RADICALS

Operations with Radicals

Common Core Standard	**Next Generation Standard**
N-RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	**AI-N.RN.3** Use properties and operations to understand the different forms of rational and irrational numbers. a.) Perform all four arithmetic operations and apply properties to generate equivalent forms of rational numbers and square roots. Note: Tasks include rationalizing numerical denominators of the form $\frac{a}{\sqrt{b}}$ where a is an integer and b is a natural number. b.) Categorize the sum or product of rational or irrational numbers. • The sum and product of two rational numbers is rational. • The sum of a rational number and an irrational number is irrational. • The product of a nonzero rational number and an irrational number is irrational. • The sum and product of two irrational numbers could be either rational or irrational.

LEARNING OBJECTIVES

Students will be able to:

1) Perform addition, subtraction, multiplication and division with radical numbers (prior skill).
2) Identify if the sum or product of two numbers is rational or irrational and explain why.

Overview of Lesson

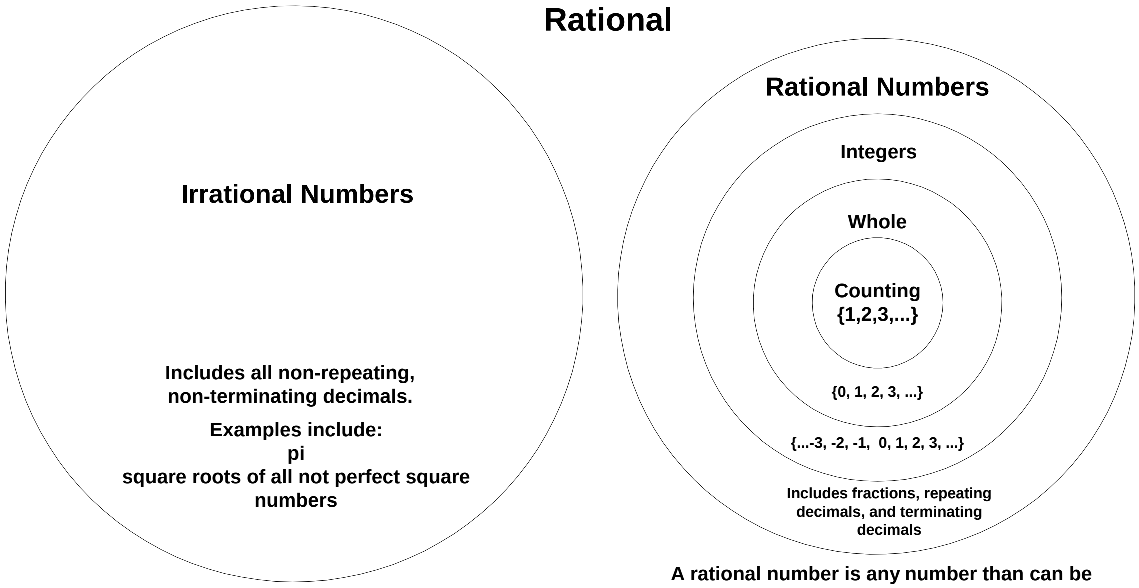
Teacher Centered Introduction	**Student Centered Activities**
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

Decimal form	Perfect square	Rational
Equivalent	Prime number	Simplest radical form
Irrational	Radical form	

BIG IDEAS

The Set of Real Numbers includes two major classifications of numbers Irrational and Rational



An irrational number is any number that cannot be expressed as the ratio of two integers.

A rational number is any number than can be expressed as the ratio of two integers.

Is a Number Irrational or Rational?

Irrational Numbers	Rational Numbers
If a decimal does not repeat or terminate, it is an irrational number. Numbers with names, such as π and e are irrational. They are given names because it is impossible to state their infinitely long values. The square roots of all numbers (that are not perfect squares) are irrational. If a term reduced to simplest form contains an irrational number, the term is irrational. .	If a number is an integer, it is rational, since it can be expressed as a ratio with the integer as the numerator and 1 as the denominator. If a decimal is a repeating decimal, it is a rational number. If a decimal terminates, it is a rational number.

Operations with Irrational and Rational Numbers

Addition and Subtraction:

When two rational numbers are added or subtracted, the result is rational.
When two irrational numbers are added or subtracted, the result is irrational.
When an irrational number and a rational number are added or subtracted, the sum is irrational.

Multiplication and Division:

When two rational numbers are multiplied or divided, the product is rational.
When an irrational number and a non-zero rational number are multiplied or divided, the product is irrational.
When two irrational numbers are multiplied or divided, the product is sometimes rational and sometimes irrational.

Example of Rational Product	Example of Irrational Product
$\sqrt{7} \times \sqrt{28}$ $\sqrt{7} \times \left(\sqrt{4} \times \sqrt{7}\right)$ $\left(\sqrt{7}\sqrt{7}\right)\sqrt{4}$ $7 \times 2 = 14$ $\frac{14}{1}$	$\sqrt{7} \times \sqrt{3}$ $\sqrt{21}$ $4.582575695......$ NOTE: Be careful using a calculator to decide if a number is irrational. The calculator stops when it runs out of room to display the numbers, and the whole number may continue beyond the calculator display.
Rational Quotient $\frac{\sqrt{20}}{\sqrt{5}} = \sqrt{\frac{20}{5}} = \sqrt{4} = 2 = \frac{2}{1}$	Irrational Quotient $\frac{\sqrt{10}}{\sqrt{5}} = \sqrt{\frac{10}{5}} = \sqrt{2}$

<u>DEVELOPING ESSENTIAL SKILLS</u>

Question	Answer	Is Answer Rational or Irrational?
1. Express the product of $3\sqrt{20}(2\sqrt{5} - 7)$ in simplest radical form.		
2. The expression $6\sqrt{50} + 6\sqrt{2}$ written in simplest radical form is:		
3. The expression $\sqrt{72} - 3\sqrt{2}$ written in simplest radical form is		
4. Express $\frac{16\sqrt{21}}{2\sqrt{7}} - 5\sqrt{12}$ in simplest radical form.		
5. Express $\frac{3\sqrt{75} + \sqrt{27}}{3}$ in simplest radical form.		
6. Express $\sqrt{25} - 2\sqrt{3} + \sqrt{27} + 2\sqrt{9}$ in simplest radical form.		
7. Express $\frac{\sqrt{84}}{2\sqrt{3}}$ in simplest radical form.		
8. Perform the indicated operations and express the answer in simplest radical form. $3\sqrt{7}\left(\sqrt{14} + 4\sqrt{56}\right)$		
9. The expression $\sqrt{90} \cdot \sqrt{40} - \sqrt{8} \cdot \sqrt{18}$ simplifies to		
10. The expression $\frac{6\sqrt{20}}{3\sqrt{5}}$ is equivalent to		

ANSWERS

Question	Answer	Is Answer Rational or Irrational?
1. Express the product of $3\sqrt{20}(2\sqrt{5}-7)$ in simplest radical form.	$60-42\sqrt{5}$	Irrational
2. The expression $6\sqrt{50}+6\sqrt{2}$ written in simplest radical form is:	$36\sqrt{2}$	Irrational
3. The expression $\sqrt{72}-3\sqrt{2}$ written in simplest radical form is	$3\sqrt{2}$	Irrational
4. Express $\frac{16\sqrt{21}}{2\sqrt{7}}-5\sqrt{12}$ in simplest radical form.	$-2\sqrt{3}$	Irrational
5. Express $\frac{3\sqrt{75}+\sqrt{27}}{3}$ in simplest radical form.	$6\sqrt{3}$	Irrational
6. Express $\sqrt{25}-2\sqrt{3}+\sqrt{27}+2\sqrt{9}$ in simplest radical form.	$11+\sqrt{3}$	Irrational
7. Express $\frac{\sqrt{84}}{2\sqrt{3}}$ in simplest radical form.	$\sqrt{7}$	Irrational
8. Perform the indicated operations and express the answer in simplest radical form. $3\sqrt{7}\left(\sqrt{14}+4\sqrt{56}\right)$	$189\sqrt{2}$	Irrational
9. The expression $\sqrt{90}\cdot\sqrt{40}-\sqrt{8}\cdot\sqrt{18}$ simplifies to	48	Rational
10. The expression $\frac{6\sqrt{20}}{3\sqrt{5}}$ is equivalent to	4	Rational

N.RN.B.3: Operations with Radicals

381) Given: $L = \sqrt{2}$

$M = 3\sqrt{3}$

$N = \sqrt{16}$

$P = \sqrt{9}$

Which expression results in a rational number?

1) $L + M$
2) $M + N$
3) $N + P$
4) $P + L$

382) Which statement is *not* always true?

1) The product of two irrational numbers is irrational.
2) The product of two rational numbers is rational.
3) The sum of two rational numbers is rational.
4) The sum of a rational number and an irrational number is irrational.

383) Ms. Fox asked her class "Is the sum of 4.2 and $\sqrt{2}$ rational or irrational?" Patrick answered that the sum would be irrational. State whether Patrick is correct or incorrect. Justify your reasoning.

384) Which statement is *not* always true?

1) The sum of two rational numbers is rational.
2) The product of two irrational numbers is rational.
3) The sum of a rational number and an irrational number is irrational.
4) The product of a nonzero rational number and an irrational number is irrational.

385) For which value of *P* and *W* is $P + W$ a rational number?

1) $P = \frac{1}{\sqrt{3}}$ and $W = \frac{1}{\sqrt{6}}$
2) $P = \frac{1}{\sqrt{4}}$ and $W = \frac{1}{\sqrt{9}}$
3) $P = \frac{1}{\sqrt{6}}$ and $W = \frac{1}{\sqrt{10}}$
4) $P = \frac{1}{\sqrt{25}}$ and $W = \frac{1}{\sqrt{2}}$

386) Given the following expressions:

I. $-\frac{5}{8} + \frac{3}{5}$ III. $\left(\sqrt{5}\right) \cdot \left(\sqrt{5}\right)$

II. $\frac{1}{2} + \sqrt{2}$ IV. $3 \cdot \left(\sqrt{49}\right)$

Which expression(s) result in an irrational number?

1) II, only
2) III, only
3) I, III, IV
4) II, III, IV

387) Determine if the product of $3\sqrt{2}$ and $8\sqrt{18}$ is rational or irrational. Explain your answer.

388) Is the sum of $3\sqrt{2}$ and $4\sqrt{2}$ rational or irrational? Explain your answer.

389) Jakob is working on his math homework. He decides that the sum of the expression $\frac{1}{3} + \frac{6\sqrt{5}}{7}$ must be rational because it is a fraction. Is Jakob correct? Explain your reasoning.

390) State whether $7-\sqrt{2}$ is rational or irrational. Explain your answer.

391) A teacher wrote the following set of numbers on the board:

$$a=\sqrt{20} \qquad b=2.5 \qquad c=\sqrt{225}$$

Explain why $a+b$ is irrational, but $b+c$ is rational.

392) The product of $\sqrt{576}$ and $\sqrt{684}$ is

1) irrational because both factors are irrational
2) rational because both factors are rational
3) irrational because one factor is irrational
4) rational because one factor is rational

393) Is the product of $\sqrt{16}$ and $\frac{4}{7}$ rational or irrational? Explain your reasoning.

<u>SOLUTIONS</u>

381) ANS: 3

$\sqrt{16}+\sqrt{9}=\frac{7}{1}$ may be expressed as the ratio of two integers.

Strategy: Recall that under the operation of addition, the addition of two irrational numbers and the addition of an irrational number and a rational number will always result in a sum that is irrational. To get a rational number as a sum, you must add two rational numbers.

STEP 1 Determine whether numbers L, M, N, and P are ratiional, then reject any answer choice that does not contain two rational numbers.

$L=\sqrt{2}$ is irrational

$M=3\sqrt{3}$ is irrational

$N=\sqrt{16}=4$ and is rational

$P=\sqrt{9}=3$ and is rational

STEP 2 Reject any answer choice that does not include $N+P$. Choose answer choice c.

PTS: 2 NAT: N.RN.B.3 TOP: Classifying Numbers

382) ANS: 1

Strategy: Find a counterexample to prove one of the answer choices is *not* always true.

Answer choice a is not always true because: $\sqrt{3}$ and $\sqrt{12}$ are both irrational numbers, but $\sqrt{3}\times\sqrt{12}=\sqrt{3\times 12}=\sqrt{36}=6$, and 6 is a rational number, so the product of two irrational numbers is not *always* irrational.

PTS: 2 NAT: N.RN.B.3 TOP: Classifying Numbers

383) ANS:

Patrick is correct. The sum of a rational and irrational is irrational.

Strategy: Determine whether 4.2 and $\sqrt{2}$ are rational or irrational numbers, then apply the rules of operations on rational and irrational numbers.

4.2 is rational because it can be expressed as $\frac{42}{10}$, which is the ratio of two integers.

$\sqrt{2}$ is irrational because it cannot be expressed as the ratio of two integers.

The rules of addition and subtraction of rational and irrational numbers are:
When two rational numbers are added or subtracted, the result is rational.
When two irrational numbers are added or subtracted, the result is irrational.
When an irrational number and a rational number are added or subtracted, the sum is irrational.

PTS: 2 NAT: N.RN.B.3 TOP: Classifying Numbers

384) ANS: 2

Strategy: Find a counterexample to prove one of the answer choices is *not* always true. This will usually involve the *product* or *quotient* of two irrational numbers since the outcomes of addition and subtraction of irrational numbers are more predictable.

Answer choice b is not always true because: $\sqrt{2}$ and $\sqrt{3}$ are both irrational numbers, but $\sqrt{2} \times \sqrt{3} = \sqrt{2 \times 3} = \sqrt{6}$, and $\sqrt{6}$ is an rational number, so the product of two irrational numbers is not *always* rational.

PTS: 2 NAT: N.RN.B.3 TOP: Classifying Numbers

385) ANS: 2

$$\frac{1}{\sqrt{4}} + \frac{1}{\sqrt{9}} = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

Strategy: Recall that under the operation of addition, the addition of two irrational numbers and the addition of an irrational number and a rational number will always result in a sum that is irrational. To get a rational number as a sum, you must add two rational numbers. Reject any answer choice that does not contain two rational numbers.

Reject answer choice a because $\frac{1}{\sqrt{3}}$ is irrational.

Choose answer choice b because both $P = \frac{1}{\sqrt{4}}$ and $W = \frac{1}{\sqrt{9}}$ can be expressed as rational numbers, as shown above.

PTS: 2 NAT: N.RN.B.3 TOP: Classifying Numbers

386) ANS: 1

Strategy: Eliminate wrong answers.

Expression I results in a rational number because the set of rational numbers is closed under addition.

$$-\frac{5}{8} + \frac{3}{5} = \frac{-25}{40} + \frac{24}{40} = \frac{-1}{40}$$

Expression II is is correct because the additon of a rational number and an irrational number always results in an irrational number.

$$\frac{1}{2} + \sqrt{2} = 0.5 + 1.414203562... = 1.914203562...$$

Expression III results in a rational number because $\left(\sqrt{5}\right) \cdot \left(\sqrt{5}\right) = \sqrt{5 \cdot 5} = \sqrt{25} = 5 = \frac{5}{1}$, which is the ratio of two integers.

Expression IV results in a rational number because $3 \cdot \left(\sqrt{49}\right) = 3 \cdot 7 = 21 = \frac{21}{1}$, which the ratio of two integers.

Expression II is the only expression that results in an irrational number, so Choice (a) is the correct answer.

PTS: 2 NAT: N.RN.B.3 TOP: Classifying Numbers

387) ANS:

$$3\sqrt{2} \cdot 8\sqrt{18}$$
$$3 \times 8 \times \sqrt{2} \times \sqrt{18}$$
$$24\sqrt{36}$$
$$144$$

The product is 144, which is rational, because it can be written as $\frac{144}{1}$, a ratio of two integers.

PTS: 2 NAT: N.RN.B.3 TOP: Classifying Numbers

388) ANS:
Irrational

$3\sqrt{2} + 4\sqrt{2} = 7\sqrt{2}$

$7\sqrt{2}$ is irrational because it is the product of a rational number and an irrational number.

7 is rational because it can be expressed as the ratio of two integers ($\frac{7}{1}$)

$\sqrt{2}$ is irrational because the square roots of all prime numbers are irrational.

PTS: 2 NAT: N.RN.B

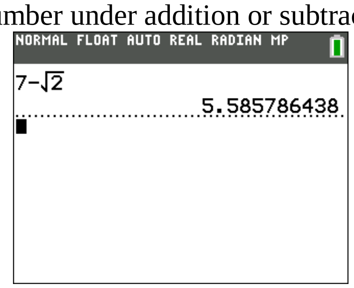
389) ANS:
Jakob is incorrect. The sum of a rational number and an irrational number is irrational.

$$\frac{1}{3} + \frac{6\sqrt{5}}{7} = \frac{7 + 18\sqrt{5}}{21}$$

Note the square root of 5 in the sum. The square root of any prime is irrational.

PTS: 2 NAT: N.RN.B.3 TOP: Classifying Numbers

390) ANS:
Irrational
A rational number and an irrational number under addition or subtraction will always be irrational.



Note that the answer does not appear to repeat or end.

is irrational because it can not be written as the ratio of two integers.

PTS: 2 NAT: N.RN.B.3 TOP: Operations with Radicals
KEY: classify

391) ANS:
The sum of *a* and *b* is irrational because the sum of an irrational number and a rational number is always irrational.

The sum of *b* and *c* is rational because the sum of a rational number and another rational number is always rational.

$\sqrt{20}$ is an irrational number that can be simplified to $2\sqrt{5}$, but cannot be expressed as the ratio of two integers or as a never-ending, never-repeating decimal.

2.5 is a rational number because it can be expressed as the ratio of two integers, such as $\frac{25}{10}$.

$\sqrt{225}$ is a rational number that can be simplified to 15 and expressed as the ratio of two integers, such as $\frac{15}{1}$.

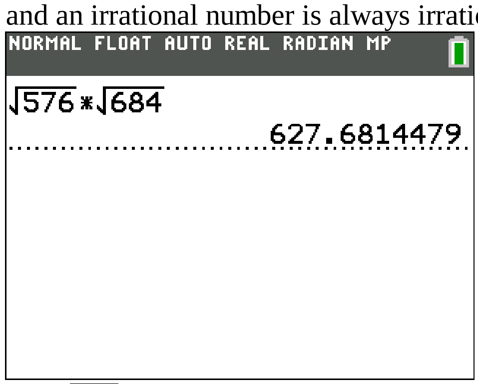
PTS: 2 NAT: N.RN.B.3 TOP: Operations with Radicals
KEY: classify

392) ANS: 3

$\sqrt{576} = 24$, which can be expressed as the ratio $\frac{24}{1}$, which means that $\sqrt{576}$ is a rational number.

$\sqrt{684}$ cannot be expressed as a rational number. It can be simplified to $6\sqrt{19}$, but it cannot be expressed as the ratio of two integers. Therefore, $\sqrt{684}$ is an irrational number.

The product of a rational number and an irrational number is always irrational.



Note that the product of $\sqrt{576}$ and $\sqrt{684}$ appears to be a never ending, non-repeating decimal, which indicates that the product is an irrational number.

PTS: 2 NAT: N.RN.B.3 TOP: Operations with Radicals
KEY: classify

393) ANS:

Answer: The product of $\sqrt{16}$ and $\frac{4}{7}$ is rational.

Explanation: A rational number is a number that can be expressed as the ratio of two intergers, in the form of $\frac{a}{b}$, where both *a* and *b* are integers. An irrational number is a number that cannot be expressed as the ratio of two integers.

$\sqrt{16}$ is a rational number because $\sqrt{16}$ can be expressed as $\frac{4}{1}$, which is a ratio of two integers.

$\frac{4}{7}$ is a rational number because it is already expressed as a ratio of two integers.

$\frac{4}{1} \times \frac{4}{7} = \frac{16}{7}$, and $\frac{16}{7}$ is a ratio of two integers.

The product of any two rational numbers will always be a rational number.

PTS: 2 NAT: N.RN.B.3 TOP: Operations with Radicals
KEY: classify