. $2(9x+25y^3)(9x-25y^3)$ c. $(6x+10y^3)(6x-10y^3)$
    b. $4(3x+5y^3)(3x-5y^3)$ d. $(18x+50y^3)(18x-50y^3)$
11. Which expression is equivalent to $64-x^2$?
    a. $(8-x)(8-x)$ c. $(x-8)(x-8)$
    b. $(8-x)(8+x)$ d. $(x-8)(x+8)$
12. The expression $9a^2-64b^2$ is equivalent to
    a. $(9a-8b)(a+8b)$ c. $(3a-8b)(3a+8b)$
    b. $(9a-8b)(a-8b)$ d. $(3a-8b)(3a-8b)$
13. The expression $100n^2-1$ is equivalent to

a. $(10n+1)(10n-1)$
c. $(50n+1)(50n-1)$
b. $(10n-1)(10n-1)$
d. $(50n-1)(50n-1)$

14. When $9x^2 - 100$ is factored, it is equivalent to $(3x-b)(3x+b)$. What is a value for *b*?
a. 50
c. 3
b. 10
d. 100

15. Which expression is equivalent to $81 - 16x^2$?
a. $(9-8x)(9+8x)$
c. $(9-4x)(9+4x)$
b. $(9-8x)(9+2x)$
d. $(9-4x)(9-4x)$

16. One of the factors of $4x^2 - 9$ is
a. $(x+3)$
c. $(4x-3)$
b. $(2x+3)$
d. $(x-3)$

17. Factor completely: $3x^2 - 27$
a. $3(x-3)^2$
c. $3(x+3)(x-3)$
b. $3(x^2-27)$
d. $(3x+3)(x-9)$

18. Written in simplest factored form, the binomial $2x^2 - 50$ can be expressed as
a. $2(x-5)(x-5)$
c. $(x-5)(x+5)$
b. $2(x-5)(x+5)$
d. $2x(x-50)$

19. Expressed in factored form, the binomial $4a^2 - 9b^2$ is equivalent to
a. $(2a-3b)(2a-3b)$
c. $(4a-3b)(a+3b)$
b. $(2a+3b)(2a-3b)$
d. $(2a-9b)(2a+b)$

20. What is a common factor of $x^2 - 9$ and $x^2 - 5x + 6$?
a. $x+3$
c. $x-2$
b. $x-3$
d. x^2

Answers

1. ANS: C
2. ANS: A
3. ANS: B
4. ANS:

$4x(x+3)(x-3)$. $4x^3 - 36x = 4x(x^2-9) = 4x(x+3)(x-3)$

5.ANS: A
6. ANS: B
7. ANS: C
8. ANS: B
9. ANS: C
10. ANS: B
11. ANS: B
12. ANS: C
13. ANS: A
14. ANS: B
15. ANS: C
16. ANS: B
17. ANS: C
18. ANS: B
19. ANS: B
20. ANS: B

REGENTS EXAM QUESTIONS (through June 2018)

A.SSE.A.2: Difference of Perfect Squares

348) When factored completely, the expression $p^4 - 81$ is equivalent to

1) $(p^2 + 9)(p^2 - 9)$
2) $(p^2 - 9)(p^2 - 9)$
3) $(p^2 + 9)(p + 3)(p - 3)$
4) $(p + 3)(p - 3)(p + 3)(p - 3)$

349) If the area of a rectangle is expressed as $x^4 - 9y^2$, then the product of the length and the width of the rectangle could be expressed as

1) $(x - 3y)(x + 3y)$
2) $(x^2 - 3y)(x^2 + 3y)$
3) $(x^2 - 3y)(x^2 - 3y)$
4) $(x^4 + y)(x - 9y)$

350) The expression $x^4 - 16$ is equivalent to

1) $(x^2 + 8)(x^2 - 8)$
2) $(x^2 - 8)(x^2 - 8)$
3) $(x^2 + 4)(x^2 - 4)$
4) $(x^2 - 4)(x^2 - 4)$

351) Which expression is equivalent to $36x^2 - 100$?

1) $4(3x - 5)(3x - 5)$
2) $4(3x + 5)(3x - 5)$
3) $2(9x - 25)(9x - 25)$
4) $2(9x + 5)(9x - 25)$

352) Which expression is equivalent to $16x^2 - 36$?

1) $4(2x - 3)(2x - 3)$
2) $4(2x + 3)(2x - 3)$
3) $(4x - 6)(4x - 6)$
4) $(4x + 6)(4x + 6)$

353) Which expression is equivalent to $16x^4 - 64$?

1) $(4x^2 - 8)^2$
2) $(8x^2 - 32)^2$
3) $(4x^2 + 8)(4x^2 - 8)$
4) $(8x^2 + 32)(8x^2 - 32)$

354) The expression $49x^2 - 36$ is equivalent to

1) $(7x - 6)^2$
2) $(24.5x - 18)^2$
3) $(7x - 6)(7x + 6)$
4) $(24.5x - 18)(24.5x + 18)$

355) Which expression is equivalent to $y^4 - 100$?

1) $(y^2 - 10)^2$
2) $(y^2 - 50)^2$
3) $(y^2 + 10)(y^2 - 10)$
4) $(y^2 + 50)(y^2 - 50)$

SOLUTIONS

348) ANS: 3

Strategy: Use difference of perfect squares.

STEP 1. Factor $p^4 - 81$

$p^4 - 81$

$\left(p^2 + 9\right)\left(p^2 - 9\right)$

STEP 2. Factor $p^2 - 9$

$\left(p^2 + 9\right)\left(p^2 - 9\right)$

$\left(p^2 + 9\right)\left(p + 3\right)\left(p - 3\right)$

PTS: 2 NAT: A.SSE.A.2 TOP: Factoring Polynomials

349) ANS: 2

Strategy: Use the distributive property to work backwards from the answer choices.

a.	c.
$(x-3y)(x+3y)$ $x^2+3xy-3xy-9y^2$ x^2-9y^2 (wrong)	$(x^2-3y)(x^2-3y)$ $x^4-3x^2y-3x^2y+9y^2$ $x^4-6x^2y+9y^2$ (wrong)
b.	**d.**
$(x^2-3y)(x^2+3y)$ $x^4+3x^2y-3x^2y-9y^2$ x^4-9y^2 (correct)	$(x^4+y)(x-9y)$ $x^5-9x^4y+xy-9y^2$ (wrong)

PTS: 2 NAT: A.SSE.A.2 TOP: Factoring Polynomials

350) ANS: 3

Step 1. Understand the problem as a “difference of perfect squares”, because the terms x^4 and 16 are both perfect squares and the operation is subtraction.

Step 2. Strategy: Use the pattern $a^2 - b^2 = (a+b)(a-b)$ to separate $x^4 - 16$ into two binomials.

Step3. Execution of Strategy

The square root of x^4 is x^2.

The square of 16 is 4.

$x^4 - 16 = (x^2+4)(x^2-4)$

Step 4. Does it make sense? Yes. You can show that $(x^2+4)(x^2-4) = x^4 - 16$ using the distributive property, as follows:

$$(x^2+4)(x^2-4)=x^4+16$$
$$x^4-4x^2+4x^2-16=x^4+16$$
$$x^4+16=x^4+16$$

PTS: 2 NAT: A.SSE.A.2 TOP: Factoring the Difference of Perfect Squares

351) ANS: 2

Strategy 1.

Recognize that the expression $36x^2-100$ is a difference of perfect squares. Therefore,

$$36x^2-100 \quad .$$
$$(6x+10)(6x-10)$$

Since this is not an answer choice, continue factoring, as follows:

$$(6x+10)(6x-10)$$
$$\left(2(3x+5)\right)\left(2(3x-5)\right)$$
$$4(3x+5)(3x-5)$$

Strategy 2.

Examine the answer choices, which begin with factors 4 and 2. Extract these factors first, as follows:

Start by extracting a 4	Start by extracting a 2
$36x^2-100$ $4\left(9x^2-25\right)$ $4(3x+5)(3x-5)$	$36x^2-100$ $2\left(18x^2-50\right)$ $(2)(2)\left(9x^2-25\right)$ $(2)(2)(3x+5)(3x-5)$ $4(3x+5)(3x-5)$

PTS: 2 NAT: A.SSE.A.2

352) ANS: 2

Strategy 1: Factor

$16x^2-36$

$4\left(4x^2-9\right)$

$4(2x+3)(2x-3)$

Strategy 2: Recognize that $16x^2-36$ appears to be a difference of perfect squares.

Recall that $a^2-b^2=(a+b)(a-b)$.

Eliminate any answers that do not take the form of $(a+b)(a-b)$, which leaves only one choice:

$4(2x+3)(2x-3)$

Check:

$$4(2x+3)(2x-3)$$

$$4\left[(2x+3)(2x-3)\right]$$

$$4\left[4x^2+6x-6x-9\right]$$

$$4\left[4x^2-9\right]$$

$$16x^2-36$$

$$\therefore\ 4(2x+3)(2x-3)=16x^2-36$$

PTS: 2 NAT: A.SSE.A.2 TOP: Factoring the Difference of Perfect Squares
KEY: quadratic

353) ANS: 3

Note that the expression $16x^4-64$ is the difference of perfect squares.

$$a^2-b^2=(a+b)(a-b)$$

$$16x^4-64=(4x^2+8)(4x^2-8)$$

PTS: 2 NAT: A.SSE.A.2 TOP: Factoring the Difference of Perfect Squares
KEY: higher power

354) ANS: 3

Note that $49x^2$ and 36 are both perfect squares. Therefore, $49x^2-36$ is the difference of perfect squares.

$$a^2-b^2=(a+b)(a-b)$$

$$49x^2-36=(7x+6)(7x-6)$$

PTS: 2 NAT: A.SSE.A.2 TOP: Factoring the Difference of Perfect Squares
KEY: quadratic

355) ANS: 3

y^4-100 is a difference of perfect squares. All polynomials in the form of a^2-b^2 can be factored into $(a+b)\left(a-b\right)$.

$$y^4-100$$

$$\left(y^2+10\right)\left(y^2-10\right)$$

PTS: 2 NAT: A.SSE.A.2 TOP: Factoring the Difference of Perfect Squares
KEY: higher power AI