M - Functions, Lesson 4, Operations with Functions (r. 2018)
FUNCTIONS
Operations with Functions

Common Core Standard
F-BF. 1 Write a function that describes a relationship between two quantities.

Next Generation Standard
AI-F.BF. 1 Write a function that describes a relationship between two quantities.

## LEARNING OBJECTIVES

Students will be able to:

1) Compose new functions from existing functions using substitution and mathematical operations.

## Overview of Lesson

| Teacher Centered Introduction | Student Centered Activities |
| :--- | :--- |
| Overview of Lesson | guided practice 飞Teacher: anticipates, monitors, selects, sequences, and <br> connects student work |
| - activate students' prior knowledge | - developing essential skills <br> - vocabulary <br> - learning objective(s) <br> - big ideas: direct instruction <br> - modeling |
| - formative assessment assignment (exit slip, explain the math, or journal <br> entry) |  |

## VOCABULARY

## LEARNING OBJECTIVES

Students will be able to:

1) Use the output of one function as the input for another function.
2) Substitute expressions from one function into another.

## BIG IDEAS

Polynomial expressions can be substituted into equations and functions.
Example:
Given that: $f(x)=g(x)-2 h(x)$ and $g(x)=3 x+4$, then $f(x)=(3 x+4)-2(5 x-6)$

$$
h(x)=5 x-6
$$

Functions can be multiplied or divided if each and every term in both expressions is multiplied or divided by the same value.

$$
\begin{gathered}
2(y=3 x+4) \\
\text { Example: } \quad 2(y)=2(3 x)+2(4) \\
2 y=6 x+8
\end{gathered}
$$

## DEVELOPING ESSENTIAL SKILLS

1. If f and g are two functions defined by $\mathrm{f}(x)=3 x+5$ and $\mathrm{g}(x)=x^{2}+1$, then $\mathrm{g}(\mathrm{f}(x))$ is
a. $x^{2}+3 x+6$
b. $9 x^{2}+30 x+26$
c. $3 x^{2}+8$
d. $9 x^{2}+26$
2. If $\mathrm{f}(x)=-2 x+7$ and $\mathrm{g}(x)=x^{2}-2$, then $\mathrm{f}(g(3))$ is equal to
a. -7
b. -3
c. -1
d. 7
3. The accompanying tables define functions f and g .

| $\boldsymbol{x}$ | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{f}(x)$ | 3 | 4 | 5 | 6 | 7 |


| $\boldsymbol{x}$ | 3 | 4 | 5 | 6 | 7 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{g}(\boldsymbol{x})$ | 4 | 6 | 8 | 10 | 12 |

What is $g(f(3))$ ?
a. 6
b. 2
c. 8
d. 4
4. If $\mathrm{f}(x)=x^{2}+4$ and $g(x)=\sqrt{1-x}$, what is the value of $\mathrm{f}(\mathrm{g}(-3))$ ?
a. $2 i \sqrt{3}$
b. 2
c. 8
d. 13
5. If $\mathrm{f}(x)=x^{2}+4$ and $\mathrm{g}(x)=2 x+3$, find $\mathrm{f}(g(-2))$.

## ANSWERS

1. ANS: B

$$
\begin{aligned}
\mathrm{f}(x) & =3 x+5 \\
g(3 x+5) & =(3 x+5)^{2}+1 \\
& =9 x^{2}+30 x+26
\end{aligned}
$$

2. ANS: A

$$
\begin{aligned}
g(3) & =3^{2}-2 \\
& =7 \\
f(7) & =-2(7)+7 \\
& -7
\end{aligned}
$$

3. ANS: C
$f(3)=5, g(5)=8$
4. ANS: C

$$
\begin{aligned}
g(-3) & =\sqrt{1-x}=\sqrt{1-(-3)}=2 \\
f(2) & =2^{2}+4=8
\end{aligned}
$$

5. ANS:
6. $\mathrm{g}(-2)=2(-2)+3=-1 . \quad \mathrm{f}(-1)=(-1)^{2}+4=5$.

## REGENTS EXAM QUESTION (through June 2018)

## A.APR.A.1: Operations with Functions

434) A company produces $x$ units of a product per month, where $C(x)$ represents the total cost and $R(x)$ represents the total revenue for the month. The functions are modeled by $C(x)=300 x+250$ and $R(x)=-0.5 x^{2}+800 x-100$. The profit is the difference between revenue and cost where $P(x)=R(x)-C(x)$. What is the total profit, $P(x)$, for the month?
435) $P(x)=-0.5 x^{2}+500 x-150$
436) $P(x)=-0.5 x^{2}+500 x-350$
437) $P(x)=-0.5 x^{2}-500 x+350$
438) $P(x)=-0.5 x^{2}+500 x+350$

## SOLUTION

434) ANS: 2

Strategy: Substitute $R(x)$ and $C(x)$ into $P(x)=R(x)-C(x)$.
Given:

$$
\begin{aligned}
& P(x)=R(x)-C(x) \\
& R(x)=-0.5 x^{2}+800 x-100 \\
& C(x)=300 x+250
\end{aligned}
$$

Therefore: $\quad P(x)=\left(-0.5 x^{2}+800 x-100\right)-(300 x+250)$

$$
\begin{aligned}
& P(x)=-0.5 x^{2}+800 x-100-300 x-250 \\
& P(x)=-0.5 x^{2}+500 x-350
\end{aligned}
$$

PTS: 2
NAT: A.APR.A. 1 TOP: Addition and Subtraction of Polynomials
KEY: subtraction

