Stability and Change in New York State Regents Mathematics Examinations, 1866-2009:

a Socio-Historical Analysis

by

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Abstract

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This dissertation illuminates relationships between micro-level practices of schools and macro-level structures of society through the socio-historical lens of New York State Regents mathematics examinations, which were administered to public school students throughout the State of New York between 1866 and 2009, inclusive. Fundamental research questions involved in this study are: 1) How has the classification, framing, and assessment of Regents level mathematics curricula in the public schools of New York changed since 1866?: and 2) How has popularization influenced the contents, structure and academic rigor of Regents mathematics examinations? Basil Bernstein's theory of educational transmissions provides a theoretical framework for the study, as does the lens of credentials theory. Expectations and beliefs based on theory and historical narrative are subjected to critical and empirical analyses using a longitudinal research sample containing 204 Regents mathematics examinations with 5,508 individual problems, representing the entire population of extant Regents mathematics examinations administered in the years 1866, 1870, 1880, 1890, 1900, 1909, 1920, 1930, 1940, 1950, 1960, 1970, 1980, 1990, 2000 and 2009.

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CHAPTER I - INTRODUCTION

The Legislation Creating the Regents Examination System of New York State

On July 27, 1864, during the midst of the American Civil War, the legislature of the pro-Union state of New York passed an ordinance creating the Regents examination system. This ordinance included the following provisions for assessment of students:

> At the close of each academic term, a public examination shall be held of all scholars presumed to have completed preliminary studies. . . .To each scholar who sustains such examination, a certificate shall entitle the person holding it to admission into the academic class in any academy subject to the visitation of the Regents, without further examination (SED, 1987, p. 1).

A careful reading of the ordinance illuminates much about the legislature's intent in establishing the Regents examination system. The central idea of the legislation was to create an educational control system that could be used to regulate the flow of funds to the well established academy system of schools that existed throughout the state of New York. This goal would be accomplished by: 1) creating a Regents examination system, which would measure student achievement through process of examination; and 2) creating a new and privileged class of students in the secondary schools of New York. The new class of students would be called the "academic class," and those students who qualified for admission to it by sustaining a process of examination would be known as "academic scholars." Academic scholars, and the institutions with which they were affiliated, would receive recognition and privilege under New York's school funding formula.

The focus of the ordinance was on assessing student achievement in the preliminary, or elementary curricula. In essence, the examinations were being positioned in the primary role of gatekeeper between the primary and secondary schools of the state of New York. The need for a gatekeeper examination system was due in part, to the state's 1864 school funding formula, which allocated public funds to private academies based on criteria that included the number of enrolled students. Typically, the academies used money distributed from the state literature fund to offset operating expenses, and any expenses in excess of funds received from the State were passed on to students and their families in the form of "rate bills." Under this system, individual academies could realize economic advantages by lowering academic standards and enrolling less qualified students. In 1864, during a time of war, the New York legislature became concerned about this issue of who was and who was not qualified to be enrolled in the common, mostly private academies of the state and also in the rare, public high schools of the state. The timing of the legislature's concern and actions in 1864 may also have been influenced by political interests associated with: 1) the military's need for young men of fighting age; and/or 2) a period of fiscal austerity in school funding, both of which were related to the ongoing Civil War.

As a state sponsored quality control system, the Regents examination system has influenced the micro-level practices of New York's public schools since 1866, when the first Regents examinations were administered. Of significant importance, this Regents examination system has detailed records of assessment and curricula practices throughout its existence. The extant historical record of Regents examinations relating only to the field of mathematics includes over 1,500 examinations in curricula ranging from arithmetic to spherical trigonometry and conics. The consistency with which these Regents mathematics examinations have been administered is also important. Mathematics examinations exist for 131 of the last 144 years and for 32 different curricula. As historical artifacts of public education in the state of New York, these consistently administered Regents mathematics examinations provide opportunities for detailed analyses of historical trends in the assessment practices of mathematics curricula in the publicly funded schools of New York State.

This dissertation uses the Regents examination system as a lens for illuminating the history of mathematics education in the public schools of New York State between 1866 and 2009 inclusive. It begins by framing the Regents examination system as a means for state regulation and control of public education and proceeds to examine the historical record left by the Regents examination system to show how state control of assessment practices in mathematics education has evolved over a span of 144 years. In doing so, it focuses on two specific research questions.

Two Research Questions

1) How has the classification and framing of assessed knowledge in the core subject area of mathematics changed in Regents level examinations administered in the public schools of New York since 1866?
2) How has popularization influenced the contents, structure and academic rigor of Regents mathematics examinations?

Throughout this dissertation, both questions are framed in Basil Bernstein's theory of educational transmissions and credentials theory. Both theories illuminate our understanding of the social stratification effects of public schools.

Why This Research is Important

This research is important for several reasons. First and foremost, no comparable historical study of any statewide assessment program in mathematics education in the United States is known to exist. Thus, any lessons learned from the historical record left by the Regents examination system relative to mathematics assessment practices in the public schools of New York over a span of 14 decades may be useful in understanding and making current decisions regarding assessment and curricula. The record of assessment practices left by the Regents examination system provides a unique opportunity for historians of education to examine the actual practices and rituals of public schools over an extended period of time. Increased knowledge of actual practices and rituals of schools could provide empirical evidence for greater understanding of the relationships between changes in the micro-level practices of schools, contemporaneous historical events, and macro-level structures of society.

This study, in adopting Basil Bernstein's view of the sociology of education, operationally defines such macro-level structures as those associated with broad historical perspectives such as the structures characteristic of agrarian, industrial and/or post-industrial societies, and in the changing compositions of social and economic classes of students being educated. Societal events, which are to be distinguished from societal structures, are operationally defined by exemplars such as: 1) the two World Wars and other wars of the 20th Century; 2) the launching of Sputnik during the Cold War in 1957, which led to the space race; 3) important documents such as "A Nation at Risk" in 1983; and 4) important legislation such as the "No Child Left Behind Act" in 2001. Such societal events are often thought of as events that influence education. Beyond rhetoric, however, little or no empirical evidence supports the proposition that societal events have typically penetrated educational bureaucracies and influenced micro-level practices of schools in the areas of assessment and curricula. As suggested by Herbert Kleibard, curriculum change appears to be continuous and ongoing in public education (Kleibard, 2004). The Regents examination system has left us a uniquely understandable historical record through which assessment and curricula can be examined over a span of 14 decades. This research focuses on developing this new approach to understanding the relationships between macro-level societal structures and events and micro-level practices of schools.

Control Systems and the Modern Social Efficiency Movement in Education

This research is also important because it is perceived as highly relevant to current issues in public education. When the No Child Left Behind (NCLB) Act promulgated high stakes testing as a means of quality control over schools, New York State simply used its long-established Regents examination system to meet new federal regulations. As will be discussed in the next few pages, the NCLB is arguably grounded in modern day social efficiency movement ideas concerning the measurement and control of performance and standards.

This researcher returned to public education and the halls of academia rather late in life, following a career and retirement from American General Corporation and its subsidiaries. During over twenty years of company service, this researcher was influenced by Harold Hook, a former President and Chief Executive Officer of American General Corporation in Houston, Texas. Harold Hook wrote a proprietary management development program known as Model-Netics, which is widely regarded in business and industry (Hook, 2009). Elements of the Model-

Netics program were used to train the administrators of the Houston Independent School District, which George W. Bush hailed as an exemplar of school improvement during his first presidential campaign. Rod Paige, the first Secretary of Education under President George W. Bush, was the superintendent of the Houston Independent School District immediately before becoming Secretary of Education, and Rod Paige is a graduate of the Model-Netics management development program and a certified Model-Netics instructor. It was under Rod Paige's tenure as Superintendent that a version of Model-Netics was introduced to the Houston Independent School District. The appointment of Rod Paige to the superintendancy of the Houston Independent School District and his subsequent introduction of the Model-Netics management development program are arguably the first steps in a series of events that culminated in what has been referred to as the "Miracle in Houston" (CBS, 2004) and also to the beginning of a control paradigm for education that would eventually be enshrined in the No Child Left Behind Act (NCLB). In 2003, Rod Paige, as Secretary of Education for the United States, made the following comments about his experiences in the Houston Independent School District (HISD) and the relevance of those experiences to NCLB during a speech:

> So there was a community-wide effort to make the school system better. The community started down the path of school reform, step by step, through a series of actions. First, HISD introduced greater accountability. There has been a long history of accountability in education, going back to the 1980s. A businessman, Charles Duncan, asked, "How will we know if our children are learning?" How indeed! The result was the Perot Report, which was the foundation of the Texas accountability system. It resulted in House Bill 72, which has been in place through several generations of state leadership. It has received bipartisan support. Second, HISD adopted an open attitude to reform, looking for the ways and means to quickly improve the quality of education. Openness included reaching out to the business community, asking them to become a partner. HISD even invited representatives from the business community to audit our books, offer suggestions to cut waste, and help develop a better managed educational system. The community looked to Al Haines, Harold Hook and the Houston Business Advisory Committee, and the Greater Houston Partnership for assistance (Paige, 2003).

Harold Hook, as the inventor of Model-Netics and Chairman and CEO of one of America’s largest insurance and financial services companies; Rod Paige, as the Superintendent of the Houston Independent School District and later as Secretary of Education of the United States; and George W. Bush, as first the Governor of Texas and then as President of the United States, were thus positioned in history as individuals whose voices were important during the development of the “No Child Left Behind Act,” and the management theories and principles embodied in Harold Hook’s Model-Netics management development program can help to illuminate and explain current issues that are often debated in public education policy.

Model-Netics is a proprietary program of the Main Event Management Corporation, and was developed by Harold Hook. The Main Event Management Corporation website describes Model-Netics as follows:

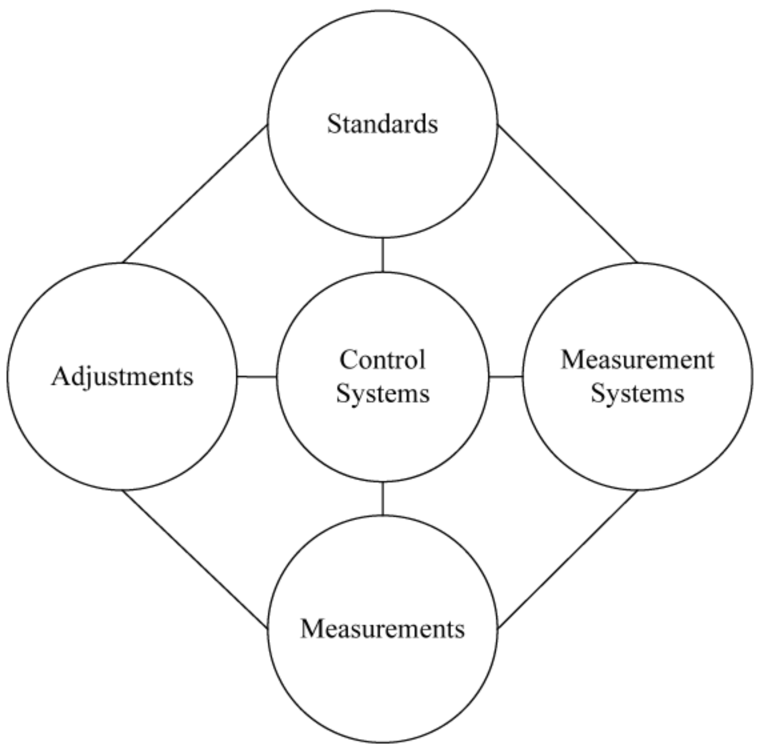
> Model-Netics is a comprehensive management training and development program. It literally means "models in action." The Basic Course in Model-Netics is composed of 151 management models that function as guides to thought and action (MEM, 2009).

The Main Event Management website goes on to define models and their use in the following words:

> The models represent key management concepts that have wide application to both organizations and individuals. The models translate these concepts into practical, operational tools. They are part of a "management alphabet"--the building blocks in a system that weds soundly conceived management theory to the practice of management (MEM, 2009).

There are parallels between the Model-Netics management development program as a “…management alphabet…” of models that function as “…guides to thought and action” and E.D. Hirsch’s book entitled “Cultural Literacy: What Every American Needs to Know.” Indeed, when Hirsch's book was initially published, Harold Hook had copies of it distributed to selected managers and directors throughout the American General Corporation that he controlled, including to this researcher (Hirsch, 1988).

An example of a Model-Netics model is the Control Diamond. The Control Diamond can be understood as positing that control systems of all types must include four essential elements: 1) standards; 2) measurement systems; 3) measurements; and 4) the willingness and ability to adjust performance that is not meeting standards. These four elements are reflected in the following representation of a Model-Netics graphic and are echoed elsewhere in this dissertation:



Hook's Model of a Control System Figure 1-1

Visual graphics are important mnemonic memory devices in the Model-Netics program. The program is premised on the idea that important concepts must be understood at levels of abstraction that can be recalled from memory in real time. Definitions of models are typically short and carefully worded. The entire Model-Netics program of 151 models is summarized in a thin “memory jogger” that is designed to fit into a suit pocket, purse, or attache. Tenets that underlie the Model-Netics program include the ideas that every model should be committed to memory, and that once memorized, an individual’s lived experiences and related knowledge will thus become more organized and retrievable in real time through a well conceived, interacting set of heuristics that serve as anchors for analogical reasoning activities involving such diverse topics as: change management; selection, evaluation and compensation of employees; delegation and motivation; planning; management processes; communications, learning and training; control; problem solving; decision making; and leadership.

As Superintendent of the Houston Independent School District, Rod Paige and Harold Hook developed a relationship, which evolved into a collaborative effort to implement the Model-Netics management development program and related Main Event Management systems in public education (MEM. 2004a, 2004b). This collaboration between business and industry and public schools is an exemplar of the social efficiency movement in public education, which attempts to influence educational practices using management practices more commonly associated with business and industry. It is within this context that we return now to our explanation of the Control Diamond, which is an anchoring model in a group of models that address control systems over individuals and institutions.

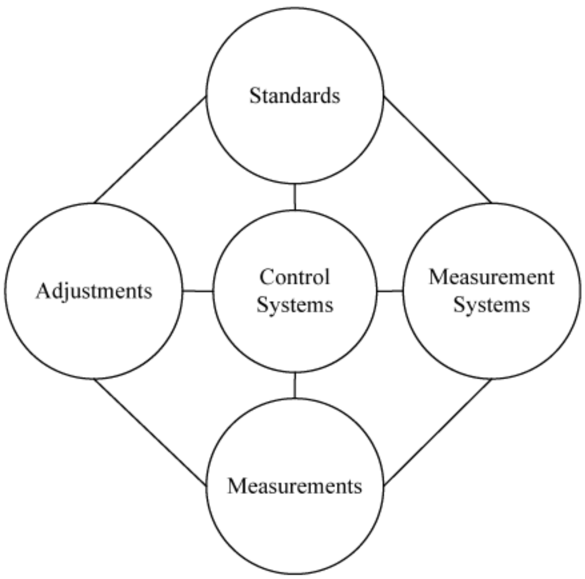
Our interest in Hook’s Control Diamond is based in its usefulness as an exemplar of sound management theory as well as the social efficiency movement’s thinking relative to standards and measurements. Its principles and ideas are also useful for identifying and understanding relationships between high stakes testing and other elements of control over public education, including elements of the Regents examination system. We begin by associating the basic definition of the Control Diamond with a real life application that is relevant to most of our lived experiences. In so doing, the model's usefulness as a grounding for analogical reasoning and metaphor is illustrated.

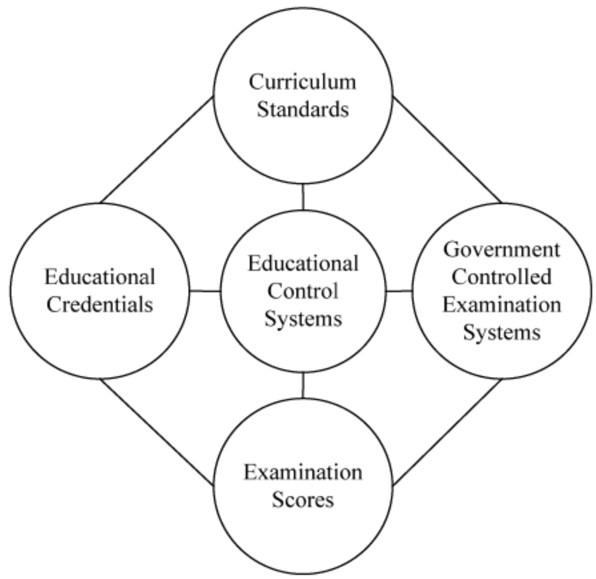
The Control Diamond can be understood through the metaphor of controlling temperature in a room. Recall that the control diamond has four elements: standards; measurements systems; measurements; and adjustments. Control of temperature in a room is first and foremost dependent on having a standard for what the temperature should be. If one does not have an expectation for a specific temperature or range of temperatures in the room, the temperature in the room is not controlled. The second necessary element is having some system for measuring the temperature, such as a thermometer. If one has no way of sensing or measuring the temperature in the room, the temperature in the room is not controlled. The third necessary element is actually using the measurement system to take a measurement. If one has a thermometer, but does not use it to measure the temperature of the room, the room temperature is not controlled. The fourth and final element necessary for control is the willingness and ability to make an adjustment. If one knows that the temperature is outside of the standard, but does not have the ability or the desire to add heat or cold to the room, the temperature in the room is not controlled. Model-Netics teaches that all four elements of a control system must be present and used for control to exist.

When the Regents examination system is understood within the framework of Hook's conceptualization of a control system, the relationship between state controls and the micro-level practices of schools are illuminated. When Hook's conceptualization of a control system is applied to the bureaucracy of public education, connections are illuminated between efforts to measure teacher effectiveness, high stakes testing, and the curriculum standards movement.

High stakes testing is the measurement system that is linked to the modern curriculum standards movement. Control over the micro-level practices of schools and teachers is the overall objective, just as it has been with the Regents examination system since 1866. Without curriculum standards, high stakes testing has no grounding. As the social efficiency advocates press their agenda to have schools and teachers controlled and managed in the same ways that businesses, industries, and private sector employees are controlled and managed, they have established themselves in a formidable position of power vis-à-vis the curriculum standards movement. On this view, the Regents examination system combines high stakes testing with a traditional classical humanist approach to education, which competes with and subverts child centered progressive education initiatives, and explains some of the resistance to high stakes testing voiced by many child centered educators. Nancy Beadie observed this threat to democratic education in the Regents examination system of the 1800s (Beadie, 1999a). The threat remains today.

The parallels between Hook's Control Diamond, the Regents examination system over a span of 144 years, and current initiatives in public education are illustrated in Figure 1-2. This is not to suggest that Harold Hook is responsible for the current social efficiency movement in education, but rather, it does suggest that Hook's conceptualization of control in modern business and industry has strong parallels in both the Regents examination system as a historical reality and in the modern high stakes testing movement as a current reality. A reasonable argument can be made that Hook's conceptualization of a control system, as reflected in the Control Diamond of the Model-Netics program, is reflected in the control paradigm that is embedded in the "No Child Left Behind Act," which was enacted when Rod Paige, a certified Model-Netics instructor and advocate for the teaching of Model-Netics to educational bureaucrats and administrators, became Secretary of Education. A second reasonable argument can be made that Hook's conceptualization of the elements of a control system was understood by the New York State Legislature in 1864, when it deliberated on and enacted legislation creating the Regents examination system.





Hook's Model of a Control System *High Stakes Testing as a Control System*

Figure 1-2

When high stakes testing is viewed through the lens of the Control Diamond, the relationship between the curriculum standards movement and high stakes testing is illuminated, as is the relationship between the curriculum standards movement and current initiatives in New York City relating to public school report cards and merit pay for teachers. Rod Paige's training in Model-Netics, the passage of George W. Bush's signature educational reform act -- No Child Left Behind -- with Rod Paige in charge as Secretary of Education, the resulting unprecedented focus on high stakes testing as a measurement system for the efficacy of public education, and the public records of political contributions made by American General Corporation to President George W. Bush's Republican presidential election campaign, suggest that the political economy of the social efficiency movement, more than scholarly research or the influence of academia, are critical to understanding the current movement towards increased reliance on high stakes testing. This is consistent with William Domhoff's views on power, politics, and who rules America (Domhoff, 2006). In 2004, Harold Hook's alma mater, the University of Missouri, accepted a $2,000,000 donation from the Hooks and honored Harold and his wife Joanne. The following passage appears in a University of Missouri publication commemorating this event.

> In the 1990s, as a leader in Houston's business community, Harold worked with the then-superintendent of the Houston school district, Rod Paige, to help turn around a school system plagued by inefficiency and underperformance. Using Model-Netics, a program Harold had created to assess management systems, the district was able to improve its performance. The project was so successful that Paige, now U.S. Secretary of Education, is using the model developed with Harold's help as the basis for the federal No Child Left Behind Act, which requires schools to meet performance standards (Missouri, Univ. of, 2004).

Tyack and Cuban convincingly argued that the many influences of the social efficiency movement on public education can be seen in the ways that public schools are typically organized and managed, and are reflected in such diverse practices as: 1) block scheduling, which resembles the production scheduling of a factory; 2) Carnegie Units and scope and sequence guidelines, which resemble the various sequences and pathways of manufacturing processes; 3) and periodic testing and grading, which resemble the accountability and measurement and control systems used in modern business and industry (Tyack and Cuban, 1995). As we begin the 21st Century, perhaps the most significant new influence of the social efficiency movement is in the growth of high stakes testing for the direct measurement of student achievement and, indirectly, for the measurement of performance of both institutions and teachers, which resembles the measurement of corporate America in the form of quarterly financial reports and merit "pay for performance" incentive systems. We now turn our attention to the genesis of the Regents examinations system that exists in New York today, and the story begins with a description of the secondary education system as it existed in New York State during and immediately after the Civil War.

The Academy System of Public Education

In 1860s New York State, few things moved faster than a horse, except possibly trains, and the fledgling railway system of the 1860s wasn't designed to get children to and from school. The lack of mobility and transportation associated with this era acted to retard the growth of public schools in a society that was primarily agrarian and had few towns with populations greater than 5,000. Smaller communities and rural areas could find enough school children to support one room schoolhouses, but there were seldom enough students within walking distance of any central location to support a public day high school. This meant that in 1864, outside of a few large metropolitan areas such as New York City and Albany, there were few or no public high schools. This should not be interpreted as suggesting there was no demand for secondary education. It means only that the geographical distance between a student's domicile and place of schooling was a significant and often overwhelming variable that influenced student choice regarding secondary education in the 1860s. This problem, which can be characterized in modern day terms as commute time, retarded the development of a statewide system of free public high schools in New York.

Though public high schools as we know them in 2010 did not exist during the Civil War, the academy system of secondary education was well established in New York State long before the Civil War began. The academy system was able to exist due in part to the fact that it was able to resolve the commute time problem. The academy system was able to address commute time issues in two ways: first, many of the academies were boarding schools; alternatively, the academies that were not boarding schools generally assisted students in finding private room and board arrangements close to the school. Both approaches eliminated commute time problems in ways that colleges and universities still use in 2010. In June 1880, the Regents Arithmetic examination included the following question, which illuminates the costs of attending an academy, and also reflects the rich historical contexts associated with questions from old Regents examinations.

> 1880_06(a)_AR_02 Proportions
> If a scholar's expenses are 90 dollars for board, 30 dollars for clothes, 12 dollars for tuition, 5 dollars for books and 7 dollars for incidentals, what would be the expenses of 27 boys at the same rate?

Nancy Beadie, a historian of education who has studied the academy system of New York, posits that the academy system of schools was widely accepted by middle class families throughout the state. Beadie estimates that 50% or more of middle class students in New York attended these academies for some period of time, though attendance was sporadic and many students never graduated. She notes that the old academy system provided middle class New York families with more than a simple secondary education. The academy system also provided middle class students with opportunities for social interaction, networking, and development of a worldview that extended beyond the confines of their local farms and villages. Students not desiring or unable to afford the perceived benefits of long stays at academies could often arrange for self-study programs at home punctuated with brief periods of attendance and tutelage at the academies -- similar in some respects to the distance learning programs of modern colleges and universities. During this era of private schools that received funding from the state, New York's school funding formula in 1864 provided financial incentives for these academies to enroll more students, and a system of accountability was perceived as necessary to protect the state from abuses of the school funding formula by academies who were willing to lower or eliminate admission standards to increase income (Beadie 1999a, 1999b).

State Control over the Micro-Level Practices of Schools and The Credentials Market

The influence of the Regents examination system was quickly felt by academies throughout the state of New York. After the implementation of the Regents examination system, the academies received per student allocations from the state literature fund only for enrolled students who had successfully sustained the state controlled process of examination and received state issued Regents examination certificates. The same academies did not receive financial support from the state for enrolling or educating non-credentialed students, though they could continue to do so without state funding. Beadie posits that under this funding formula, students who were certified by process of examination as academic scholars were preferable to students who were not credentialed, and a competition for credentialed students arose in the academies (Beadie, 1999a, 1999b).

Beadie also illuminates contemporary events of the 1860s that may have influenced the legislative decision to create the Regents examination system. She notes that during the years immediately preceding the creation of the Regents examination system, there was considerable public sentiment for unity and standardization through centralized control in many areas of government. The Civil War itself is sometimes framed as a struggle between those who advocated for a stronger central government and those who advocated for decentralized states rights, tantamount to autonomy at a more local level. One result of the Union's victory and the defeat of the Confederacy was a shift in the locus of power toward the national government and away from the state governments. Similarly, new federal laws authorized the national banking system and the use of gold to back United States currency, thus ending the era of free banking in which autonomous states chartered their own banks. Federal legislation also was enacted during this period that prohibited states from issuing their own currencies, thus restricting practices of local autonomy that predated the American Revolution. It was within this period of strong preference for centralized rather than localized authority that the New York State Legislature in 1864 deliberated on and voted for a single, state-controlled system of educational testing and credentials, which was intended to eliminate the local autonomy of the academies over these matters and establish a gold standard for educational credentials in the public schools of New York.

Beadie documents a drop in the total number of students attending high schools and academies in New York immediately after the creation of the Regents examination system and lasting until the 1890s. She attributes the drop in academy enrollments in New York following the Civil War to the influence of the Regents examination system in preventing students from qualifying for admission to the academic classes of students within the academies that received state funding when she writes,

> The restriction of access to a level below existing demand had the effect of creating competition for credentials that mediated that access. It was precisely because academies had already built a broad clientele for higher level study, in other words, that the raising of academic standards produced a competitive student culture and a valuable academic credential (Beadie, 1999a, p.27).

Because of this competition for credentials, Beadie opines that the creation and implementation of the Regents examination system in New York was not supportive of democratic equality in education, noting that the immediate effect of the Regents examination system was that fewer students were educated.

The "certificates" envisioned in the 1864 ordinance were issued to students who passed the preliminary Regents examinations. These certificates quickly became a new form of educational credentials that were like currency in the sense that they had economic value. Through these certificates, the state could influence the micro-level practices of any academy that desired to receive state funding, and any elementary schools that desired to prepare students for matriculation into the academy system. Hence, the Regents examination system assumed dual roles associated with both quality control and gatekeeping at the intersection of the elementary and secondary school systems of the state of New York. When viewed as a state sponsored educational control system, the Regents examination system provided: 1) the ability to establish curriculum standards at the state level; 2) the ability to measure performance against curriculum standards by process of examination; 3) the ability to issue certificates of academic credentials associated with uniform curriculum standards; and ultimately 4) the ability to link school funding and secondary school diplomas to standards, assessments, and earned credentials. These four characteristics of the Regents examination system have a one-to-one correspondence with Hook's four required characteristics of management control systems, which are embedded in NCLB. Accordingly, when NCLB was implemented in the public schools of New York, the state adapted the Regents examination system without significant modification to meet federal high stakes testing requirements.

The Expansion of the Regents Examination System

to Regulate Secondary School Curricula

The success of the early Regents examinations in regulating enrollment into the academies and creating uniform standards for academic scholars was quickly acknowledged, The influence of the Regents examination on the academies notwithstanding, perhaps more important to a longer view of history is the idea that the Regents examination system exerted a similar controlling influence on the curricula and pedagogical practices of the still small, but growing, number of public high schools in the state. This small number of public high schools would grow and eventually supplant the academy system, becoming the network of public high schools that currently dominates New York secondary education – still under the influence of the Regents examination system.

Contemporaneous with the early successes of the preliminary examinations in controlling admission to the “academic class” of students in the academies of New York, a new problem was being recognized in schools throughout the state. Colleges and universities were expressing concerns about the uneven quality and lack of uniformity in the curricula of the independent academies. Teachers within the academies were complaining that they did not know what to teach, since each college or university had its own examinations with different academic standards. One university might require the study of Virgil and another might require the study of Homer, and if two universities each required the study of Virgil, each university might have significantly different approaches to evaluating prospective students. Teachers and administrators in the academies were subjected to criticism from students, parents, colleagues, and others if a graduate was found during a particular college or university’s entrance examinations to be inadequately prepared in a required subject matter (SED, 1987). Clear and uniform standards were needed and wanted, and the Regents, being (then as well as now) the overseers of New York's public and private colleges and universities, as well as New York's elementary and secondary schools, were empowered to resolve the dilemma. (See Appendix A for an overview of the broad authority that today's New York State Board of Regents hold over education and other matters in the state of New York.) Embedded deep within the bureaucratic structures associated with the Board of Regents are those systems and procedures used by the state to develop and administer Regents examinations within the scope of the legislation authorizing the Regents examination system. Since the institution and power of the Board of Regents is defined by the New York State Constitution, it is reasonable to believe that the Board of Regents has always been a powerful force responsible for controlling the micro-level practices of public schools. The Regents examination system, from this perspective, has long been an important management tool for state control of public education.

In 1878, Dr. John E. Bradley, a high school principal in Albany, New York, at one of the few public high schools in the state at the time, commented in a speech about the decision to create a new set of academic examinations. He noted that these examinations would be in addition to the preliminary examinations, and that they would assess the knowledge acquired by students after they were admitted to the academic classes of students in the academies, rather than their qualifications for admittance to such academic classes. Dr. Bradley's remarks indicate that educators and politicians were very much aware of the power of the state controlled process of examination to influence the micro-level practices of schools, and he suggested that this characteristic of the Regents examination system was a primary factor in the Board of Regents' decision to expand it. In 1878, Dr. Bradley was quoted as saying,

> The salutary influence of the primary examinations in stimulating both teachers and pupils to thoroughness in the acquisition of the elementary branches suggested the extension of the system to academic studies. It was argued that the Regents exhibited great solicitude with reference to the admission of pupils to high schools and academies, but took no interest in the kind of instruction they received there, or the amount of knowledge with which they graduated. If there was danger of neglecting the elementary branches and advancing schools prematurely, the danger of superficiality and misdirection in the range of secondary study was still greater (SED, 1987).

Beady posits a slightly different, but complementary view to Dr. Bradley's when she argued more than 100 years later that the Regents examination system has evolved since its creation. Beady argues that the Regents examination system was originally created to rationalize the educational system throughout New York State, but later evolved to include a credentialing function with significance for the workforce. Beadie refers to the 1878 expansion of the Regents examination system and the evolving nature of Regents credentials when she writes:

> In effect, the arena of competition in higher schooling would change, from one in which institutions competed for students to one in which students competed for access to higher schooling and for the acquisition of academic credentials….This account differs from other explanations of the rise of credentialism in that it locates much of the impetus for the creation of credentials markets with the education system itself rather than in external market forces (Beadie, 1999a, p.29).

Beadie's position is deeply grounded in credentials theory. The academic examinations, which were introduced in 1878 to assess student achievement in the secondary schools of New York, and the Regents academic diplomas associated with them, heralded the beginning of: 1) state controlled credentialism in the public schools of New York; and 2) the Regents examination system that still exists in the year 2010.

In 1878, when the Regents examination system was expanded, the first academic examinations assessed student achievement in 24 subject areas that were taught in the academic curricula. Students were allowed to take as many as fifteen examinations during their years of secondary schooling. To earn a Regents diploma, students had to pass examinations in the seven core subjects of: 1) algebra; 2) plane geometry; 3) physiology; 4) natural philosophy (physics and astronomy); 5) rhetoric and English composition; 6) history (American and general); and 7) chemistry. This expansion in the scope of the Regents examination system can be better understood in terms of the number of curricula assessed and the number of examinations that needed to be created for each examination period. In 1877, prior to the advent of the academic examinations, only five elementary curricula were assessed. In 1878, 29 different subjects were assessed through preliminary and academic examinations. Thus, while the Regents academic examinations were initially created to rationalize relationships between educational institutions in the state of New York, they later became a means of regulating competition for access to higher education and labor markets.

The Enshrinement of the Regents Examination System as a Ritualistic Hallmark of Public Education in the Secondary Schools of New York

In 1965, on the 100th anniversary of the Regents examination system, the New York State Education Department published a celebratory booklet entitled, *Regents Examinations – 100 Years of Quality Control in Education: 1865-1965*. The following passage from this celebratory booklet summarizes the New York State Education Department's 1965 reflection on the creation of the Regents examination system.

> The Regents examination system began in New York State in November 1865 (sic) as a plan of high school entrance examinations. The amount of state aid to public academies was based on the number of pupils enrolled in each academy. To discover who were bona fide academy students, the Board of Regents established admission examinations, and a State certificate was awarded to successful candidates. The plan of uniform and impartial entrance examinations was immediately successful, and there soon arose a strong demand for similar

> safeguards and standards for high school graduation and college admission. In June 1878, therefore, the Regents administered the first of the academic or high school examinations….From these beginnings, the modern system of high school achievement examinations developed. In a relatively short time, “Regents credit” became universal academic currency (SED 1965, p. 4).

The celebratory tone of the State Education Department’s 1965 booklet commemorating the 100 year anniversary of the Regents examination system was echoed on the first page of every academic examination administered during June of that year. The banner read, “Centennial of Regents Examinations 1865 — 1965.” One can imagine that for examinees in 1965 there might have been a distinct lack of joy in this centennial celebration. What was important for the examinee was not the centennial celebration, but participation in a process of examination, which by 1965 had become a necessary and ritualistic ordeal for academically elite middle class students in their pursuit of valued credentials that were associated with passage into higher reaches of the educational system.

The Evolution of the Regents Examination System

The Regents examination system has evolved in many ways since the introduction of the early preliminary and academic examinations, reflecting in part the growth of public education as a bureaucracy. In 1875, total enrollments in academies and the academic departments of union schools in New York State was 12,000. By 1900, as the number of academies decreased and the number of free public high schools increased, total enrollment had increased to about 100,000. By 1925, total enrollment had increased to 350,000, and in 1965, total enrollment was approaching one million students (SED, 1965). Today, the total enrollment in secondary schools is approaching 1.5 million students.

The growth in public school enrollments was paralleled by the development of the modern system of high schools in New York State, which displaced the former system of private and public academies, and also by the growth of an educational bureaucracy involving layers of administration and the development of education as a profession. Eileen Donoghue writes about the growth of mathematics education as a profession within education and locates this development in time as occurring between 1890 and 1920 (Donoghue, 2003a). Donoghue's assertions generally coincide with the development of the normal school system for the training of teachers in New York State.

New York's system of normal schools for the preparation of teachers began in 1844 with a single normal school in Albany. No other normal schools were created in New York before the end of the Civil War. However, during the thirty years that followed the Civil War, additional normal schools were developed by New York State in the towns of Oswego, Brockport, Fredonia, Cortland, Buffalo, Geneseo, Potsdam, New Paltz, Oneonta, Plattsburgh, and Jamaica (Queens). These normal schools would, in the early 20th Century, become the core schools of a network of state colleges and universities known as the State University of New York, which is more commonly referred to by its acronym, SUNY. As the structures of public education grew and evolved, so too did the Regents examination system.

In its 1965 celebratory publication, the State Education Department commented on two major developments of particular significance with respect to the evolution of the Regents system of examinations during their first one hundred years. The State Education Department called specific attention to the following changes:

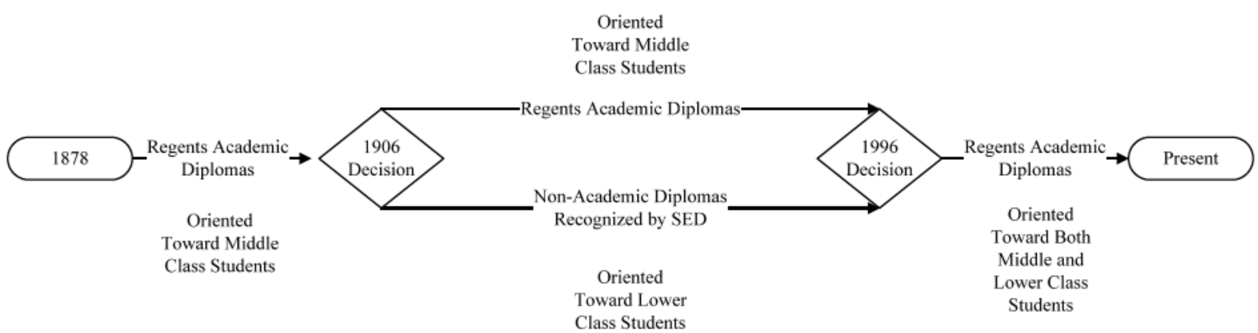
> First, Regents examinations have been transformed from narrowly viewed college preparatory tests into broad evaluation instruments…The increasing variety and broadened scope of high school subjects have led to a corresponding change in the examinations.…Second, the purposes and functions of Regents examinations

> have been reoriented….Local schools have been encouraged to recognize that Regents examinations constitute only a partial basis for evaluating pupil progress or the effectiveness of the school program, and have been assisted in developing well-balanced local testing programs for those purposes (SED, 1965).

The idea that the tests had evolved by 1965 away from college admissions examination and into broad evaluation instruments can be empirically validated by the movement towards integrated curricula and fewer numbers of examinations. The second observation made in 1965, that the examinations had been reoriented to accommodate other methods for evaluating pupil progress, can also be supported with evidence from the historical record. Prior to 1906, the only diploma recognized by the State Education Department was the Regents Diploma. In 1906, the state responded to progressive voices in education and began recognizing non-academic diplomas, which were not associated with Regents academic examinations. The "reorienting" of the Regents system of examinations and diplomas to allow recognition of non-academic diplomas within the educational bureaucracy of public schools created new opportunities for tracking students into different curricula, and such tracking became a fundamental feature of public schools. This bifurcation of the student body into academic and non-academic tracks arguably preserved the high standards of the Regents examinations during a time when increasing numbers of lower class students were being enrolled in the educational bureaucracy. Toward this end, the State Education Department explicitly stated in 1965 that, "….Regents examinations….are designed for pupils of average and above-average ability…." The reality of who received Regents academic diplomas in 1965 suggests that Regents examinations may not have been as popular with students of average ability as the designers intended. With total high school enrollment in 1965 approaching one million students, only 65,000 students actually earned Regents academic diplomas.

In 1996, the State Education Department decided again to reorient the Regents examination and diploma system. After ninety years of recognizing local option diplomas together with Regents academic diplomas, the State Education Department decided to return the Regents examination and diploma system to its pre-1906 roots, and will once again require all students to earn high school diplomas by passing Regents examinations. From this historical perspective, the educational credentials recognized by the New York State Education Department can be summarized in figure 1-3.

Diplomas Recognized by the State of New York



Three Eras of Diplomas Figure 1-3

This graphic illuminates the present research opportunity to analyze the influence of popularization on Regents examinations. The general characteristics of the pre-1906 population of students in New York's academies and secondary schools has been the subject of published research and is generally recognized as primarily middle class (Beadie, 1999b). Between 1906 and 1996, the number of lower class students attending schools increased significantly, but Regents diplomas and the Regents academic examinations associated with them were optional and were targeted toward students of average and above average ability (SED, 1965). Research on tracking by Jeannie Oakes and others has repeatedly shown that disproportionate numbers of lower class students were tracked into less demanding, non-academic curricula (Hallinan, 1990, 1994a, 1994b) (Hallinan and Sorenson, 1987) (Kubitshek and Hallinan, 1998) (Kulik & Kulik)

(LeTendre, Hofer and Shimizu, 2003) (Ma, 2002) (Oakes, 1994) and (Useem, 1992). Since 1996, more and more New York State students have been moved into tracks that culminate in commencement level Regents academic examinations, and the academic rigor of these examinations, originally intended for average and above average students, has been reduced to accommodate the politicalyl driven need to graduate students with less than average abilities. A historical analysis of the Regents examination system and the classes of students taking Regents examinations would thus be expected to show that popularization of the Regents diploma as an educational credential leads to lower levels of academic rigor and to deteriorations in credential prestige and value. Such a hypothesis is consistent with other studies that have shown marked declines in test scores when the population of test takers increases (Madaus, 2003). Chapter V explores this hypothesis against empirical evidence left by Regents academic examinations in mathematics.

Today's Regents Examination System

As the first decade of the 21st Century ends, the Regents examination system continues to evolve. The Regents diploma is no longer viewed as a hallmark of academic excellence. Regents examinations are no longer associated with academically elite students. Instead, Regents examinations are administered to all students, and the Regents diploma is the lowest level of general diploma offered in the secondary schools of New York. Secondary school students entering general education classes in the public secondary schools of New York State in 2010 must take and pass Regents examinations in English, Math, Science, Global Studies and United States History before they can graduate. Consideration is being given to reducing the number of Regents examinations to the minimum required by NCLB. As decisions about the future of the Regents examination system in New York and elsewhere are made, it is important to illuminate and interpret the history of the Regents examination system in the public schools of New York.

Summary

In the preceding pages, the Regents examination system is introduced as a control system for public education, which has been in existence throughout the state of New York since the Civil War. This control system for public education features a highly ritualized process of examination and produces educations credentials with purported values within education and within the workforce. The Regents process of examination in mathematics has always relied on print-based paper examinations, and there is an abundant population of extant Regents examinations in the historical record. Understanding the story of these examinations is important to the history of public education and may illuminate areas of state control of the micro-level processes of public schools.

In this chapter, the 1864 origins and subsequent 144 years of evolution of the Regents examination system are highlighted, and comparisons are made between the Regents examination system and the modern day control paradigms associated with NCLB. Chapter II examines the methodology used in gathering and synthesizing: 1) the historical records of mathematical assessments left by the Regents examination system over a span of 144 years; and 2) popular discourses affecting mathematics education. Chapter III is a historical narrative that synthesizes popular discourses of history and mathematics education with contemporaneous mathematics assessment practices observed in the research sample of Regents mathematics examinations. Chapters IV and V respond to the two research questions using information from the research sample, the synthesized narrative, and two theoretical lenses, which are: 1) Basil Bernstein's theories of educational transmissions and the relationships between micro-level practices of schools and macro-level structures of society; and 2) credentials theory.

CHAPTER II - METHODOLOGY

Overview of the Methodology

The research methodologies described in the following pages evolved from a simple belief that historical Regents mathematics examinations represent a window, through which historical practices in mathematics education can be illuminated. The structures of these examinations, plus the individual problems they contain, constitute historical artifacts of actual assessment practices created and administered by a state education department for the control of matriculation into and graduation from public secondary schools, and the historical record is largely intact over a span of 144 years. Through the study and evaluation of the extant artifacts of this examination system, inferences can be made about the micro-level practices of public schools, such as when different assessment topics were added to or deleted from the assessed curricula, which topics were assessed in different curricula, and how assessments of an individual topic changed over time.

This chapter focuses on the methodologies and difficulties associated with interpreting the historical record left by the Regents examination system. It attempts to provide a modest foundation for a longer term project to understand the history of mathematics education in New York, and to provide fellow researchers and interested parties with information concerning the capabilities and limitations of interpreting history through lenses crafted from past assessment practices in mathematics. Once appropriate disclosures and confessions of methodology are completed, this vetted interpretive lens will be used to respond to the two research questions that are associated with this study. These two research questions are:

1) How has the classification and framing of assessed knowledge in the core subject area of mathematics changed in Regents level examinations administered in the public schools of New York since 1866?; and
2) How has popularization influenced the contents, structure and academic rigor of Regents mathematics examinations?

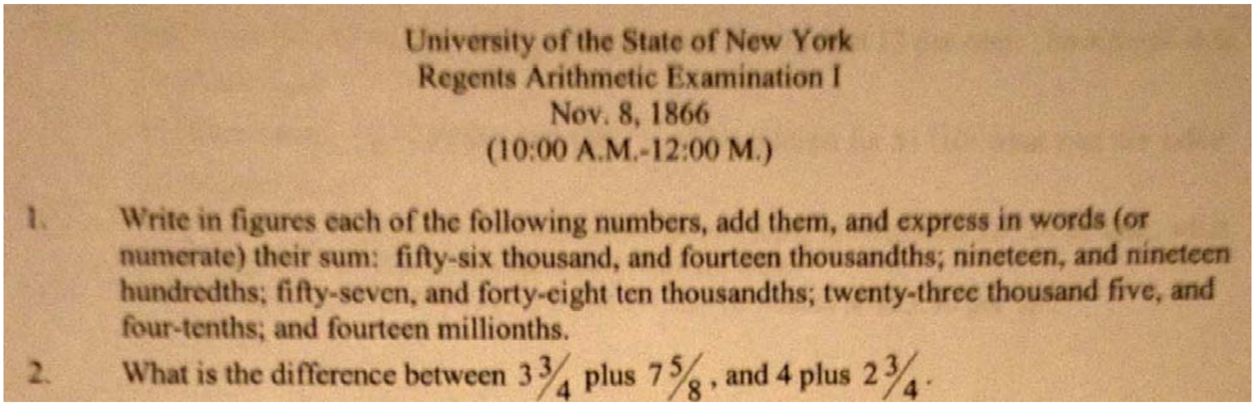
Three General Categories of Source Materials

Historical research involves the study and interpretation of a defined segment of the past, and generally requires the collection and analysis of some type of record of the event or events being studied (Wiersma, 1969, p. 234). For the purposes of this research, the record of mathematics assessment practices in the public schools of the state of New York contains source materials that may be classified into three general categories: 1) original historical artifacts and digital images of historical artifacts; 2) primary sources that reflect a transcription, summarization or interpretation of original historical artifacts or their digital images based on direct, first person interaction between interpreters and artifact; and 3) secondary sources that tend to reinterpret primary sources. A goal in developing the methodology used in this study was that the methodology should result in the creation of a primary source of information in the form of a database, directly transcribed and interpreted by a single researcher from public domain digital images of original historical artifacts. This primary resource should then be used to explore and summarize the 144 year long historical record, and to draw reasonably accurate inferences about the micro-level assessment practices of public schools in the state of New York at various points in time. Accordingly, significant effort was made during the course of this study to locate original historical artifacts and digitally record them for publication in the public domain of the internet and subsequent transcription into the database. Primary and secondary sources relating to the Regents examination system were also collected during the course of this research and are used to support and clarify various interpretations of the historical artifacts.

Digital images of 1534 old Regents mathematics examinations were collected. (See Appendix B.) Approximately 1450 of these digital images are of original source documents. The remaining digital images, which were not taken directly from extant examinations, are images of 19th Century transcriptions of then extant examinations, each of which was transcribed and published under the authority of the New York State Board of Regents.

All of the digital images were obtained from three general collections of historic Regents examinations. These three collections are located in: 1) New York City public libraries in the boroughs of Manhattan and Brooklyn; 2) the digital collections of modern Regents examinations on the website of the New York State Education Department; and 3) the Rare Manuscripts Division of the New York State Library and Archives in Albany, New York. The vast majority of the digital images were obtained from the last of these three collections.

An exemplar of a digital image of a portion of an original examination appears below, and is generally representative of the digital images created during the course of this research effort.



University of the State of New York
Regents Arithmetic Examination I
Nov. 8, 1866
(10:00 A.M.-12:00 M.)

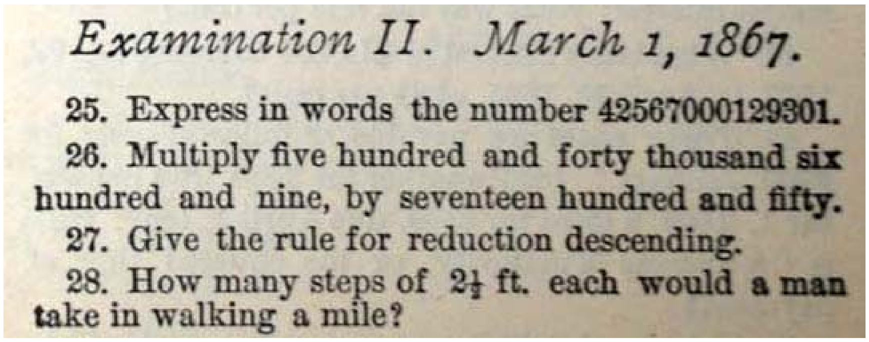
1. Write in figures each of the following numbers, add them, and express in words (or numerate) their sum: fifty-six thousand, and fourteen thousandths; nineteen, and nineteen hundredths; fifty-seven, and forty-eight ten thousandths; twenty-three thousand five, and four-tenths; and fourteen millionths.
2. What is the difference between $3\frac{3}{4}$ plus $7\frac{5}{8}$, and 4 plus $2\frac{3}{4}$.

Digital Image of An Original Examination Figure 2-1

Note that this digital image is not perfectly focused, but it is generally good enough for a reasonably accurate transcription. All but a handful of the digital images of examinations are of sufficiently high quality to ensure that the resulting transcriptions are representative of the original historical documents. When digital quality was poor, an annotation was appended to the transcription in the research sample to indicate that the transcription was based on a digital image that was questionable or unclear. These notations appear in less than 1% of all transcribed problems and do not significantly influence the overall reliability of the research sample. Unfortunately, some original examinations are missing in the extant historical record, including all academic examinations administered before 1890. Fortunately, however, 19th Century transcriptions of some of the preliminary examinations exist.

The idea of transcribing and publishing collections of questions from previously administered examinations dates back to at least 1877, when John Pratt, the Secretary of the New York State Board of Regents, began writing and publishing bound transcriptions of Regents examination problems. While Pratt's transcriptions of the original examinations are considered to be primary source documents, and therefore less credible than digital images of original examinations, they are nonetheless generally reliable as evidence of what was actually assessed.

This said, Pratt's transcriptions do not show the overall appearance and structures of the examinations. An example of Pratt's transcription of the second Regents mathematics examination is shown in figure 2-2.



Examination II. March 1, 1867.

25. Express in words the number 42567000129301.

26. Multiply five hundred and forty thousand six hundred and nine, by seventeen hundred and fifty.

27. Give the rule for reduction descending.

28. How many steps of 2½ ft. each would a man take in walking a mile?

Digital Image of a Transcription of an Examination Figure 2-2

Note that Pratt's transcription of the second Regents mathematics examination is significantly different in appearance than the digital image of the first examination. The two exemplars use different kinds and sizes of paper, different fonts, different margins, etc. Also, Pratt's transcription is not perfectly representative of the original examination because it has no header information and the first question is number 25. These are intentional inaccuracies introduced into the transcription by Pratt to facilitate the publication of scores of examination questions in a single, bound manuscript, which was intended for sale to schools and teachers throughout the state of New York. Pratt's books of Regents examination problems were probably used by both students and teachers as they prepared for the Regents mathematics examinations. Even though Pratt's transcription loses some of the structure of the original examinations, Pratt's books nonetheless retained enough information that they were of use to students and teachers for many years and several reprintings during the 19th Century. Several comparisons of original source documents with Pratt's transcriptions have satisfied this researcher that Pratt's books are suitable as evidence of historical assessment practices in the absence of digital images of the original examinations.

Pratt's purpose in transcribing and publishing problems from previously administered Regents mathematics examinations was probably to help teachers to: 1) understand the curriculum; and 2) prepare students for future Regents examinations. It can be assumed that since Pratt was the Secretary of the Board of Regents, and since he published compilations of Regents questions on several occasions in the late 1800s, his publications carried the full imprimatur of the New York State Board of Regents. The full title of the 1878 edition of Pratt's book reinforces this belief. It reads as follows: *The Regents questions: 1866 to 1878 being the questions for the preliminary examination for admission to the University of the State of New York prepared by the Regents of the university and participated in simultaneously by nearly 250 academies forming a basis for distributing nearly a million dollars*.

In the preceding analysis of Pratt's book as a primary source document, it was noted that Pratt's transcription loses the structure and format of the examinations. This is a weakness that is also associated with the current research effort. In deconstructing each examination selected for inclusion in the research sample, and transcribing each problem from each selected examination, the structures and formats of the original examinations are lost. While this loss of format and structure is a matter for disclosure, it did not deter Pratt in the 19th Century and it does deter the current effort. On this view, Pratt's books and the various compendia of Regents examination problems generated from the research database are nearly synonymous, except the compendia developed in this research effort cover longer spans of time and have more problems in them. The differences notwithstanding, Pratt created the basic paradigm for this research in the 19^{th} Century. Pratt's idea was simple: you can learn something if you collect and study questions asked on previous Regents examinations.

Pratt's books and the compendia of Regents problems created during this research effort are also comparable in the sense that both may be viewed as first person interpretations of original artifacts in the historical record, and thus as primary records. As primary records, they should not be confused with the original source documents from whence they came, and any modern day inferences made from the database and its output should be understood as containing the subjective biases of this researcher. Accordingly, it is important for any reader of this research paper, or any future users of the database, to understand its capabilities and limitations.

Secondary sources constitute the third general classification of sources that are used in this research effort, and are considered less credible than digital images of original examinations or primary sources relating to such examinations. The secondary sources used in this research effort include historical newspaper accounts, State Education Department publications, old mathematics textbooks, and other printed publications relevant to the research agenda. Secondary sources are typically removed from direct contact with the actual examinations and usually involve interpretations of interpretations. Dr. Bradley's prescient comments in 1878, described in Chapter 1 of this manuscript, are taken from a newspaper account of his speech and are representative of the secondary sources of information used in this research effort. Similarly, the words of Dr. Nancy Beadie, the educational historian quoted in Chapter 1, are also classified as secondary source materials. While secondary sources are used throughout this research effort, they are unequivocally excluded from the database. The goal in developing the database was to develop a primary source of data, taken directly from digital images of historical artifacts whenever possible, and from unimpeachable primary sources when digital images of historical artifacts could not be obtained. From this viewpoint, the historical database so created is tantamount to a primary source document. We now return to our discussion of the methodology used in the creation of the database and to the many subjective interpretations made during its creation.

The Collection and Evaluation of Source Materials for the Database

In order to obtain the source materials necessary to complete this research, it was necessary to identify as many Regents mathematics examinations as possible. When the research began in 2007, there were two repositories with relatively large collections of Regents mathematics examinations. One repository was the online database of the New York State Education Department, which contained digital images in portable document format of original source documents (Regents Mathematics Examinations) dating back to the 1960s. (http://www.nysedregents.org/). The other was the New York State Library and Archives in Albany, New York. The New York City public libraries, by comparison, had relatively few historical examinations.

The State Education Department's online database contained several hundred digital images of actual Regents mathematics examinations, which were retrieved in their entireties via internet downloads. The collections of the New York State Library and Archives include more than a thousand original Regents mathematics examinations that have been preserved in twelve large boxes, which are accessible only through the Rare Documents and Manuscripts section of the library. Almost all of the original paper examinations in this collection have been organized and bound in volumes by academic school years. In many instances, the documents contained in these volumes are fragile, and it was decided early in the research effort that they should not be subjected to traditional photocopying when making the digital images necessary for this study. After considering alternatives, it was decided that these old examinations would be photographed in situ using a high quality 35 millimeter digital camera.

Three trips were made to the Rare Manuscripts division of the State Archives during the course of this research. During the first trip, in 2008, a survey was made of what examinations were available and how they might be accessed. During the second trip, which also occurred in 2008, digital images of more than 1,000 old Regents Mathematics Examinations were obtained from original source documents. After the digital images from these examinations were processed and organized, it was determined that a representative sampling of all examinations administered since 1866 could be created by focusing on examinations administered in the calendar years that ended in zeros. A third trip to the State Archives was made in 2009. During this third trip, special attention was given to locating and photographing examinations that might be missing from the developing database of Regents mathematics examinations from the years 1870, 1880, 1890, 1900, 1910, 1920, 1930, 1940, 1950, 1960, 1970, 1980, 1990, and 2000.

The original objective of collecting every Regents mathematics examination from every year ending in a zero was not accomplished in this research effort. Diligent searches of all known repositories of historic Regent examinations produced no historical artifacts or primary sources that could be used to identify the questions that were asked on the earliest academic examinations between 1878 and 1889. Additionally, examinations were not found for calendar year 1910. It is possible, but not proven, that the missing examinations for these years were destroyed by a significant fire in the State Archives, which occurred in 1911. It is also possible that these examinations never resided in the State Archives. Regardless of the reason for their being missing, it was decided that the research could continue using the examinations from 1909 as proxies for the 1910 examinations, and without the academic examinations administered in 1880. Thus, while the database includes preliminary examinations dating back to 1866, the academic examinations date only to 1890. The absence of the academic examinations from 1880 in the database means that inferences about the academic curricula that existed prior to 1890 are less reliable than are inferences about the academic curricula from 1890 to 2009, inclusive. The inclusion of the academic examinations from the year 2009 was done so that the dissertation might be as comprehensive as possible when submitted.

Altogether, 1534 Regents Mathematics Examinations from 1866 through 2009, inclusive, were collected for the purposes of this research. From this known population of Regents Mathematics Examinations, a subset of 204 examinations, representing all known examinations from the years 1866, 1870, 1880, 1890, 1900, 1909, 1920, 1930, 1940, 1950, 1960, 1970, 1980, 1990, 2000 and 2009, were selected for further analysis and study. Each of these examinations exists in the form of a digital image of a historical artifact or primary source document, which can be accessed online at www.jmap.org/archives in what is believed to be the world's largest repository of digital images of old Regents mathematics examinations. (Appendix B summarizes the extant population of 1534 Regents mathematics examinations that are in this online repository, and Appendix C provides a timeline of the various curricula with which these digital images are associated.)

The Selection and Transcription of a Representative Sample of Examinations

After reviewing several software options, ExamView software, version 6.0, created by Design Sciences, Inc., was selected as the best available platform for the database that would be created from the research sample of 204 Regents mathematics examinations. Exam View was selected because it contains a relatively powerful equation editor for the transcription of mathematical problems, can handle an unlimited number of questions with minor limitations, and can produce outputs in various formats that could be useful to educators and researchers. A total of 5508 questions was transcribed from the 204 examinations and input into the database. After all problems were input into the database, each individual problem was encoded with two pieces of data:

1) The first piece of encoded data is a reference number that specifies the year, month, curriculum, and question number of the individual problem. This reference number allows chronological analysis and quick reference to original source documents.
2) The second piece of information encoded for each problem was a subjective interpretation/determination of the mathematical topic being assessed in each problem. Altogether, a total of 264 different mathematical topics were identified. This number may change as future refinements are made in the taxonomy.

The taxonomy of assessed Regents topics developed during the course of this research, and reflected in the resulting database, warrants additional critical analysis and is discussed further in a later section of this chapter.

The main database of 5508 questions is supplemented by a much smaller database containing 204 transcriptions of metadata from the first pages of the examinations in the research sample. The amount of metadata that appeared on individual examinations evolved over time and varies significantly by year of administration. The metadata for a given examination in a year might include: the date and time of the examination; instructions to the student to answer the questions; clarifications of permissions concerning the use of tables, slide rules and calculators; required attendance at recitations prior to the process of examination; and for academic examinations from 1890 through 1950, information about the minimum number of credits necessary to pass the examination. This second database recovers much, but not all of the structure and formatting of the original examinations, which is missing from the larger database of problems transcribed from examinations in the research sample. Together, and with some refinements in design and content of the metadata, these two databases could conceivably be used simultaneously to reproduce reasonable facsimiles of the original examinations. However, such a goal is a technical goal associated with desktop publishing and beyond the scope of the current academic research effort.

In addition to the Regents mathematics examinations that are used as source documents in this research effort, numerous other historical documents were identified and used. These include publications of the New York State Board of Regents and the New York State Education Department that relate specifically to the Regents examination system and to its history. These documents, while of lesser value to the research effort than the Regents mathematics examinations themselves, provide useful information that helps to frame and interpret the actual examinations. Other categories of documents collected and referenced in this dissertation for framing and interpretation of the examinations include old mathematics textbooks, which were used in the schools of New York during the 1800s and 1900s; annual reports of the New York Board of Regents; newspaper clippings relating to the Regents examination system; and general histories of education in New York State and mathematics education in the United States. These additional sources are presented and critiqued individually throughout this dissertation when relevant to the analysis and interpretation of actual records of assessment practices left by the Regents examination system.

Internal and External Criticism of Source Materials

> External criticism in historical research evaluates the validity of the document – that is, where, when, and by whom it was produced.…Internal criticism in historical research evaluates the meaning, accuracy, and trustworthiness of the content of the document (Wiersma, 1969, pp 238-239).

With the exception of the primary source documents obtained from Pratt's transcriptions of the early preliminary examinations in arithmetic, the source documents used in the creation of the database are digital images of historical artifacts. These historical artifacts were produced by the New York State Education Department, under the supervision of the New York State Board of Regents, and administered to students in the public schools of the state of New York on the dates and at the times reflected on the examinations. No evidence has been found of attempts by the New York State Library and State Archives or the State Education Department to misrepresent the historical record regarding these examinations. Thus, all databases used in this study are thoroughly and completely grounded in sources that are relatively unimpeachable to the extent that they purport to represent the actual assessment practices associated with the Regents examination system in the public schools of the state of New York between 1866 and 2009, inclusive. Furthermore, the resulting database is robust in terms of its size. It includes over 13% of all extant Regents mathematics examinations administered over a 144 year period and includes 5508 individual mathematics problems. Thus, a strong prima facie argument can be made that the database, excluding any subjective interpretations introduced by the taxonomy, represents an historical picture of the micro-level mathematics assessment practices of the Regents examination system without significant bias. The historical research criterion of external validity is well met. Attention thus shifts to internal criticism, which is central to our interpretations of the meaning of the information within the documents.

The Advantages and Disadvantages of Taxonomies

Within each Regents mathematics examination, there exists a collection of eight to sixty individual mathematical problems. Presumably, each problem was associated with one or more specific learning standards associated with the curriculum being assessed. Since 24 different curricula over a span of 144 years are represented in the database, 24 different sets of standards are also represented, and no single set of standards is perceived as superior to or warranting preference over all the other sets of standards. Thus, the taxonomy of mathematical knowledge necessary for the current longitudinal analysis of curriculum change over a period of 144 years had to be developed from study and analysis of the individual problems selected for inclusion in the database.

Any taxonomy can be viewed as situated in time as well as in the general purposes for which it is developed. When taxonomies are used to classify historical artifacts, an element of subjective interpretation emanates from the taxonomy, and this interpretation is not inherent to the artifacts being interpreted. While subjective interpretations associated with taxonomies may help to illuminate and understand the artifacts being studied, they can also lead to errors and wrong conclusions. An understanding of the taxonomy and how it was developed is therefore useful in evaluating the types of inferences that can reasonably be made from a taxonomically organized database such as the one developed from the research sample. From the outset of this research, it was presumed that the taxonomy would have to be developed from the questions in the database. This is common in historical research, but the process by which it occurred in this research effort nonetheless deserves mention and critique. Several general approaches were evaluated for developing the taxonomy, and the strengths and weaknesses of these approaches are discussed in the following paragraphs.

The first general approach was to start with the earliest examinations of the 1860s and analyze and encode the individual problems in the general order in which the examinations were administered. The perceived advantages of this chronological approach were: 1) that it was free of the bias of the current curriculum; 2) it most clearly reflects the actual creation process of the historical record; and 3) it would be easy to keep the encoding project organized. These advantages notwithstanding, a simile can be used to illustrate a major problem with this approach. In essence, this approach is like planting an acorn to see what kind of oak tree it might grow into, all the while feigning a lack of awareness that the oak tree is already fully matured and providing shade for the research effort. In short, this researcher cannot escape his own historical situatedness.

A second general approach considered for encoding the database was to start with the most recent examinations and work backwards to the oldest examinations. The perceived advantages of this approach were: 1) it reflects the general approach used by genealogists in starting with the present and working backwards to discover the history and roots of the present generation; 2) this researcher is well versed in a taxonomy of the present curriculum as a practicing secondary school mathematics educator, an adjunct professor of mathematics education at the City College of New York, and a cofounder of the Jefferson Math Project (www.jmap.org), which provides taxonomically sorted Regents curricula resources to several thousand teachers and students every day via the internet; and 3) it biases the interpretation of history in a way that privileges current views of mathematics education and disadvantages the perceptions of earlier generations of educators. The benefits of this approach notwithstanding, it was perceived that some effort was necessary to distance the taxonomy from the biases of the current curriculum, and so a compromise was developed.

The compromise strategy, which was followed during the creation of the main database involved in this research effort, was to start from both ends of the timeline and work toward the middle, analogous to the building of the first transcontinental railroad, which started simultaneously in the east and in the west, and met in the middle at Promontory Summit, Utah. In the case of the research database developed for this project, the meeting in the middle occurred during the encoding of the examinations from the 1960s. This compromise strategy helped to sustain awareness throughout the encoding process of the longitudinal nature of the challenge, and also to identify connections and differences between assessment topics and practices as they were concurrently identified and labeled from opposite ends of the timeline.

An exemplar of a connection between a topic represented in the old examinations and a similar topic represented in the new examinations may be found in the two topics originally called “reductions” and “conversions.” These two distinct topical categories were eventually merged into a single topical category called “conversions.” Prior to their merger, however, they were perceived as separate topics, thus illustrating the subjective nature of taxonomies. The term “reduction” is an archaic term that was used extensively in mathematics education during the 1800s and into the early 1900s. Students studied the subjects of reduction ascending and reduction descending, and historical mathematical textbooks from the 1800s devoted entire chapters to these topics (Quackenbos, 1869). Typically, reduction problems involved archaic or specialized measurement systems that are no longer included in the curricula. Equally important, reduction problems almost always begin with the word "reduce." This was the view when starting in 1866 and working forward. From the other end of the timeline, when starting in 2009 and working toward the past, similar problems were being classified as conversions. When the encoding from both ends of the timeline met in the middle, the two different topical names for similar sets of problems were reconciled, and the reduction problems were relabeled as conversion problems. What follows are exemplars of problems in the database that illustrate the commonality of reduction problems and conversion problems.

1866_11_AR_06 Conversions (formerly reduction)
Which one of the fundamental operations (or ground rules) of arithmetic is employed in reduction ascending?

1870_02_AR_13 Conversions (formerly reduction)
Reduce 6 fur. 8 rd. to the decimal of a mile.

1870_06_AR_17 Conversions (formerly reduction)
Reduce 10 oz. 18 pwt. 9 gr. to the decimal of a pound Troy.

1900_01_AAR_03 Conversions (formerly reduction)
Reduce to a common fraction .39285714

1960_01_AA_21 Conversions (formerly conversions)
Express the repeating decimal 0.636363 . . . . as a common fraction.

Over the years, various measurements systems that were once routinely taught in the schools of New York were dropped from the curricula. Additionally, language and terminology changed so that, over time, what was once taught as reductions morphed into a new topic called conversions. The last two questions in the above set of five conversion problem exemplars are essentially instructing the examinee to convert a decimal to a common fraction. The question from 1900 uses the language of reduction, while the question from 1960 uses more modern language. Differences in language notwithstanding, both problems are instructing the examinee to perform the same task. Hence, the topics were merged.

The preceding interpretation of how reduction problems morphed into conversion problems is subjective by any standard, and is representative of a bias in the taxonomy that favors current terminology over archaic terminology. The final taxonomy reflects literally dozens of subjective decisions like the decision to combine reduction and conversions, and thus must be viewed as a primary source document that is highly influenced by the subjective interpretations of the researcher. Users of the database are encouraged to remember that it reflects subjective interpretations and to add to and/or change these interpretations as necessary.

A reoccurring problem associated with classifying and encoding many of the older examination problems was the researcher's lack of familiarity with archaic terms and pedagogies. To overcome this obstacle, numerous old mathematics textbooks from the Civil War to the present were collected and analyzed (Quackenbos, 1859) (Welchons and Krickenberger, 1950). These old textbooks constituted historical artifacts that proved invaluable to the current research effort. For example, the mathematical terminology in Quackenbos' *A Practical Arithmetic*, published in 1869, was found to correlate almost perfectly with the terminology used in the old examinations. In addition to reduction ascending and reduction descending, other archaic terms included evolution (square roots), involution (powers), allegation (an arithmetic approach for solving mixture problems), and mensuration (measurement) using various conversion tables associated with time measure, dry measure, liquid measure, paper measure, linear measure, cubic measure, avoirdupois weight, troy weight, apothecary weight, circular measure, and surface measure. Many of these topics are still taught in today's curriculum, but using different language and terminology and, in some cases, different methods. When the two strategies of encoding from both ends of the database met during the encoding of the examinations from the 1960s, it became necessary to reconcile the language differences. This reconciliation process resulted in approximately 360 topical categories being reduced to 264 topical categories, which are shown in Appendix D. Appendix D also shows the different curricula in which each assessment topic was observed in the research sample.

Once all of the problems were encoded and the language differences reconciled, the resulting taxonomy was compared to an independent taxonomy created under an initiative of the National Science Digital Library in Mathematics and a consortium of interested parties from academia and organizations; including … "the College Board (AP Mathematics and Statistics), Eisenhower National Clearinghouse, iLumina, MAA *(Mathematical Association of America)*, Math Forum, MathDL *(National Science Math Digital Library)*, JOMA *(Journal of Online Mathematics and its Applications)*, MERLOT *(Multimedia Educational Resource for Learning and Online Teaching)*, and NCTM *(National Council of Teachers of Mathematics)*" (Mathematics Taxonomy Committee, 2002, p.1)[1]. (See Appendix E.) This process resulted in further recognition of the subjective nature of taxonomies and the fact that taxonomies are typically created for specific purposes. The taxonomy created by academics from major universities and representatives from different organizations, including the National Council of Teachers of Mathematics (NCTM) was not designed to illuminate historical assessment practices in secondary school mathematics. Rather, its scope went far beyond the mathematics taught in the secondary schools of New York, and it attempts to encompass all branches and strands of

[1] *(Italics)* not in original.

mathematics taught in public schools and colleges and universities, a much broader scope than the public schools of New York (Mathematics Taxonomy Committee, 2002).

When the two taxonomies were compared, it became obvious that many mathematical topics have never been taught in the secondary schools of New York, that certain types of mathematical knowledge are given disproportionate emphasis in the public school curriculum, and that schools reproduce only part of a larger universe of knowledge. In reflecting on the differences between the two taxonomies, it is now clear that the taxonomy created for this research effort is unique to historical assessment practices in secondary school mathematics education. The differences between these two taxonomies are discussed in greater detail in Chapter IV of this dissertation.

Rigid versus Fluid Classification Schema and Different Shades of Gray in the Taxonomy

There is nothing sacred about the 264 topical categories in the taxonomy. It simply represents this researcher's subjective interpretations of the extant historical record at a point in time. Some of the topical categories could reasonably be consolidated or split into additional categories, thus changing the total number of categories. It is probable that users of the database may look at several topics in unison to find the kind of information that is sought. To facilitate this process, and to further emphasize the fluidity of the taxonomy, the names of the different topical categories were modified after the taxonomy was created so that like-topics are grouped together. For example, seven topical categories are associated primarily with the field of mathematics known as Solid Geometry. The general organizing schema and the actual names of these seven categories of problems in the taxonomy were taken from the table of contents of a solid geometry textbook available in secondary schools of New York during the 1930s, 1940s and 1950s (Welchons and Krickenberger, 1950). After the database was created and all problems sorted, it was decided that the problems in these seven categories should be grouped together for easier reference. Thus, each of the seven categories was renamed to include a higher level of abstraction, which is perceived to be helpful to persons who are not well versed in the taxonomy, but nonetheless wish to casually inspect the database or find specific problems. This process of renaming the topical categories to facilitate both groupings of related problems and searches of the database is illustrated in the table labeled Figure 2-3, and occurred with most topical categories in the taxonomy.

Original Topical Name	Revised Topical Name
Dihedral and Polyhedral Angles	Solid Geometry: Dihedral and Polyhedral Angles
General Polyhedrons	Solid Geometry: General Polyhedrons
Lines and Planes in Space	Solid Geometry: Lines and Planes in Space
Prisms and Cylinders	Solid Geometry: Prisms and Cylinders
Pyramids and Cones	Solid Geometry: Pyramids and Cones
Spheres	Solid Geometry: Spheres
Spherical Polygons	Solid Geometry: Spherical Polygons

Revisions Made to the Taxonomy Figure 2-3

A conscious decision was also made to review any category with less than five problems to determine if it could be combined with another topical category. This was possible in many, but not all cases.

The boundaries and distinguishing features of individual topics in the taxonomy are a mix of rigid and fluid classifications. The following problem is taken from the June 1890 Advanced Algebra examination. It provides a first preview of how the 5508 problems in the database are encoded, and serves as an exemplar of the subjectivity of the encoding challenge. The first line contains (from left to right in smaller font) the date, month, curriculum, problem number, and topic of the problem. The second line, in larger font, contains the actual transcription of the problem from a digital image of the original examination.

> 1890_06_AA_07 Relationships Between Arithmetic and Geometric Progressions
> Three numbers whose sum is 18 are in arithmetical progression; if 1, 2, and 7 be added to them respectively they are in geometrical progression. Required the numbers.

When this problem was first encountered, it was classified in the topical category of "series," which at the time included both arithmetical and geometrical series. Later, it became obvious that arithmetical progressions and geometrical progressions occurred frequently enough to warrant individual topical categories, and the "series" topical category was split into three sub-categories: series, arithmetical progressions and geometrical progressions. After most of the series problems were reclassified as either arithmetical or geometrical progressions, it was determined that 1890_06_AA_07 did not fit neatly into any of the three new categories, but rather, was assessing a slightly different area of knowledge, which was the relationship between the two different types of progressions. This awareness resulted in the creation of a new category "Relationships Between Arithmetic and Geometric Progressions." In this way, the process of encoding followed a general pattern of first classifying the problem into a broad topical category, and later identifying fine differences that often led to reclassification into new topical categories. The refinement of topical categories and the recoding of problems proved to be an iterative and highly subjective process, which was influenced not only by the content of each individual problem, but also by this researcher's bias and experiences as a mathematics educator.

An example of the researcher's bias as a mathematics educator is the decision to create and include in the database the topical category of "constructions," which contains 67 problems, the first of which appeared in 1890 and the most recent of which appeared in 2009. Construction problems have long been a hallmark of geometry courses, and they typically require the use of compass and straight edge to demonstrate various forms of mathematical knowledge that can also be taught without compass and straightedge. For example, the following problem appeared in the June 1900 Plane Geometry examination.

> 1900_06_PG_14 Constructions
> Given a line a; construct a line x so that $x = a\sqrt{2}$

It can be argued that the topical category of constructions is unnecessary from a pure mathematics perspective, and that each construction problem can be reclassified into another category that is independent of construction as a pedagogical method. In the above example, the problem could easily be reclassified into the topical category of iscosceles right triangles, in which the ratio of sides must always be $1:1:\sqrt{2}$. While the argument for classification of this problem in the topical category of isosceles right triangles has merit, the current research effort focuses on understanding secondary school mathematics as it has most likely been taught in secondary schools. Toward this end, it is the experience of this researcher as a secondary school mathematics educator that constructions are typically taught separately, and a separate category of constructions is therefore justifiable in the taxonomy.

The inherent subjectivity of the encoding process was recognized early in the research process and was compounded by two phenomena. The first was the researcher's natural bias to view all problems as they are viewed in mathematics education today. This bias to view the past through the lens of modernity is a common problem in historical research, though one can also argue that the genealogy of the present is most easily discovered by starting in the present and working backwards. The second problem influencing this researcher's subjectivity was at times the incomprehensible nature of some of the very old problems. Despite the fact that all examinations in the research sample were produced and administered in the English language, the historical situatedness of the early examinations and the differences in the use of the language and mathematics terminology made some of the early problems truly incomprehensible on first viewing. On numerous occasions, old mathematics textbooks had to be found so that precise meanings of language could be understood. In classifying individual problems, it was often necessary to solve the problem to better understand what was being assessed. This was a delightful, but often time consuming distraction from the overall challenge of sorting and classifying each of the problems, and also one in which this researcher's own past education and acquired understandings of mathematics undoubtedly contributed to subjectivity. One problem that created particular difficulty for this researcher was the following, in which the words "two numbers" were used differently 100 years ago than they would be used today.

> 1909_01_IN_11 Writing Systems of Equations
> What two numbers whose difference is d are to each other as $a{:}b$?

Hours were spent by this researcher trying to interpret and solve this particular problem, so that it could be topically classified. Eventually, this researcher submitted the problem to a larger mathematics community consisting of the members of the list serve of the Association of New York State Mathematics Teachers. The consensus of opinion from responders on the list serve was that the problem asks for a representation of a set of numbers, and can be solved as a system of equations in the following manner:

Let x and y represent the two numbers.
Write two equations:

$x - y = d$ and $\dfrac{x}{y} = \dfrac{a}{b}$

Solve:

$y = \dfrac{bd}{a-b}$

$x = \dfrac{ad}{a-b}$

Answer: $\dfrac{bd}{a-b}$ and $\dfrac{ad}{a-b}$

For modern day mathematics educators, the solution shown above may not intuitively fall within the general parameters of what is meant by the words “two numbers.” This problem illustrates one other aspect of 100 year old Regents examinations, which is the fact that no answer keys exist for them. The old examinations were sent to Albany for scoring, and any scoring keys or rubrics associated with the old examinations are missing from the historical record.

Another difficulty encountered in the transcription and encoding of the problems is that of the multiple part problem. Some problems, particularly in the older examinations, had multiple parts that evaluated knowledge of several different topics. To facilitate encoding and analysis, these questions were separated into their component parts. For example, the following problem from the June 1940 Intermediate Algebra Examination was entered into the database as five separate problems:

1940_06_IN_34
Each of the following statements is sometimes true and sometimes false. In each case give one illustration in which it is true and one illustration in which it is false.

a) The positive square root of a number is less than the number. [2]
b) The graphs of two equations of the first degree intersect in one point. [2]
c) If *a, b* and *c* are each greater than 1, the graph of the equation $ax^2 + by^2 = c$ is a circle. [2]
d) A root of a negative number is an imaginary number. [2]
e) If *y* is a function of *x*, *y* increases as *x* increases from 0. [2]

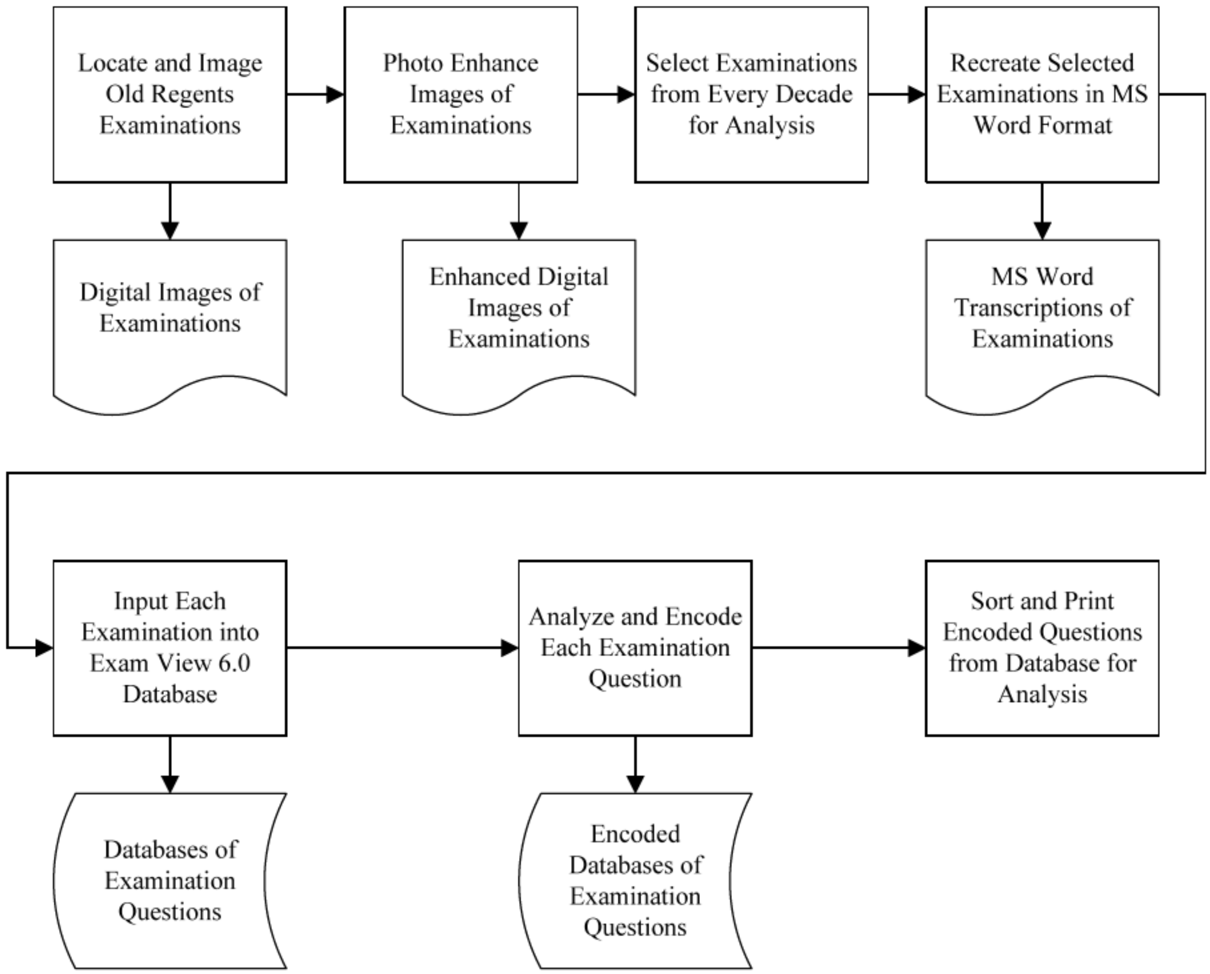
Whenever questions like these were split into their various subtopics during transcription and encoding, the instructional stem was modified to facilitate reader understanding. For example, part a) of the above question appears in the database as follows:

> 1940_06_IN_34a Square Roots
> The following statement is sometimes true and sometimes false. Give one illustration in which it is true and one illustration in which it is false.
> The positive square root of a number is less than the number. [2]

One result of the above process of breaking multiple part problems into their component parts is that the total number of problems in the database is higher than the actual number of problems that appeared on the original examinations. All charts and tables purporting to compare numbers of questions on examinations are taken from the actual numbering of questions on the examinations and not the numbers of questions as encoded in the database.

Summary of the Data Collection and Analysis Process

Figure 2-4 summarizes the general sequence of activities in this research effort that resulted in the ability to sort and print excerpts from the historical record of Regents mathematics problems in the research sample.



Flowchart of the Methodology Figure 2-4

During the course of this research, a total of 1,534 Regents mathematics examinations from 131 different calendar years were collected, digitally imaged and photo-enhanced. No Regents mathematics examinations were located for 12 calendar years, including the contiguous years 1883 through 1889. The search for the missing examinations from these years continues. From this database of 1,534 examinations, every extant examination given during 1866, 1870, 1880, 1890, 1900, 1909, 1920, 1930, 1940, 1950, 1960, 1970, 1980, 1990, 2000, and 2009 was selected for analysis and encoding. A total of 5508 questions from 204 examinations and 26 mathematics curricula are represented in the completed database of Regents questions, which is designed to answer the first research question: How has the classification and framing of assessed knowledge in the core subject area of mathematics changed in Regents level examinations administered in the public schools of New York since 1866? The second database, which contains metadata from examinations in the database, is designed to illuminate the second general research question: How has popularization influenced the contents, structure and academic rigor of Regents examinations?

An Overview of Appendices and Data Used in Analyzing the Research Sample

Selected data compiled using the methodology described in the preceding pages of this Chapter are summarized in appendices, which are introduced here and referred to throughout the remainder of this dissertation.

- Appendix A consists of two graphics: 1) the first provides an overview of the organizational structure and scope of authority of the New York Board of Regents; and 2) the second provides a general flowchart for the creation of a modern Regents mathematics examination.
- Appendix B provides: 1) a summary of the 32 known mathematics curricula assessed by extant Regents examinations and the abbreviations used for each curricula when encoding the database and writing this dissertation; 2) a chronological index by calendar years showing each of the 1534 extant Regents mathematics examination collected and digitized during the course of this research; and 3) summary data on the 204 examinations taken from calendar years 1866, 1870, 1880, 1890, 1900, 1909, 1920, 1930,

1940, 1950, 1960, 1970, 1980, 1990, 2000, and 2009 for the creation of the representative research sample.

- Appendix C provides a timeline for the 26 different mathematics curricula represented in the research sample.
- Appendix D provides an alphabetical listing of the 264 topics in the taxonomy of assessed mathematical topics together with a listing of the various curricula in which each assessed mathematical topic is observed in the research sample.
- Appendix E juxtaposes: 1) the taxonomy of knowledge reproduced in New York Schools between 1866 and 2009 inclusive, with 2) a taxonomy of all mathematics knowledge reproduced in public schools, colleges, universities and libraries, thus illuminating the role of the state in selecting mathematical knowledge for reproduction in the public schools of New York.
- Appendix F provides a longitudinal census by calendar year of observed topics in the Mathematics Curricula of the Public Schools of New York State, as reflected in the completed research sample.
- Appendix G uses census data from Appendix F to create a decade by decade analysis of the research sample showing: 1) an alphabetical listing of all mathematical topics assessed during each decade; 2) an alphabetical listing of all new topics observed in the research sample for the first time that decade; and 3) an alphabetical listing of topics observed in the research sample for the last time that decade.
- Appendix H is a set of Regents mathematics problems associated with warfare. World Wars I and II are the only instances of historical events that are repeatedly and systematically used as evoking contexts for mathematics assessment, suggesting that

societal events, as opposed to societal structures, generally do not influence the mathematics assessment practices of the Regents examination system.

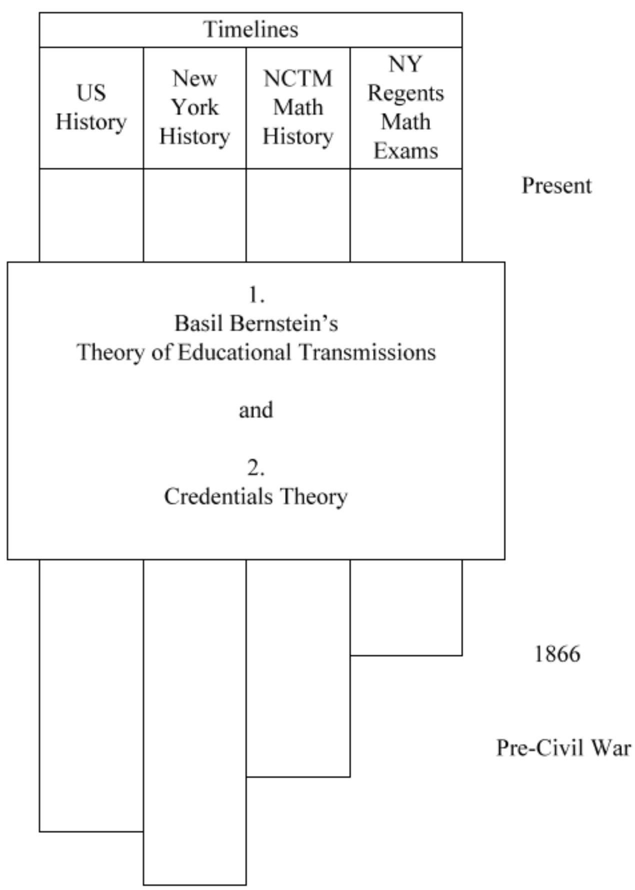
The following appendices are available only in electronic formats. Because of their length, which is approximately 475 pages for each appendix, and environmental concerns, they are not typically reproduced in hard copy. These appendices are available in portable document format (pdf) from ProQuest/UMI Dissertation Publishing. The Exam View version 6.0 databases from which these appendices are made are also available from ProQuest/UMI Dissertation Publishing, and permission is given for the use of these databases, with attributions, in other scholarly research.

- Appendix I is a chronological listing of 5508 mathematical problems in the research sample. It is useful for developing understandings of how assessment practices in mathematics evolved over time.
- Appendix J is a listing by curriculum of the 5508 mathematical problems in the research sample. This appendix is useful for understanding how curricula evolved over time and how different curricula relate to one another.
- Appendix K is a listing by assessed mathematical topic of the 5508 mathematical problems in the research sample. This appendix is useful for analysis of any or all of the 264 mathematical assessment topics in the taxonomy, and allows researchers to study the evolution of assessment practices on specific topics over relatively long periods of time.

CHAPTER III -- POPULAR DISCOURSES IN EDUCATIONAL HISTORY AND THE RESEARCH SAMPLE

A Synthesis of the Research Sample and Other Source Materials

This chapter focuses on synthesizing data from the research sample and other sources to create a historical narrative that juxtaposes historical events and macro-level structures of society with micro-level mathematics assessment practices in the public schools of New York on a decade-by-decade basis. In so doing, it generally follows a model first developed in the dissertation proposal. This model is presented as Figure 2-5.



A Synthesis of Vertical Histories and Horizontal Theories Figure 3-1

The model reflects the general socio-historical design of this research effort. The research model depicted above requires that the synthesized historical narratives in the vertical columns be interpreted through the lenses of the two theories of the sociology of education that are shown in the horizontal row. The first theory in the horizontal row is Basil Bernstein's theory of educational transmissions. The second theory is credentials theory. Both theories illuminate our understanding of the social stratification effects of public schools. The synthesis of historical information reflected in the vertical columns is presented in the following pages of this chapter. The theoretical lenses to be used in the interpretation of the synthesized historical narrative that follows are presented in Chapters IV and V of this dissertation.

The research sample being introduced in the following historical narrative was originally developed to represent the entire population of extant Regents mathematics examinations administered in the public schools of New York during one year in every decade since the Civil War. Hence, this narrative is also organized on a decade-by decade basis. Within each decade, two subcategories are juxtaposed. The first subcategory concerns a general discussion of popular discourses and historical markers relevant to mathematics education, and the second subcategory contains highlights of changes in Regents mathematics assessment practices, which are observed in the research sample.

The summaries of popular discourses described in this chapter constitute a bricolage of secondary sources. Attributions are presented as appropriate. Special attribution is given to: 1) Angela Walmsley, whose decade-by-decade analysis of a century of mathematics education in the United States provides the structure and much of the content that is reflected throughout the synthesized narrative that follows; 2) the National Council of Teachers of Mathematics (NCTM) for their sponsorship of numerous publications on the history of mathematics education in the

United States; 3) Forest Chester Ensign, whose 1921 study of New York's 19th and early 20th century compulsory school attendance and child labor laws serve as critical milestones in the following synthesized narrative; and 4) James D. Folz, director of the New York state archives, who has chronicled and written much about the history of education in New York State. This researcher, having been born in 1950, knows nothing of what occurred before the early 1950s, except for that which has been learned through the words of others, and is deeply indebted to those who came before and recorded the various histories that are repeated and echoed in the following synthesis.

While there are numerous other educational historians cited in the following pages, their individual and collective interpretations of history are typically general in nature and have heretofore not been juxtaposed against a historical record of mathematics assessment practices left by the Regents examination system. Such juxtaposition is important because it allows us to make inferences about the extent to which popular discourses concerning the history of public education actually correspond to changes in mathematics assessment practices over time.

In focusing on New York, this researcher is aware that the public school mathematics curricula and the relationships between schools and society in the state of New York are not necessarily representative of curricula and relationships between schools and society in other states. Indeed, the Regents examination system, which produced the original historical artifacts concerning mathematics assessment practices from which the research sample introduced in this chronology was drawn, makes public education in New York different than any other state. Throughout this chapter, the Regents examination system is operationally defined as a quality control system for public education in New York. It has been in continuous operation since the first state sponsored Regents examinations in 1866. Thus, it provides a significant and unique opportunity to understand what actually happened with mathematics assessment practices at almost any point in time during the past 144 years.

In operationally defining the Regents examination system as a control system for public education, this researcher adopts a view long embraced by the educational bureaucracy of the state of New York. In 1965, James E. Allen, New York State Commissioner of Education, wrote,

> That the high schools of New York now enjoy wide prestige and respect throughout the nation must be recognized as due in no small part to the system of high school achievement tests popularly known as Regents examinations. Regents examinations have been an integral feature of the secondary education program in New York State for 100 years. Generally regarded as a hallmark of the New York State educational system, Regents examinations have played a major role in developing and maintaining the high standards of instruction and achievement found in our high schools (SED 1965, preface).

We now turn to our synthesis of historical records, which juxtaposes popular discourses concerning the histories of mathematics education and public schooling against historical records of micro-level mathematics assessment practices left by the Regents examination system.

Pre – 1860[2]

Popular Discourses of the Pre-1860 Era

Prior to the United States Civil War (1861-1865), the United States was primarily a village centered agrarian society, and schools were typically village centered institutions with one or a few rooms, and without uniform standards for curriculum and assessment. In New York, a law was passed in 1805 to provide financial support for common schools, and in 1812, the Common School Act established a framework of common school districts under the control of locally elected trustees. James Folts, director of the New York State Archives, summarized the huge impact of the 1812 legislation as follows:

> The 1812 common school act shaped the future of public education in New York by establishing that 1) common schools are a state function under state control; 2) funding of public schools is a joint state-local responsibility; and 3) the school district -- not the county or the town -- is the primary administrative unit for public education (Folts, 1996).

By the start of the Civil War, the number of common school districts in the state of New York had grown to over ten thousand (Folts, 1996). This said, not all school age children attended public schools.

In 1921, Forest Ensign wrote a doctoral dissertation concerning the history of compulsory school attendance and child labor laws for the faculty of philosophy at Columbia University. He

[2] The large font used as decade markers in this synthesis of the research sample and the historical record is intentional and facilitates navigation through and reference use of the narrative, thus justifying a deviation in style.

begins his historical narrative on New York with the following paragraph, which illuminates much about the social structures and public schools of New York in the 1830s.

> The first compulsory education law in the state of New York was a special measure enacted in 1831 requiring that all children between five and sixteen years of age detained in county poor-houses "be taught and educated in the same manner as children are now taught in the common schools of this state, at least one-fourth part of the time said paupers shall remain in said poor-houses." In order to meet the requirements of this law schools were established within the poor-houses themselves. They were, as might be expected, of inferior grade, yet were maintained for many years, affording the sole means of instruction to thousands of children (Ensign, 1921, p. 115).

From these rather modest beginnings, Ensign traces the history of New York's compulsory school attendance and child labor laws through a series of ineffective legislative efforts, opining that effective standards and enforcement of school attendance and child labor laws did not occur in New York until after 1903, when the New York Child Labor Committee created a coalition of political interests that caused the legislature to enact a modern and effective regulatory framework. While school attendance was not required by law for most students prior to the Civil War, there was a general public sentiment that schooling was good for children, and those not forced by circumstances to work at very early ages were typically encouraged to attend public schools to learn reading, writing and arithmetic.

In 1853, Ensign reports that the first truancy law was enacted in New York, which was arguably the first attempt at requiring school aged children from the general population to attend schools. Ensign notes the following provision from the 1853 school truancy law:

> Children between the ages of five and fourteen found wandering in the streets or lanes of any city or incorporated village, idle and truant, without any lawful occupation, might be restrained from wandering about and might be required to remain upon the premises of parent or guardian, or caused to engage in some lawful occupation, and might be required to attend school for at least four months each year until fourteen years of age (Ensign, 1921, p.118).

Ensign reports that this attempt at regulation was generally ineffective and seldom enforced.

Throughout the fledgling nation, there were relatively few qualifications required of secondary school teachers. Diane Ravitch reports generally that prospective school teachers simply needed to convince local school boards that they were of good moral character and had enough knowledge to get the job done (Ravitch, 2002, p. 1). Folts described the situation in New York State as follows, "Between 1814 and 1856, town school officials had the authority to examine and license teachers; the law prescribed no qualifications except good moral character and ability to teach" (Folts, 1996). The unfettered freedom of villages and towns throughout the nation to establish their own teacher education and teacher certification requirements began to erode during the decades preceding the Civil War. Pennsylvania adopted what is believed to be the first statewide teacher certification requirements in 1834. The Pennsylvania teacher qualification examination covered the "three R's," reading, writing, and arithmetic. In 1856, New York removed the authority to license teachers from all rural and small town school districts and consolidated the authority in the state's Superintendent of Public Instruction. However, major cities such as Buffalo, Syracuse, Albany, and New York City retained the right to examine and certify their own teachers – a practice that continued in New York City until 1990.

The common schools of New York prior to the Civil War were typically elementary schools, and they focused primarily on teaching the three R's. Advanced instruction was typically, but not always, provided by private secondary schools, which were referred to as "academies" or seminaries. In 1850, there were over 10,000 common schools and 165 academies and seminaries in the state of New York. A small, but unknown number of these

10,000 plus common schools were high schools, and the Board of Regents held the authority to monitor both common schools and private academies and seminaries to determine their eligibility to receive state aid from the "Literature Fund." In the 1840s, the proliferation of common school districts had grown so large, and the administrative requirements associated with having a Board of Trustees for each school so burdensome, that the legislature authorized the consolidation of common schools into union school districts. The structure and governance of today's public high schools in New York State can trace their origins to the common high schools of these union school districts (Folts, 1996).

During the years preceding the Civil War, pedagogy was not well developed in the United States, and local school districts established their own pedagogical expectations. Nonetheless, there was a deepening interest in the development of public schooling and in the preparation of teachers. In New York, this interest was expressed in legislation passed in 1827 and 1834, which provided financial aid from the state's "Literature Fund" to support private academies that were approved by the Regents to educate new teachers. In what was arguably the first effort by the Board of Regents to control curricula in the state of New York, the Regents specified the texts and the subjects that these academies must teach to be eligible for state aid. In 1844, the New York legislature approved the creation of a "model normal school" in Albany for the training of secondary school teachers (Folts, 1996). This model normal school was the first of twelve normal schools that would eventually be established by the New York legislature, and the normal school at Albany was the only functioning normal school in the state of New York prior to the Civil War.

The Research Sample Prior to 1860

There were no Regents examinations prior to 1866.

1860-1869

Popular Discourses in the 1860s

The defining event of the 1860s was the Civil War. The war is generally viewed as beginning on April 12, 1861, when Confederate forces began the bombardment of Fort Sumter in South Carolina, and it concluded with General Lee's surrender at Appomattox Courts House, Virginia, on April 9, 1865. Three months before the end of the war, New York's Governor, Reuben E. Fenton, delivered his first annual message to the legislature. A copy of his message is available in the rare documents and manuscripts room of the New York state archives in Albany, New York. In his remarks, he references the report of the Superintendent of Public Instruction. Governor Fenton reported,

> The Report of the Superintendent of Public Instruction shows our schools to have been, during the year, in a prosperous condition. Notwithstanding the extraordinary demands occasioned by the war, upon the attention and resources of our people, it is apparent that there is no diminution in the interest manifested in the progress of public education, or in the fidelity of school officers in the discharge of their duties (Fenton, 1865).

Governor Fenton then included a listing of income, expenses, and statistical data about the status of education in the state of New York, portions of which are reproduced in figure 3-2.

Expenditures	
For supervision (rural districts)	$ 56,000. 00
For teachers' wages	3,093,460.48
For libraries	28,890 .51
For apparatus	137,613 .49
For building and repairs	647,301. 23
For colored school	30,468 .33
Miscellaneous	614,036. 64
Total	$4,605,770 .66
Number of children in the State between the ages of five and twenty—one years	1,307,822
Number of children who have attended school at any time during the year	881,144
Teachers employed at the same time for six months or more	15,801
Whole number of male teachers	5,707
Whole number of female teachers	21,754
Total number of school districts	11,459
Total number of school houses	11,457
Whole number of months school	82,727
Number of months school taught by qualified teachers	82,389
Average number of months school	74
Number of volumes in district libraries	1,111, 438
Number of pupils attending the Normal School during the year,	299
Number of teachers instructed in Teachers' Institutes	7,349
Classes in academies	1,683
The amount of money to be apportioned by the Superintendent of Public Instruction, for the support of common schools for the current fiscal year, is	$1,445,749. 90

Educational Statistics from 1865 Figure 3-2

The above report provides considerable information about the status of public education in the state of New York in 1865, which was the last year in history that the state of New York did not administer a Regents examination in mathematics. In that year, there were 11,457 school houses with just 299 pupils at the state's only functioning Normal School in Albany, which was created as a model for the training of secondary school teachers, as opposed to elementary school teachers. These facts suggest: 1) the relative scarcity of secondary schools; 2) the idea that the secondary school curriculum was perceived as warranting additional studies beyond what was required for teaching elementary school; and 3) that the normal schools were not a significant source of teachers for the average common school district in New York. A second Normal School, to be located at Oswego, New York, had been approved by the New York State legislature in 1863, but was not yet functioning in January 1865, when Governor Fenton made this report to the state legislature. Subsequent to this message to the legislature, normal schools were approved for the towns of Brockport in 1867; Fredonia in 1868; and Cortland in 1869. These and other normal schools were the origins of the modern State University of New York, which is commonly known by its acronym, SUNY.

In 1865, teacher institutes were probably more important than normal schools for the preparation of teachers. Governore Fenton's report shows that 7,349 teachers attended teachers' institutes. These teachers' institutes were single-day, in-service training sessions that were held annually in each county of the state, beginning in 1847 and continuing through 1912. Another significant form of training for teachers appears to have been in the form of classes (1,683) at academies, which were private secondary schools that were approved for state subsidies by the Board of Regents." (Folts, 1996).

Governor Fenton's report continued, noting that,

> The consolidated act in regard to public instruction, passed last May, is regarded favorably, and the mode therein provided for the apportionment of public moneys, is causing a large increase in the number of pupils at the schools, and in the daily average attendance. To give full force and effect to that act, and to increase parental solicitude for the proper instruction of the young, the propriety of making more ample provision for an annual supply of thoroughly qualified teachers is suggested. Creditable provision for this purpose has already been made in the Normal School, teachers' classes in the academies, teachers' institutes, and the Oswego Training School for primary teachers; but these, as now supported, are manifestly inadequate to supply so great a demand. I desire to repeat the suggestion made by my predecessor, urging upon the public the duty of a proper recognition of the important services of the teachers, on whose intelligence and

> fidelity the welfare of their children in such great measures depends (Fenton, 1865).

The consolidated act of 1864, which is referred to above by Governor Fenton, was accompanied by an 1864 decision of the legislature to establish a system of public examinations of all students. This second act included the following provisions establishing the Regents examination system:

> At the close of each academic term, a public examination shall be held of all scholars presumed to have completed preliminary studies. . . .To each scholar who sustains such examination, a certificate shall entitle the person holding it to admission into the academic class in any academy subject to the visitation of the Regents, without further examination (SED 1987).

This was the beginning of the Regents examination system that continues to this day in the state of New York.

Beginning the Synthesis: The Research Sample from 1866

The first Regents examination in mathematics was presented on November 22, 1866, and it was the only Regents mathematics examination administered that year. This first examination had 24 problems assessing mathematical knowledge in 17 topical areas, all of which were being assessed for the first time. (See Appendix G.)

1870-1879

Beginning the Decade: The Research Sample from 1870

Three Regents mathematics examinations were administered in 1870, all of them covering the Arithmetic curriculum. The research sample contains 71 problems from 1870 and these 71 problems assessed a total of 27 mathematical topics, 11 of which appear as new topics.

The 1870 problems were not distributed evenly amongst the 27 topical areas, but rather, the assessed curriculum was heavily weighted toward consumer mathematics and business arithmetic. (See Appendix G.)

Popular Discourses of the 1870s

The 1870s began with former Civil War General Ulysses Grant in the White House and the ratification of the fifteenth amendment, which prevented the denial of voting based on race, color, or previous servitude (slavery). Federal troops still occupied some of the former Confederate States of America. In 1872, President Grant was re-elected to a second term. The second industrial revolution was beginning to grow in strength in the United States and the economy and society were recovering and reconstructing following the Civil War. Manufacturing jobs in the cities were drawing people away from the agrarian lifestyle that had predominated since the American Revolution, thus creating demographic trends that would facilitate the growth of public high schools in larger towns and cities. In New York, the legislature continued it focus on teacher preparation by approving in 1871 additional normal schools for the cities of Buffalo, Geneseo, and Potsdam. This brought the total number of normal schools created by the legislature to eight.

Of significant importance to this research study, during the 1870s, college enrollments were increasing rapidly and colleges in New York were unhappy with the level of preparation of students coming to them from an increasing number of secondary schools. These colleges, all under the supervision of the Board of Regents, also had their own entrance examinations, which differed from college to college, thus creating problems for secondary school teachers who needed to prepare students using different curricula for the different entrance examinations of different colleges. The Regents addressed this problem by supporting legislation approved in 1877 that authorized the expansion of the examination system to high schools. Under the early academic examination rules, a student who passed a secondary school Regents academic examination in any subject area was exempt from further examination in that subject at any college or university under the supervision of the Board of Regents. Folts reports that the new "….Regents exams were quickly adopted because they embodied high scholastic standards, and because academies and high schools had to use them to qualify for aid from the Literature Fund" (Folts, 1996). The first Regents academic examinations were presented at the end of the 1878-1879 academic year. However, no copies of the first twelve years of academic examinations have been found.

An important event occurred in 1874, which may have indirectly influenced New York's decision to expand the Regents examination system to secondary schools. That event was the Kalamazoo decision of the Supreme Court of the State of Michigan. The Kalamazoo decision established in Michigan the simple idea that public funds could be used to support secondary education. The Court rejected the argument that governments could support elementary schools, but had no right to use public moneys for the support of secondary schools. This legal case, which originated in Kalamazoo, Michigan, was widely publicized throughout the nation and is viewed by many historians as setting the stage for growth in public support for secondary education throughout the nation prior to World War I (Pulliam, 1994, p. 90) (Sadovnik, Cookson, and Semel, 2001). These general facts notwithstanding, this researcher has found no evidence that the Kalamazoo decision was directly associated with New York's decision to expand the Regents examination system to assess the curricula of secondary schools. Closer to home, and perhaps more relevant to the 1878 decision to expand the Regents examination system, was the enactment in 1874 of New York's first compulsory school attendance law, which also contained restrictions on child labor. Ensign reports,

> The law of 1874 provided:
>
> 1. That those having control of children between eight and fourteen years of age, of proper mental and physical capacity, should cause them to attend some public or private school for at least fourteen weeks each year, unless regularly taught at home in the common school branches for a like period.
> 2. That no child under fourteen was to be employed in any business whatever unless, during the preceding year, he had received instruction as required by law.
> 3. That a child on going to work should deliver to the employer a certificate of schooling signed by a teacher or a school trustee, this to be preserved by the employer and exhibited on demand of the proper examining officer (Ensign, 1921, p. 120).

Ensign went on to comment that,

> This law might have been enforced had school trustees and others entrusted with its administration set themselves resolutely to the task. Experience in many states has shown, however, that no such general compulsory law has ever functioned further than to register public opinion. This measure was no more effectual than others of its type. After a full decade of trial, the proportion of children attending school was actually less than before its enactment (Ensign, 1921, pp. 120-121).

1880-1889

Beginning the Decade: The Research Sample from 1880

The research sample for 1880 is known to be incomplete and non representative of the academic examinations administered that year, since no academic examinations have been found for calendar years prior to 1890. This fact notwithstanding, the record of preliminary examinations shows that four Regents mathematics examinations were administered in 1880, all of which assessed the Arithmetic curriculum. The research sample contains 107 problems from

1880 and these 107 problems assessed a total of 33 mathematical topics, only 3 of which appeared as new topics in the Arithmetic curriculum, which was then 14 years old. This suggests a fundamental characteristic of the database, which is the idea that looking at two consecutive decades of data develops a better approximation of what was assessed and what was not assessed in a particular curriculum than a single year of data. This attribute of the research sample appears to hold true throughout the 144 year historical span of the database and can be interpreted and inferring that all mathematics topics in a given curriculum were not assessed in any given year. The 1880 problems reflect the same general distribution pattern as the 1870 problems with respect to the topics assessed, and continued to be heavily weighted toward consumer mathematics and business arithmetic. (See Appendix G.)

Popular Discourses of the 1880s

The population of the United States reached 50 million in 1880, and increasing numbers of schools and teachers were necessary to educate the growing number of school children. The growth of public education was widespread throughout the nation, and the decade of the 1880s is notable for the founding of two great institutions for the preparation of teachers. The Tuskegee Institute was founded in 1881 as a normal school for free blacks, former slaves, and their children. Teachers College, now a part of Columbia University, was founded in New York City in 1887 as the New York School for the Training of Teachers. The New York legislature also approved normal schools to be located in the towns of New Paltz in 1886 and Oneonta in 1889, thus bringing the number of state sponsored normal schools to ten.

The Board of Regents issued their first high school syllabus in 1880, and during the next ten years, a program of examinations, certificates, and diplomas was created for all students. It would not be until 1910 that separate syllabi were issued for individual subjects (Folts, 1996). The absence of separate syllabi for individual subjects suggests that questions from past Regents mathematics examinations were important resources for educators facing decisions of what to teach, and may explain why the Regents occasionally published examination problems in bound booklets. During these years, all secondary school students pursued Regents diplomas. According to Folts, “The early curricula emphasized learning and reciting of facts, lots of them, with the aim of instilling ‘mental discipline’ (if nothing else)” (Folz, 1996).

1890-1899

Beginning the Decade: The Research Sample from 1890

In calendar year 1890, the extant record of Regents academic examinations relating to mathematics assessed curricula named Advanced Arithmetic, Algebra, Higher Algebra, Plane Geometry, Solid Geometry and Plane Trigonometry. In calendar year 1891 New York also administered Regents academic examinations in Analytical Geometry, Conic Sections and Spheric Trigonometry. However, these additional curricula from 1891 appear to have been short lived, suggesting that competing interests were struggling for control of the mathematics curriculum and the Board of Regents was receptive to change.

The research sample selected for the database includes 20 different examinations from 1890 with 253 problems. This number includes the preliminary examinations in Arithmetic, A total of 93 mathematical topics were assessed in 1890, 66 of which were observed for the first time that year. The large number of new assessment topics is almost entirely due to the inclusion of problems from the new academic examinations in the research sample for the first time, despite the fact that academic examinations had begun approximately twelve years earlier, in 1878. (See Appendix G.)

Popular Discourses of the 1890s

The 1890s saw continued growth in public education in New York, both in terms of numbers of students and in the number of schools for teacher education. In 1890, New York's normal college at Albany became a degree granting institution, thus becoming the first normal school established by the New York legislature to award regular academic degrees in academic areas other than education. Normal schools were also approved for Plattsburgh in 1890 and for Jamaica (now a part of the Borough of Queens in New York City) in 1893. These two schools were the last of the twelve normal schools established by the New York legislature for the preparation of teachers. By 1942, two of these institutions would become degree-granting institutions and the remaining ten would become teachers colleges. In 1948, these colleges would become the founding institutions of the State University of New York, more commonly known as SUNY.

Though much of present day higher education in New York is built upon the foundations of the normal schools that were originally developed for the preparation of educators, there is a long standing gap between schools of pedagogy and other academic disciplines. In 1894, Teachers College moved to 120th Street in Manhattan, and in 1898, it became affiliated with Columbia University. Ravitch writes:

> After Teachers College was created in the late nineteenth century, it was often said that 120^{th} street, which separates Teachers College from the rest of Columbia University, is "the widest street in the world." The price of professionalism unfortunately was the split between pedagogy and the traditional disciplines of the liberal arts and sciences. The new leaders of the profession took charge of teacher certification. Certification became, increasingly, dependent on taking courses in pedagogy and in passing tests of pedagogical theory. State education departments and the colleges of education agreed that longer periods of formal training in pedagogy were required for future professionals of education. Teacher certification eventually came to be identified with the completion of teacher education programs rather than with the receipt of local certificates or the passing of subject-matter examinations. (Ravitch, 2002).

This view of when teaching began its evolution into a profession is important, and is supported by the research of Eileen Donoghue, which locates the rise of mathematics education as a profession between 1890 and 1920 (Donoghue, 2003a). This is important because it shows that the secondary school mathematics curriculum was already established and being regulated by process of examination prior to the widespread rise of teaching as a profession. The standards for the assessment of the secondary school mathematics curriculum that these professionally educated teachers would eventually teach were already anchored in the assessment practices of the first academic examinations administered in 1878, when the Regents examination system was expanded from a system of preliminary examinations to a system of both preliminary and academic examinations. The assessment topics associated with the secondary curriculum did not share the heavy applications emphasis associated with consumer arithmetic and business mathematics problems, which were common in the preliminary examinations of the Arithmetic curriculum. Rather, the new academic examinations in mathematics were deeply grounded in classical humanist traditions.

In 1892, the National Education Association (NEA) appointed a “Committee of Ten” to help standardize the curricula of secondary schools amidst growing differences of opinions between interest groups that advocated for more traditional curricula practices and those advocating for more progressive curricula practices. It can be assumed that the lack of standardization of the high school curricula in places other than New York State caused the NEA to form the Committee of Ten. This is because New York had already recognized similar problems within the vast system of education governed by the New York Board of Regents and had standardized its secondary school curricula approximately 14 years earlier by expanding the Regents examination system to include academic examinations.

The Committee of Ten was arguably biased from the beginning toward a classical humanist agenda, based on its appointed members. John Pulliam and James van Patten report

> Commissioner W.T. Harris and Harvard president Charles W. Eliot were well-known members of the Committee. There were four other college presidents, two headmasters, one professor, and one high school administrator, but no high school teachers. College interests dominated in the Committee of Ten, and the report was a bastion of educational conservatism (Pulliam and Patten, 1994, p. 91).

The secondary schools of New York during the 1890s were likewise bastions of educational conservativism. Only one diploma – the Regents Diploma -- was recognized by the state education department, and the mathematics assessment practices of the Regents examination system were solidly grounded in classical humanism, a conservative educational archetype. The research sample shows a significant increase in the number of new assessment topics observed in 1890, the first year of Regents academic examinations, but the number of new topics in 1900 is significantly less, suggesting that the “Committee of Ten” had very little impact on what topics were assessed in the mathematics curricula of the public schools of New York.

Educational historians have given much time and effort to interpreting the history of the Committee of Ten (Sadovnik, Cookson and Semel, 2001) (Kleibard, 2004) (Ravitch, 2000). This is arguably due to the perception that it was a milestone in educational debates between traditionalists and progressives. The Committee of Ten had argued, in effect, that all secondary school children should receive an education that was in many ways consistent with the Regents diploma standards already in place throughout the state of New York long before the 1890s. Indeed, the five windows of the soul espoused by Commissioner Harris of the Committee of Ten were: 1) arithmetic and mathematics; 2) geography; 3) history; 4) grammar; and 5) literature and art. The Regents examination system had been looking into the "souls" of students in New York's secondary schools through these windows years before the Committee of Ten was created.

When the Committee of Ten released its report, most secondary school teachers in New York and throughout the United States were not formally educated with university credentials. This meant that educators and school administrators were often left to their own devices in terms of discovering and learning about pedagogies. This vacuum of pedagogical strategies was filled for many with the teachings of Johan Herbart (1776-1841), a German philosopher whose pedagogical theories were disseminated through quarterly journals during his lifetime and following his death.

> Herbart's followers in America used his insistence upon association and interests to develop a very rigid educational program. This program came to be known as the Five Formal Steps of Teaching and Learning. They were: (1) preparation, in which old ideas useful in learning new materials are called to the learner's mind; (2) presentation, or the actual giving of the new material; (3) association, in which new material is compared with and related to the old; (4) generalization, in which rules, definitions, or general principles are drawn from specific cases; and (5) application, in which general principles are given meaning by reference to specific examples and practical situations (Pulliam, p. 103).

The Herbartian influence dominated American education in the 1890s and can still be observed in many mathematics classrooms in the year 2010, in which the sequence of instruction often begins with a "Do Now" that is followed by a presentation of new materials, which are modeled and explained for students, whereupon the students are expected to solve specific problem sets in which the new concepts are embedded. The Herbartian approach was particularly well suited to delivering the canonical texts associated with classical humanism, but were not universally embraced by more progressive educators.

Ravitch provides a detailed account of the public debate that occurred after the release of the Committee of Ten's recommendation. She describes a portion of the debate as follows:

> Nothing Eliot said could satisfy G. Stanley Hall, president of Clark University in Massachusetts, who was the report's most caustic critic. Hall was relentless in his efforts to tarnish the report. Renowned at the turn of the century as the founder of the child study movement, Hall derided the proposal that every subject "should be taught in the same way and to the same extent to every pupil so long as he pursues it." Calling this "a masterpiece of college policy," Hall declared that "this principle does not apply to the great army of incapables, shading down to those who should be in schools for dullards or subnormal children, for whose mental development heredity decrees a slow pace and early arrest, and for whom by general consent both studies and methods must be different" (Ravitch, pp. 45-46 > Hall, 1904, p.510).

Hall's argument was part of a chorus of opposition to classical humanist curricula and traditional pedagogy for all students, a movement in education that would eventually lead the New York Board of Regents in 1906 to adopt a dual track diploma system, with classical humanism standards controlled by the state, and control over progressive education standards being ceded back to local schools and school districts. Included in this rising chorus for progressive education was one of America's greatest voices in education, John Dewey, who had founded his Laboratory School at the University of Chicago in 1896 and would arrive in New York City during the next decade. Dewey published *School and Society*, his first book on education, in 1899, thus ushering out the 18th century with a preview of things to come in the 20th century.

1900-1909

Beginning the Decade: The Research Sample from 1900

During calendar year 1900, there were 18 examinations with 235 problems administered to assess curricula named Arithmetic, Advanced Arithmetic, Algebra, Plane Geometry, Solid Geometry, and Plane Trigonometry. Only 28 of the 89 mathematical topics assessed in 1900 were observed for the first time in the research sample in 1900, and a significant number of these new topics are associated with the previously reported limitation of the research sample, which is that all topics in the curricula are never assessed in any single year. Thus, many and perhaps most of these 28 new assessment topics were not new to the curricula in 1900, even though they first appear in the research sample during that year. (See Appendix G.)

Popular Discourses of the 1900-1909 Decade

The public education system in the United States was growing rapidly at the beginning of the 20th century, and teacher education was developing its own identity through an expanding number of normal schools and education departments in colleges and universities. This fact notwithstanding, educational historian David Angus noted that taking a subject matter examination was, “At the opening of the century…far and away the primary means of determining the competence of aspiring teachers” (Angus, 2001, p. 2). Thomas Good reports that some teachers were still beginning to teach with no training and only a high school diploma (Good, 2000), but Phillip Jones and Arthur Coxford had shown earlier that the trend in teacher education was clearly beginning to move toward two and four year diplomas at the turn of the century (Jones and Coxford, 1970). About this time, normal schools throughout the nation began to convert themselves from one and two year schools to four year colleges, and the three interrelated and interdependent components of almost all modern undergraduate teacher education programs emerged. These components were: 1) the study of academic areas to develop subject matter expertise; 2) the study of pedagogy to develop teaching expertise; and 3) student teaching under an experienced master teacher (Jones and Coxford, 1970). Ravitch noted that,

> The turn of the century was a time in which relatively small departments of pedagogy expanded into undergraduate and graduate schools of education. These institutions developed numerous specializations, such as school administration, educational psychology, educational sociology, and curriculum. Experts and professionals sought to create an education profession, which had its own preparation programs and its own technical language (Ravitch, 2002).

It was during this decade that the Teacher Institutes mentioned by Governor Fenton in his 1865 address to the New York legislature were mortally wounded by the increasingly powerful professional education movement that was centered in colleges and universities. Angus noted, “In the nineteenth century, county superintendents ran teacher training institutes lasting from a few days to a month or more. Professional educators despised these institutes because they threatened the image of professionalism” (Angus, 2001, p. 7). Teacher Institutes in New York declined during the decade from 1900 to 1909 and ceased altogether in 1912 (Folts, 1996).

The public was interested in public education and the numbers of students and teachers were growing. Concern about teacher quality was widespread, and with regards to mathematics, Donoghue noted that most mathematics teachers during this decade had only about one year of mathematics education beyond the level of mathematics they were teaching (Donoghue, 2003). With the increasing professionalization of teacher education, pedagogy became a field of interest in and of itself, and progressive education methods began to gain traction as newer and better than the older, traditional teaching methods. Folts noted that New York State instituted formal laboratory work in high school science courses in 1905, thereby acknowledging the value of experiential learning (Foltz, 1996). Also in this decade in New York, "…a parallel, non-Regents secondary school program emerged. Starting in 1906 high schools were authorized to issue a local diploma to students who had not taken and passed Regents exams" (Folts, 1996).

The 1906 decision to establish a dual track diploma system was the beginning of an unbroken 90 year period of tracking students into different education curricula in the state of New York. Not until 1996 would a decision be made to revert to a single Regents diploma with minimum quality controls over the credentials value of the diploma tied to a single set of academic standards for all students. Accordingly, 1906 represents an important marker in the evolution of the Regents examination system. Prior to 1906, New York State had a one-standard-fits-all approach to commencement level diploma requirements, and all students were subjected to the Regents process of examination. Then for 90 consecutive years, New York State used a dual diploma system, with one diploma associated with a classical humanist approach to mathematics education, and another diploma associated with local option diplomas and typically more progressive curricula. In 2010, as this dissertation is being written, public secondary school education in New York State is almost completely returned to a single diploma system. Concurrently, a national debate is developing over high stakes testing and diploma requirements as Congress prepares to rewrite the federal No Child Left Behind (NCLB) Act.

Just prior to the establishment of a dual track diploma system, an important event occurred that would over time greatly influence the public schools of New York. That event was the adoption in 1903 by the state legislature of new regulations for compulsory school attendance and child labor. Ensign provides the following statistics for the years 1901, 1902 and 1903 to show the general lack of enforcement and the inadequacies of New York's child labor laws prior to the enactment of the new legislation.

	1901	1902	1903
Violations of the child labor laws…	33,766	49,538	50,572
Convictions………………………	70	7	39
Fines……………………………	$2,010	$215	$1,060

(Ensign, p. 131)

Note that the probability of an employer being convicted in the courts after being cited for a child labor violation was highest in 1902, when approximately two out of every one thousand citations resulted in employer convictions. Clearly, the regulations and the judicial system were not functioning as an effective control system. In the context of longstanding ineffectiveness of compulsory school attendance and child labor laws, Ensign makes the following comments regarding the 1903 legislation.

> Now for the first time the requirements of the child labor laws and the compulsory attendance laws were in harmony. Heretofore, there had been no adequate basis for cooperation between the boards of education throughout the state and the boards of health charged with the duty of issuing working papers….New York was in advance of any other state in the Union at the time in requiring of the working child not only evidence of a minimum age, but a definite school-attendance record. Besides these data, the law required that the child possess a certain ability to read and write as exhibited in an examination to be given by the officials issuing working papers (Ensign, pp 134-135).

Hence, the 1906 decision to create a dual diploma system in the public schools of New York occurred within the context of the enactment of more effective and more efficient state controls over school attendance and child labor, which could arguably have the effect of changing the demographics of students attending public schools.

1910-1919

Beginning the Decade: The Research Sample from 1909

No extant examinations have been found from calendar year 1910. Accordingly, the research sample includes 16 examinations with 179 problems from calendar year 1909. The names of the curricula assessed by these topics, however, changed. The Algebra curriculum was not assessed in 1909, having been replaced by the Intermediate Algebra curriculum. Also, spherical trigonometry was discontinued as a separate curriculum and merged into the Trigonometry curriculum. All other curricula were the same as in 1900. The 179 problems administered during calendar year 1909 assessed 90 different topic areas, which includes 10 topics observed for the first time in 1909. It is interesting to note that by 1909 the total number of assessed mathematical topics assessed by the Regents examination system in a given year had become stabilized. In 1890, there were 93 assessed topics. In 1900, there were 89 assessed topics. In 1909, the count was 90 assessed topics, and in 1920, the count would rise slightly to 96 assessed topics. Throughout this period of time, the highest number of assessed topics dropped from the curricula never exceeded five. This supports the idea that the secondary school mathematics curriculum, being grounded in classical humanism, was larger than could be assessed in any one calendar year, but highly stable. (See Appendix G.)

Popular Discourses of the 1910s

The defining event of this decade was World War I, also known as the Great War and the War to End All Wars. The War is generally thought to have begun with the assassination in June

1914 of Archduke Franz Ferdinand of Austria-Hungary. The United States joined the war in April 1917, and the war was concluded at the Treaty of Versailles in June 1919.

During the years preceding the war, there was tremendous growth in secondary education and in the number of high schools. Moreover, progressive education was the subject of much discussion. In 1916, John Dewey Published *Democracy and Education,* which featured ideas from Dewey's laboratory school at the University of Chicago, where Dewey had "…experimented with democratic organization, nontraditional methods and equipment, and a curriculum based on the natural needs and interests of children … (Pulliam, 1994, p. 137). In 1917, the Smith-Hughes National Vocational Education Act was enacted by Congress and encouraged the establishment and development of vocational schools. Near the decade's end, in 1919, the Progressive Education Association was founded.

The cumulative impact of these progressive trends in education notwithstanding, they were mostly separate and apart from the Regents examination system. The classical humanist mathematics curricula associated with the Regents examination system had been insulated from the progressive education movement in 1906, when the dual diploma system was created, and significant control over the progressive agenda in education had been ceded by the state of New York back to schools and local school districts. These facts notwithstanding, the Regents examination system came under serious attack in the early years of this decade, so much so that the passing score on most examinations was lowered to 60% for a few years. By the end of the decade, however, passing scores were returned to the traditional level of 75%. Harlan Horner, Chief of the Examination Division of the University of the State of New York, provided useful information concerning the Regents examination system during these early years of the 20th century, when he authored a 1915 article for the prestigious journal *Education Administration* *and Supervision*, in which he argued that Regents examination scores should be a basis for the rating and promotion of teachers. Horner noted,

> The passing mark in all Regents examinations was formerly 75 percent. There is no appreciable change in the total per cent. of papers rejected now that the passing mark is 60 per cent. In January and June, 1914, 425,986 papers were written in the secondary schools of the State, of which 341,673, or 80.3 per cent, were claimed by the schools for acceptance at the University. Of the papers claimed, 297,390, or 87 per cent., were accepted by the University examiners. It will thus be seen that out of the total of 425,986 papers written, 69.8 per cent. were finally accepted. This means that 30.2 per cent. were rejected. The experience of the central office proves that it is reasonably safe to judge in large measure the efficiency of a given school by the relation which its total per cent. of papers finally rejected bears to the average per cent. of rejections for the entire State (Horner, 1915, pp. 380-381).

Horner's arguments of 1915 are remarkably similar to popular discourse concerning the appropriate uses of Regents examination scores in the year 2010. Perhaps more important, the debate over the advantages, disadvantages, and appropriate uses of the Regents examination have been part of the discourse concerning public schools in New York for at least a century.

A milestone in the progressive era was the adoption in 1918 of the Cardinal Principles, which was a report of the Commission on the Reorganization of Secondary Education. The commission was appointed by the National Education Association and Walmsley described its impact on mathematics education as follows:

> *The Cardinal Principles* was a report in 1918 that basically stated that not all students should be required to take the "traditional" mathematics courses of algebra and geometry. Because more students were going to high school, and many were training in vocational tracks, the recommendations were made that those students could take just one year of mathematics - a generic mathematics course-to graduate from high school. While the *Cardinal Principles* took a position for education for all, the focus on higher level mathematics in schools declined. Those college bound still took algebra, geometry, and trigonometry, but those not college bound usually found themselves floundering in these courses if they elected to enroll. Therefore, despite any initial progressive ideals, what eventually became associated with the progressive movement was a shift in mathematics content to only subject matter that the average citizen may need in an industrial job (Walmsley, 2007, pp 8-9).

The Seven Cardinal Principals stated that the curricula of secondary schools should address the subjects of: 1) health; 2) command of fundamental processes; 3) worthy home membership; 4) vocation; 5) civic education; 6) worthy use of leisure; and 7) ethical character. There is little or no evidence in the historical record left by the Regents examination system that any of these seven Cardinal Principles, other than command of fundamental processes, was ever a continuing focus of mathematics assessment practices in Regents curricula. This supports the idea that the dual diploma system effectively insulated the classical humanist curricula associated with the Regents examination system from the progressive agenda associated with local option diplomas.

The push from the developing education profession for a four year baccalaureate degree was strong, and normal schools responded by developing four year programs or, in some cases, by closing (Jones and Coxford, 1970). Many colleges and universities developed departments of education and supplemented regular academic courses with additional courses in pedagogy (Donoghue, 2003). Professional development for in-service teachers increased as the new teaching methods of the progressive movement became popular, and normal schools and colleges began offering summer courses for ongoing professional development (Walmsley, 2007). Questions began to arise about teacher competencies in the field of mathematics, and it was common to hear debates about the relative importance of educational courses versus subject matter courses. Walmsley again reports

> … in 1918, a report stated that the United States could not offer high levels of mathematics in schools because it lacked teachers highly trained in mathematics as well as individuals with strong mathematical backgrounds who wanted to become teachers. Furthermore, there was a constant complaint that with the increase in secondary schools, the focus of education of teachers was on secondary teachers and not elementary teachers (Walmsley, 2007, page 9).

1920-1929

Beginning the Decade: The Research Sample from the 1920s

In calendar year 1920, Regents academic examinations were administered to assess student achievement in Arithmetic, Elementary Algebra, Intermediate Algebra, Advanced Algebra, Plane Geometry, Solid Geometry and Plane Trigonometry. A total of 21 examinations with 257 problems were administered. These 21 examinations assessed a total of 96 different mathematical topics, 13 of which were new. Only one of the 96 topics assessed in 1920 was not seen in subsequent decades in the research sample, thus providing additional evidence of the stability of the assessed curricula. (See Appendix G.)

Popular Discourses of the 1920s

The "roaring 20's" was a decade of increasing prosperity for the nation, increasing urbanization, and strong support for, and growth in, public education. Many of the existing high schools in New York City were built during the 1920s. The National Council of Teachers of Mathematics (NCTM) was formed in 1920. Its mission was to be a "….public voice of mathematics education, providing vision, leadership and professional development to support teachers in ensuring equitable mathematics learning of the highest quality for all students (NCTM website, 2008). New ideas developed by educational researchers were transforming public education. Psychological testing enabled the efficient assessment and labeling of thousands of school children, and educational research was widespread, if not always accurate. Ravitch reports that "In the 1920s, reading researchers advised teachers that children should avoid oral reading, and they advised parents not to read to their children, on the grounds that children were supposed to read with their eyes, not their ears" (Ravitch, 2002).

The impact of the universities was also being felt in the area of mathematics education. University leaders and teachers worked together to better understand learning and to develop curriculum. New ideas from academia found their way into the classroom (Ravitch, 2000) Still, there was no general agreement between various groups competing for control of the curricula. Douglas Grouws and Kristine Cebulla report that calls were made upon mathematics educators to focus more on mathematical understanding and less on the drill methods that had been used for years (Grouws and Cebulla, 2000). Melinda Smith found that different programs for the education of secondary school mathematics teachers had distinct differences in the amount of focus on pure mathematics, applied mathematics, and mathematics pedagogy (Smith, 2004). Jeremy Kilpatrick noted that the focus on pedagogy was so great in some schools that there was little time left for the study of mathematics (Kilpatrick, 1992).

In the state of New York, major changes occurred in curriculum and assessment. In 1922, the state authorized high school administrators in city and village school districts to use assessment alternatives to the now ritualized Regents process of examination (Foltz, 1996), though such replacement assessments did not carry the same weight as the state developed assessments when applying for admission to colleges and universities. In allowing alternative assessments, the state was preparing the stage for the life adjustment education programs associated with the 1930's, 1940's, and 1950's. In 1927, approximately one third of the Regents examinations were discontinued (Folts, 1996). Also in 1927, the State Education Department published a document entitled, "Cardinal Objectives of Elementary Education" (Folts, 1996), which paralleled the seven Cardinal Principles for secondary education published a decade earlier. Progressive education had arrived in New York, but it could typically be found only in public school curricula not regulated by the Regents examination system. The decade of the 1920s was among the greatest decades of change in the history of the Regents examination system, and is seen in the research sample in the form of significant differences in the process of examination in 1920 and the process of examination in 1930.

1930-1939

Beginning the Decade: The Research Sample from 1930

The research sample shows that 21 examinations with a total of 539 problems were administered during calendar year 1930. A total of 149 topics were assessed, of which 29 assessment topics were observed for the first time. This total number of assessment topics represented a new high for any calendar year thus far in the research sample, and would remain relatively stable over the next eighty years. A decade earlier, during calendar year 1920, 21 examinations were administered with a total of 96 assessment topics and 257 problems. These numbers show that the average number of problems per examination doubled in a period of only ten years. Fundamental changes in Regents assessment practices occurred during the decade of the 1920s, and these changes are first reflected in the research sample during calendar year 1930. Each examination assessed more questions and more topics, and new ways of asking questions are evident in the examinations of 1930. The first use of a coordinate grid (a Cartesian plane) is observed. The first yes/no questions are observed. The first true/false questions are observed. The first compass and straightedge construction is observed. And finally, the first fill-in-the blank questions are introduced.

Regents academic examinations were administered in 1930 to assess student achievement in Arithmetic, Commercial Arithmetic, Elementary Algebra, Intermediate Algebra, Advanced Algebra, Plane Geometry, Solid Geometry and Plane Trigonometry. All of these examinations with the exception of the Commercial Arithmetic examination are included in the research sample. Only three of the 144 topics assessed in 1930 were not seen in subsequent decades in the research sample, again providing evidence of the stability of the assessed curricula. See (Appendix G.)

Popular Discourses of the 1930s

During the 1930s, the prosperity of the 1920s seemed to disappear and the United States sank into what is now called the Great Depression. Progressivism became an antidote for what was seen as the failures of capitalism, and progressive education entered what David Tyack and Larry Cuban would later refer to as a "golden age." Growth in the number of students attending secondary schools continued. Tyack and Cuban report that there was widespread belief amongst the white middle classes that America's public schools were, "…good and getting better…," even though "…there were wide discrepancies in access to and quality of educational opportunities based on race, class, and gender during the so-called "golden age" of schooling" (Tyack & Cuban, 1995, pp.23-26).

Ravitch commented on the progressive movement's focus on making education more utilitarian as opposed to the traditional humanist's focus on classical understanding of a body of knowledge in her discussion of the Progressive Education Association (PEA). Ravitch wrote,

> The PEA tried to show how every academic subject could be converted to meet the "needs of youth." For example, Science in General Education maintained that

> science teaching should center on practical problems that young people were likely to encounter in their daily lives, especially problems of health, homemaking, sex, sanitation, living conditions, and understanding how familiar machines work….The point of these curricular reorganizations was to replace logically organized academic subject matter with contemporary social issues, exchanges of opinion, or useful information. (Ravitch, 2000, p.275).

Widespread support for the progressive education movement's curricula and teaching methods can be inferred from the promulgation of New York State's "Cardinal Objectives," for elementary schools by the New York State Education Department. These cardinal objectives echoed the "Cardinal Principles" for secondary schools. The state also adopted in 1934 new rules for basic and elective courses of studies in secondary schools (grades 7-12). English, social studies, health, and physical education were required of all students (Folts, 1996). Mathematics and science were conspicuously absent from the list of required courses for local option diplomas during this "Golden Age" of education, though rigorous courses in mathematics and science were still offered to students pursuing Regents academic diplomas.

In 1937, a decision was made to move toward more comprehensive and integrated Regents examinations, rather than the narrowly focused examinations of the past. The impact of this decision would not be seen in the research sample until 1950. Consideration was also given to the idea of discontinuing the elementary examinations, but this recommendation met with resistance from school administrators and the elementary examinations were not discontinued until 1959 (Folts, 1996).

In terms of rising standards for teacher certification and teacher education, Jaime Grinberg described the 1930s as an era of "aggressive professionalism" (Grinberg, 2003). The desire for higher standards notwithstanding, there was disagreement as to how higher standards should be measured. Ravitch notes

> When the American Council on Education established a National Teachers' Examination in the 1930s, spokesmen from the nation's schools of education vociferously attacked it. The exams tested subject matter mastery. They were offered a few times and seemed to be very popular with urban school districts. Unfortunately, with the outbreak of World War II, there was a severe national teacher shortage; school superintendents hired anyone they could get and lost interest in the Council's external subject-matter examinations. (Ravitch, 2002).

With regards to mathematics education, the NCTM reported in 1933 growing concerns about the quality of teacher education. They argued that elementary school teachers in the seventh and eighth grade should study more algebra and geometry since these subjects were increasingly being taught in the growing number of junior high schools (NCTM, 1933). Jones and Coxford report that similar demands for increased subject matter training were called for in 1935 for secondary school mathematics teachers (Jones and Coxford, 1970). These movements and calls for reform notwithstanding, the historical artifacts of the Regents examination system in the research sample suggest that very little changed in assessment practices between 1930 and 1940.

1940-1950

Beginning the Decade: The Research Sample from 1940

A total of 18 examinations containing 502 problems were administered during calendar year 1940, and are included in the research sample. During 1940, 145 different mathematical topics were assessed, and 14 assessment topics were observed for the first time. Only 5 topics assessed in 1940 were observed in the research sample for the last time than year. Examinations were administered to assess curricula with names of Arithmetic, Business Arithmetic, Intermediate Algebra, Advanced Algebra, Plane Geometry, Solid Geometry and Plane

Trigonometry. Evidence suggests that Regents mathematics examinations were also administered in Elementary Algebra, but no samples of these tests have been located. (See Appendix G.)

Popular Discourses of the 1940s

The decade of the 1940s saw the entry of the United States into World War II, following a December 7, 1941 Japanese airstrike against United States naval forces at Pearl Harbor, Hawaii. From the United State's perspective, the war ended first in Europe on V-E Day (Victory in Europe), which occurred on May 7 and 8, 1945. The war ended in the Pacific theater of operations on V-J day (Victory over Japan), September 14 and 15, 1945, following the United State's nuclear bombings of the Japanese cities of Hiroshima and Nagasaki. Though the end of World War II was cause for great celebration, the cessation of overt hostilities was accompanied by a reshaping of the political map of the world, with the creation of the United Nations and the emergence of the Union of Soviet Socialist Republics and the United States of America as two great superpowers, thereby setting the stage for approximately 45 years of superpower conflicts, in what would be called the "Cold War." World War II was a "total war," meaning that it influenced almost every aspect of American life and American society, including mathematics education. Only two societal events in the history of the United States are known to have had a significant impact on Regents mathematics assessments in the public schools of New York State. These two events were World Wars I and II, and the historic Regents examinations from the war years contain dozens of mathematics problems embedded in evoking contexts associated with warfare. Exemplars of the Regents examination system being used to support the war effort of the United States are provided in Appendix H.

At the beginning of the war, millions of men in the United States volunteered for military service. Eventually, almost all men between the ages of 18 and 45 (inclusive) were called to military service. Jobs once performed almost exclusively by men were filled with women who previously had never worked outside the home, the upshot being significant changes to the lived experiences of both men and women. Concurrently, advances in technology and manufacturing associated with the war effort greatly expanded the industrial base, setting the stage for significant advances in non-military technology and manufacturing following the war. From these perspectives, World War II can also be viewed as a milestone in the evolution of macro-level structures of American society.

World War II is associated with several important events that would influence mathematics education in New York. Immediately prior to entry into the war, mathematics education in the United States was studied by several national committees. Two important reports were released in 1940: the Report of the Committee on the Function of Mathematics in General Education; and the Report of the Joint Commission to Study the Place of Mathematics in Secondary Education. Additionally, a “War Preparedness Committee” was appointed by the American Mathematics Society and the Mathematical Association of America. In 1941, the War Preparedness Committee issued its report, entitled "On Education for Service" (NCTM, 1970). The amount of mathematics being taught in secondary schools was a public concern throughout the 1940s. Early in the decade, Admiral Chester Nimitz, who would become the United States’ highest WWII naval commander, complained that high school graduates coming into the Great Lakes Naval Training Center were not adequately prepared in mathematics and had to be retrained by the Navy (NCTM, 1970, pp. 58-59). There were also concerns about teacher preparation and the amount of mathematics that teachers knew (Jones and Coxford, 1970) and teacher certification requirements, especially those for elementary school teachers, were criticized as not requiring enough mathematics (Stanic and Kilpatrick, 2003). These complaints were already being voiced at the beginning of the decade. They would grow stronger as the decade passed.

In public elementary and secondary schools throughout the nation, the decade of the 1940s saw the culmination of a nearly fifty year movement in mathematics education towards a progressive life adjustment philosophy, in which public education was seen as responsible for preparing students to assume specific roles in society. The Educational Policies Commission of the National Education Association (NEA) published a book in 1944, which argued that students should be sorted into academic and vocational tracks according to their dispositions and abilities, and that their educational experiences in public schools should be tailored to their likely roles in society (Educational Policies Commission, 1944). With regard to mathematics, the net result of this argument was that all students did not study the same mathematics, and some students studied little or no mathematics at all. Critics complained that standards were being lowered for children in non-academic tracks (Raimi, 2000). All of these changes, however, had little influence on the mathematical topics actually assessed in the classical humanist curricula assessed by Regents mathematics examinations.

Although teacher certification requirements had typically increased to include a four year degree from a state accredited institution, there was widespread concern with the content that mathematics teachers knew (Tozer, Violas, and Senese, 1998). The joint commission of the Mathematics Association (MAA) of America and the National Council of Teachers of Mathematics (NCTM) was created to study secondary school mathematics education in the United States. It issued the first of several reports in 1940. The 1940 report asserted that a serious problem existed in the training of secondary school mathematics teachers, and that higher level mathematics courses should be added to strengthen teacher certification requirements. (NCTM. 1940). The Report of the Joint Commission was also an attempt by the Mathematics Association of America and the National Council of Teachers of Mathematics to convince the public of the importance of mathematics. The report was published as the fifteenth yearbook of the NCTM and entitled *The place of mathematics in secondary education, Fifteenth Yearbook* (NCTM, 1940). The report gave support to the goals of lifestyle education by advocating different levels of mathematics for students in academic tracks that led to college and vocational tracks that did not lead to college (Garrett and Davis, 2003). Traditional courses in algebra, geometry, and trigonometry were recommended for students on academic tracks while new courses in consumer mathematics were recommended for others (Grouws and Cebulla, 2000). This was a pattern already established in New York's public schools. Between 1944 and 1947, the NCTM published several reports detailing post-war recommendations, and these reports called for three tracks instead of two. The recommended highest track was the traditional academic track, which was reserved for the most capable students. A recommended middle track was for those who would use applied mathematics in their vocations, and the recommended lower track consisted of consumer mathematics for those students who were unlikely to use mathematics except for routine, everyday functions (Garrett and Davis, 2003). These reports were significant outside of New York because they coincided with the end of a half century of movement toward practical and vocational mathematics for most students at the expense of traditional, academic mathematics for students of higher abilities (Kliebard and Franklin, 2003). In New York, the dual option diploma system provided local schools with flexibility to adopt these recommendations and create appropriate progressive curricula leading toward local option diplomas, while simultaneously insulating the classical humanist Regents mathematics curriculum as a optional curriculum for academically elite students.

A second influence of World War II on public education was associated with the return of servicemen from the war and the displacement of millions of women from jobs outside the home, which they had been performing while the men were at war. The return of the servicemen from war was followed by a dramatic increase in the birthrate in America, giving rise to a new generation that would be called the "baby boomers." Public attitudes in America following World War II were characterized by a general level of optimism and beliefs in technology and education. During the war years, many women had been exposed to lived experiences and roles outside the home and many men had been exposed to foreign travel with new cultures and experiences and to new technologies, such as airplanes, ships, and motor vehicles. They were profoundly changed by these experiences, and not eager to embrace their pre-war, great depression, lifestyles. Concurrent with the end of the war, the G.I. Bill made money available to veterans for post-secondary education, and by the fall of 1945, almost half of college students were veterans (Walmsley, 2007).

The life adjustment philosophy of education was under attack by the end of the decade of the 1940s, and the pendulum of educational reform was beginning to move in the direction of more traditional academics. Diane Ravitch reports,

> By mid-century, the schools had become agencies dedicated to socializing students, teaching them proper attitudes and behaviors, and encouraging conformity to the norms of social life and the workplace. Educators at the national, state, and local levels who subscribed to life adjustment education thought that the schools were meeting the needs of their students and of democratic society admirably (Ravitch, 2000, p. 343).

Teacher satisfaction with the schools notwithstanding, the experiences of the forties led to growing concerns about the purpose of schools and the quality and amount of academics being taught in the schools. Many groups believed that the schools were not doing an adequate job of preparing students for the demands of a changing society. By the middle of the next decade, the Progressive Education Association (PEA) would officially disband (Kilpatrick, 1992).

Test design also changed during the 1940s. Multiple choice questions and mechanical scoring were considered to have significant advantages over open ended questions (Stanic and Kilpatrick, 2003). These changes are first reflected in the research sample during calendar year 1950.

1950-1960

Beginning the Decade: The Research Sample from 1950

A total of 20 Regents mathematics examinations were administered in 1950, and these 20 examinations contained 598 problems. A total of 153 different mathematical topics were assessed, only nine of which were observed for the first time. Sixteen topics were observed in the research sample for the last time in 1950. Most of these assessment topics observed in 1950 for the last time were associated with the elimination of the preliminary examination system in 1959 and the phasing out of solid geometry proofs from the curricula.

In 1950, Regents mathematics examinations assessed curricula named Preliminary Mathematics, Intermediate Algebra, Advanced Algebra, Plane Geometry, Solid Geometry and Trigonometry. These curricula names, with minor modifications, dated to the origins of the Regents academic examinations. Two new curricula names appear in the research sample for the first time in 1950, signaling the advent of the integrated mathematics curricula that would dominate the second half of the 21st Century. These new curricula were simply called Tenth Year Mathematics and Eleventh Year Mathematics. (See Appendix G.)

Popular Discourses of the 1950s

The decade of the 1950s in the United States was characterized by an expanding post WWII economy, increasing living standards for the middle class, including significant growth in post secondary education, and continuing escalation of tensions between the two major superpowers that emerged from WWII. The decade began with the war in Korea (1950-1953) and growing fears of communism. Dwight Eisenhower, Commander in Chief of Allied Forces during WWII, was elected President in 1952 and again in 1956. Significant events also occurred in the developing Civil Rights movement. In the 1954 Brown versus Board of Education decision, the United States Supreme Court unanimously agreed that separate educational facilities for black and white children were inherently unequal, thus overturning the long standing precedent for separate but equal government services established in 1896 in Plessy versus Ferguson.

During the early years of the 1950s, increasing demands were placed on secondary schools to offer more advanced courses in mathematics, especially college preparatory courses in algebra, geometry, and calculus (Jones and Coxford, 1970, pp. 78-79). This was due in part to the increasing numbers of secondary school students who were planning to attend college, and also to increasing awareness of the importance of science, technology, and mathematics in a post WWII society. Concerns over the importance of mathematics education as a foundation for the rapidly expanding fields of science and technology during the 1950s led to numerous calls for reform of mathematics education, and numerous commissions and studies addressed the question of what mathematics should be taught in our nation's secondary schools (Jones and Coxford, 1979, pp. 235-300). This perception of an increased need for mathematics for the support of defense, technology, and science, was reinforced by concerns that the progressive education movement had gone too far in life adjustment education and that many students were not receiving a basic education in traditional curricula (Berube, 1994). Thus, the stage was set for the pendulum to swing back to the basics (Rury, John L. 2005b) and away from what many considered to be the anti-intellectualism excesses of the progressive life adjustment curricula (Hofstadter, 1963).

It was during the decade of the 1950s that a "new math" movement was created, with its genesis arguably being in the work of the University of Illinois Committee on School Mathematics, which began in 1951 (Raimi, 2000). Another "new math" initiative was the School Mathematics Study Group (SMSG), which began in 1954 (Jones and Coxford, 1970, pp. 269-271). Both initiatives shared: 1) the idea that academic mathematics should be taught to more students in secondary schools; 2) the idea that mathematics instruction should be changed to add increased emphasis on understanding and problem solving; and 3) widespread support and involvement of university level mathematicians and mathematics educators who received federal and private foundation funding. And even though the genesis of the new math movement can be traced to a backlash against progressive education and the lack of traditional mathematics content in the curriculum, much of the pedagogy advocated by the new math movement was reflective of progressive pedagogy (Grouws and Cebulla, 2000). Discovery learning was emphasized and the NCTM argued that technology was increasingly doing complex calculations, thereby reducing the need for drilling students to increase skills in algorithms and symbol manipulation and increasing the need for deeper understanding of mathematics. The basic idea was that "We must teach our students to do the work that machines cannot do" (NCTM, 1957, p.424). From this view, the new math movement can be viewed as a call for increased content knowledge and academic rigor in mathematics combined with teaching methods long advocated by progressive educators.

The new math movement was also seen as an opportunity to address a long standing perception that traditional mathematics had been taught for many years without unifying concepts that traversed the boundaries of classical strands of mathematics. Various unifying concepts had been tried in different experimental curricula over several decades, and the new math movement generally agreed on set theory as the unifying concept (NCTM. 1957). This resulted in textbooks and problems that often appeared quite different from the mathematics studied a generation earlier by the students' parents. Walmsley reports that additional subjects in the new math curricula "…included: set theory, deductive methods, vector analysis, limits and functions, and probability and statistics" (Walmsley, 2007, p.26). The influence of the new math movement is not seen in the research sample until 1970, when set theory is first observed and a special SMSG examination was administered, reflecting the influence of the School Mathematics Study Group that was founded in 1958 with National Science Foundation support (NCTM 1970).

The back to basics and new math movements shared the goal of more mathematics for more students, and both movements were significantly influenced by the October 1957 launch of the Soviet satellite, Sputnik, which was the first man-made object to orbit the Earth. The reaction of the United States to Sputnik included a belief by many Americans that their cold-war enemy, the Soviet Union, was getting ahead of the United States in military technology. They reasoned that the Soviet Union had superior science and technology that enabled them to be the first in space, and they saw Sputnik as a serious threat to the security of the United States. Moreover, many critics blamed the schools, reasoning that public education in the United States was not doing an adequate job of teaching the mathematics, science, and technology necessary for American supremacy (Rury, John L. 2005b). Congress reacted in 1958 to the Sputnik crisis by creating the National Aeronautics and Space Administration (NASA), and by passing the National Defense Education Act. The latter addressed the perceived shortcomings of the schools and provided funds to improve science and mathematics education, thus leading to a golden age of support for mathematics and science education in secondary schools. Money was readily available to teachers for additional training and education in mathematics and science and more students wanted to take these courses in preparation for careers in science, technology, and mathematics. Public support for increased mathematics education was very high (Garraty and Carnes, 2000).

As support for life adjustment education eroded, and the demands for increased academic rigor in traditional subject areas increased, a common solution was for schools in the 1950s to segregate students into academic and non-academic tracks, with rigorous academic mathematics being offered in the higher academic tracks and practical, applied or no mathematics offered in the lower non-academic tracks (Ravitch, 2000). The New York State Regents examinations in mathematics were designed primarily to define and assess the curricula offered to students in the higher level academic tracks. In similar tracking situations, students tracked into the higher academic levels were typically middle class and academically elite students. Large numbers of minority and lower social economic class students were placed in non-academic tracks, thus further reducing the probability that they would meet college entrance requirements that increasingly required academic mathematics (Hallinan, 1994a, 1994b) (Hallinan and Soreneson, 1987) (LeTendre et al. 2003) (Ma, 2002) (Oakes, 1994).

More than 90 years after the New York Regents established the Regents academic examinations as uniform tests for admissions to academies and universities in the state of New York, College Entrance Examination Board (CEEB) established a program of Advanced Placement examinations in 1959 (Stanic and Kilpatrick, 2003). Concurrently, CEEB issued a report in 1959 in which specific recommendations were made concerning what students should know in order to be admitted to college. Arguably, the CEEB was doing for the schools of the nation what the Board of Regents had done for the schools of New York 93 years earlier.

The Regents preliminary examination system had originally been created as a quality control system to regulate admissions to the old academy system of secondary education. This need no longer existed, since a modern system of public high schools had long since replaced the old academy system of boarding schools. After the last examination administered by the Regents preliminary examination system in 1959, testing of public school students continued to occur at the transition from elementary school to secondary school, but the new tests were no longer associated with the Regents examination system and are therefore are not included in this research effort. At the beginning of the next decade, the Regents examination system showed little evidence of being influenced by the events of the 1950s.

1960-1970

Beginning the Decade: The Research Sample from 1960

A total of 17 Regents mathematics examinations were administered in 1960, and these 17 examinations contained 630 problems. The total number of different mathematical topics assessed in 1960 was 144, a slight decrease from 1950 when 153 different mathematical topics were assessed. A total of 18 topics were observed in the research sample for the last time in 1960, and these last observed topics were associated with some of the highest levels of mathematics ever assessed by the Regents examination system. Examples of topics last observed in the research sample in 1960 include: differential calculus; integral calculus, higher order equations; matrices; arithmetic and geometric progressions; proofs involving dihedral and polyhedral angles and spherical polygons; and advanced trigonometric topics such as polar form and factoring trigonometric expressions. Interestingly, there were only 10 new mathematical assessment topics first observed in the research sample in 1960, and two of these new topics are never again seen in the research sample. (See Appendix G.)

Popular Discourses of the 1960s

The decade of the 1960s was one of radical change in mathematics education and politics. The decade began during the last year of the presidential administration of Dwight Eisenhower, who was replaced by John F. Kennedy in 1961. Throughout the decade, the Cold War between the United States and the Union of Soviet Socialist Republics continued to escalate. During the Cuban missile crisis of 1962, the United States and the Union of Soviet

Socialist Republics were arguably closer to nuclear war than at any time since atomic weapons were first created. In schools, these increasing international tensions were reflected in rehearsals of ‘duck and cover” drills, during which children were supposed to learn to protect themselves from nuclear weapons. In some school districts, children who lived close enough to their schools were allowed to run home during duck and cover drills, presumably so they could die with their mothers in the event of an actual nuclear attack. In a perhaps unrelated development, 1962 was also the year when the Supreme Court of the United States, in Engle vs. Vitale, determined that school prayers were unconstitutional, thus ending many long standing practices in public schools requiring students to recite approved prayers as part of daily educational rituals.

In 1963, President John F. Kennedy was assassinated. Lyndon Johnson assumed the presidency, promising to continue many of the initiatives of Kennedy’s *New Frontiers* agenda in what Johnson would call the *Great Society*. Of particular importance to schools, the legacy of the Johnson administration includes a rewrite of the nation’s immigration laws, shifting the tide of immigration away from Europe and toward Asia, Korea and Latin America (Walmsley, 2007). The enduring legislative accomplishments of the Johnson administration notwithstanding, America was also heavily involved in fighting the Cold War, whose focus for the United States had been transferred from Cuba to a growing and prolonged war in South Vietnam.

The year 1968 was pivotal. President Johnson had decided not to seek reelection for a second full term. During April, Martin Luther King Jr. was assassinated. Two months later, in June, Robert F. Kennedy was assassinated. Robert F. Kennedy was the brother of slain President John F. Kennedy, and was himself a presidential candidate to replace Lyndon Johnson. The nation was stunned by these back-to-back assassinations of prominent liberals, and race riots and protests were commonplace across the nation. With these events as background, Richard M.

Nixon won a narrow victory over Hubert Humphrey. Nixon ran on a conservative platform that promised to restore law and order to America, and to end the war in Vietnam.

In 1969, under the administration of President Nixon, America put its first astronaut on the moon, thus marking an important technological win over the Soviet Union in the Cold War. The decade of the 1960s began and ended with a Republican in the Whitehouse, but during the intervening years of Democratic presidencies, the nation experienced dramatic change. The situation was similar in mathematics education.

In 1961, the National Council of Teachers of mathematics held a series of eight regional conferences throughout the United States. These conferences were focused on new ways of teaching mathematics, which were summarized in a new NCTM publication entitled *The Revolution in School Mathematics*. The publication asserted four requirements which must be included in the new mathematics education paradigm. These four requirements were: 1) in-service retraining of teachers; 2) better pre-service training of teachers; 3) improved teaching techniques; and 4) sufficiently large high schools (NCTM, 1961, pp. 13-14.) This call for revolution came in the aftermath of Sputnik and during a period of high public concern over the Cold War space race. There was lots of money available from the federal government for efforts to improve mathematics education in secondary schools, and the NCTM was advocating a move toward more progressive pedagogies and away from more traditional pedagogies whose genealogies could typically be traced back to the American Herbartianism pedagogy of the 1890s.

In 1961, the NCTM sponsored what might be considered a book tour for *The Revolution in School Mathematics.* The 97 page book could be purchased for 50 cents and was arguably a call to arms for progressive mathematics educators, signaling the beginning of what many would soon begin referring to as "The Math Wars." The war metaphor is useful. As is typical of war, there are two sides: the progressives, who want to pursue mathematics curricula associated with child development and/social meliorist agendas; and the conservatives, who typically advocate a classical humanist agenda, (i.e. the math that educated people have learned forever and ever, or so it seems). Differences in the opinions of progressive and traditional mathematics educators typically can be classified into two categories: 1) differences about what mathematics should be taught in public schools; and 2) differences about how mathematics should be taught.

During the 1960s, the progressive mathematics lobby became a powerful voice in many academic institutions and in state and federal educational administrations. Some states responded to the call for change by designing new curricula that reflected the new instructional paradigms of the New Math movement. New York was one state that designed a curriculum based on the New Math movement. The new curriculum was called "Special (SMSG) Geometry" and three Regents mathematics examinations have been found that were used to assess the "Special (SMSG) Geometry" curriculum. These examinations were administered between 1970 and 1976, and the 1970 examination is included in the research sample. The SMSG acronym in the name of the examination is short for School Mathematics Study Group.

The School Mathematics Study Group (SMSG) was a think tank of top academics with a large National Science Foundation Grant in the aftermath of Sputnik. It was often criticized as lacking credibility because of under-representation of public school teachers in its membership. The educational philosophies promulgated by the SMSG are at the core of *The Revolution in School Mathematics* and establish much of the message of the New Math movement. Interestingly, SMSG placed a significant emphasis on students learning mathematics by reading well illustrated textbooks, arguably a further signifier of concern in academia about the ability of mathematics teachers in the 1960s to implement the new pedagogical paradigms being advocated for mathematics education.

Recommendation number 4 in *The Revolution in School Mathematics* seems particularly relevant, and perhaps antithetical, to the new small school movement in New York City as this dissertation is being written in 2010. It is quoted in its entirety in the following paragraph.

> SUFFICIENTLY LARGE HIGH SCHOOLS
> A final requirement for the mathematics education which I have described as adequate for our times is that the high school itself be sufficiently large. A small high school cannot provide the mathematics courses and the teachers I have described above as necessary; James B. Conant has suggested that a high school with a graduating class of 100 is the minimum size. Students in a smaller school almost certainly are denied proper mathematics courses. The nation cannot waste its limited supply of good mathematics teachers by placing them in schools where they teach their specialty to less than full capacity. The nation cannot afford the waste of talent that results from sending gifted students (they occur also in small schools) to schools with poor mathematics programs and poor teachers. Fortunately, many states are solving the problem of the small high school by consolidating small schools into large schools (NCTM, 1961, p.14).

From the perspective of the NCTM in 1961, the modern movement in New York City during the first decade of the 21st Century toward small schools is diametrically opposed to the large school recommendations of the new mathematics movement.

Walmsley reported in her commentaries concerning mathematic education during the 1960s that,

> The Commission on Mathematics of the College Entrance Examination Board (CEEB) stated the following for college-bound students in mathematics: logic, statistics, and probability should be a part of school mathematics; plane and solid geometry should be integrated into one course; trigonometry should be taught with a second level algebra course; and pattern seeking should unify all mathematics. Many of the "new math" projects and movements that produced materials for schools followed these recommendations for the new teaching materials. These recommendations are what led to what many high schools offer presently (Walmsley, 2007, 30-31).

Walmsley went on to describe the School Mathematics Study Group of the 1960s, whose special geometry curriculum would be assessed during the 1970s in secondary schools throughout the state of New York by the Regents examination system. She wrote,

> A variety of projects constituted the "new math" movement; with the most popular being SMSG-the School Mathematics Study Group. Most projects developed course materials around the CEEB recommendations led by mathematicians at universities and some mathematics educators and teachers…. Many of the projects had hoped teachers would use discovery learning in teaching the material, but few projects presented content in this manner….While the "new math" gained national attention and was present in many schools, the fact remains that in the entire United States school population, very few students were exposed to "new math." In fact, one researcher stated that, "It was possible at the time to walk into almost any school in the United States and see mathematics teaching that was little different from typical teaching before World War II" (Walmsley, 2007, p.31).

Indeed, by the end of the 1960s, the new mathematics movement of the early 1960s was being challenged by what many would call the back to basics movement of the 1970s. Walmsley's observations concerning the continuity of mathematics teaching practices are supported by a cursory comparison of the Regents mathematics examinations of 1960 and the Regents mathematics examinations of 1970. This comparison shows that 108, or 88%, of the 123 mathematical topics assessed in 1970 had been assessed in previous decades, suggesting that when measured by the impact on the assessed Regents mathematics curricula, the modern mathematics movement was not overwhelming.

1970-1979

Beginning the Decade: The Research Sample from 1960

A total of 10 Regents mathematics examinations were administered in 1970, and these 10 examinations contained 352 problems. The number of different curricula assessed was the lowest since the Regents academic examination began in 1890, and with the exception of the Special (SMSG) Geometry examination, the only curricula assessed were associated with the three-year integrated Ninth, Tenth and Eleventh Year Mathematics curricula. The total number of different mathematical topics assessed in 1970 was 123, a decrease from 1960 when 144 different mathematical topics were assessed and 1950 when 153 topics were assessed. This decrease in the number of assessed topics appears to be associated with fewer mathematics curricula being assessed in 1970. Only 7 topics were observed in the research sample for the last time in 1970. There were 15 new mathematical assessment topics first observed in the research sample in 1970, and these were: Absolute Value; Absolute Value Equations; Absolute Value Inequalities; Defining Functions; Domain and Range of Functions; Compositions of Functions; Graphing Systems of Equations; Locus with Equations; Logical Reasoning; Biconditional Statements; Undefined Rationals; Set Theory; Replacement Sets; Transformations; and Simple Equations with Decimals. (See Appendix G.)

Popular Discourses of the 1970s

The 1970s began with Richard Nixon in the Whitehouse and U.S. troops fighting the Cold War in Vietnam. The war in Vietnam arguably ended for the United States in 1975 during the Fall of Saigon. Richard Nixon was reelected in 1972, but resigned in 1974 during the Watergate political scandal. Gerald Ford became President when Nixon resigned, promptly pardoned Nixon, and ran for election to the Presidency in 1976. Ford was defeated in the general election by Jimmy Carter, who served as President throughout the remainder of the decade.

In mathematics education, the Regents examination system began assessing student achievement in the new Three Year Sequential Mathematics Curricula, with successive years of instruction named simply Course I, Course II and Course III. Nationally in the 1970s, a chorus of voices for more traditional practices in mathematics education mounted a counter-offensive against the New Math movement. This counteroffensive has been called the “back-to-the-basics movement” in mathematics education, and perhaps it was in response to this back-to-the-basics movement that New York discontinued the Special (SMSG) Geometry curriculum during the second half of the 1970s. Regardless of the precise reasons for the discontinuation of the Special (SMSG) Geometry curriculum, the fact is that it went away in the late 1970s, and following the demise of this curriculum, there is scant historical evidence of the New Math movement in the extant historical record of mathematics assessment practices left by the Regents examination system. One possible reason for the failure of the New Math movement to leave a significant imprint on Regents assessment practices may be that the Regents examination system was redefined in 1906 in such a manner as to insulate it from external forces. The dual diploma system had long allowed a relatively peaceful coexistence of progressive and traditional educators in New York.

With respect to mathematics education, this bifurcated diploma system allowed progressive educators to follow their child development and social meliorist philosophies in numerous ways while the traditionalists clung to a classical humanist agenda that was integrated with a highly ritualized process of state controlled examination and state sponsored credentialism. When the progressive ideologies of the New Math movement were introduced in the public schools of New York, they co-opted the Regents process of examination, but did not co-opt the standards setting mechanism associated with the traditional secondary school mathematics curricula. Hence, before, during and after the demise of the Special (SMSG) Geometry curriculum of the New Math movement in New York State, the core Regents level mathematics curricula of secondary schools of New York remained relatively unaffected and highly stable. What the research sample shows for secondary education in mathematics in the state of New York is generally consistent with the following summary by Walmsley of the impact of the New Math movement on secondary mathematics education in the nation as a whole:

> For most students graduating from high schools in the 1970s, they only needed to take one course in mathematics, and many of these students never took a course beyond the traditional one offered in ninth grade. The 1970s saw a "back to basics" movement in mathematics as many Americans were not happy with the "new math" movement of the previous decade that seemed to produce children who were weaker in computational skills than they had hoped. In fact, very few students actually saw the "new math" of the 1950s and 1960s, so when there was a call for "back to basics" most schools were back to teaching the curriculum and way they always had. Students were seen capable in mathematics once they could master these basics which were defined as computational or pencil-and-paper skills. Another reason that the "back to basics" movement became popular was the drastic cuts in federal funding to the "new math" movements of the previous two decades. The focus became on basic arithmetic operations with little emphasis on problem solving or applications. Words were taken out of textbooks and replaced by numerous mathematics exercises stressing the same content. (Wamsley, 2007, p. 36).

1980-1990

Beginning the Decade: The Research Sample from 1980

A total of 16 Regents mathematics examinations were administered in 1980, and these 16 examinations contained 559 problems. The number of different curricula assessed in 1980 was almost double that of a decade before. This increase in the number of different curricula assessed was because the Ninth, Tenth and Eleventh Year curricula introduced in the 1940s was being phased out and replaced by the curricula of the new Sequential Mathematics Courses I, II and III. Also, the Special (SMSG) Geometry examination, which appeared in the research sample only in calendar year 1970, and which was associated with the New Mathematics movement of the 1960s, was ended prior to 1980. The total number of different mathematical topics assessed in 1980 was 151, an increase from 1970 when 123 different mathematical topics were assessed. Only 11 topics were observed in the research sample for the first time in 1980, while ten were observed for the last time. Thus, approximately 93% of the assessed curriculum in 1980 was the same as that assessed in 1970. This pattern of stability in the curricula assessed by the Regents examination system is observed repeatedly throughout the research sample. (See Appendix G.)

Popular Discourses of the 1980s

In 1980, the NCTM published "An Agenda for Action," which made eight generally progressive recommendations for school mathematics in the 1980s. The full text of these recommendations are as follows:

The National Council of Teachers of Mathematics recommends that

1. problem solving be the focus of school mathematics in the 1980s;
2. basic skills in mathematics be defined to encompass more than computational facility;
3. mathematics programs take full advantage of the power of calculators and computers at all grade levels;
4. stringent standards of both effectiveness and efficiency be applied to the teaching of mathematics;
5. the success of mathematics programs and student learning be evaluated by a wider range of measures than conventional testing;
6. more mathematics study be required for all students and a flexible curriculum with a greater range of options be designed to accommodate the diverse needs of the student population;
7. mathematics teachers demand of themselves and their colleagues a high level of professionalism;
8. public support for mathematics instruction be raised to a level commensurate with the importance of mathematical understanding to individuals and society (NCTM, 1980, p.1).

These generally progressive recommendations of the NCTM, which were released during the last years of Jimmy Carter's presidency, were soon overshadowed by the 1983 release of *A Nation at Risk*, which reflected a new conservativism that underlay Ronald Reagan's administration. The single recommendation from *An Agenda for Action* that is readily apparent in the record of mathematics assessment practices left by the Regents examination system is the required use of calculators on Regents mathematics examinations, which is first observed in the research sample in calendar year 2000.

In many ways, the 1980s belonged to Ronald Reagan, who defeated Jimmy Carter in 1980 and assumed the Presidency in 1981. The genesis of the contemporary testing movement at the beginning of the 21st Century can arguably be traced to Ronald Reagan's administration and the 1983 report of the National Commission on Excellence in Education, which was entitled *A Nation At Risk - The Imperative For Educational Reform* (US Government, 1983). *A Nation at Risk* provided two important recommendations that influenced states to adopt high stakes testing and teaching quality initiatives. Recommendation "A" posited

> …State and local high school graduation requirements (*should*) be strengthened and that, at a minimum, all students seeking a diploma (*should*) be required to lay the foundations in the Five New Basics… (*English, Mathematics, Science, Social Studies, and Computer Science*)[3] (US Government, 1983).

Recommendation "B" concerned itself with standards and expectations and included the language, "We recommend that schools, colleges, and universities adopt more rigorous and measurable standards, and higher expectations, for academic performance and student conduct…" Recommendation "B" also included the following specific language about standardized testing,

> Standardized tests of achievement (not to be confused with aptitude tests) should be administered at major transition points from one level of schooling to another and particularly from high school to college or work. The purposes of these tests would be to: (a) certify the student's credentials; (b) identify the need for remedial intervention; and (c) identify the opportunity for advanced or accelerated work. The tests should be administered as part of a nationwide (but not Federal) system of State and local standardized tests. This system should include other diagnostic procedures that assist teachers and students to evaluate student progress (US Government, 1983).

The very title of "A Nation at Risk" suggests that its r*aison d'etra* may have been Social Meliorism, though its goals and methods were clearly traditional and grounded in economics. The first paragraph of the report supports this conclusion by asserting,

> Our Nation is at risk. Our once unchallenged preeminence in commerce, industry, science, and technological innovation is being overtaken by competitors throughout the world. This report is concerned with only one of the many causes and dimensions of the problem, but it is the one that under girds American prosperity, security, and civility (US Government, 1983).

The commission believed that American Society was in danger, that one cause of the problem was the condition of public schools, and that significant changes in public education were necessary to make society what it should be. Though undeniably traditional in focus, the

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[3] *Italics* not in original.

commission's recommendations were cloaked in progressive rhetoric. The five basic subject areas recommended for the curriculum (*English, Mathematics, Science, Social Studies, and Computer Science)* loudly echo the Essentialist's calls from the 1930s for a return to classical humanism and its core "windows of the soul," first enumerated by the Committee of Ten in the 1890s.

In New York State, the basic reforms called for in "A Nation at Risk" (i.e. those of classical humanist standards and accountability through testing) were already in place, and had been in place for more than 100 years. With federal funding moving in the direction of higher standards, more traditional curricula and accountability through high stakes testing, the stage was being set for expansion of the Regents examination system and a return to a single diploma system, decisions which would be made in the following decade.

Near the end of the decade of the 1980s, the NCTM published the first of several publications on standards, which was entitled *Curriculum and Evaluation Standards for School Mathematics.* As with other public discourses concerning change in mathematics education, there is little evidence in the research sample that these standards influenced the content of what was taught in the Regents curricula of the public schools of New York State or the mathematics assessment practices associated with the Regents examination system.

1990-1999

Beginning the Decade: The Research Sample from 1990

A total of nine Regents mathematics examinations were administered in 1990. These examinations assessed the Eleventh Year curriculum and the Sequential Mathematics Courses I, II and III. Altogether, these nine examinations contained 375 problems. The total number of different mathematical topics assessed in 1990 was 136. Only 6 assessment topics were observed in the research sample for the first time in 1990. Seven topics were observed in the research sample for the last time during 1990. (See Appendix G.)

Popular Discourses of the 1990s

The decade of the 1990s began with George Herbert Walker Bush, who had been Vice President under Ronald Reagan, in the White House. William Jefferson Clinton, a Democrat, won the presidential elections of 1992 and was sworn in as President of the United States in January 1993. Clinton was reelected in 1996 and served for the remainder of the decade. However, the new conservatism of Ronald Reagan and George Herbert Walker Bush was not vanquished by Clinton's election to the Presidency. The Republicans gained control of both the Senate and the House of Representatives in the 1994 elections, a position of power that the Republicans had not enjoyed for almost 40 years. In the Presidential elections of 2000, George Walker Bush, son of George Herbert Walker Bush, would defeat Al Gore and succeed Bill Clinton as President.

In mathematics education, the influence of the NCTM's publication in 1989 of *Curriculum and Evaluation Standards for School Mathematics* was arguably significant. The *Standards*, as they were called, continued NCTM's encouragement of progressive problem solving approaches and understanding in mathematics education, which was sometimes interpreted as a de-emphasis on basic skills and computation. The introduction to the Standards provided the following comments concerning their background and intended purpose:

> These standards are one facet of the mathematics education community's response to the call for reform in the teaching and learning of mathematics. They reflect, and are an extension of, the community's responses to those demands for change. Inherent in this document is a consensus that all students need to learn more, and often different, mathematics and that instruction in mathematics must be significantly revised.
>
> As a function of NCTM's leadership in current efforts to reform school mathematics, the Commission on Standards for School Mathematics was established by the Board of Directors and charged with two tasks:
>
> 1. Create a coherent vision of what it means to be mathematically literate both in a world that relies on calculators and computers to carry out mathematical procedures and in a world where mathematics is rapidly growing and is extensively being applied in diverse fields.
> 2. Create a set of standards to guide the revision of the school mathematics curriculum and its associated evaluation toward this vision.
>
> The Working Groups of the commission prepared the *Standards* in response to this charge (NCTM, 1989).

In New York, the general philosophical underpinnings of NCTM's *Curriculum and Evaluation Standards for School Mathematics* were arguably significant. Evidence for this opinion can be seen in the adoption of new requirements that students be provided with calculators during Regents mathematics examination. The collection of 1534 extant Regents mathematics examinations used in this research has no examinations for 1991 and 1992. However, the examinations of 1990 do not require that students be provided with calculators and the examinations from 1993 onward do require that students be provided with calculators. Hence, the required use of calculators during Regents examinations is shown by the extant historical record as occurring during the early 1990s, immediately following the NCTM's 1989 call for calculator use in mathematics education. By the end of the decade, the requirement of a calculator for student use during the Regents process of examination would evolve into a requirement for a scientific calculator for student use.

The *Standards* also posited that "all" students should learn more mathematics. Within the spirit of this recommendation, and a scope larger than mathematics education alone, New York State in 1996 decided to eliminate its dual diploma system and move to a single diploma system, with quality control over all diplomas to be safeguarded by the Regents examination system. This move toward egalitarianism in New York's secondary school mathematics curricula would thus be implemented throughout the state of New York within a larger plan that promoted egalitarianism in all core subject areas, which were defined as English, global history, mathematics, science, and United States history. Interestingly, this decision in New York occurred before the No Child Left Behind (NCLB) Act of 2001 made high stakes testing a requirement for certain types of federal funding in education. Since the late 1800s, when the Regents academic examinations were first introduced, and for more than a century thereafter, Regents level mathematics curricula in the secondary schools of New York had been tied to high academic standards that were grounded in classical humanism. Thus, the 1996 decision to eliminate the dual diploma system, which had long permitted progressive educators to pursue progressive pedagogies through local option diplomas, effectively forced all children in the state of New York to sustain a traditional mathematics curriculum grounded in classical humanism before graduating from high school. Perhaps equally important, the insulation of the traditional classical humanist agenda and the Regents examination system from students and educators associated with progressivism, would be lost. All students and all educators within the state of

New York would be educated to the same standards and assessed using the same quality control system. During the next decade, as this new approach to secondary education was being implemented, and as more and more students were diverted from local option diplomas to Regents diplomas, the minimum number of raw score points necessary to sustain a traditional Regents academic examination in mathematics would plummet.

The 1990s saw a continuation of the “Math Wars,” which had last re-emerged in the 1960s, and before the 1960s in the arguments of the pioneers of progressive mathematics education during the first decades of the 20th Century. The NCTM standards were seen by some as focusing too much on problem solving and not enough on basic skills acquired though drill and practice. The 1989 Standards were derisively referred to by detractors as the “New New Math Movement” and as “fuzzy math” (Walmsley, 2007, p. 43). Within this context, New York State embarked on a bold, but somewhat short lived curricula, which was known as the Mathematics A/B Curricula.

The Mathematics A/B curricula was different from all previous curricula, in that it was a three-year, two examination program of study. All previous curricula had typically had examinations at the end of each semester of study or at the end of each annualized year of study. The Mathematics A curriculum was designed so that the most academically gifted students could complete it in one full year of study, average students could complete it in three semesters of study, and slower students could take up to two years. In some schools, academically weak students could take two years of double-period math classes every day to prepare for the Mathematics A examination. Mathematics B was then taken by students who needed additional mathematics credits for graduation, or those who sought the advanced Regents academic diploma, which required sustaining both the Mathematics A and the Mathematics B Regents examinations. Most students never completed the Mathematics B curriculum, and if they did complete the Mathematics B curriculum, they did so only for academic credits toward graduations, and they did not sit for the Mathematics B Regents examination. What is interesting about the Mathematics A/B curricula, and particularly the pacing guides associated with the Mathematics A portion of the curricula, is the idea that the curricula was specifically designed and implemented within the framework of the 1996 decision to expand the Regents examination system to all students. Thus, it was the first mathematics curricula designed by the state of New York for all students in secondary schools since before the 1906 decision to implement a dual diploma system. On this view, the pacing options associated with the Mathematics A curriculum can be interpreted as accommodations for general education students of differentiated abilities, all of whom would be expected to master the classical humanist mathematics curriculum and sustain the Regents process of examination, else they would not graduate.

In addition to the differentiated pacing options of the Mathematics A curriculum, the state of New York also implemented a new way of determining the minimum number of raw score points necessary to sustain a Regents mathematics examination. Prior to the Mathematics A curriculum, minimum passing scores were expressed in terms of a constant percentage of raw score points. In the earliest academic examinations of the late 1800s, 75% of raw score points was required to sustain an examination. This 75% standard was first eroded when students were given limited choices over which problems they would attempt during the process of examination.[4] Later, the 75% standard was reduced to a minimum of 65% for a Regents diploma and 55% for a local option diploma. When the Mathematics A curriculum was implemented, the fundamental paradigm for determining whether a student sustained an examination was changed to a curve, and conversion tables were used to convert raw score percentages to scaled scores, with the latter being recorded on student transcripts. Under this system, during the following decade, when more and more students of differentiated abilities began taking Regents mathematics examinations, the minimum raw score percents required to sustain each Mathematics A examination dropped to 43% for a Regents diploma and 32% for a local option diploma.

2000-2009

Beginning the Decade: The Research Sample from 2000

A total of 12 Regents mathematics examinations were administered in 2000, and these 12 examinations contained 494 problems. Four different curricula are observed in the research sample. These are: the Sequential Mathematics Courses I, II and III and the first observation of the Mathematics A portion of new three-year, two-examination curricula known as the Mathematics A/B curricula. The number of different mathematical topics assessed in 2000 was 154, seven of which were observed in the research sample for the first time in 2000. Thirty-five topics were observed in the research sample for the last time in 2000, but this number is a

[4] For a few years circa 1914, the minimum passing score was lowered to 60%, but soon restored to 75%.

statistical anomaly associated with the metrics of the research sample and should not be interpreted as indicating any lack of stability in the assessed curricula. (See Appendix G.)

Popular Discourses of 2000 – 2009

The arrival of the new millennium found Bill Clinton in the White House for the end of his second term as President, as well as a new NCTM publication in 2000 entitled *Principles and Standards for School Mathematics*, which generally updated the NCTM's *Curriculum and Evaluation Standards for School Mathematics* published in 1989. Walmsley reports that the "*Principles and Standards for School Mathematics…*stressed basic skills and computational skills more than …the previous *Standards*" (Walmsley, 2007, p. 39).

In 2001, George Walker Bush was inaugurated as the 43rd President of the United States. His father, George H.W. Bush, was the 41st President of the United States, as well as Vice President of the United States under Ronald Reagan. During his first year in office, and before the events of September 11, 2001, George W. Bush's administration successfully maneuvered the No Child Left Behind (NCLB) Act through Congress. This major legislative initiative was approved by the House of Representatives just four months after President Bush was sworn into office, and the NCLB Act arguably reflects the culmination of what was probably a long series of events that occurred prior to the Presidential elections of November 2000.

In the United States, a defining event of the 2000-2009 decade was September 11, 2001, commonly referred to as 9/11. On that day, Al Queda terrorists attacked the twin towers in New York City and the Pentagon in Washington, D.C., using passenger airlines with innocents aboard as tools of terror. These attacks led to the "War on Terror," in which the United States became engaged in ground wars in both Iraq and Afghanistan, which continued throughout the remainder of the decade. Unlike World Wars I and II, these wars are not reflected in the evoking contexts of mathematics assessment in the Regents examination system, suggesting that the wars in Afghanistan and Iraq, like the wars in Vietnam and Korea, have had relatively little impact on mathematics education in the state of New York. The Presidency of George Bush, who was reading a book to schoolchildren in Florida when he learned of the terrorist attacks in New York City and Washington, DC, was forever altered. Coincidentally, Rod Paige was with President Bush promoting the NCLB agenda in Florida when the President learned that the United States was under attack. The President was reading to a class of Florida school children when he was interrupted by an aide who informed him of the attacks. The President continued the lesson and was subsequently moved to the protective safety of Air Force One.

One of the more significant events in public discourse during the 2000-2009 decade was the rise of the Internet as a new medium for discourse, and scholars at all levels from elementary schools to graduate schools must now address issues of when, how, and if information retrieved from discourse on the Internet should be used in scholarly endeavors. The safest action path in most situations throughout the first decade of the 21st Century has been to practice problem avoidance by not quoting from the Internet. However, not doing so now presents a conundrum for the current research effort. This dissertation is grounded in public discourse as well as historical mathematics assessment practices, and public discourse during the past decade has embraced the new technology of the Internet. Hence, it seems necessary and appropriate that discourses taken from the Internet must be admissible in this research effort as evidence of public discourse during the past decade. With these concerns acknowledged, and with a further reminder that the historical narratives for every decade in this synthesis constitute secondary sources of information about public discourses, we turn to Wikipedia, a first-decade-of-the-21st Century phenomenon in education, and a reminder that the internal and external validity of all sources must always be critically examined.

Wikipedia describes itself as an online encyclopedia, and posits that its articles tend to become more accurate over time, as they are constantly monitored and updated by contributors. Wikipedia's perception of time is relative, as are all perceptions, and a long time for a Wikipedia article is a very short time in the history of the Regents examination system. Wikipedia was founded in 2001. As this dissertation is written, Wikipedia has yet to survive an entire decade. The following online encyclopedia article from Wikipedia, was initiated in 2004. By generally accepted standards for historical research, it meets general criteria for the external validity of a source, i.e. there is little or no doubt that is represents a public discourse in the first decade of the 21st Century. By Wikipedia's own standards, it also meets a general criterion for internal validity, because in has withstood the test of time and is therefore more reliable. Critical readers can decide for themselves. Hyperlinks have been removed.

> The No Child Left Behind Act of 2001 (often abbreviated in print as NCLB and sometimes shortened in pronunciation to "nicklebee") is a United States Act of Congress that was originally proposed by the administration of President George W. Bush immediately after taking office. The bill, shepherded through the Senate by Senator Ted Kennedy, one of the bill's sponsors, received overwhelming bipartisan support in Congress. The House of Representatives passed the bill on May 23, 2001 (voting 384-45), and United States Senate passed it on June 14, 2001 (voting 91-8).[President Bush signed it into law on January 8, 2002.
>
> NCLB is the latest federal legislation that enacts the theories of standards-based education reform, which is based on the belief that setting high standards and establishing measurable goals can improve individual outcomes in education. The Act requires states to develop assessments in basic skills to be given to all students in certain grades, if those states are to receive federal funding for schools. The Act does not assert a national achievement standard; standards are set by each individual state (Wikipedia, 2010a).

The above Wikipedia narrative concerning the history of the NCLB is arguably an excellent representation of popular discourse in education in the year 2010. Furthermore, it points to additional discourse in another Wikipedia article about standards-based education reform. The public discourse on Wikipedia concerning standards-based education reform begins as follows:

> Education reform in the United States since the 1980s has been largely driven by the setting of academic standards for what students should know and be able to do. These standards can then be used to guide all other system components. The standards-based reform movement calls for clear, measurable standards for all school students. Rather than norm-referenced rankings, a standards-based system measures each student against the concrete standard, instead of measuring how well the student performed compared to others. Curriculum, assessments, and professional development are aligned to the standards (Wikipedia, 2010b).

Together, these two excerpts from Wikipedia articles on educational reform reflect contemporary public discourses on quality control systems associated with billions of dollars in federal aid for public education. In Chapter I of this dissertation, a theory of the genesis of the control paradigm used in the NCLB Act is presented, and is thus incorporated as a part of this synthesis. Against this public discourse stands the record of mathematics assessment practices left by the Regents examination system.

When the NCLB became law in 2001, New York State's 1996 decision to migrate to a single diploma system for all students was already being implemented. Likewise, the new Mathematics A/B curricula, with pacing flexibility for differentiated instruction and a new norm-based grading scale, was also in place. The Regents examination system, which had long been used as a quality control mechanism for public education in the state of New York, was thus well structured to meet the standards and high-stakes testing requirements of the NCLB Act. Hence, a formal link was established between the Regents examination system and federal funding of public education in the state of New York. The Regents examination system was thus subjected to new environmental pressures, even though it had not yet evolved to survive in this new environment.

In January 2004, the last Regents examination in the Three Year Sequential Mathematics Curriculum was administered. January 2004 also marked the introduction of a revised process of examination for the Math A curriculum, which became a political spectacle in June 2003 when approximately two-thirds of the school children who took the examination failed it. No Math A examination was administered in August 2003, and the revised examination that emerged in January 2004 reflected a 50% increase in the number of multiple choice questions, a significant decrease in the number of open-end questions, and a new conversion table with significantly lower thresholds for sustaining the examination, which was being phased-in as a high school graduation requirement for increasing numbers of students.

Contemporaneous with the decision to revise the Mathematics A process of examination, a separate decision was made to altogether abandon the relatively new Mathematics A/B curricula and to return to a more traditional three-year, three examination mathematics curricula, which would be known in year one as Integrated Algebra, in year two as Geometry, and in year three as Algebra 2 and Trigonometry. This move toward a more traditional paradigm for mathematics curricula and assessment arguably moved the curriculum and standards for secondary school mathematics education in New York into a close alignment with the NCTM's *Principles and Standards for School Mathematics.* Testing in the new curricula began with the Regents examination in Integrated Algebra in June 2008 and the Regents examination in Geometry in June 2009. The first Regents examination in Algebra 2 and Trigonometry is scheduled for June 2010, after this dissertation is finalized.

The Regents examination system was born in a perceived need of the state to regulate a mostly private academy system of secondary schools recognized by the Board of Regents in the state of New York. The high-stakes testing requirements associated with NCLB were born of a perceived need of the federal government to regulate public schools and what the neo-conservative architects of NCLB had hoped would be a growing number of charter schools in the states. Both the state of New York and the federal government tied regulatory control systems involving high stakes testing to school funding formulas. However, the New York Regents examination system, throughout its history, had only been used to measure student achievement for academically elite students, typically from the middle class. In New York, the assessed mathematics curricula of the Regents examination system had always been grounded in academic standards associated with classical humanism. Students interested or forced into progressive education tracks that were not associated with this classical humanist agenda were not subjected to the Regents process of examination, and thus were assessed against different standards, which often used different and more progressive assessment paradigms. Under these conditions, in June 2003, the Regents Mathematics A examination was administered to secondary students throughout the state of New York, including many students who were not academically elite, and who could have graduated from high school without taking any Regents examinations at all if they had been born between one decade and one century earlier.

When the non-academically elite began taking the Regents examinations, the average number of raw score points necessary to pass the examination went down, and traditional pedagogical practices associated with the classical humanist standards of the Regents mathematics curricula were highlighted against the more progressive pedagogical practices advocated by the NCTM and others. Internet discussion boards and list-serves were flooded with lamentations and wailings over the loss of value associated with the Regents diploma, which for over a century had been a hallmark of the academic diploma and a classical humanist education, but was now being offered to students of markedly lower academic abilities. The Regents diploma is in 2010, after all is said and done, quickly becoming the only game in town. It is tempting to look at how these events will shape the course and direction of the national debate that will occur over the next several months over the renewal of the NCLB Act, however, such speculation about the future is beyond the scope of the research agenda set out for this dissertation, and is thus left for another day.

In the Presidential elections of 2008, President Barack Obama was elected, thus ending an American dynasty in which a member of the Bush family was in the White House as President or Vice President for 16 of the preceding 24 years. During upcoming months of 2010, the second year of the Obama administration, the NCLB Act will be debated and a new federal policy on public education will almost certainly be enacted. Lobbying efforts and political debate will almost certainly be focused on proposed changes in curricula standards. Likewise, another focus will be on high stakes testing as a control paradigm for the regulation of public education. Within the context of this new federal legislative environment, the Regents examination system has arrived at a crossroad. It has evolved during the first decade of the 21st Century into a tool for demonstrating compliance with federal regulation, which means that it has also evolved from a regulatory system for academically elite students into a regulatory system for all students. Similarly, it has evolved into a quality control system for public education that is aligned with the curricula classification schema recommended by the NCTM for all students (NCTM 2000). These facts notwithstanding, the basic Regents diploma is no longer associated with students of average and above average academic abilities. Rather, the

Regents examination system is now associated with the lowest commencement level graduation standards permitted by the state of New York in publically funded secondary schools.

2009

The Current Position: The Research Sample from 2009

A total of nine Regents mathematics examinations were administered in 2009, and these nine examinations contained 333 problems from four different curricula. These included: the Mathematics A/B curricula and the new Integrated Algebra and Geometry curricula. The total number of different mathematical topics assessed in 2009 was 148, and nine of these topics were assessed for the first time. Because 2009 was the last year of the research sample, all 148 topics assessed in 2009 were last observed in the research sample that year. This reinforces the general idea that caution should be exercised when making inferences based solely on metrics associated with a single decade in the research sample. This shortcoming noted, the research sample appears reliable and stable whenever two or more decades are used as a basis for inferences. (See Appendix G.)

A Summary of the Different Eras of Mathematics Assessment Practices in New York State

The history of the Regents examination system as a control system over mathematics education in the state of New York can be divided into numerous different eras, some of which overlap one another. Some of the more significant eras and dates associated with them are:

1. The Era of the Preliminary Examinations (1866-1959);
2. The Era of the Academic Examinations (1878-2010);
3. The First Era of a Single Diploma System (1878-1906);
4. The Era of the Dual Diploma System (1906-c.1996);
5. The Second Era of a Single Diploma System (c.1996-present);
6. The Scoring Era of 75% Minimums with No Choice (c.1866-c.1906)
7. The Scoring Era of 75% Minimums with Choice (c.1906-c.1950)
8. The Scoring Era of 65% Minimums with Choice (c.1960-2004)
9. The Modern Scoring Era of 34% Thresholds (c.2008-Present)
10. The Era When Regents Examinations were for Academically Elite Students (1866-2002)
11. The Era When Regents Examinations were for All Students (2003-Present)
12. The Era of Slide Rules and Reference Tables (1866-1990)
13. The Era of Electronic Calculators (2000-Present)

Throughout each of this incomplete list of eras, the research sample suggests that assessed Regents mathematics curricula at the secondary school level have remained grounded in a classical humanist agenda. The research sample also reflects a general decade-to-decade stability in the topics that are assessed, with incremental change during each decade. The examinations of long ago and the examinations of today are quite similar, as are the examinations administered during the intervening years. This observation suggests that the ongoing struggle between progressive and traditional forces for control of mathematics education in the state of New York has not penetrated the curriculum and assessment practices of the Regents examination system. Rather, the Regents examination system has historically insulated the classical humanist agenda from the agendas of more progressive educators.

A Summary of Progressive versus Traditional Approaches to Mathematics Education

During the last century, progressive educators and traditional educators have differed in their approaches for dealing with educational issues. Using the classification schema of humanism, child development, social efficiency, and social meliorism, as espoused and explained by Kliebard in The Struggle for the American Curriculum, progressives have generally focused on solutions associated with child development and social meliorist-driven pedagogies, while traditionals have generally focused on solutions associated with humanist and social efficiency-driven pedagogies (Kliebard, 1995). Child development advocates believe the child’s interests and developmental considerations should drive curriculum making decisions while social meliorists advocate the position that schools should address the problems of society and strive to make society better. Classical humanism is associated with traditional approaches to both subject matter and teaching methods in curricula while social efficiency is associated with the idea that scientific methods and management principles should be applied to the field of education, much as they are applied in business and industry. Progressives, in the spirit of John Dewey, have generally viewed schools as places where the needs of the child and society can both be addressed. Programs such as Head Start, federally subsidized free-lunch programs, and in-school nurses and healthcare are illustrative of progressive solutions to urban problems that have become standard features of today's urban schools. Traditionalists, on the other hand, have long been associated with classical humanism and, in New York, they long ago adopted a high stakes testing as a control paradigm. This control paradigm is now associated with the social efficiency movement, but has long been associated with the classical humanist agenda in New York.

Although the Progressive Education Association attempted to convert classically organized academic subjects into new curricula organized around the daily lives of students (Ravitch, 2000, p.275), they were generally unsuccessful in changing the Regents level mathematics curricula as evidenced by the stability of assessed topics in the research sample. Inherent in the progressive agenda for revising classically organized curricula is the progressive belief that every child deserves an education fitted to his or her particular needs. In later writings which arguably support traditional pedagogies and structures, Ravitch suggests that traditionals are also driven by the idea of a quality education for all. Ravitch writes,

> In the early decades of the century, progressives had derided the knowledge taught in school as useless or aristocratic; late-twentieth-century critics called it arbitrary or trivial. The counter-argument, however, remains valid: Knowledge is power, and those who have it control the debate and ultimately control the levers of power in society. A democratic system of education, as Lester Frank Ward wrote a century earlier, disseminates knowledge as broadly as possible throughout society" (Ravitch, 2000, p.451).

On this view, both traditionals and progressives are driven by the same idea that every child deserves a good education. Traditionalism, with its belief that knowledge is power, seems to have broad appeal across all social-economic classes, and humanist pedagogies are sometimes preferred over child development and social meliorist pedagogies specifically designed to address the problems of disadvantaged groups (Cuban, 1993). The experience of progressive educator Caroline Pratt, founder of City and Country School in Manhattan, illustrates this point. Pratt's early emphasis on educating immigrants and the poor quickly changed, not because of any change in Pratt's social consciousness, but rather, because poor families were unwilling to send their children to a school where fundamentals (reading, writing, and arithmetic) were embedded in progressive "play" activities and not explicit. Pratt's poor and lower class students wanted a more traditional, less progressive education, and the poor and lower class students in her school were quickly replaced by children of the intelligentsia of Greenwich Village (Semel & Sadovnik, 1999, Ch.5).

Inherent to the problem of defining exactly what constitutes good education and good curriculum is whether one holds a traditional or a progressive worldview, and praxis, not rhetoric, is the key to differentiating traditionalists from progressives. The Regents examination system, and the Regents curricula with which it is associated, reflect a traditional worldview that has survived without significant change over a span of 14 decades, arguably because both have been insulated from the rising chorus of progressive educators during the early years of the 20th Century. This insulation, however, appears to be eroding at the beginning of the 21st Century.

Locus of Control

The history of the Regents examination system reflects important decisions and accommodations between competing interest groups for control of mathematics curricula and assessment practices in public schools. Prior to the Civil War, the state of New York did not seek direct control of curricula and assessment practices in the public schools of New York. When the typically private academies of New York were perceived as abusing state funding by lowering academic standards during the midst of the Civil War, the State moved to establish more rigorous regulatory control over both curricula and assessment practices. Thus were born the Regents preliminary examinations. Since the advent of the Regents academic examinations in 1878, the state of New York has controlled the classical humanist agenda in mathematics education and given it preference over more progressive agenda through state sponsored academic credentials known as Regents diplomas. When progressive voices called for alternative approaches in public education, the state of New York ceded control over progressive education practices to local schools and school districts by creating local option diplomas. Thus the era of the dual diploma system came into being that would last for approximately 100 years. During the last decade of the 20th century, as conservative and progressive voices both argued for more and better mathematics education for all students, the state eliminated the local option diploma that was associated with progressive education practices and once again required all students to submit to state control and pursue a Regents curriculum. This second era of a single diploma system was significantly different than the first era, which ended in 1906, because the very nature of public schools and the students who attend them had changed during the century of the dual diploma system, which featured shared state and local control over curricula and assessment practices in public education. As the first decade of the 21st Century ends, with local control almost gone from the curricula and assessment practices of New York's public schools, the Regents examination system continues to assess a classical humanist mathematics curriculum which is not significantly different than curricula of decades long past. What is being assessed has not changed. Who is being assessed has changed.

Egalitarianism and Standards Erosion

As more and more students attended high school and participated in the process of examination known as the Regents examination system, the minimum percentage of raw score points required to sustain an examination has declined. The general decline in standards can easily be observed when comparisons are made over long periods of time. For example, in 1878 every student had to obtain 75% of all possible points on an examination to pass it and qualify for a Regents diploma. During the most recent administration of the Regents Integrated Algebra in 2010, the minimum percent of raw score points necessary to sustain the examination and qualify for a Regents diploma was 34%. Most of this deterioration in minimum percent of raw score points necessary to sustain a Regents academic examination has occurred since the 1996 decision to revert to a single diploma system, providing strong evidence that the movement toward egalitarianism in education and the lowering of standards are related phenomena.

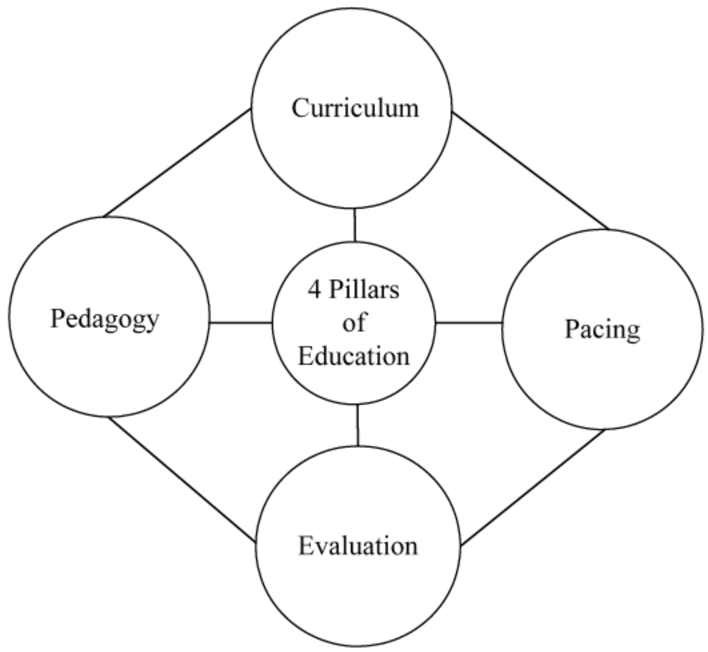
CHAPTER IV – A LONGITUDINAL ANALYSIS OF STABILITY, CHANGE AND EDUCATIONAL TRANSMISSIONS

Overview

This chapter uses Basil Bernstein's theory of educational transmissions, credentials theory, and the synthesized history of the Regents examination system (from Chapter III), in order to respond to the following research question: How has the classification and framing of assessed knowledge in the core subject area of mathematics changed in Regents level examinations administered in the public schools of New York since 1866?

An Introduction to Basil Bernstein's Theory of Educational Transmissions

Theorists of the sociology of education can be broadly classified into two groups. The first group, the phenomenologists, views education as a social process that occurs primarily in the interactions of students and teachers. The second group is more concerned with the relationships between micro-level processes of education: such as curriculum, pedagogy, and evaluation; and macro-level structures of society: such as the economy, politics, and the division of labor. The latter are the structuralists. While not in opposition to the phenomenologists, the projects of the structuralists generally attempt to show how education relates to, is influenced by, and sometimes serves other social structures (Sadovnik, 1991). Sociologist Basil Bernstein presented a structuralist view of education when he posited that there are four pillars of public education, these being: curriculum; pedagogy; evaluation and pacing.



Bernstein's Four Pillars of Education Figure 4-1

In explaining the first three of these pillars, Bernstein wrote,

> Formal educational knowledge can be considered through three message systems: curriculum, pedagogy, and evaluation. Curriculum defines what counts as valid knowledge, pedagogy defines what counts as valid transmission of knowledge, and evaluation defines what counts as a valid realization of this knowledge on the part of the taught (Bernstein 1977. P. 85).

The fourth pillar, which is pacing, might also be viewed as an aspect of pedagogy (framing), but it is sufficiently important to warrant its own status in a graphic representation of Bernstein's ideas. Bernstein notes that

> …frames may be examined at a number of levels and the strength can vary as between the levels of selection, organization, pacing and timing of the knowledge transmitted in the pedagogical relationship (Bernstein 1977, P. 89).

In this study, Bernstein's theory of educational transmissions provides numerous lenses, through which can be examined the micro-level practices of schools over a span of 144 years. To better understand why Bernstein described each element of the four pillars of education as a message system, we briefly turn to a discussion of his concept of codes.

Codes

Bernstein believed that "… a code is a regulative principle, tacitly acquired, which selects and integrates relevant meanings, forms of realizations, and evoking contexts" (Bernstein, 1990, p101). The tacit acquisition of codes derives from their socio-linguistic origins, in which cultural ways of knowing are passed via linguistic structures through social interactions with families and social identity groups, such as schools (Bernstein, 1971, pp.173-174). Bernstein used his theory of codes to focus attention on the interaction of school codes with the codes of different social classes, and thus to better understand how some individuals are privileged and others disadvantaged as a result of the ways that schools organize, transmit, and evaluate knowledge.

Bernstein's theory of codes involves two types of codes, restricted and elaborated, which are associated with linguistic interactions and personal experience. With restricted codes, "The speech is epitomized by a low-level and limiting syntactic organization and there is little motivation or orientation towards increasing vocabulary" (Bernstein 1971, p157). Elaborated codes provide greater support for both syntactic organization and increased vocabulary. Bernstein posits that

> …elaborated codes orient their users toward universalistic meanings, whereas restricted codes orient, sensitize, their users to particularistic meaning…. Elaborated codes are less tied to a given or local structure and thus contain the potentiality of change in principles. In the case of elaborated codes the speech may be freed from its evoking social structure and it can take on an autonomy...Where codes are elaborated, the socialized has more access to the grounds of his own socialization, and so can enter into a reflexive relationship to the social order he has taken over…..One of the effects of the class system is to limit access to elaborated codes (Bernstein 1971, p.200).

The above quote is particularly important to understanding Bernstein's general position relative to educational transmissions, and underscores the idea that different social classes tend to use different linguistic codes.

Bernstein posits that elaborated codes are more typical of middle class communications and that restricted codes, which are more likely to be embedded in evoking social structures, are more typical of lower class communications. He draws attention to the elaborated codes of schools and their inter-relationships with social class, and argues that the interaction of class codes with institutionalized school codes tends to advantage the middle class and disadvantage the working class. For example, the following question appeared on the January 2001 Mathematics A Regents examination.

> 2001_01_MA_01
> There are 461 students and 20 teachers taking buses on a trip to a museum. Each bus can seat a maximum of 52. What is the *least* number of buses needed for the trip?
>
> (1) 8 (3) 10
> (2) 9 (4) 11

The only correct answer, according to the scoring rubric that accompanied the examination, is choice (3), which is to say that *at least* 10 buses are needed for the trip to the museum. Implicit assumptions that appear to underlie the correct answer choice include: 1) there are no student absences on the day of the trip to the museum; 2) there are no other schools or classes sharing the buses for the trip to the museum; 3) there are no small children that might crowd three or more into seats designed for two, thus increasing the carrying capacity of the buses; 4) there are no handicapped persons in wheelchairs who might reduce the carrying capacities of the buses; and 5) every bus, except the last one, is filled to the specified carrying capacity. Bernstein's theory would suggest that these assumptions are part of an elaborated and autonomous code associated with mathematics tests, and are free from restricted social contexts or experiences associated with bus travel. Any of these five assumptions in the elaborated and autonomous code might be questioned and altered by an experienced person with responsibility for ordering buses, and any of the children answering this problem are likely to have had lived experiences with buses that negate one or more of these assumptions. The elaborated code of the Regents mathematics curriculum, however, suggests that any such real-life lived experiences with buses are irrelevant to the problem. What appears at first look to be a realistic context for a mathematics problem is, upon further analysis, merely a façade for determining if a student understands an implicitly coded message to first add 461 and 20 to get 481, then divide 481 by 52, and then round up the resultant 9.307692308 to the nearest integer, which is 10. Any other realities are unnecessary and are scored as wrong.

An example of what Bernstein might consider an elaborated code "…freed from its evoking social structure…" is a mathematical algorithm used to solve a class of Regents mathematics problems, which are upon reflection, shown to be independent of their evoking social structures. In the following set of problems, which can all be solved using standard algorithms for rate, time and distance problems, numerous evoking social structures are used to elicit an elaborated code response. One can imagine that this problem set might disadvantage students whose code orientations are focused more toward the evoking social structures of the problems and privilege those students whose code orientations are toward the more elaborated codes of the algorithms that underlie these problems. Bernstein posited that code orientations were class-related.

1880_02_AR_02 Rate, Time and Distance
Two men started from different places, distant 189 miles, and traveled toward each other; one goes 4 miles, and the other 5 miles an hour; in how many hours will they meet?

1930_06_EA_24 Rate, Time and Distance
Two towns, M and N, are 200 miles apart. A truck leaves M for N at the same time that an automobile leaves N for M. The truck averages 16 miles an hour, the automobile 24 miles an hour. How far from M will they meet? [8,2]

2000_01_MA_27 Rate, Time and Distance
A truck traveling at a constant rate of 45 miles per hour leaves Albany. One hour later a car traveling at a constant rate of 60 miles per hour also leaves Albany traveling in the same direction on the same highway. How long will it take for the car to catch up to the truck, if both vehicles continue in the same direction on the highway?

1920_09_IN_10 Rate, Time and Distance
A boatman trying to row up a river drifted back at the rate of 2 miles per hour, but when rowing down the river his rate was 12½ miles per hour; find the rate of the current.

1930_08_AA_25 Rate, Time and Distance
A man can row 24 miles down a river in one hour less time than he requires to row 12 miles down and back; he can row 12 miles down and back in exactly the same time he needs to row 20 miles upstream. Find his rate of rowing in still water and the rate of the current. [7,3]

1940_06_IN_35 Rate, Time and Distance
Two points move at different but constant rates along a circle whose circumference is 150 feet. Starting at the same time and from the same point, when they move in opposite directions they coincide every 5 seconds; when they move in the same direction they coincide every 25 seconds. Find their rates in feet per second. [10]

2000_08_MA_19 Rate, Time and Distance
A girl can ski down a hill five times as fast as she can climb up the same hill. If she can climb up the hill and ski down in a total of 9 minutes, how many minutes does it take her to climb up the hill?

1) 1.8
2) 4.5
3) 7.2
4) 7.5

Bernstein’s theory would suggest that lower class students with restricted linguistic codes are disadvantaged by questions such as these not because they are ignorant or less capable, but rather, because their code orientation toward evoking social structures is not aligned with the more autonomous codes of school mathematics, which are necessary to fully understand the bus problem in the way intended by the test designer. Said differently with specific reference to the bus problem, Bernstein’s code theory would suggest that the code orientations of lower class students would channel lower class student effort toward understanding the context of the bus problem whereas the code orientations of middle class students would facilitate a focus on finding algorithms and autonomous mathematical constructs, which are independent of the busing context of the problem. In the bus problem, the scoring rubric is clearly oriented towards the student who focuses more on the autonomous algorithms and mathematical constructs than on the evoking context. Similarly, students who focus on the evoking contests of the rate, time and distance problem set are presumably disadvantaged.

Algorithms and mathematical constructs are among the highest levels of abstraction routinely used by the human mind, and pure mathematicians have sometimes been caricatured as living in an abstract world devoid of real-world connections. Keith Devlin, when describing mathematics as part of the highest level of human abstraction, writes, “Mathematical objects are entirely abstract; they have no simple or direct link to the real world, other than being abstracted from the world….” (Devlin, 2000, p.121). Thus, the teaching of high school mathematics can be understood as one in which abstract algorithms and mathematical constructs are articulated as elements of an elaborated code that is autonomous of evoking contexts. On this view, the evoking context of a problem on a Regents mathematics examination is seen as relevant primarily as a means through which the student is instructed to retrieve from memory a more abstract, hence more autonomous, elaborated code of an algorithm or mathematical construct. The context of the problem is not a call for the student to use past experiences in the articulated context to solve the problem. Accordingly, one would expect to find in a study of past Regents examinations that algorithms and mathematical constructs are embedded in numerous social contexts, and that the elaborated and autonomous codes of the algorithms and mathematical constructs are independent of their evoking social structures. This situation is verified through literally scores of sets of problems found in the research sample that are characterized by specific algorithmic solutions, but elicited by significantly different evoking contexts. On this view, the research sample provides empirical evidence that Regents mathematics assessment practices have historically used various evoking contexts as stimuli for desired examinee responses involving elaborated codes.

$$\frac{\text{Stimulus}}{\text{Various Evoking Contexts}} \Rightarrow \frac{\text{Response}}{\text{Single Elaborated Code}}$$

Evoking Contexts and Elaborated Codes Figure 4-2

Figure 4-2 suggests that code theory is a useful lens through which to examine the research sample. Thus, code theory offers a plausible lens for understanding how assessment practices in mathematics education might result in social stratification.

Constructivism, Codes and Pedagogies

Catherine Fosnot is an exemplar of a progressive educator and author who advocates the framing of mathematics curricula through constructivist learning theory. In so doing, she embraces a commonly held belief amongst professors of mathematics education that all learning builds upon the child's pre-existing knowledge and understandings, and she advocates classroom environments that encourage children to build upon their own knowledge and experiences when solving mathematics problems (Fosnot, 2005). Constructivism, despite being a learning theory, is widely associated with the pedagogical practices of inquiry and discovery based instruction, in which the evoking contexts of stimuli are used to elicit pre-existing knowledge of students. Support for such progressive pedagogical practices is also found in various publications of the National Council of Teachers of Mathematics and in various textbooks used in mathematics education methods courses in colleges and universities. Exemplars of textbooks advocating such progressive pedagogies in mathematics education may be found in John Van de Walle's popular methods textbook, *Elementary and Middle School Mathematics: Teaching Developmentally*, and in Key Curriculum Press's high school mathematics textbooks entitled, *Discovering Algebra*, *Discovering Geometry*, and *Discovering Advanced Algebra*. All are grounded in the more widely know learning theories of Jean Piaget and William Bruner. John Van de Walle posits four guiding principles as foundations for his progressive and developmental mathematics pedagogy. These four principles are:

1. Children construct their own knowledge and understanding; we cannot transmit ideas to passive learners.
2. Knowledge and understanding are unique for each learner.
3. Reflective thinking is the single most important ingredient in effective learning.
4. Effective teaching is a child-centered activity (Van de Walle, 2004, pp. 331-32).

Despite the fact that inquiry and discovery based teaching methods are widely embraced in colleges and universities that prepare future mathematics teachers for their roles as reproducers of school knowledge, Larry Cuban found that mathematics education in the secondary schools of New York is largely expository in nature and rarely lives up to the ideal pedagogies advocated by progressive educators (Cuban, 1993).

The validity and merit of constructivist ideologies and beliefs, and their associated pedagogical practices, notwithstanding, item analyses of problems from historic Regents mathematics examinations suggest that teachers are encouraged by the structure and problems of the Regents examination system to avoid emphasizing real life experiences of students with the evoking contexts of an examination problem as a basis for problem solving, and to instead focus on student understanding of what Basil Bernstein would describe as the elaborated codes for the kind of academic mathematics taught in schools and measured by Regents mathematics examinations. From this view, teachers preparing students for examination would be encouraged by the examination itself to teach students to focus more on rapidly identifying and executing an underlying algorithm or mathematical concept called into relevance by evoking the context of the problem. In regards to the examination itself, the context and any lived experiences of the student are secondary in importance to the correct identification and execution of the intended algorithm or mathematical concept. Such is the elaborated code of school mathematics.

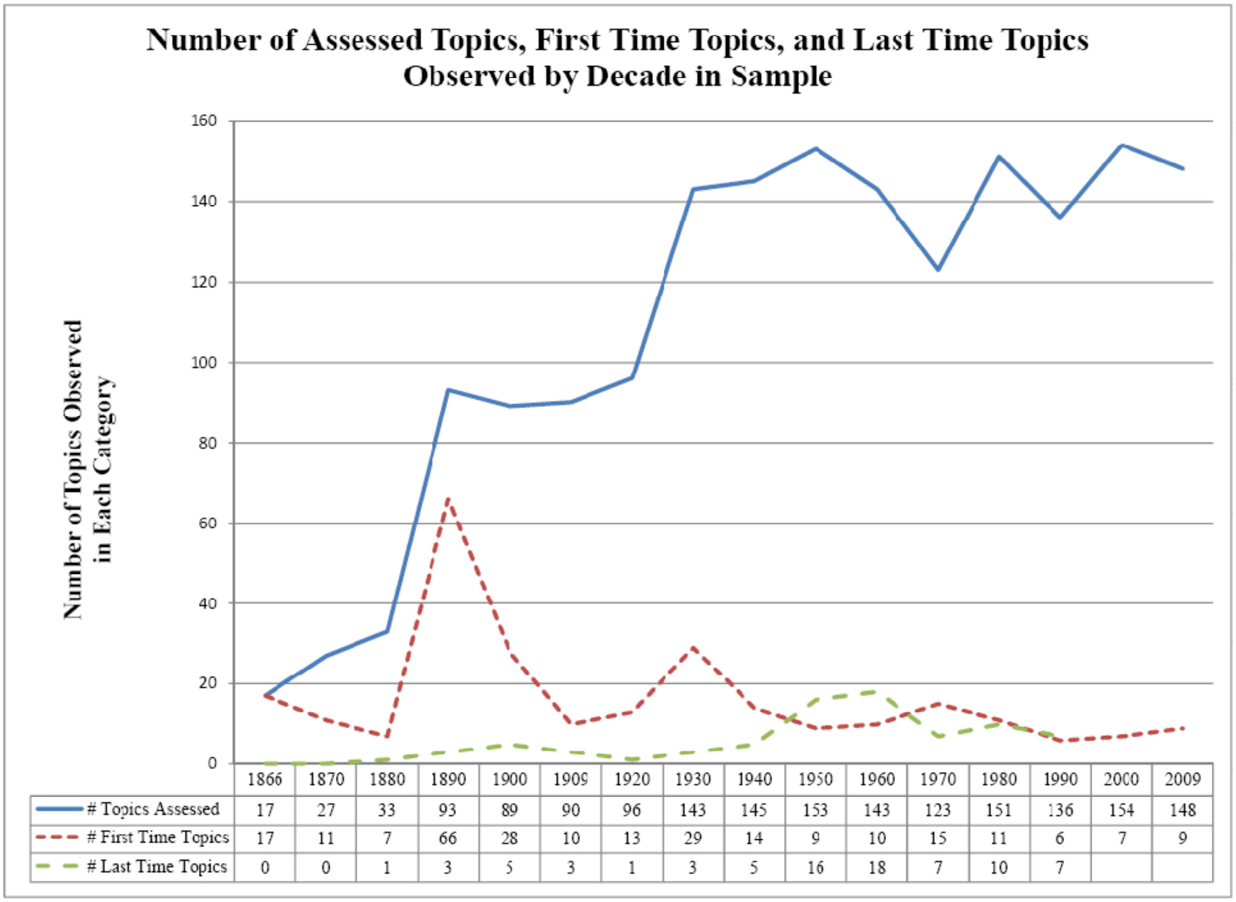
Successful schools and teachers, when measured by student achievement in solving problems like the previously mentioned bus problem, are those whose pedagogies transmit the elaborated codes necessary to decipher and understand what the test maker wants. Hence, the examination process is inextricably linked to the pedagogical process, and when the examination is not grounded in realities and lived experiences of students, the pedagogical practices of teachers are pushed away from such realities and lived experiences, and pulled towards the teaching of elaborated codes, algorithms, and mathematical concepts necessary for student achievement on examinations. In this scenario, a great schism has developed between the pedagogies advocated by academia and the pedagogies practiced by teachers preparing students for high stakes examinations. This schism will be discussed in greater detail later in this paper. We turn now to an overview of the research sample as it relates to 144 years of assessed mathematics curricula.

Overview of Changes Observed in the Research Sample

Analysis of data from the research sample suggests that curricular reform in mathematics education in the public schools of New York since the Civil War, as measured by changes in assessment practices in the Regents examination system, is metaphorically comparable to pouring old wine into new glasses. Despite numerous curricula changes over a span of 14 decades, there has been relatively little change from decade to decade in assessment practices and the mathematical topics that are assessed. The cumulative total of all incremental changes over the decades has never moved the Regents mathematics curricula away from its early fundamental grounding in classical humanism. Notwithstanding the general theme that old wine continues to be poured from new bottles, there are subtle nuances associated with the different vintages of the wines that have been served. What follows is a deeper look at the mathematics assessment practices as seen through analysis of the research sample.

The Assessed Topics Census

Figure 4-3 is a graph that summarizes a census of decade-by-decade observations of 264 different mathematical topics in the research sample. It is important to understand that the taxonomy associated with these 264 topics reflects a subjective classification system developed from the historical record, and is thus heavily biased by the experiences of the researcher and the objectives of the current research effort. These points notwithstanding, the total count of 264 different mathematical topics assessed over a span of 144 years is a starting point for interpreting the synthesized historical record. Throughout the fourteen decades of examinations reflected in the research sample, the maximum number of topics assessed in any given year was never more than 153. Additionally, it should be noted that the number of mathematical topics in the curricula during any given year was always greater than the number of mathematical topics assessed during the same year. Nothing has been found in the historical record of the Regents examination system to suggest that any attempt was ever made to assess every topic in the curricula in a given year. Said differently, the Regents examination system, throughout its history, has always assessed each year only a subset of all topics in the curricula.



	1866	1870	1880	1890	1900	1909	1920	1930	1940	1950	1960	1970	1980	1990	2000	2009
# Topics Assessed	17	27	33	93	89	90	96	143	145	153	143	123	151	136	154	148
# First Time Topics	17	11	7	66	28	10	13	29	14	9	10	15	11	6	7	9
# Last Time Topics	0	0	1	3	5	3	1	3	5	16	18	7	10	7		

Number of Assessed Topics, First Time Topics, and Last Time Topics Observed in the Research Sample Figure 4-3

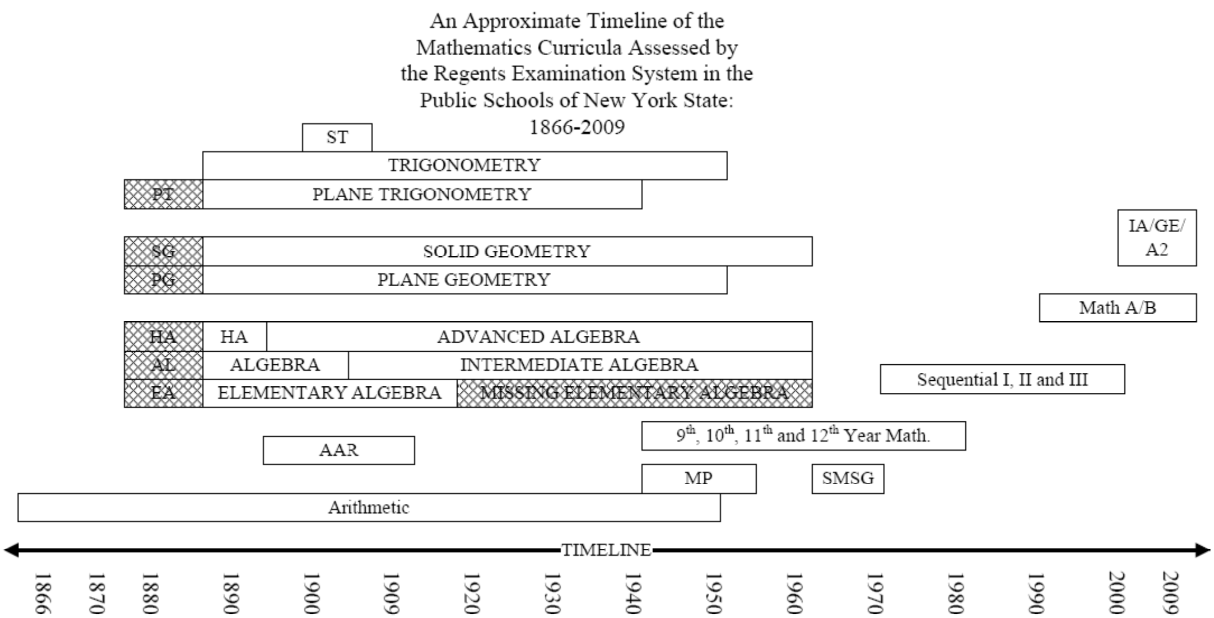
Figure 4-3 summarizes descriptive statistics taken from Appendix F, which is entitled, *A Longitudinal Census of Observed Topics in the Mathematics Curricula of the Public Schools of New York State: 1866-2009*. The methodology and rationale underlying this census are comparable to the methodology and rationale for the National Audubon Society bird counts, except that instead of enumerating bird species observed in a given year, the census behind the chart enumerates observations of different mathematical assessment topics in different calendar years. Many things can be learned from a general census of this type, including: 1) how many different assessment topics were observed in a given calendar year; 2) how many assessment topics observed in a given year were observed for the first time ever that year; and 3) how many of the different assessment topics observed in a given year were observed for the last time ever that year. Appendix G refines the data from the census and shows three lists for each year of examinations in the research sample. These three lists are: 1) a list of all topics assessed that year; 2) a list of all topics observed in the research sample for the first time that year; and 3) a list of all topics observed in the research sample for the last time that year. The counts of topics in each list are represented in the three charted lines of Figure 4-3. Numerous inferences about curriculum standards and assessment practices in years past can be made using the information in the census of observed topics and these lists.

Figure 4-3 can also be interpreted as reflecting three general eras in the history of Regents mathematics assessment practices. The first era begins in 1866 and continues through 1880. It is the era when the research sample was associated only with the Arithmetic curriculum, and there were relatively few topics associated with this single curriculum. The second era actually began in 1878, but is not reflected in the research sample, and hence the chart, until 1890. This second era began with the introduction of the Regents academic examinations in the secondary schools. The impact of the academic examinations is first observed in the research sample in 1890, when both the total number of all assessed topics increases and the number of new assessment topics increases. This upward trend in both metrics was accentuated by the fact that there were more topics assessed in the secondary curricula than in the elementary school Arithmetic curriculum, and can be interpreted as providing evidence that the pacing of the curricula increased for elite students in the academic classes of academies and high schools who were taking these examinations, relative to the pacing of the elementary school Arithmetic curriculum. Following the introduction of the academic examinations, the number of assessed mathematics topics stayed relatively constant for thirty years. Then, sometime during the 1920s, the Regents examination system fundamentally changed, and the number of topics increased again, this time to its present level of around 140 to 150 assessed topics per year.

Throughout the history of the Regents examination system, the addition and deletion of assessed mathematical topics from the Regents examination system has typically been a relatively slow and incremental process. If one looks at the broken lines near the bottom of the chart, it can be seen that the number of topics going into the curricula and the number of topics going out of the curricula have remained pretty much in balance over the 144 years of the research sample, except of course for the upward trend in “last time topics” observed during the last ten years. The upward spike in this metric during calendar years 2000 and 2009 is a function of the way the metric is defined, and not representative of a general trend beginning in calendar year 2000. Therefore, the upward trend in last observations for calendar years 2000 and 2009 are not shown in figure 4-3. Based on a cursory analysis of the numbers of topics assessed, and the number of topics added to or deleted from the assessed curricula, it can be argued that Regents level mathematics curricula have been very stable for a long time. We now move from the level of assessed topics to the level of assessed curricula.

Figure 4-4, which appears on the next page, is a timeline with the names and dates of the different curricula in the research sample. Figure 4-4 suggests more change than Figure 4-3, but at a higher level of abstraction.

The Names of Mathematical Curricula



A Timeline of Mathematics Curricula in the Research Sample Figure 4-4

This chart, unlike figure 4-3, suggests that there have been fundamental changes in the ways that the relatively stable set of assessed mathematical topics are organized into curricula. In the immediate aftermath of World War II, the Regents examination system stopped assessing curricula with traditional names like algebra, geometry, and trigonometry. The decision to do so had occurred in the late 1930s, just prior to WWII, and an 80 year old established taxonomy for organizing mathematics curricula in the public schools of New York was abandoned in favor of a new taxonomy of curricula that purported to integrate rather than differentiate curricula.

From the outset of the Regents examination system in 1866 through the early 1940s, mathematics curricula in New York State were traditionally organized into curricula with names based on differentiated fields of mathematics. The curricula associated with these differentiated fields of mathematics had straightforward names, like arithmetic, algebra and plane geometry. The names of these curricula were typically, but not totally, descriptive of the topics taught and assessed within them. During the 1940s, World War II climaxed and then ended, and before the 1950s arrived, the names of the curricula were changed and most of the old assessment topics were rearranged in new scope and sequence arrangements. Though rearranged, the list of assessed topics didn't change much. The only new topics that appear in the research sample for the first time in 1950 are the nine assessment topics (out of a total of 264) that follow:

Calculus: Differential (1950-1960)
Distance (1950-2009)
Functional Notation (1950-2000)
Graphic Representation: Histograms and Tables (1950-2009)
Midpoint (1950-2009)
Parallel Lines: Angles Involving (1950-2009)
Probability: Mutually Exclusive Events (1950-2000)
Quadratics: Axis of Symmetry (1950-2009)
Quadratics: Difference of Perfect Squares (1950-2009)

Any significance of the changes in curriculum names notwithstanding, it appears obvious from the data that few teachers would have needed to return to school to learn new mathematical content.

After the 1950s, the names of the secondary school curricula changed, so that it was no longer possible to identify the topics associated with a curriculum by the curriculum's name. As we begin the 21st Century, the newest mathematics curricula, first assessed in 2008 through 2010, and consisting of Integrated Algebra, Geometry, and Advanced Algebra/Trigonometry, appear to reflect a qualified return to an old way of naming curricula.

When interpreted together, Figures 4-3 and 4-4 suggest that although the Regents mathematics curricula sometimes changed names, the changing of curricula names was not accompanied by significant changes in aggregate assessment practices. When curricula names changed, the Regents examination system continued to assess most of the same topics that were assessed in the previous curricula. This does not mean, however, that the topics assessed within a specific curriculum did not change. Rather, it means that when change did occur, it was often a matter of moving a given topic from one curriculum into another curriculum. Interpreted through a slightly different lens, mathematics teachers who have taught Regents level mathematics courses in the public schools of New York have been teaching the same topics for a long time, but sometimes in different sets and different sequences. Further analysis of the research sample shows that integration of curricula is a concept that is best understood as the mix of mathematical topics within specific curricula, and not: 1) the total number of topics within a specific curriculum; or 2) the amount of overlap between curricula.

Bernstein's Theories Concerning Curriculum

Curriculum, which Bernstein defined in terms of "…what counts as valid knowledge...," is associated with the classification of knowledge and the boundaries or lack of boundaries between classifications of knowledge. Traditionally defined subjects and rigid boundaries and insulation between areas of knowledge are characteristic of what Bernstein refers to as strong classification of knowledge. When strong classification of knowledge exists, learners differentiate themselves by accumulating credits in differentiated subjects, and the educational system is grounded in what Bernstein calls a collection code. Weak classification of knowledge is associated with more integrated subject areas, boundaries between subjects that are less clearly demarcated, and what Bernstein calls integrated codes, in which learners are focused more on belonging to a community of learners and less on individual differentiation through the accumulation of specific academic credits (Sadovnik, 1991). Bernstein associated traditional education with strong classification and strong framing, whereas he associated progressive education with weak classification and weak framing. When classification alone is considered, Bernstein associated strong classification with differentiated curricula and weak classification with integrated curricula.

Bernstein observed a trend in public education toward integrated curricula that is generally confirmed through cursory analysis of the research sample. During the first 13 years of Regents mathematics examinations, there was only one curriculum, Arithmetic, and it was associated with determining whether a student qualified as an “academic scholar” (Beadie, 1999a). In 1878, when the Regents examination system was expanded to include exit examinations for the academic subjects taught in the academies and the newly developing secondary schools, the secondary school mathematics curriculum was introduced as a collection code with individual examinations in such narrowly defined categories as: 1) advanced arithmetic; 2) algebra; 3) higher algebra; 4) plane geometry; 5) spherical geometry; and 6) plane trigonometry. After World War II, a trend began in secondary school mathematics to replace the traditional, rigid boundaries between subject areas with more integrated classifications. This trend began with the Ninth, Tenth, and Eleventh Grade mathematics curriculum introduced circa 1947, progressed through the Sequential I, II, and III mathematics curriculum introduced circa 1977, and culminated at the turn of the millennium with the introduction of the two exam, three year Math A/B curriculum, which was introduced in 1999. In 2009, as this paper is being written, the mathematics curriculum is reverting back to a three exam, three year sequence, with names that appear to be an attempt at reconciling the ongoing struggle between those who advocate for a traditional collection codes and those who advocate for integrated curricula.

Based on cursory analysis of the research sample, there is empirical support for Bernstein’s observation of a general trend in curriculum design toward integrated curricula and away from collection curricula. However, this trend is observed through cursory analysis only in the names of the curricula, and not in the content assessed. Bernstein's observation of a general trend away from differentiated curricula and towards integrated curricula appears validated when only curricula names are considered, as in Figure 4-4. However, the research sample allows deeper analysis of this phenomenon at the topics within curricula level. At this level, two questions arise. These are: 1) to what extent are topics in one curriculum also assessed in another contemporaneous curriculum; and 2) to what extent are curricula created using topics from multiple fields of mathematics. As noted previously, Bernstein saw differentiated curricula as being characterized by traditionally defined subjects with rigid boundaries and insulation between areas of knowledge. Bernstein saw integrated curricula as having boundaries between subjects that are less clearly demarcated. The two questions associated with our mini-investigation address these ideas of rigid boundaries and insulation between areas of knowledge.

To answer the first question, the research sample was sorted and analyzed by year, topic and curriculum. Then, each curriculum was described in terms of the different mathematics topics assessed in it. A metric was created to evaluate the degree of overlap in contemporaneous related curricula, with the hypothesis being that differentiated curricula should show higher ratios of unique topics to total topics than integrated curricula. This mini-investigation using the research sample is summarized in the Figure 4-5. The data in Figure 4-5 suggests that Bernstein's ideas about borders between curricula, when measured by the amount of overlap between curricula, is either a misinterpretation of Bernstein's thinking, or Bernstein's thinking must be mediated by other factors. If the latter is the case, two obvious candidates for these unknown mediating factors might be advances in psychometrics and accountability. Figure 4-5 was produced by taking the total number of unique topics assessed in two or more related curricula during a given year, and dividing it by the total number of topics assessed in both curricula. The result is a percentage that arguably measures how much overlap exists between the boundaries of the assessed curricula.

Measuring the Percent of Overlapping Topics Assessed in Different Curricula	
The Differentiated Curricula	The Integrated Curricula
Intermediate Algebra (80 Topics) Advanced Algebra (80 Topics) $\frac{\text{Total \# of Unique Topics}}{\text{Total Number of Topics}} = \frac{105}{160} = 66\%$ Conclusion: The boundaries between the Intermediate Algebra and Advanced Algebra curricula were loosely defined and flexible. Plane Geometry (39 Topics) Solid Geometry (19 Topics) $\frac{\text{Total \# of Unique Topics}}{\text{Total Number of Topics}} = \frac{56}{58} = 97\%$ Conclusion: The boundaries between the Plane Geometry and Solid Geometry curricula were precisely defined and inflexible. Plane Trigonometry (40 Topics) Trigonometry (35 Topics) $\frac{\text{Total \# of Unique Topics}}{\text{Total Number of Topics}} = \frac{47}{75} = 63\%$ Total # of Unique Topics – 47 Total Number of Topics – 75 % Unique to Total – 63% Conclusion: The boundaries between the Plane Trigonometry and the Trigonometry curricula were loosely defined and flexible.	Ninth Year Mathematics (55 Topics) Tenth Year Mathematics (53 Topics) Eleventh Year Mathematics (80 Topics) Twelfth Year Mathematics (58 Topics) $\frac{\text{Total \# of Unique Topics}}{\text{Total Number of Topics}} = \frac{169}{246} = 69\%$ Conclusion: The boundaries between the Ninth, Tenth, Eleventh and Twelfth Year Mathematics curricula were loosely defined and flexible. Sequential Course I in Mathematics (76 Topics) Sequential Course II in Mathematics (76 Topics) Sequential Course III in Mathematics (66 Topics) $\frac{\text{Total \# of Unique Topics}}{\text{Total Number of Topics}} = \frac{163}{218} = 75\%$ Conclusion: The boundaries between the Sequential I, II and III Courses in Mathematics curricula were more precisely defined and less flexible than in the predecessor curricula. Mathematics A (78 Topics) Mathematics B (69 Topics) $\frac{\text{Total \# of Unique Topics}}{\text{Total Number of Topics}} = \frac{133}{147} = 90\%$ Conclusion: The boundaries between the Mathematics A and Mathematics B curricula continued the trend of being more precisely defined and less flexible than in the predecessor curricula.

Overlap of Assessed Mathematics Topics in Curricula Figure 4-5

The data in this table suggest that the variation in the % Unique/Total" metric, which arguably measures the preciseness of what Bernstein referred to as boundaries between curricula, is totally mediated by other variables in curriculum or accountability systems design, and the trend during the last 50 years is toward increasingly unique and non-overlapping sets of topics. On this view, Bernstein's observation of a movement towards more integrated curricula cannot be observed through a metric that simply monitors the degree of overlap between different contemporaneous curricula, and the metric is considered inappropriate for either validating or invalidating Bernstein's theories concerning curriculum trends. What may be inferred from the metric, however, is the idea that boundaries between curricula are being more precisely defined. Whether subjects within a defined curricula are becoming more diverse and inclusive of subjects from multiple strands of mathematics is a new and different question.

The new question addresses the extent to which an individual curriculum assesses topics drawn from multiple fields of mathematics. Bernstein theory might suggest that differentiated curricula encompass assessed topics from fewer areas of mathematics, while integrated curricula encompass assessed topics from more areas of mathematics. To answer the research question, a taxonomy of extant mathematical knowledge taught in schools was compared to the taxonomy of the research sample. The taxonomy of extant mathematical knowledge taught in schools was created as an initiative of the National Science Digital Library in Mathematics using a consortium of interested parties from academia and organizations; including … "the College Board (AP Mathematics and Statistics), Eisenhower National Clearinghouse, iLumina, MAA *(Mathematical Association of America),* Math Forum, MathDL *(National Science Math Digital Library),* JOMA *(Journal of Online Mathematics and its Applications)*, MERLOT *(Multimedia Educational Resource for Learning and Online Teaching),* and NCTM *(National Council of Teachers of Mathematics)*" (Mathematics Taxonomy Committee, 2002, p.1)[5]. It was designed to

[5] *(Italics)* not in original.

encompass all known fields of mathematical knowledge taught in schools, and divides this universe of mathematical knowledge into ten general areas. (See Appendix E.)

The 264 original assessment topics associated with the 5508 individual mathematics problems in the research sample were then further encoded to include the ten major categories of the second taxonomy. This methodology allowed each curriculum in the research sample to be evaluated in terms of the distribution of different major areas of mathematics assessed in it. Figure 4-6 shows the ten areas of mathematics from the second taxonomy and the various curricula in New York State that assessed each of these major area of mathematics.

		Differentiated Curricula													
		AR	AAR	MP	BA	AL	HA	EA	IN	AA	PG	SG	PT	TR	ST
1	Numbers and Computation	+	+	+	+	+	+	+	+	+	+	+	+	+	
2	Logic and Foundations										+		+		
3	Algebra and Number Theory	+	+	+	+	+	+	+	+	+			+	+	
4	Discret Mathematics														
5	Geometry and Topology	+	+	+	+			+	+	+	+	+	+	+	+
6	Calculus									+					
7	Analysis	+		+	+	+	+	+	+	+			+		
8	Differential and Difference Equations														
9	Statistics and Probability	+		+	+		+	+	+	+					
10	Applied Mathematics	+	+	+	+	+		+	+	+					
	Total Number of Topics Assessed	6	4	6	6	4	4	6	6	7	3	2	5	3	1

		Integrated Curricula											
		NY	TY	EY	TW	S1	S2	S3	SMSG	MA	MB	IA	GE
1	Numbers and Computation	+	+	+	+	+	+	+	+	+	+	+	+
2	Logic and Foundations	+	+	+		+	+	+	+	+		+	+
3	Algebra and Number Theory	+	+	+	+	+	+	+	+	+	+	+	
4	Discret Mathematics												
5	Geometry and Topology	+	+	+	+	+	+	+	+	+	+	+	+
6	Calculus				+								
7	Analysis	+		+	+	+	+	+		+	+	+	+
8	Differential and Difference Equations												
9	Statistics and Probability	+	+		+	+	+	+		+	+	+	
10	Applied Mathematics	+										+	
	Total Number of Topics Assessed	7	5	5	6	6	6	6	4	6	5	7	4

Differentiated and Integrated Curricula Topics Figure 4-6

Analysis of the vertical columns of Figure 4-6 suggests that many of the early curricula with differentiated sounding names, which existed from 1866 through the mid-20th Century, were in fact integrated curricula, if differentiated curricula are operationally defined as curricula that assess topics from relatively restricted as opposed to relatively diverse numbers of mathematical subject areas. Of the early curricula with differentiated sounding names, only those curricula associated with the topical areas of geometry and trigonometry were relatively restricted in the total numbers of areas of mathematics assessed. All other curricula, throughout a span of 144 years, show evidence of diversity and integration of assessed topical areas. This suggests that the movement toward integrated curricula in mathematics education was primarily a function of eliminating the highly differentiated curricula associated with geometry and trigonometry. On this view, the curriculum is not getting more integrated, but it is getting less differentiated. From this perspective, and with these qualifications, the historical record clearly supports Bernstein's observations that curricula are trending toward integration. We turn our focus now to the different types of Regents examinations that exist in the historical record and the mathematics curricula with which each type of examination was associated.

The Preliminary Examinations: (1866 - 1953)

From the first Regents Arithmetic examination in 1866 through 1877, the Regents examination system had no other purpose than to regulate admission to the class of academic scholars in the secondary schools of New York. These examinations typically occurred at the end of elementary schooling or the beginning of secondary schooling, and they were designed to assess student achievement in the elementary curricula as opposed to the secondary curricula. In the core subject area of mathematics, there was only one elementary curriculum, and that curriculum was simply called Arithmetic.

The earliest examinations for the Arithmetic curriculum typically had 24 to 27 problems, and each examination assessed student knowledge in areas chosen from a list that would over time include about 52 topics. The assessed topics of this particular curriculum are important because they represent what are probably the first standards for mathematics assessment established by the state of New York.

At least 165 Regents Arithmetic examinations were administered between 1866 and 1940, inclusive. Twenty-two of these examinations, containing 469 mathematics problems, were administered during the years of 1866, 1870, 1890, 1900, 1909, 1920, 1930 and 1940, and are the basis for the following observations. Over a lifespan of at least 74 years, the Regents Arithmetic examinations included questions relating to the following topics:

Arithmetic Operations
Arithmetic: Addition
Arithmetic: Division
Arithmetic: Multiplication
Arithmetic: Numeration
Arithmetic: Place Value
Arithmetic: Subtraction
Bills and Receipts
Brokerage and Commission
Central Tendency: Averages
Circles: Area of
Circles: Center, Radius and Circumference
Conversions
Cost
Decimals
Definitions: Arithmetic
Equations and Expressions: Modeling
Equations and Expressions: Using Substitution in
Equations: Simple
Exponents: Operations with
Factors: Greatest Common
Factors: Least Common Multiples
Factors: Prime
Fraction Madness
Fractions
Fractions: Complex
Longitude
Mensuration
Notes and Interest
Numbers: Comparing Reals
Numbers: Prime and Composite
Order of Operations
Percent
Perimeter
Polygons and Circles: Inscribed
Polygons: Area of
Profit and Loss
Progressions: Arithmetic
Proportions
Radicals: N-Roots
Radicals: Square Roots
Rate
Rate, Time and Distance
Ratio
Solid Geometry: Lines and Planes in Space
Solid Geometry: Prisms and Cylinders
Special Quadrilaterals: Rectangles and Squares
Systems: Writing
Triangles: Equilateral
Triangles: Pythagoras
Valuation
Volume

All of the topics in the above list were not assessed equally. Significant emphasis was placed on consumer and business mathematics, and about 36% of all Arithmetic questions in the research sample are associated with just six of the 52 topics. These consumer and business mathematics topics include: 1) Bills and Receipts; 2) Brokerage and Commisions; 3) Costs; 4) Notes and Interest; 5) Profit and Loss; and 6) Valuation. On this view, the preliminary examinations were focused more on applied mathematics and the elementary mathematics curriculum could be considered as a generally progressive curriculum. When the Regents Arithmetic curriculum ended around 1940, these six consumer and business mathematics topics were included in the relatively short lived Preliminary Mathematics curriculum. When the Preliminary Mathematics curriculum ended around 1953, these applied mathematics topics, with the single exception of Notes and Interests, also disappeared.

Based on an examination of 1534 Regents mathematics examinations, it is a verifiable fact that consumer and business mathematics were of significant importance to the Regents examination system from the Civil War through the first half of the 20th Century. It is also a verifiable fact that consumer and business mathematics all but disappeared from the Regents examination system after approximately 1953. What is not fact is why this de-emphasis of consumer and business mathematics occurred.

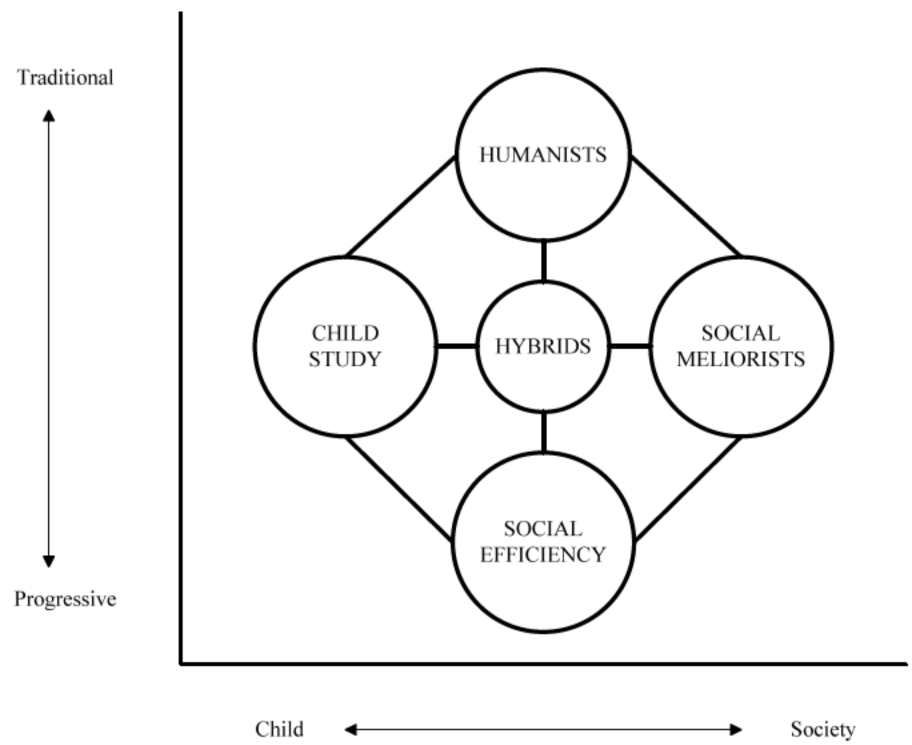
One possibility is that consumer and business mathematics topics became threatened or extinct in the Regents examination system because these topics were perceived to be elementary school topics, and the Regents examination system was evolving in the direction of quality control of academic standards in public high schools only. The old academy system of education through typically private schools was largely gone, thus obviating the need for preliminary examinations as a quality control system through which the state could regulate elementary education, and admission to and funding of the academies. The demise of the academy system was undoubtedly facilitated by the rise of public high schools, and it was this new arena of secondary education in the high schools that required the focus and resources of the Regents examination system. Under this scenario, the State Education Department did not necessarily turn its back on consumer and business mathematics education, but merely refocused its energies on the academic curricula of the secondary schools. The Regents examination system chose to leave the teaching of consumer and business mathematics to the elementary schools, which would have their own quality control methods and their own examination system. This is an entirely plausible scenario, but it is mostly conjecture. What we know for certain is that consumer and business mathematics largely went away in all curricula assessed by the Regents examination system during the decade of the 1950s.

Another plausible explanation, also mostly conjecture, is that the state decided to de-emphasize consumer and business mathematics because doing so was in the interests of one or more powerful interest groups. Under this explanation, changes in the topics covered by the Regents examination system are reflections of macro level changes in society. When the Arithmetic curriculum began in 1866, the Civil War had just ended and New York was primarily an agrarian society. The students being assessed by the early Regents examination system were typically children of middle class farmers and merchants, who presumably had needs for basic skills in consumer and business mathematics. When the Regents Arithmetic and Preliminary Mathematics examinations ended around 1959, New York was much different than it was in 1866. World War II was over. New York had acquired the accoutrements of an industrial society, and far more students from far more social classes were being educated in public high schools in almost every city, town and county in the state. On this view, it is conceivable that economic and social changes in society led to the demise of consumer and business mathematics in the Regents examination system. This scenario is consistent with Bernsteinian theory, but not proven by the current research effort. We now focus our attention on the Regents "academic" examinations, which began in 1878.

The Academic Examinations (1878 - 2009)

The academic examinations were first administered in 1878. They were created to regulate the academic curricula that occurred in the academies and public high schools of New York, as opposed to the regulatory purpose of the preliminary examinations, which was to regulate entry into a state defined class of academic scholars who were privileged under school funding formulas. In the 1870s, the curricula studied in the academic classes of the academies under the purview of the Board of Regents was much different than the curricula of the one room elementary schoolhouses across the state. The earliest academic examinations in the research sample are from 1890, and an inspection of the mathematical topics assessed in 1890 shows them to be exemplars of a classical humanist agenda. The assessment practices were likewise traditional. What was less traditional was the elementary school agenda, which was more progressive and pragmatic, with emphases on both consumer and commercial arithmetic.

Herbert Kliebard provides a framework for classifying curricula as traditional or progressive and enumerates four forces that have struggled for control of the American curriculum over the past 100 years (Kliebard, 1995). Kliebard's four forces are those of: 1) the humanists, representing traditionalism in both subject matter and teaching methods; 2) the developmentalists, representing those who believe the child's interests and developmental considerations should drive curriculum making decisions; 3) the social efficiency advocates, representing the idea that scientific methods and management principles should be applied to the field of education, much as they are applied in business and industry; and 4) the social meliorists, representing the viewpoint that schools should address the problems of society and strive to make society better. Figure 4-7 graphically organizes these forces.



Kliebard's Curricula Classification Schema Figure 4-7

Two dualities, or dialectics, are represented in this arrangement of Kliebard's four forces.

- Vertically, there is the duality of traditionalism versus progressivism.
- Horizontally, there is the duality of the child's needs versus societal or community needs.

Kliebard also posits a fifth general category, that of hybrids, which often results when one or more primary forces fuse together to form amalgamations in specific environments. This category is shown at the center of the diagram. Kliebard's schema is useful for looking at long term change in the curricula, and is particularly useful for understanding the role of the Regents examination system in New York's system of public education. Using Kleibard's classification schema as a window through which to view the Regents examination system, it can be observed that the Regents examinations system assessed more progressive, applied mathematics curricula with the preliminary examinations, and more traditional, classical humanist mathematics curricula in the secondary schools. From a Bernsteinian perspective, these differences are illustrative of the differences between the instrumental and expressive orders of elementary and secondary schools. On this view, the Regents examination system used one set of examinations to regulate a more progressive elementary curricula, and another set of examinations to regulate a more traditional classical humanist agenda.

Compulsory school attendance laws in the state of New York trace their origins to 1831, when a law was enacted requiring poor houses to provide schooling for orphans and paupers under their care. The first attempt at compulsory school attendance for the greater population was enacted in 1874, but was generally regarded as ineffective. In 1903, however, the state built an improved control system that would signal the beginning of a more effective era when compulsory school attendance laws and child-labor laws were synchronized and public health, law enforcement and education officials began cooperating to get children into the schools and out of the workplace or off the streets (Ensign, 1921). As school enrollments began to increase at faster rates during the first decade of the 20th century, the voices of progressive educators also grew.

In 1906, the Board of Regents recognized concerns about: 1) the appropriateness of the Regents classical humanist agenda for all students; and 2) a growing demand for progressive education at the secondary school level. The state responded by creating a new class of diplomas called local option diplomas. Control over curricula and assessment practices associated with these new local option diplomas was ceded back to schools and school districts. Meanwhile, the Regents examination system continued controlling the standards, measurement systems, and measurements associated with classical humanist mathematics curricula and Regents academic diplomas. In this way, mathematics education practices in the state of New York became bifurcated. The traditional mathematics education curricula continued to be controlled by the state and associated with the Regents diploma, while control over other more progressive forms of secondary education were ceded by the state back to localities. These generalizations notwithstanding, there were isolated examples of relatively progressive curricula assessed by Regents academic examinations, but these were uncommon and typically short-lived curricula. The research sample at both the curricula naming level and the topics assessed level supports a general conclusion that the mathematics curricula assessed by the Regents academic examinations have remained significantly grounded in its traditional classical humanist origins over a span of 132 years. This occurred through state control of the Regents mathematics curricula and assessment practices, which was separate from the local controls associated with local option diplomas.

Pedagogies and Pedagogical Practices

Pedagogy, as defined by Bernstein, is concerned with the framing of the curriculum and what counts as the valid transmission of knowledge. Sadovnik asserts that "…framing is related to the transmission of knowledge through pedagogic practices." Atkinson is quoted by Sadovnik to illuminate the relationship between pedagogy and curriculum. Atkinson states

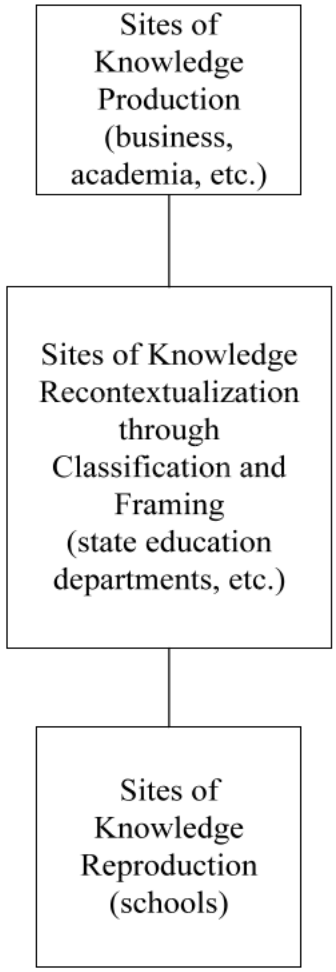
> At the heart of the "pedagogic device" is the coding of power whereby the "thinkable" is discriminated and demarcated, in a fashion which corresponds to the function of "classification." In modern, complex societies the contrast between the "sacred" and the "profane" is formally paralleled by the classificatory principles emanating from the higher reaches of the education system. The pedagogic device is a mechanism for the distribution of the "thinkable" among different social groups, for the identification of what may be thought simultaneously implies who may think it. Social order is thus equivalent to the cosmological order of legitimate categories of consciousness (Sadovnik, 1991. p. 10).

From this passage, it can be inferred that Atkinson believes that pedagogy involves not only the phenomenological activity of transmitting knowledge within teacher/student relationships, but also educational systems and procedures that frame decisions concerning who will and will not participate in the distribution and acquisition of certain forms of knowledge. On this view, both phenomenological and structural aspects of the sociology of education can be illuminated through analysis of Regents mathematics examinations over a time span that encompasses major changes in both schools and society. In doing so, issues of inclusion/exclusion, equality/inequality, and suitability/non-suitability associated with the social classes of students are theorized to influence pedagogical practices. We turn now to the structuralist view of pedagogy expressed by Atkinson, which focuses our attention on the interplay of curriculum, assessment practices and social class. In particular, we focus on Atkinson's view that the "…pedagogic device is a mechanism for the distribution of the "thinkable" among different social groups…."

In *Discourse and Reproduction, Essays in Honor of Basil Bernstein,* edited by Atkinson, Parlo Singh, a protégé of Bernstein, discusses the pedagogic device, which must be distinguished from a pedagogical practice. Singh notes that, "The pedagogic device provides the generative rules of the privileging texts of school knowledge through three inter-related rules: *distributive,* *recontextualizing, and evaluative* (Singh 2002, p 573). Singh then discusses Bernstein's concept of fields, which is conceptually similar to Pierre Bourdieu's concept of fields, as

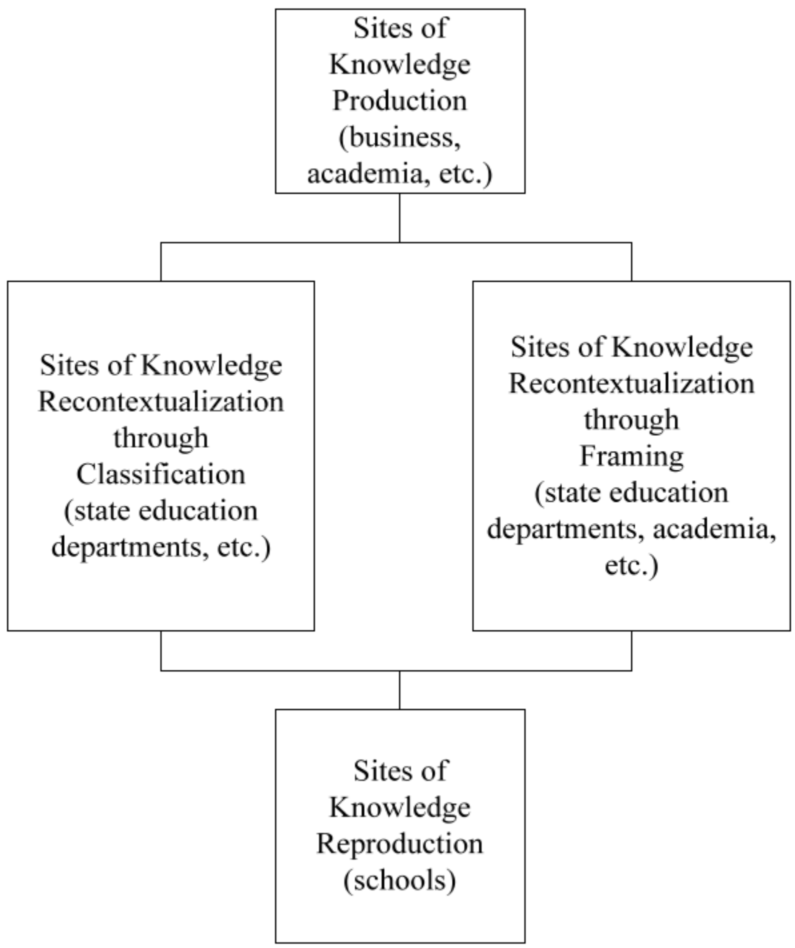
> …a social space of conflict and competition, an arena in which participants vie to establish monopoly over the species of capital effective in it…and the power to decree the hierarchy and "conversion rates" between all forms of authority in the field of power (Singh 2002. p. 573).

Singh describes the fields of the pedagogic device as: 1) fields of production of knowledge; 2) fields of recontextualization – both official and pedagogic; and 3) fields of reproduction – schooling institutions. Using Singh's view as a guide, it can be posited that: 1) the production of knowledge in the field of mathematics occurs in fields associated with academia and specialized institutions/places other than the public schools where mathematics is taught; 2) the New York Board of Regents and the New York State Education Department are separate fields where the knowledge is recontextualized and the curriculum is defined as official knowledge; and 3) the public schools where mathematics is taught are fields where the official knowledge is merely reproduced. These relationships are depicted in Figure 4-6.



Singh's Conceptualization of the Pedagogic Device Figure 4-8

Singh's conceptualization of the pedagogic device is particularly relevant to a historical study of mathematics education in New York State. When the Regents examination system began in 1866, mathematics education in New York State had not yet risen to the level of a profession, and the recontextualization of pedagogical knowledge was solely in the hands of the Board of Regents and the State Education Department. However, Singh's view requires some enhancement and/or clarification to better reflect the rise of mathematics teaching as a profession and the accompanying rise of university and college programs devoted to mathematics pedagogy. The rise of mathematics teaching as a profession, which Donoghue reports as occurring between 1890 and 1920, was accompanied by general trends of: 1) more and more students from different social classes attending schools; and 2) more and more mathematics educators being influenced by the academic philosophies of schools and universities concerning proper pedagogies (Donoghue, 2003a). This led to the current situation in New York State, in which the field associated with recontextualization of mathematics knowledge has become bifurcated between the State and academic institutions of higher learning, as shown in figure 4-9.



The Pedagogic Device As It Exists in New York State in 2010 Figure 4-9

The pedagogic device as it currently exists for mathematics education in New York State is characterized by bifurcated fields for the recontextualization of pedagogical knowledge. On the left side of the model depicted in Figure 4-9 is the State Education Department, which continues to influence mathematics pedagogy through assessment and curriculum decisions and practices that predate the rise of teaching as a profession. On the right side of Figure 4-9 are the colleges and universities, which are separate and independent from the State Education Department, that teach pedagogies and prepare new mathematics teachers for the classroom. This bifurcation of the fields for recontextualizing pedagogical knowledge has resulted in different and sometimes competing messages being transmitted to mathematics teachers concerning what pedagogical practices should be used for knowledge reproduction in public schools. These competing messages are embodied in an ongoing controversy in mathematics education that is sometimes referred to as the “math wars,” in which traditional teaching methods are pitted against more progressive inquiry and discovery based teaching methods (Willoughby, 1968). As these “math wars” continue, the Regents mathematics curricula can be understood as a set of insulated and privileged curricula and evaluation practices that are controlled by the state and used to credential students and to differentiate access to the higher realms of the educational system. (See pages 105 and 118.) We turn now to the ideas that: 1) the state recontextualizes knowledge through curriculum design and assessment practices; and 2) public schools merely reproduce the knowledge chosen by the state for reproduction.

The Regents examination system has been a state sponsored quality control system for a classical humanist agenda in mathematics education in the public schools of New York for 144 years, and as such, it is a primary means through which the state of New York exercises its governmental prerogative to recontextualize knowledge and control curricula in public schools. In this regard, it is important to note that the state of New York does not control curricula through textbook selection and approval, as for example does Texas (Apple, 1995). In New York, the fundamental control system is the Regents examination system, which establishes specific learning standards and objectives and then links them to assessment practices associated with a ritualized process of examination.

The two different taxonomies used in this research project led to two important observations about the research sample. These are: 1) certain broad areas of mathematical knowledge, notably discrete mathematics, calculus, and differential and difference equations, have never or rarely been included in the Regents level curricula of public schools in the state of New York; and 2) New York State slowly, incrementally and actively manipulates curriculum and assessment standards, thus exerting control over the micro-level practices of schools. The theories of Basil Bernstein and Parlo Singh are empirically supported by the research sample.

Evaluation

Bernstein's life project can be summarized as an attempt to connect the micro-level educational practices of schools to the macro-level structures of society. In 1977, Bernstein acknowledged the ongoing and unfinished nature of his project, and the need for empirical studies, when he wrote the following words:

> The evaluative system places an emphasis upon attaining states of knowledge rather than ways of knowing. A study of the examination questions and format, the symbolic structure of assessment, would be, from this point of view, a rewarding empirical study. Knowledge thus tends to be transmitted, particularly to elite pupils at the secondary level, through strong frames which control the selecting, organization, pacing, and timing of the knowledge. The receipt of the knowledge is not so much a right as something to be won or earned. The stronger the classification and the framing, the more the educational relationship tends to be hierarchical and ritualized, the educand seen as ignorant, with little status and few rights. These are things which one earns, rather like spurs, and are used for the purpose of encouraging and sustaining the motivation of pupils. Depending on the strength of the frames, knowledge is transmitted in a context where the teacher has maximal control or surveillance, as in hierarchical secondary school relationships (Bernstein 1977. p. 98).

The historical record of evaluation practices left by the Regents examination system over a period of 144 years provides excellent research opportunities to better understand the role of evaluation in the sociology of public education, and this research effort is guided throughout its entirety by the opportunities suggested by the above quote by Bernstein. The research sample in the current study provides opportunities to critically evaluate and empirically validate several of Bernstein's observations.

Bernstein argued that the evaluative system places an emphasis upon attaining states of knowledge rather than ways of knowing. The Regents examination system was created by legislative act in 1864 to determine if a student had attained a state of knowledge sufficient for credentialing as an "academic scholar," and thereby deserving of a privileged status under the school funding formula. The state of knowledge required in mathematics for credentialing by the state as an academic scholar was assessed using paper examinations with sets of mathematics problems. Over a span of 144 years, the Regents examination system never stopped using this paper approach to assessment. This fact alone inhibited the Regents examination system from assessing numerous other ways of knowing. Accordingly, the Regents examination system in mathematics qualifies as an evaluative system that has long placed an emphasis on attaining states of knowledge rather than ways of knowing.

For the first 135 years of its history, the Regents examination system was associated with academically elite students. Because the Regents examination system was associated with credentials in the form of certifications for sustaining examinations and with Regents academic diplomas, the Regents examination system became a hallmark of distinction for academically elite secondary school students in the state of New York. This coincides nicely with Bernstein's theory that knowledge tends to be transmitted to elite pupils at the secondary level through strong frames, which control the selecting, organization, pacing, and timing of the knowledge. The Regents examination system has long been such a strong frame in the public schools of New York, but during the past decade, it has become somewhat disassociated with academic elitism and is now being used to assess students of all academic abilities. This transforms the very nature of the Regents diploma from something to be won or earned, as suggested by Bernstein, to what many would argue is a right, or at least a goal, for all good citizens, which is the right of each child to a public high school education.

Bernstein argued that evaluation practices with strong classification and framing tended to be hierarchical and ritualized, with students being viewed as ignorant and having little status and few rights. He observed that such evaluation practices are used for the purpose of encouraging and sustaining student motivation. The Regents examination system is all of these. It is hierarchical in that it comes to students, teachers, administrators, schools, and school districts from a higher level, which is the level of the state. It is highly ritualistic, and Regents exam week is a celebrated rite of passage and a marker of status attainment for public secondary school students throughout the state of New York. It is relatively non-negotiable, and it has been since 1866. This unbending, non-negotiable characteristic of the Regents examination system was challenged in 1906 by progressive education forces, and the historical records indicates that a decision was made to preserve the classical humanist nature of the Regents examination system with respect to its classical humanist agenda in mathematics, and cede authority to local authorities for local option diplomas. Thus, there was opportunity for alternative agendas with local option diplomas. The dual diploma system allowed for a relatively peaceful coexistence of traditional and progressive curricula for over a century, until the implementation of the 1996 decision to have all secondary school students, regardless of academic ability, take Regents examinations.

The 1996 decision to expand the Regents examination system, with its strong classification and framing of knowledge, into a mandatory credentialing system for all students changes many equilibriums between traditional and progressive educators that were created in 1906, when the State adopted a dual track diploma system, which allowed: 1) the traditional classical humanist agenda to be controlled by the State in a hierarchical manner; and 1) more progressive educational agendas to be controlled locally. On this view, the Regents examination system is seen as favoring assessment of states of knowledge associated with classical humanism while the local option diplomas would have been more likely places to find examples of assessing different ways of knowing. This has relevance to the current alliances between the standards movement and the high stakes testing movement.

William Tyler has written and spoken about how Bernstein's ideas concerning evaluation are reflected in two different international tests of mathematics in secondary schools. These two international tests are the Trends in International Mathematics and Science Study (TIMSS) and the Program for International Student Achievement. Tyler writes that,

> …TIMMS is age-graded and aimed at testing mastery of science and mathematical curricula. PISA aims to capture the students' abilities to use their knowledge and skills in the challenges of real-life situations at the end of their primary schooling….The styles of questioning, particularly in mathematics, emphasize different approaches (textbook knowledge vs. context), and a different balance and coverage in each area….Their marking schemes also differed widely…(Tyler, 2006).

Tyler's comments regarding the TIMMS and PISA international examination systems suggests that each is associated with its own assessment standards and process of examinations, thus illustrating that standards and evaluative measures of performance against standards are linked, but malleable under the influence of different agendas. Thus, the link between the Regents examination system and the classical humanist agenda is seen as only one of many links that can be made between innumerable sets of educational standards and various forms of evaluation. Tyler also notes that,

> The surface features of the different emphases on the PISA and the TIMSS testing regimes therefore appear to resonate with Bernstein's distinction between the common sense knowledge of everyday life and experience and that of the school test of the lecture theatre (Tyler, 2006).

The fact that the Regents examination system has survived for 144 years suggests that, as a control system over curriculum and evaluation, the Regents examination system has long been perceived by the State as adding value to the public schools of the state of New York. This value can be characterized as strong framing of the curriculum and evaluation of the most coveted secondary school diplomas in the public schools of New York. Beneath this academic class of scholars, where the majority of students have resided since 1906, lies the realm of the local option diploma. By controlling the standards associated with academic elitism and coveted academic credentials, the state of New York found it unnecessary to exert direct control over the standards associated with minimum requirements for high school graduation. Accordingly, non-academically elite students were tracked into educational curricula typically associated with local option diplomas over which the state exercised less control and awarded lesser status.

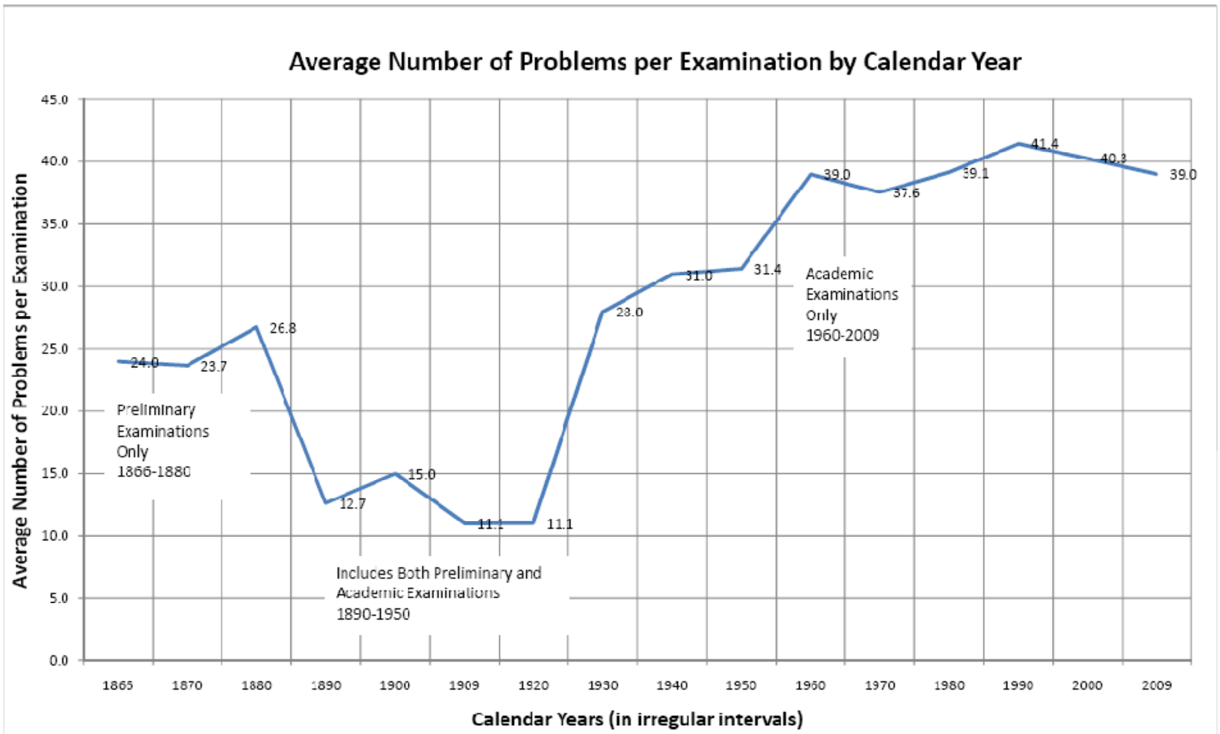
In Chapter I of this dissertation, the Regents examination system was operationally defined as a control system for public education. Control systems were theorized as having four necessary elements are: 1) standards; 2) measurement systems; 3) measurements; and 4) adjustments. These four elements of a control system are reflected in the No Child Left Behind Act. These elements of a control system are also reflected in Tyler's comments regarding international mathematics tests, which support the idea that standards, processes of examination, measures of student achievement, and status are interrelated. Thus, elements of control systems are observable in the Regents examination system in respect to mathematics; the control paradigm in the No Child Left Behind Act, the TIMMS and PISA international assessments, and in Bernstein's observations about how evaluative systems work in public education. We return now to the original research question and a discussion of changes in the Regents examination system since its inception, and then on to a discussion of social stratification associated with New York's dual diploma system.

The Regents examination system has undergone numerous changes since its inception in 1866. Originally intended as a system of entry credentials for secondary schooling, the examination system was expanded during the 1878-1879 school year to include exit examinations for courses of study in secondary schools. Different types of academic and non-academic Regents diplomas have also been introduced over the years, and the qualifications for the different diplomas have evolved. The procedures for grading Regents examinations have also changed. Initially, all Regents examinations were sent to Albany for scoring by the State Education Department. As the number of examinations became prohibitive, scoring rubrics were implemented and a system developed for local scoring by classroom teachers who were also responsible for teaching the curricula being assessed. Advances in testing theory and design led to the introduction of multiple choice questions to facilitate consistency and ease in scoring. These and other changes suggest that although there has been change in the Regents examination system since 1866, the essential structures associated with evaluation and control, as posited in different terms by: 1) the state of New York in 1864; 2) Bernstein in the 1970s; 3) Hook and Page in the first decade of the 21st century; and 4) countless others, are enduring features of evaluation in public schools. Accordingly, further study of the long historical record left by the

Regents examination system promises to shed new light on assessment practices in public schools and the desires of the state. Accordingly, these studies illuminate Bernstein's larger objective of understanding the relationships between the micro-level processes of public schools and the larger societal structures that surround them.

The Average Number of Questions per Examination

The average number of questions per examination is another example of how Regents examinations have changed. The earliest Arithmetic examinations contained 24 to 27 problems. During the 1890 to 1920 time period, following the introduction of the academic examinations, the average number of questions per examination dropped. In 1909 and 1920, the average number of problems on all examinations, including the academic examinations, was 11.1, and the number of problems on the Arithmetic examinations dropped to 15. No historical documentation explaining this drop has been found during the course of this research effort and it is interesting to speculate why the average number of problems per examination dropped between 1880 and 1890. One fact is that the new academic examinations, as a group, had a lower average number of problems per examination than did the earliest Arithmetic examinations. This explanation, however, does not explain why the average number of questions on Arithmetic examinations also dropped. One unproven hypothesis is that views of what constitutes good assessment were evolving, and shorter tests were perceived as better tests. Another possibility is that the tests became shorter because there were increasing numbers of students taking an increasing number of examinations, and the resources of the State Education Department in Albany were inadequate for creating and grading longer examinations. Under this explanation, the State Education Department in Albany simply did not have the manpower to grade long examinations, so they made short examinations. Later, when longer examinations returned and the population of students taking the examinations continued to grow, the State Education Department would resolve the "manpower necessary for grading problem" by having the Regents Examinations graded locally by teachers in the schools where assessments occurred.



Average Number of Problems per Examination by Calendar Year Figure 4-10

Figure 4-10 shows the average number of questions on Regents mathematics examinations given in specific calendar years. It is derived from the numbering of questions actually used on the Regents mathematics examinations in the research sample, and not from the number of questions as they were entered into the database. This research cannot explain the decline in the average number of test questions that occurred between 1880 and 1890, and interpretations of this historical record during this period are hampered by the absence of the academic examinations administered in 1880. Whatever the reason, the period of short examinations lasted for over thirty years and ended abruptly during the decade of the 1920s, in which the average number of questions per examination went from 11.1 to 28. By 1990, the average number of questions per examination had increased to 41.4. Most of the dramatic increase in the average number of questions per examination occurred between 1920 and 1930, a period of time that: 1) follows the rise of mathematics education as a profession between 1920 and 1930 (Donoghue, 2003); and 2) coincides with the rise of psychometrics and testing, which was heavily influenced in the 1920s and 1930s by Edward L. Thorndike of New York City's Columbia University Teacher's College. A comparison of Figures 4-3 and 4-10 suggests that the increase in the number of new topics between 1920 and 1930 was contemporaneous with an increase in the average number of questions per examination. The examinations were not just getting bigger, but the taxonomy of assessed topics was getting longer.

The average number of problems on Regents examinations, and the number of different topics assessed by the Regents examination system, after increasing dramatically between 1920 and 1930, have become increasingly stable since the 1950s. Much of what changed in the 1950s is associated with the elimination of preliminary examinations from the Regents examination system after 1959. Beginning in 1960, the research sample consists only of academic examinations associated with the secondary school curricula. The increases in the stability of these two metrics (average number of problems and total number of topics) after the 1950s may also be associated with the movement toward integrated curricula designs.

Changes in Question Types and Formats

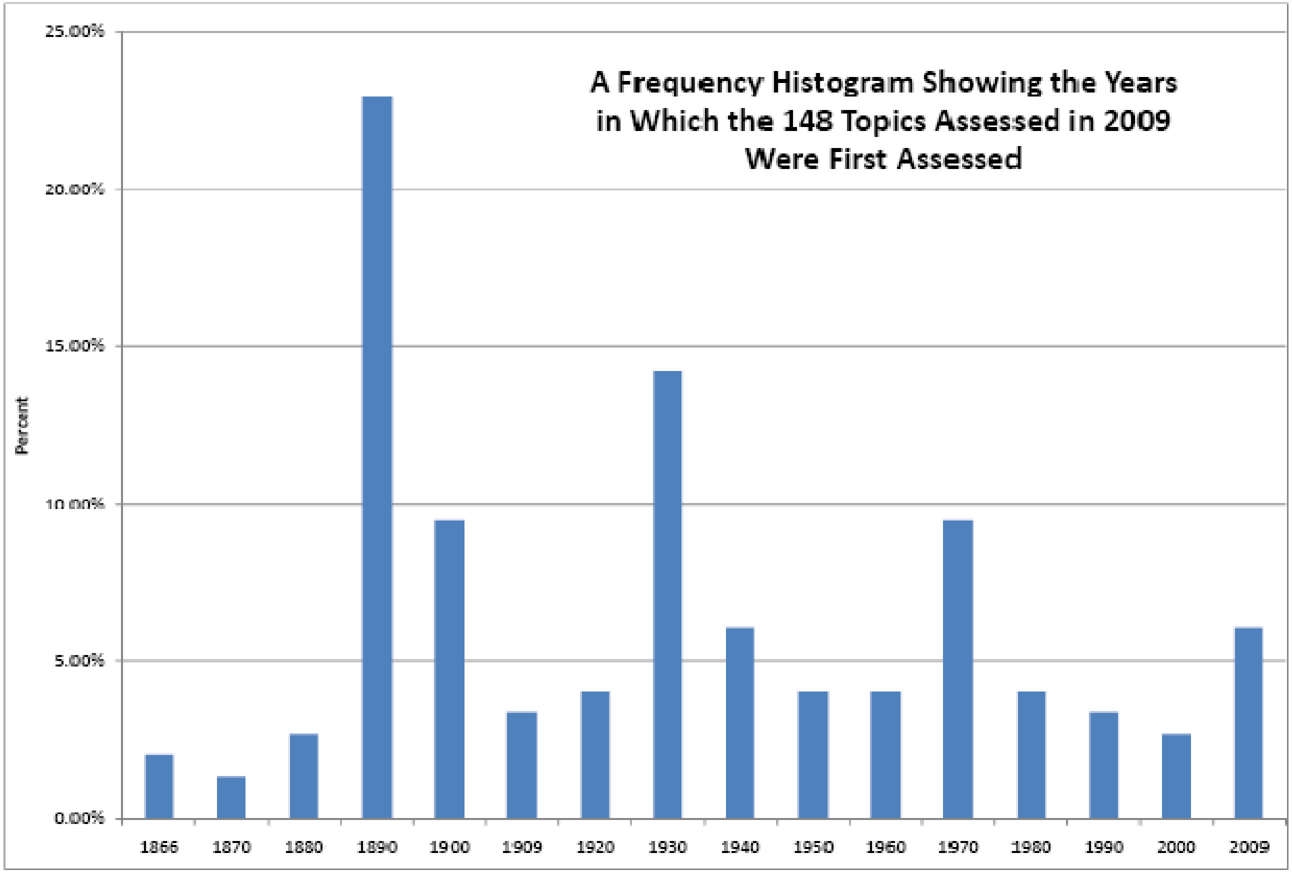
When the problems in the research sample are sorted in chronological order, it is a simple task to scan through the history of mathematics assessment practices left by the Regents examination system to see when changes occurred in the processes of examination, which includes the structure and format of examination questions. The following list shows when specific types of questions are first observed in the research sample.

1920_01_PG_07	First Illustration of any kind is observed.
1930_01_EA_20	First Cartesian plane (coordinate grid) is observed.
1930_01_IN_07	First Yes/No questions are observed.
1930_06_EA_25	First True/False questions are observed.
1930_01_PG_18	First compass and straightedge construction observed.
1930_01_SG_01	First fill-in-the-blank questions observed.
1950_08_IN_21	First multiple choice questions observed.

When looking at this list of “first observations,” it is important to note the number of “firsts” that occurred in the research sample in 1930. This list, together with Figures 4-3 and 4-10, show that the 1920s were particularly important years in the history of mathematics curriculum design and assessment in the secondary schools of New York.

Curricula Changes and a Genealogy of the 2009 Assessed Curricula

The research sample provides a sound basis for exploring the genealogy of the mathematics curricula of the public schools of New York. Using various sorts of the 5508 problems in the research sample, it is possible to reconstruct chronologies, groupings of problems by curricula, and groupings of problems by mathematical topic. From these sorts, it is possible to answer questions such as, "When did the mathematics that we teach in New York public schools today get started?" and "What is the genealogy of our current curriculum?" The answers to these questions may be approximated by looking at the topics assessed by the Regents examination system in 2009 and determining when each topic was first observed in the research sample. While it is possible that any given topic was a part of the curriculum prior to the topic being assessed on a Regents mathematics examination, it is likewise obvious that the topic was included in the curriculum at or before the time it was first observed in the research sample. During 2009, a total of 148 different mathematics topics were assessed by a total of nine different examinations that were administered in the Math A, Math B, Integrated Algebra, and Geometry curricula. When the first observation of each topic assessed in 2009 is located in the research sample, the following graph relating to the genealogy of the current curricula emerges from the historical record.



Frequency Histogram: the Genealogy of the Current Curricula Figure 4-11

Figure 4-11 suggests that change in the curriculum is not evenly distributed over the years. Indeed, on first look, there appears to be a forty year repeating cycle associated with the introduction of new topics that have survived to become a part of the current mathematics curricula. Deeper analysis of the data, however, suggests that this cycle is coincidental as opposed to the result of some lurking variable. Approximately 22% of the current curricula can be traced in the research sample to first observations during 1890. This was the year of the first academic examinations included in the research sample. The next interval in the observed forty year cycle occurs in 1930, which is a year that reflects growth in the average number of questions per examination combined with a significant increase in the number of different mathematics topics being assessed. This appears to be associated with improvements in the field of psychometrics and test design being reflected in the structures of the Regents examination system. Thus, the peaks in 1880 and 1930 have plausible explanations. The peak that occurs in 1970 reflects the first observations of the following topics in the Regents examination system, only one of which was a short lived assessment topic:

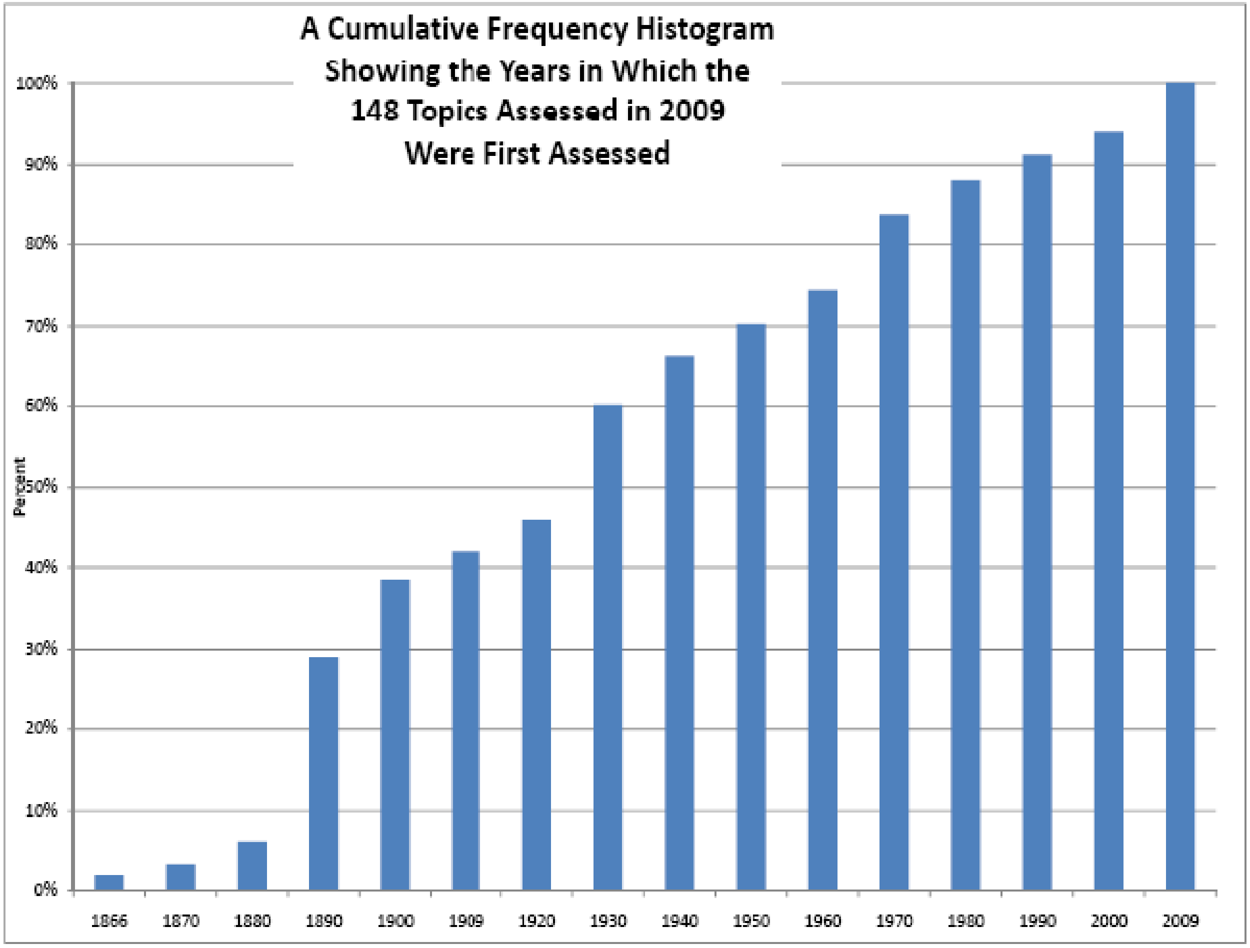
Absolute Value (1970-2009)
Equations: Absolute Value (1970-2009)
Equations: Simple with Decimals (1970-2009)
Functions: Compositions of (1970-2009)
Functions: Defining (1970-2009)
Functions: Domain and Range (1970-2009)
Inequalities: Absolute Value (1970-2009)
Inequalities: Graphing Systems of (1970-2009)
Locus with Equations (1970-1980)
Logical Reasoning (1970-2009)
Logical Reasoning: Biconditional (1970-2009)
Rationals: Undefined (1970-2009)
Set Theory (1970-2009)
Sets: Replacement (1970-2009)
Transformations: Reflections (1970-2009)

These topics reflect the influence of the modern mathematics movement on the curricula, including the presence of the Special Geometry (SMSG) Examination in the research sample. The SMSG acronym, which was also used by the School Mathematics Study Group, a leader in the modern mathematics movement, also appears as the title of a curriculum examined by the Regents examination system between 1970 and 1976. The School Mathematics Study Group was financed by the National Science Foundation under the direction of Edward G. Begle and the origins of the School Mathematics Study Group can be traced to events in the Cold War, the Space Race and the perceived crisis in the aftermath of Sputnik. While the list of new topics observed for the first time in 1970 is higher than in other years, it should also be understood that this chart shows that the new math movement was responsible for at most, eight percent of the curriculum assessed in 1970, which is only about four percent more than what is typical from decade to decade.

The final spike in what appears to be a forty year cycle due to coincidence occurs in 2009, and is associated with the first observations of the following assessment topics:

Analysis of Data (2009-)
Error (2009-)
Exponential Growth (2009-)
Graphing Functions and Relations (2009-)
Polygons and Circles: Compositions of (2009-)
Probability: Conditional (2009-)
Regression: Linear (2009-)
Regression: Logarithmic (2009-)
Regression: Power (2009-)

These topics fall into two general trends in mathematics education that are discernible in the research sample. The first general trend is found in the generally growing number of topics relating to statistics and data analysis in recent decades, which is arguably a reflection of the increasing roles of these two strands of mathematics in modern society. The second general trend, which is related to the first, involves the increasing use of technology in mathematics education. The three regression topics, as well as various topics relating to statistics and data analysis, are arguably more accessible to secondary school students because of advances in computer and graphing calculator technology. In this sense, the new topics represent new fields of knowledge made accessible to secondary school students by technology, which is now provided by the state and mandated for student use during modern Regents mathematics examinations.



Cumulative Frequency Histogram: Genealogy of the Current Curricula Figure 4-12

Figure 4-12 is created from the same data that was used to create Figure 4-11, but the data are organized as a cumulative frequency histogram rather than as a frequency histogram, thus showing the percent of the current curriculum that existed during any previous decade in the research sample. This graph shows that the topics assessed by the Regents examination system are relatively stable over time. Over ninety percent of the topics assessed in 2009 were also assessed in 2000 and 1990. About 75% of the topics assessed in 2009 have been around for fifty years. Interestingly, almost 30% of the current curriculum was assessed in 1890. When interpreting this graph, it is important to understand that the research sample is only 13% of the overall population of extant examinations, and not every assessment topic in the curricula was assessed every year. Thus, it is quite likely that the chart under-represents the actual percentage of current assessment topics that were in the curricula of the 1890s. If one looks at whether the present curricula was assessed not by year, but by curricula with unbroken lineage into the 19th Century, as much as two-thirds of the current curricula traces its origins to vintage, differentiated curricula. Change overall is relatively slow and incremental, and the current curricula have deep, strong roots.

Stabilities of Regents Examination System Structures and Rituals

The relative stability of topics assessed by the Regents examination system notwithstanding, there is another element of stability that is important to understanding the Regents examination system. The Regents examination system has existed as a right of passage in the public schools of New York since 1866. The system was created and exists today for purposes that include assessing student achievement by process of examination and awarding credentials and privileges to those who sustain the process of examination. The process of examination is and always has been organized around discrete units of curricula. With very few exceptions, the examinations are and always have been administered to all students throughout the state on the same dates and at the same time. The calendar dates on which examinations are administered are always determined by the state. Typically, they occur at the end of each semester and at the end of summer school. Over a span of 144 years, certain features of the process of examination have become predictable, if not ritualistic. For example, all past examinations appear grounded in an overarching belief that student learning can be measured by instructing students to "answer" specific questions. This can be verified by a cursory examination of the metadata from past examinations. Of the 204 examinations included in the research sample, 197 begin with instructions that include a directive to the examinee to “answer” the problems that follow. The seven examinations that do not include the word “answer” have no instructions whatsoever, and the Regents examination system appears built upon the assumption that knowledge can be assessed using paper and pencil examinations, in which the student is expected to demonstrate knowledge on command. This approach to assessment has long been associated with classical humanism.

Summary

The historical record of mathematics assessment practices in the public schools of New York, which was left by the Regents examination system, provides solid empirical support for several critical areas of Basil Bernstein’s theory of educational transmissions. Bernstein’s ideas concerning code theory appear to be reflected in the various contexts of mathematical problems that are associated with elaborated codes. His ideas concerning the classification and framing of knowledge in differentiated curricula are supported by the historical record to the extent that highly differentiated curricula have been absent from public schools of New York for approximately fifty years. Bernstein’s ideas concerning pedagogies and pedagogical practices, as elaborated on and augmented by by Atkinson and Singh, are thoroughly supported by the historical record. Perhaps most importantly, support for Bernstein’s ideas that schools privilege social classes differently are seen in the instrumental and expressive orders of schools associated with tracking academically elite students into Regents academic tracks and others into tracks leading to local option diplomas. The classical humanist agenda remains thoroughly ensconced in the public schools of New York, but as more and more students from different social classes are diverted from more progressive curricula and forced to engage in the classical humanist mathematics curricula associated with the Regents examinations system, the academic standards once associated with elitism and high quality education are being dramatically eroded.

CHAPTER V- POPULARIZATION OF THE REGENTS EXAMINATION SYSTEM

Overview

This chapter addresses the research question: How has popularization influenced the contents, structure and academic rigor of Regents examinations. In response to this question, the contents, structure and academic rigor of Regents examinations in mathematics administered in the public schools of New York State over a span of 144 years is analyzed through theoretical lenses associated with systems theory, control theory, credentials theory and Basil Bernstein's theory of educational transmissions.

A Brief Summary of Methodology

Evidence of the micro-level practices of schools between 1866 and 2009 was obtained from the form, structure and individual mathematics assessment problems associated with 204 Regents mathematics examinations administered to public school children in the state of New York in calendar years 1866, 1870, 1880, 1890, 1900, 1909, 1920, 1930, 1940, 1950, 1960, 1970, 1980, 1990, 2000 and 2009. From a collection of 1534 extant Regents mathematics examinations administered in 131 of the 143 years between 1866 and 2009, every examination administered from one year in each decade was used to create a representative sample of the historical record of assessment practices. The problems from these 204 examinations were then transcribed and entered in a database, which was subsequently encoded with: a topic for each problem; the name of the curriculum it was used to assess; and the date and month in which the problem was administered to students. This methodology allows reasonable inferences to be drawn about what topics were assessed within different curriculums, when specific topics were assessed in different curricula, and how mathematics assessment practices changed over a span of 14 decades.

In response to the current research question, evidence was sought in the research sample of changes in content, structures and rigor of Regents mathematics examinations. Changes in content are operationally described as changes in the mathematical topics assessed by the Regents mathematics examinations. Changes in structures are defined as including variations in either: 1) the forms and processes of examination; or 2) the instrumental and expressive orders of schools. Changes in rigor are operationally defined as variation in the ratios of raw score points, expressed as a percentage of total points possible, that are necessary to sustain an examination and earn credit toward an Regents academic diploma. On this view, a Regents mathematics examination requiring a minimum passing score of 75% would be considered more rigorous than is a Regents mathematics examination requiring a minimum passing score of 25%. This straightforward approach to the assessment of academic rigor must be cautiously applied, since it assumes that the overall rigor of individual problems across the decades and within specific curricula remains stable, and this research effort did not attempt to validate tthis assumption by individually assessing or otherwise quantifying the level of academic rigor associated with each of the 5508 individual problems transcribed from the research sample. Rather, general beliefs were developed over the course of the research effort that: 1) problems taken from the academic examinations were generally consistent in rigor across the decades; and 2) any variations in rigor are probably trending downward, which is to say that problems from examination of long ago are a little harder to solve than are problems from more recent examinations. This latter observation is very difficult to quantify, and may be attributable in part to the archaic language and methods used in the older mathematics problems.

The response begins with a general discussion of popular education and credentials theory, and then proceeds to a synthesized historical narrative of: 1) the history of assessment topics and practices associated with Regents level mathematics courses; 2) changes in the instrumental and expressive orders of schools that are associated with the Regents examination system; and 3) observed changes in the academic rigor of Regents mathematics examinations. Throughout the synthesized history of the Regents examination system, references are made to various theoretical lenses associated with systems theory, control theory, credentials theory and Basil Bernstein's theory of educational transmissions.

Popular Education

In 1990, Lawrence A. Cremin wrote *Popular Education and Its Discontents*, in which he elaborated on three Inglis and Burton Lectures he had earlier presented in March, 1989 at the Harvard Graduate School of Education. In commenting on the lecture, *Popular Schooling,* Cremin posited the belief of some that education is elitist by nature – to be educated is to set oneself apart from the common. Cremin then went on to posit the idea that popular education is an oxymoron, arguing that when education applies to all, elitism no longer exists (Cremins, 1990). Though Cremin was talking about public education in general, he could easily have been talking about the Regents examination system.

When the Regents examination system was established by an act of the New York legislature in 1864, the legislature arguably embraced the idea that education was elitist by nature. The 1864 legislation specifically established an elite class of academic scholars within academic institutions throughout the state. Membership in this elite class of academic scholars was controlled by the State, and the school funding formula privileged students who were certified by this state controlled process of examination as academic scholars. In this way, the state privileged students and institutions by connecting school funding with academic elitism (SED, 1965).

When the Regents preliminary examinations were debated in the legislature of the state of New York in 1864, secondary schooling was less common than it is in calendar year 2010, and there were no statewide standards for what a student should know in order to obtain a secondary school diploma. The debate was about whether or not all students had the right to a secondary school education at state expense. The essence of the legislature's decision was that secondary education was a privilege, which could be extended to some and not extended to others, and that qualification to receive state subsidies for secondary education should be based on a meritocratic process of examination. On this view, the Regents examination system was originally intended to regulate the extension of privileges by the state. Said another way, the Regents examination system was created to ensure that only those worthy of a state subsidized secondary school education received such privilege. Today, in calendar year 2010, a state subsidized secondary school education is available to all students throughout the state of New York until the student reaches the age of 21.

In 1875, there were only about 12,000 students in the secondary schools of New York, and effective compulsory school attendance and child labor laws did not exist. Secondary schooling was considered optional for most of the school age population, and those who chose to attend secondary schools were primarily middle class. In 2010, there are approximately 1.5

million students in the secondary schools of New York, and relatively effective and efficient enforcement of compulsory attendance and child labor laws has been in place throughout the state of New York for approximately 100 years. Since schooling itself was optional for most students in the 1870s, participation in the Regents examination system was also optional.

The optional nature of secondary schools for most school age children in the state of New York began to change in the first decade of the 20th century, when effective compulsory school attendance and child labor laws were enacted. The social class diversity of high schools increased during these years as students of lower class families, who had previously opted out of secondary education, were compelled by more and more effective and efficient regulations to attend public high schools. In the midst of increasing enrollments of students from different social classes, progressive voices in education argued that the classical humanist agenda of the old academy system of secondary schools did not meet the needs of a growing number of secondary school students. In response to these trends in public education, the Regents examination system was made optional in 1906, and local option diplomas were created. Thus, the optional nature of the Regents examination system evolved from: 1) being optional because schooling itself was optional; to 2) being optional because there was another secondary school diploma that could be obtained without taking Regents examinations. Throughout all of the years that the Regents examination was optional, it was associated with academically elite students, typically from middle class families who valued the classical humanist agenda in secondary education.

In 1996, a decision was made to popularize the Regents examination system, and to make all general education students take five Regents examinations in English, Global Studies, Mathematics, Science, and U.S. History. The implementation of this decision is nearing completion in the year 2010, and all general education students are now taking Regents examinations. On this view, the Regents examination system was associated with an optional, academically elite curricula, throughout most of its many years of existence, but during the last decade, has become popularized. On Cremin's view, now that the Regents examination system applies to all students, the elitism once associated with the Regents examination system no longer exists.

Control Theory, Credentials Theory

and Basil Bernstein's Theory of Educational Transmission

Every state that desires to ensure quality in public education exercises some form of regulatory oversight. Regulatory oversight is tantamount to control, and control systems of all types have four elements in common: 1) standards; 2) measurement systems; 3) measurements; and 4) the willingness and ability to make adjustments (Hook, 2000) The Regents examination system is a control system, which: 1) was created by the legislature of the state of New York in 1864 to regulate academy admissions; 2) expanded in 1878 to regulate secondary school curricula and diplomas; and 3) has been in continuous operation in the public schools of New York for 144 years.

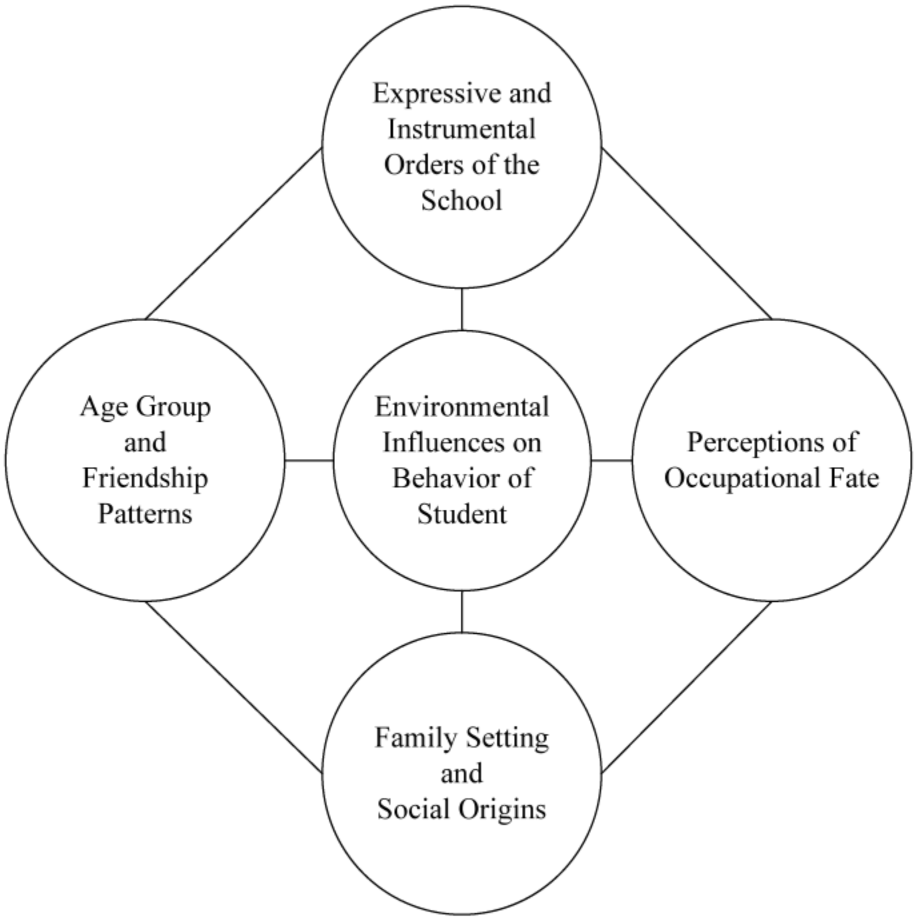
Credentials may be thought of as testimonials or documents attesting to the legitimacy of a person's or institution's entitlement to privilege or recognition, and credentials extended by the state are typically associated with licensed professions such as doctors and schoolteachers. In New York State, the New York Board of Regents are responsible for bestowing credentials on doctors and teachers, and also on students in public schools. Thus, there are professional credentials and academic credentials in New York State, and the Regents examination system is associated with state sponsored academic credentialism. In exercising the state's prerogative of quality control over public education, the Regents examination system has exerted a shaping influence over the instrumental and expressive orders of New York's public schools.

With respect to such instrumental and expressive orders of schools, Basil Bernstein defined these two orders of schools when he described their respective characteristics. He wrote,

> There are two distinct, but in practice inter-related, complexes of behavior which the school is transmitting to the pupil: that part concerned with character training and that part which is concerned with more formal learning. In the one hand, the school is attempting to transmit to the pupil images of conduct, character and manner; it does this by means of certain practices and activities, certain procedures and judgments necessary for the acquisition of specific skills: these may be skills involved in the humanities or sciences. These specific skills are often examinable and measurable by relatively objective means. The image of conduct, character and manner is not measurable in the same way. Among the staff there may be a fair degree of agreement about the learning, but there is more room for doubt and uncertainty about the image of the conduct, character and manner which the school is trying to transmit….I propose to call that complex of behavior and activities in the school which is to do with conduct, character and manner the *expressive* order of the school , and that complex of behavior, and the activities which generate it, which is to do with the acquisition of specific skills the *instrumental* order (Bernstein, 1970, p. 38).

When the instrumental and expressive order of schools are used to frame a longitudinal analysis of mathematical assessment practices and credentialism, based on historical evidence left by the Regents examination system, the intersections of student social class and micro-level practices of schools are illuminated.

Basil Bernstein argued that the structures of public education and of society interact in ways that serve to replicate the social stratification of society. He posited that children of different social classes interact differently with public education, and that the experience of public education is not the same for all children because of different environmental influences. These environmental influences are summarized as: 1) the expressive and instrumental orders of the school; 2) student age groups and friendship patterns; 3) family settings and social origins; and 4) perceptions of occupational fate, and are reflected in the following model (Bernstein, 1997, Ch.1).



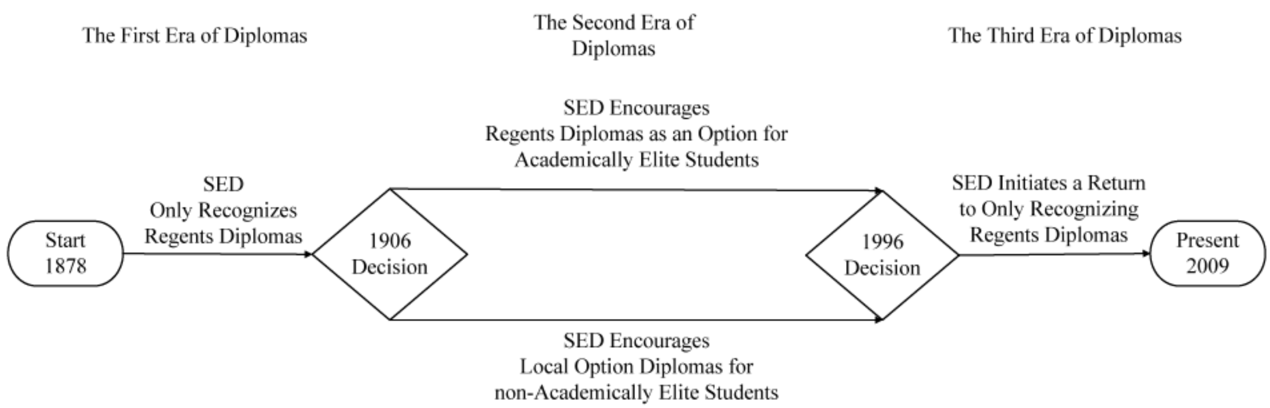
Environmental Influences on Student Behaviors Figure 5-1

Bernstein argues that because the interaction of schools with children from differing environments are not always the same, the outcomes of their interactions will also differ, and that schools will tend to replicate the social stratifications of society, even when it is their expressed desire not to do so. On this view, egalitarianism and the goal of educating for a just and democratic society may be beyond what is possible within the constraints of existing public schools and existing societal structures, thereby providing one possible theory of the high failure rate of so many well-intentioned efforts at school improvement and reform.

An Overview of How Popularization Has Influenced the Regents Examination System

The Regents examination system has exerted a continuous shaping influence on the instrumental and expressive orders of New York's secondary schools for 144 years. The most visible influence of the Regents examination system throughout its long history has been the segregation of students into two categories: 1) those who pursued Regents academic credentials; and 2) those who did not pursue Regents academic credentials. This classification schema, embedded within the instrumental and expressive orders of public schools through curricula and evaluative controls associated with the Regents examination system, shaped student friendship patterns as well as student perceptions of occupational fate. As Bernstein's theory would suggest, this classification schema, which helped to define the instrumental and expressive orders of public schools throughout the state of New York, was also reflective of the family settings and social origins of the students who were classified and credentialed.

The overall history of the influence of Regents credentials on the public schools of New York can be divided into three eras: 1) the first era of the single diploma system, which started during the era of the academy system and ended as the era of public high schools began; 2) the era of both Regents diplomas and local option diplomas, which began in 1906 during the first decade of the 20th Century and continued into the first decade of the 21st Century; and 3) the second era of the single diploma system, which began in the last decade. Academic elitism was associated with the Regents diplomas of the first and second eras of diplomas, classification and academic tracking are associated with the second era of diplomas, and popularization is associated with the third era of diplomas. During the first decade of the 21st Century, which coincides with the beginning of the third era of diplomas, the Regents examination system was reoriented in such a way that all general education students now pursue Regents academic credentials. Thus, the option of tracking into a non-Regents curricula leading to a local option diploma no longer exists. The three eras of the Regents diploma system are graphically represented in Figure 5-2.



Three Eras of Diplomas in New York Secondary Education Figure 5-2

In response to the research question, How has popularization influenced the contents, structure and academic rigor of Regents examinations, the preceding graphic is associated with the following general observations.

1. Throughout the first, second, and third eras of diplomas, the Regents examination system has been used to define and evaluate a stable set of mathematics curricula that is associated with a classical humanist agenda in education. Most of the mathematics topics assessed in the third era of diplomas were also assessed in the first and second era of diplomas.
2. During the first era of diplomas, schooling was optional and secondary school students were typically from the middle class.

3. During the first and second eras of diplomas, the Regents examination system was optional: first because schooling itself was optional; and later, because local option diplomas were widely available.
4. During the second era of diplomas, academically elite students were tracked into traditional curricula leading to the Regents academic diplomas, and non-academically elite students were tracked into more progressive curricula leading to local option diplomas. Lower class students tended to be over-represented in tracks leading toward local option diplomas while middle class students tended to be tracked into academic tracks leading towards Regents academic diplomas. This era ended in a spirit of egalitarianism as New York attempted to establish high academic standards in all public schools for all students.
5. During the third era of diplomas, all general education students are required to pursue the traditional Regents mathematics curricula, and their achievements are evaluated using traditional Regents academic examinations in mathematics. As increasing numbers of lower class students previously exempt from taking Regents academic examinations in mathematics were tested, the academic rigor required to pass Regents mathematics examinations and qualify for Regents academic diplomas has plummeted. This lowering of academic rigor is attributed to popularization of the Regents examination system.

The decision to popularize the Regents examination system has been implemented over the last ten years, and has resulted in a significant deterioration in the academic rigor necessary to sustain an examination. The empirical evidence points to a conclusion that popularization of the Regents examination system has led to a deterioration of quality associated with the micro-level practices of public schools. We now turn to a more in-depth look at the first era of Regents diplomas.

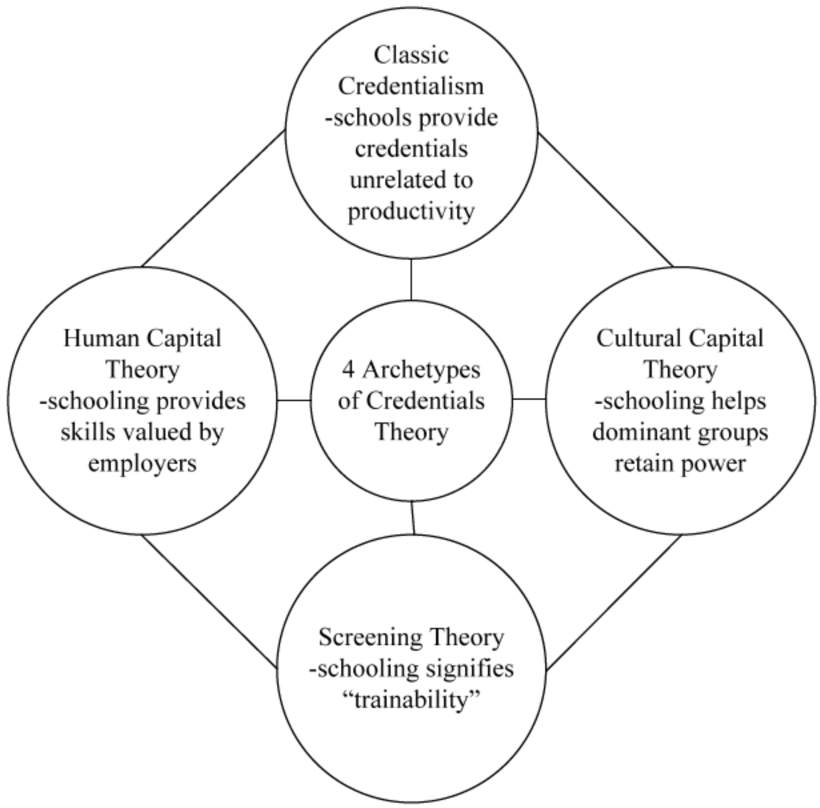
As a control system, the Regents examination system has directly influenced the instrumental and expressive orders of secondary schools in the state of New York since 1878, when the Regents academic examinations were first administered. The analysis that follows explores these ideas through the lens of mathematics assessment practices in the public schools of New York between 1866 and 2009, inclusive. Before exploring each of the three eras of diplomas in the public schools of New York, however, a closer look at credentials theory is needed.

An Introduction to Credentials Theory

Credentials theory provides a theoretical lens for examining the historical record left by the Regents examination system relative to questions of: 1) how differentiated sets of students have been credentialed and thus privileged or disadvantaged by the Regents examination system; and 2) how the Regents examination system itself has been influenced by differentiated sets of students.

Basil Bernstein and many credentials theorists are in general agreement that schools tend to replicate social structure in a process called social stratification. They differ, however, in their foci and in their theories of how different mechanisms contribute to social stratification. Bernstein theorizes that social stratification results from: 1) the interactions of class codes of students and class codes of schools, through which lower class students are disadvantaged and middle class students are privileged, and 2) the instrumental and expressive orders of schools. Theorists such as Randall Collins also believe that schools tend to replicate social structure in a process called social stratification, but Collins would argue that the credentialing function of schools is what mediates social stratification. On this view, the social stratification of society, which is facilitated by schools, is associated with educational credentials, through which individuals differentiate themselves and acquire either human or cultural capital. The Regents examination system is an exemplar of a control system that uses educational credentials to shape and influence the instrumental and expressive orders of public schools, thus creating different experiences of public schools for academically elite and non-academically elite students. Historically, in the state of New York, this has meant that academically elite students were tracked into traditional humanist mathematics curricula and non-academically elite students were tracked into more progressive curricula. Credentials associated with the traditional humanist mathematics curricula of the Regents examination system were controlled by the State, whereas credentials associated with the typically more progressive local-option-diplomas were controlled by local schools and school districts.

David Bills, a modern academic, describes four distinct variations, or themes, in modern credentials theory, which are referred to as archetypes of credentials theory in Figure 5-3.



Four Archetypes of Credentials Theory Figure 5-3

The four archetypes of credentials theory are further described by Bills as follows:

- In classical credentials theory, schools are viewed as bestowing credentials for academic skills that are not necessarily associated with increases in job productivity. Ivar Berg is an exemplar of theorists in this category.
- In human capital theory, schools are viewed as producing both general and specific job skills that are useful to employers. Proponents of human capital theory include theorists such as Gary Becker, Samuel Bowles, and Herbert Gintis.
- In cultural capital theory, schools are viewed as providing forms of cultural capital that help dominant groups retain their powers and privileges. Bills cites Randall Collins as an exemplar of a credentials theorist who might be sub-classified as a cultural capitalist.

- In screening theory, schooling is viewed as a signifier of "trainability" for occupational skills that are primarily acquired on the job. Lester Thurow, Harold Wilenski, and Anne Lawrence are theorists in this category (Bills, 1988a).

Classic credentialism, as reflected in figure 3-5, is often used to theorize credentialing systems that are not associated with the workforce. The other three archetypes share a common focus on the valuation of credentials in job markets and how credentials influence access to and status within the workforce. Most but not all current literature regarding credentialism involves some consideration for the value of credentials in the workforce. Human capital theorists and cultural capital theorists debate whether credentials should be understood as: 1) individual-level variables; or 2) the embodiment of social class relations embedded in history and political struggles. The fundamental question of whether educational degrees signify individual skills or valued social culture leads both human capital and cultural capital theorists to address issues relating to mass education and credentials inflation. Modern writers often combine different elements of the four archetypes to define new hybrids, which can be used to explain observed phenomena. David Brown is an exemplar of such writers.

Brown posits that the symbolic meanings of credentials are different in bureaucratic and professional labor markets. In bureaucratic markets, degrees signify technical competence in certain routine tasks, such as reading, writing and arithmetic. In professional markets, degrees are used by professional groups to control the recruitment process, and thus to control access to privileged occupational positions. Brown's view reflects elements of both human capital and cultural capital theories. Brown also notes that governments can play significant roles in developing and maintaining credentials systems to promote state objectives, which is consistent with classic credentialism. Brown shows an overall leaning toward cultural capital theory when he summarizes modern credentialing theory as having the following four primary features.

1. The content and occupational significance of credentials are more cultural and exclusionary than technical and efficacious. Correspondingly, degree thresholds are more important in credentialed labor markets than are years of schooling or technical knowledge.
2. The formality of credentials (i.e., the information in the degree itself) is an abstraction from the actual substantive knowledge of degree holders that delimits which authorities may question the substantive competence of degree holders. Thus, formal qualifications are linked to positional power in jobs.
3. Credentials are: (a) monopolized by occupational status groups as exclusionary, cultural entry barriers to positions; and (b) used by hiring parties as measures of a candidate's trustworthiness in positions that embody discretionary powers. Professional and bureaucratic labor markets are end points on a continuum of credential usage from (a) to (b) respectively.
4. Historical credential inflation at the top of credentialing hierarchies drives educational expansion. Credentialing crises may occur in credentials markets, and states may be more or less involved in the regulation of credentials markets and crises (Brown, 2001).

The Perspective of Randall Collins

Randall Collins, in 1979, argued that education in our society serves a gatekeeper role, which is linked to an ultimate goal of social stratification. He posited that educational credentials are forms of cultural capital and have currency value, and that the lack of educational credentials inhibits mobility into and between social and occupational categories. Occupational groups establish minimum educational credentials for group members as a means of promoting cultural similarity and limiting access to and competition for coveted positions (Collins, 1979).

Collins divides the world of work into two general categories: productive labor and political labor. He then argues that educational credentials have little to do with preparing persons for productive labor, and that knowledge and skills associated with productive labor are as easily learned through apprenticeship and on-the-job training as through education. Collins also argues against the so-called “myth” that education is needed to meet the needs of an increasingly technology driven society, and he posits that the minimum education requirements for occupations have more to do with limiting access to those with desirable forms of social and cultural capital. On Collins’ view, the rise of the educational credentials system in the United States is correlated with a desire to protect middle and upper social classes from competition, especially from immigrants, and that as competition increases, the costs of educational credentials become inflated, so as to preserve the self interests of higher social and economic groups in our society. Collins believes that educational credentials provide access to the political sector of the workforce, which is characterized by its control function over the production workforce, and in which a primary goal is the sinecure -- a position with title and recompense, but little or no requirements for productivity.

The Genealogy and Characteristics of Modern Credentials Theory

David Brown traces the genealogy of modern credentials theory to Max Weber’s theories of status attainment and even further back to the Marxist struggle between the bourgeoisie and the proletariat. Brown contrasts the evidence for the more Marxian theories of Antonio Gramsci, Samuel Bowles and Herbert Gintis, and Michael Apple, and concludes that the evidence for the Marxian hypothesis, which is primarily associated with human capital theory, appears stronger at lower levels of credentialing systems, whereas Weberian perspectives of status attainment offer more plausible explanations with regards to higher degrees and for the inflation of credentials as a whole (Brown 2001 pp. 21-22). The Marxist hypothesis notwithstanding, all four archetypes of credentials theory in figure 3-5 are arguably grounded in status attainment theory.

Status attainment theory attempts to explain how different factors influence the attainment of individual status. Archibald Haller and Alejandro Portes report that two categories of statistical models have evolved for exploring and understanding status attainment theory and the effects of schooling. These are: 1) the Blau Duncan model; and 2) the Wisconsin model. Both models are based on path analysis and typically involve analyses of large longitudinal data sets. In research conducted by Haller and Portes, both models produce similar coefficients for paths of influence, despite their use of data sets from significantly different samples.

- The Blau-Duncan model posits the simple idea that parental occupational status and education have significant influence over a child's educational attainment, which in turn has significant influence over the child's occupational attainment, thus suggesting that parental influence over occupational attainment is mediated by educational attainment.
- The Wisconsin model seeks to understand the social psychological mediating dynamics through which parental status influences occupational attainment, and adds consideration for such factors as mental ability, academic performance, and the influence of peer groups and significant others.

Both models attest to the primacy of educational attainment as a correlate of occupational attainment, and both show educational attainment to be a function of parental status. However, educational attainment is shown in the Wisconsin model to be related not only to the influence of parents, but also to the formation of status attainment aspirations that are influenced by significant others. Thus, the mechanism by which parental status influences educational attainment and later occupational status appears to be mediated through status attainment aspirations that are formed in social psychological interactions, which are themselves highly influenced by parental status. Haller and Portes interpret the Wisconsin model as suggesting that the,

> …negative impact of low parental status on children's educational and occupational attainment could well be altered if a set of counterbalancing influences – at school and within the peer group – were brought to bear at the time status aspirations were formed and at the point they were enacted into relevant behavior (Haller 1973, pp. 65-66).

This interpretation of the Wisconsin model is consistent with Basil Bernstein's theories concerning the different environmental influences in the lives of children that lead to differentiated experiences of schooling (Bernstein 1977).

The Wisconsin model, grounded in status attainment theory and path analysis, is analogous to and provides support for Basil Bernstein's theory of environmental influences on the behaviors of students. Beginning with family setting and social origins, the Wisconsin model shows how these factors correlate with peer groups and significant others, and with perceptions of occupational fate, which are then correlated with occupational status.

What is missing from the Wisconsin model and from the work of Haller and Portes is consideration for what Bernstein describes as the expressive and instrumental orders of schools. In the secondary schools of New York State, these orders have long been shaped and influenced by the Regents examination system, which has historically been associated with: 1) the tracking of academically elite students into Regents academic tracks leading to academic credentials that facilitate access to higher levels of the educational system of New York State; and 2) the tracking of non-elite students into non-academic curricula, which by comparison retard access to the higher reaches of the educational system of New York State. On this view, the Regents examination and Regents diploma credentialing systems can be viewed as micro-level practices of schools that influence the career aspirations of secondary school students in public education, and ultimately lead to stratification of society. Accordingly, the current effort to interpret the historical record of curricula and assessment practices left by the Regents examination system and the types of Regents diplomas associated with specific examinations is well framed by both status attainment theory and Basil Bernstein's theory of educational transmissions. We return now to the three eras of diplomas that are associated with the Regents Examination System.

The First Era of Diplomas

The Regents examination system was approved by the New York legislature in 1864 as part of an educational credentialing and funding plan that would regulate student admissions to a class of "academic scholars" in secondary schools throughout the state of New York. When the Regents examination system was created, secondary education in New York was much different than the modern system of public high schools that we know today. In 1864, secondary education was associated with the old academy system of schools, most of which no longer exist, having long ago been replaced by the public high schools. After the first Regents examinations were administered in 1866, the Regents examination system was expanded in 1878 to include academic examinations in 27 subject areas. Concurrent with this expansion of the Regents examination system, a new academic credential – the Regents diploma – was created. In this way, and during a period of less than fifteen years, the Regents examination system grew from a system that only administered preliminary admission examinations into a system that administered both preliminary admissions examinations and commencement level academic examinations.

The preliminary examinations, which regulated entry into the academic class of scholars, did not totally control entry into the academies. After the Regents examination system started, an academy could still enroll students who were not certified by process of examination as academic scholars. However, the amount of state money distributed to each academy from the state's Literature Fund was conditioned on the number of its enrolled academic scholars, and non-credentialed enrollees were not funded. Under this funding formula, competition arose within the private academy system of secondary schools for credentialed students. The success of the early Regents examination system was almost certainly associated with the school funding formula (Beadie, 1999a, 1999b).

When the Regents examination system was subsequently expanded to include examinations of the academic curricula taught in the secondary schools, the intent was to rationalize a cacophony of curricular practices and standards that existed within the academy system, and also to establish a system of state imposed credentials of recognized exchange values between secondary schools and colleges and universities throughout the state. The advent of the academic examinations was accompanied by a new kind of diploma, and student achievement efforts were reoriented toward the Regents examination system and the acquisition of Regents academic diplomas.

Between 1878 and 1906, the Board of Regents of the State of New York recognized only one kind of secondary school diploma, and that was the Regents diploma. If a secondary school student in New York wanted to attend a college or university in the state of New York, almost all of which were also regulated by the Board of Regents, a Regents diploma was generally sufficient to meet admission requirements. Because of the value of the Regents diploma in obtaining access to higher reaches of the expanding educational system in the state of New York, the Regents examination system became associated with academic distinctions for elite secondary school students. It also served as a quality control system through which the state could regulate the micro-level practices of secondary schools.

In 1866, when the first Regents preliminary examination were administered, New York State was a patchwork of farms and small villages with a few large cities, including Buffalo, Syracuse, Albany, and New York. These large cities were capable of supporting public day high schools, but outside these large cities, the academy system was the primary provider of secondary schooling. The lack of mobility and transportation associated with this era acted as natural restrictions to the growth of public schools in a society that was primarily agrarian. Smaller communities and rural areas could find enough school children to support one room schoolhouses, but there were seldom enough students within walking distance of any central location to support a public day high school. The academy system was able to exist due in part to the fact that it was able to resolve the commute time problem in two ways: first, many of the academies were boarding schools; second, the academies that were not boarding schools generally assisted students in finding private room and board arrangements close to the school. A limited number of students enrolled in some academies undoubtedly lived within walking distance of their schools. There was probably considerable variation in the number of lucky, or perhaps unlucky, students who lived within walking distance of their academies.

Nancy Beadie, a historian of education who has studied the academy system of New York, posits that the academy system of schools was widely accepted by middle class families throughout the state. Beadie estimates that 50% or more of middle class students in New York attended these academies for some period of time, though attendance was sporadic and many students never graduated. She notes that the old academy system provided middle class New York families with more than a simple secondary education. The academy system also provided middle class students with opportunities for social interaction, networking, and development of a worldview that extended beyond the confines of their local farms and villages (Beadie, 1999a, 1999b) Students not desiring or unable to afford the perceived benefits of long stays at academies could often arrange for self-study programs at home punctuated with brief periods of attendance and tutelage at the academies -- similar in some respects to the distance learning programs of modern colleges and universities.

When the old academy system of education in New York is viewed through the lens of Basil Bernstein’s conceptualization of environmental influences on the behavior of students, the following observations emerge: 1) the expressive and instrumental orders of the academies were oriented towards student achievement by the Regents examination system; 2) the age group and friendship patterns of students within the academies were influenced by geographic distances and boarding school atmospheres; 3) the family settings and social origins of most students were predominantly middle class in the sense that these students could delay entering the labor force and afford to pay the expenses of a secondary education not covered by the state; and 4) the perceptions of occupational fate of the students enrolled in the academies were associated with managing farms and businesses in an agrarian society, and for which a secondary school diploma was entirely adequate, else attending college or university for which a Regents diploma was then necessary. All of these assumptions about the characteristics of students in the old academy system are consistent with Bernsteinian theory.

In a separate study of the demographics of who attended secondary schools in the 19th Century, David Labaree studied the Philadelphia school system. Labaree's longitudinal study of a Philadelphia high school is an example of the type of study advocated by Bernstein to demonstrate the relationships between the micro-level processes of schools and the macro-level structures of society. Labaree uses the lenses of status attainment theory and credentialism to explain observed relationships between curricular practices and class structures. Central High School in Philadelphia was founded in 1838, and Labaree's study examines the curricula and the student population from 1838 through 1920. Student social class was identified via parental occupations obtained through examination of federal censuses taken every ten years from 1840 to 1920 (Labaree, 1986).

Labaree's method of correlating student names with parental occupation through federal census data is an interesting approach to identifying the social class of early high school students and suggests that approximately two-thirds of urban high school students between 1840 and 1920 were from the middle class, while one-third were from working class families. Being more or less a meritocracy with admission based on performance on a test, Central High School's graduation rate of approximately 27% was relatively undifferentiated across all social classes studied. Graduation rates by cohort gradually increased from the low teens in the 1850s to 1870s to 30% in 1920. These graduation rates notwithstanding, approximately one in every 50 first graders enrolled in Philadelphia schools were later admitted to Central High School, and only one in every 200 admitted first graders eventually received a high school diploma. Under these circumstances, the high school diploma was highly valued as an academic credential (Labaree, 1986).

Labaree's observations of the relationships between schools and social class in a 19th Century urban Pennsylvania high school and Beadie's observations regarding students who attended secondary school in the old academy system in New York are quite consistent. Both assert that most students were middle class, that students competed for credentials in a meritocratic educational system; and that relatively few students earned diplomas. What is important for the current research effort is the premise, which is supported by both Beadie and Labaree, that 19th Century secondary school students came primarily from the middle class. On this view, the Regents examination system in New York was designed as a means of assessing middle class students and of regulating mostly private academies that served the middle class. It was in this capacity of regulating the academic credentials earned by middle class students that the Regents examination system became a hallmark of New York public education in secondary schools.

During these early years of the Regents examination system, the mathematics examinations were of two types: the preliminary examinations and the academic examinations. The preliminary examinations in the Arithmetic curriculum were used by the state for the regulation of admissions into the academic class of scholars in the academies, and almost 36% of the assessed topics in this curriculum were associated with business math and commercial math. This emphasis on business math and commercial math is illustrated in the following exemplars from 19th Century Regents Arithmetic examinations.

1866_11_AR_07 Mensuration

In exchanging gold dust for cotton, by what weight would each be weighed?

1866_11_AR_16 Cost

Find the cost of the several articles, and the amount of the following bill:

To 16750 feet of boards at $12.50 per M.,

"	1750	"	"	24.00	"
"	3500	"	"	25.00	"

Received Payment, $

SAMUEL PALMER

1890_01_AR_05 Cost

Find the cost of each of the following:

5 gals. 3 qts. 1 pt. of vinegar at 20 cents a gallon

10 acres, 50 sq. rods of land at $48 an acre

1890_01_AR_07 Mensuration

Write the table of linear measure.

1890_01_AR_08 Bills and Receipts

James Jones buys of John Wilson for cash Jan. 1, 1890, 5 gals. Vinegar at $.20; 27 lbs. sugar at 10 cents; 5 lbs. oat meal at 5 cents. Make out a bill of the above and receipt it for Wilson.

1890_01_AR_13 Notes and Interest

Find the proceeds, bank discount and date of maturity of a note for $2,000 at 90 days at 5%, dated and discounted July 1, 1889.

The preceding exemplars reflect the types of mathematical knowledge and assessment practices associated with the earliest Regents preliminary examinations.

The academic examinations, which began in 1878, had a different regulatory purpose. The academic examinations were used as a quality control system to influence secondary school curricula and to regulate the conferring of Regents diplomas. The academic examinations did not share the same emphasis on applied mathematics relating to business and commercial mathematics. Instead, the academic examinations were solidly grounded in classical humanism. Students were expected to know and perform the types of mathematics that were associated with the ancient civilizations of Greece and Alexandria. The following brief collection of questions from Regents academic examinations administered in the schools of New York in 1890 reflects the influence of classical humanism on the early secondary school mathematics curricula.

1890_01_AL_08 Equations and Expressions: Modeling
What number is that which being multiplied by 7 gives a product as much greater than 20 as the number itself is less than twenty?

1890_01_HA_01 Radicals: Square Roots
Find the square root of $28+10\sqrt{3}$.

1890_01_PG_06 Proofs: Pythagorus
Prove that the square described on the hypothenuse of a right-angled triangle is equal to the sum of the squares described on the other two sides.

1890_03_HA_08 Progressions: Arithmetic and Geometric
Show that if, in a geometrical progression, each term be added to or subtracted from that next following, the sums or the remainders will form a geometrical progression.

1890_03_PG_b_03 Proofs: Polygon
Prove that diagonals of a parallelogram bisect each other.

1890_03_PG_b_06 Proofs: Circle
Prove that when two chords intersect in a circle the angle thus formed is measured by one-half the sum of the intercepted arcs.

By the early 1900s, the very nature of secondary education in New York had begun to change. The industrial revolution was reshaping and enlarging villages and cities across the state. As population density increased in cities and villages, demands for secondary education led to the building of more public high schools. A new system of public education was ascending and the old academy system was slipping away. Contemporaneous with this evolution of society, new ideas about schooling were being developed and the discourse concerning the proper relationship between schools and society was growing louder. The progressive movement in education was making its agenda known.

In 1905, John Dewey arrived at Teacher's College of Columbia University in New York City, having already made a name for himself at the University of Chicago. John Dewey is today viewed as a great prophet of the progressive movement in education, and though he is not directly connected to any particular change in the Regents examination system by this researcher, his arrival in New York City as an important professor in the flagship of private New York universities regulated by the Board of Regents almost guarantees that John Dewey's arrival and his views on progressive education were known to the Board of Regents, which oversaw the Regents examination system. In fact, the Board of Regents probably knew the views of John Dewey and any number of other academics, including those who had different ideas about what public schools should do. The presence of these competing interest groups notwithstanding, the Board of Regents appears to have been influenced by the progressive movement.

The Second Era of Diplomas

During the first decade of the 20th century, the demographic characteristics of students in secondary schools in New York were beginning to change. As the old academy system was dying and being replaced by the modern system of public high schools, new laws concerning compulsory school attendance and child labor were enacted. Schooling was evolving from a privilege for some into a right for all, or at least for more. Many voices argued that the classical humanist agenda created for the regulation of the old academy system was inappropriate for the new classes of students being admitted to the public schools. Under assault from progressive educators and in the midst of a changing student demographic, the state in 1906 created a new local option diploma, and ceded control over the curricula standards and evaluation practices to local schools and school districts. Thus, the Regents academic curricula and the Regents examination system became a means of segregating public school students into two groups: 1) a group that studied the classical humanist agenda carried over into the high schools from the dying academy system; and 2) everybody else.

The creation of the local option diploma meant that the Regents examination system would no longer control secondary school graduation standards by process of examination for all students. In essence, the regulation of secondary school graduation standards was being ceded by the state back to local schools and school districts, which would be permitted to design curricula based on criteria that were different than classical humanism. As more schools were built and more and more students began attending secondary schools, secondary education came to a point where it was no longer seen as something for middle class students. Rather, secondary education was seen as beneficial for all classes of students, and the micro-level practices of schools evolved to accommodate the increasing numbers and increasing class diversity of students. The decision to cede control over high school graduation requirements back to local schools and school districts was significant because it allowed secondary schools to offer curricula other than those grounded in the classical humanism of the academy system. This accommodated the needs of students who were not interested in a classical humanist education while simultaneously preserving the classical humanist agenda and its status symbol, the Regents diploma, for academically elite, typically middle class students. This new approach to education in the secondary school of New York, which involved a bifurcated system of: 1) Regents diplomas associated with classical humanist educations for academically elite students; and 2) local option diplomas for non-academically elite students, existed until 1996, when the state decided to once again recognize only one type of diploma for all general education students – the Regents diploma.

Under Bernstein's theories concerning the instrumental and expressive orders of schools, the relative orientations of the new curricula with respect to Kliebard's four curricula shaping forces can be viewed as manifestations of the instrumental orders of schools. However, Bernstein's theory was that the instrumental and expressive orders of schools are inter-related, thus creating the theoretical expectation that the expressive orders of schools with these new progressive curricula were also impacted by the Regents examination system. When viewed through the lens of Kleibard's classification schema, control of the classical humanist agenda was retained by the state while control of the progressive agenda was ceded to local schools and school districts, and the more highly valued Regents academic diploma was only associated with the state controlled classical humanist agenda. We proceed now in search of evidence of how the instrumental orders of secondary schools under the Regents examination system related to the expressive order of secondary schools.

Between 1906 and 1996, students in the secondary schools of New York were routinely sorted into academic and non-academic tracks. The academic track, with curricula and assessment practices regulated by the state, led to the more prestigious Regents diplomas. The non-academic tracks, with curricula and assessment practices governed by local authorities, led to less prestigious local option diplomas. Under this dual track system, Regents diplomas became widely recognized as hallmarks of excellence in classical humanism, and progressive education became widely associated with local option diplomas.

In 1965, on the 100th anniversary of the Regents examination system, the New York State Education Department published a celebratory booklet entitled, *Regents Examinations – 100 Years of Quality Control in Education: 1865-1965*. The following passage from this celebratory booklet summarizes the New York State Education Department's 1965 reflection on the creation and impact of the Regents examination system.

> The Regents examination system began in New York State in November 1865 (sic) as a plan of high school entrance examinations. The amount of state aid to public academies was based on the number of pupils enrolled in each academy. To discover who were bona fide academy students, the Board of Regents

> established admission examinations, and a State certificate was awarded to successful candidates. The plan of uniform and impartial entrance examinations was immediately successful, and there soon arose a strong demand for similar safeguards and standards for high school graduation and college admission. In June 1878, therefore, the Regents administered the first of the academic or high school examinations….From these beginnings, the modern system of high school achievement examinations developed. In a relatively short time, "Regents credit" became universal academic currency (SED 1965, p. 4).

This same 1965 publication of the New York State Education Department described the students who took the academic examinations as students with "…average and above average academic abilities" (SED 1965, p. 6). Thus, any students perceived to have lower than average academic abilities were presumably tracked into the lower, non-academic curricula that led to local option diplomas.

The vast body of literature on tracking is characterized by a sometimes acrimonious debate over the merits and problems inherent in the segregation of students based on academic achievement. The opposing sides of this debate typically frame their positions along two general beliefs: 1) the idea that the effectiveness and efficiency of instruction is increased when it is delivered to homogeneous groups of students sorted by academic abilities into separate classrooms, which is a pro-tracking argument; and 2) the idea that sorting criteria purportedly based solely on academic achievement are inevitably confounded by extraneous lurking variables including race, ethnicity and social economic status, which is an anti-tracking argument. Jeannie Oakes and others have convincingly argued that when such assessments of student ability were made, minorities and lower socio-economic classes were often tracked toward and over-represented in lower level curricula (Hallinan, 1994a, 1994b) (Hallinan and Soreneson, 1987) (Kubitshek and Hallinan, 1997) (LeTendre et al. 2003) (Ma, 2002) (Oakes, 1994) and (Useem, 1992). There are strong arguments, but relatively little empirical, quantitative research, supporting both sides of the debate.

Under the new dual track diploma system implemented in 1906, the expressive and instrumental orders of schools changed. Excellence in classical humanist studies, as determined by process of examination, was no longer the sole arbiter of success in secondary schools. Curricula based on more progressive themes and pedagogical approaches were introduced in the public schools of New York, and non-academically elite students were routinely tracked away from the classical humanist curricula and toward more progressive curricula. Age group and friendship patterns of students were in turn influenced by practices associated with grouping of students into differentiated tracks and the segregation of these different tracks of students from one another through the use of separate classrooms and separate teachers. Students in the upper track of secondary schools, the academic track, were expected to master a mathematics curricula steeply grounded in classical humanism, and for which the state conferred academic credentials. Students in the lower track were expected to pursue what might collectively be referred to as progressive curricula (Kleibard, 2004). The progressive curricula was without associated state credentials in the diploma system of the state of New York. Students in the upper tracks were preparing for different kinds of futures and occupational fates than were students in the lower tracks. And whether a student was in the upper track or the lower track was viewed as being influenced by variables other than academic ability.

The Third Era of Diplomas

The dual track diploma system, which started in the public high schools of the state of New York in 1906, existed uninterrupted, but sometimes challenged, until 1996. In 1996, a decision was made to revert back to a single track diploma system, and efforts began soon thereafter to dismantle the dual track system that was at the time ninety years old. It is still being dismantled in 2010, but most of the work is done. The transition to a single track diploma system for all students, under state control through the Regents examination system, is nearly complete. Unless exempted from process of examination by an individual education plan (IEP), all general education students from all social classes are now participating in the Regents examination system. The consequences of this decision are: 1) that all general education students in New York's public schools must now take Regents examinations in English, Mathematics, Science, Global History and U.S, History; and 2) the remnants of the progressive curricula of the past century, oriented to what Kliebard referred to as child centered and social meliorist educational agendas, are almost completely vanquished.

Tracking still exists in some schools, but the instrumental and expressive orders of schools have returned to the days of the academy system, when one set of state standards was perceived as appropriate for all students. On this view, the first and third eras of diplomas in the public schools of New York are quite similar in terms of the Regents examination system, especially when viewed through the lens of what has been assessed on Regents mathematics examinations. What is not identical between the first and third eras is the demographic profiles of the students taking Regents examination, and at this point we return to the research sample to look for changes in the examination structures that reflect changes in the demographics of students taking the examinations. By examining the historical record of mathematics assessment practices left by the Regents examination system during each of these three eras of diplomas, this research effort attempts to illuminate differences between examinations administered to academically elite students and non-academically elite secondary school students. The difference in assessment practices could then be attributed as evidence of the influence of the instrumental and expressive orders of schools on students of different social classes, thus providing opportunities for empirical validation of Bernstein's theories of how the micro-level practices of schools interact with social class.

Four Phases of Academic Rigor Observed in the Research Sample

Four identifiable phases of academic rigor have been identified in the research sample. These four phases of academic rigor are highly correlated with the three eras of academic diplomas discussed in the preceding sections of this Chapter.

- Scoring Phase I is first observed in the research sample with the Regents academic examinations administered in 1890. Presumably, these standards also applied to the twelve years of missing academic examinations between 1878 and 1889, but this presumption is unsupported. This phase continued at least through calendar year 1990. During this phase, Regents academic examinations in mathematics simply required that a student obtain 75% of the possible raw score points on the examination. Evidence of this scoring phase can be found in the research sample on the first pages of Regents mathematics examinations administered during 1890 and 1900.
- Scoring Phase II presumably began before calendar year 1909, when the minimum scores necessary to sustain an examination are not observed in the research sample on examinations administered that year. During this phase of academic rigor, the general scoring standard was typically maintained at 75% of the raw score points allowed on the examination, but each examination was structured to allow examinees limited choices over which problems would be attempted. The introduction of choice can arguably be interpreted as a lowering of

examination standards. Secondary sources not associated with the research sample indicate that the standard was dropped to 60% in 1914 (Horner, 1915), but the research sample clearly shows that the standard had returned to 75% by 1920.

- Scoring Phase III is associated with the post World War II implementation of the Sequential I, II, III mathematics curricula, and lowered the percentage of raw score points to 65%, while continuing to allow each examinee limited choices over which problems would be attempted. Exemplars of this Phase include all of the examinations of the Sequential I, II and III curricula.
- Scoring Phase IV was implemented subsequent to the 1996 decision by the Board of Regents and the State Education Department to require all general education students to take Regents examinations and earn Regents diplomas (with exceptions for students with individual education plans) The conversion charts associated with this scoring phase allow examinees to sustain an examination with as few as 34% of the possible raw score points. Exemplars of scoring phase IV can be seen in the examinations of the Math A/B curricula and the Integrated Algebra, Geometry and Algebra 2/Trigonometry curricula currently being implemented. Evidence from the historical record shows that the significant deterioration in academic rigor associated with this phase is related to increased numbers of non-academically elite students being forced to take Regents examinations.

Throughout each of these four phases, the number of students enrolled in schools and taking Regents examinations continued to increase.

The exact number of students sitting for Regents academic examinations in various years is difficult to ascertain, since the examinations are no longer forwarded to the state and consolidated state records do not exist. This fact notwithstanding, reasonable estimates can be inferred from the historical record. Because most enrolled students in 1875 never graduated from secondary schools, and many students in the academies were not qualified for Regents academic classes, it can be inferred that only a small subset of the 12,000 enrolled students sat for commencement level Regents mathematics examinations. The situation is similar in the year 1900, when an estimated total of 100,000 students were enrolled in academies and high schools, but a much smaller number completed their examinations and graduated. By 1965, the situation had changed due to the dual track system leading either to Regents diplomas or local option diplomas. Considering that an estimated one million students were enrolled in secondary schools in New York in 1965, the fact that only 65,000 Regents diplomas were awarded is an excellent indicator of the elitist nature of the Regents examination system. Between 1906 and 1996, the number of lower class students attending schools increased significantly. However, Regents diplomas and the Regents academic examinations associated with them were optional and were targeted toward students of average and above average ability (SED 1965). Today, the number of students enrolled in high schools and pursuing Regents diplomas is approximatey1.5 million. It is within this context of increasing numbers of students taking the Regents examinations in mathematics that the drop in academic rigor observed in scoring phase IV is next analyzed.

Phase IV began during the implementation of the 1996 decision to expand Regents testing to all students and revert to a single diploma system. Thus, in the first decade of the 21st Century, the Regents examination system was evolving from a quality control system for the humanist curricula and elite middle class students into a quality control system for humanist curricula and all classes of students. When the numbers of non-elite students taking the Regents mathematics examinations began to increase, the minimum percentage of raw score points necessary to sustain the examination plummeted. As this dissertation is written, it takes only

34% of the total possible raw score points to sustain the Regents Examination in Integrated Algebra, which is the only Regents mathematics examination necessary to earn a Regents diploma. This analysis begins with a look at the scoring conversion charts used with the examinations of the Mathematics A and Integrated Algebra curricula. A table summarizing the minimum percentages of raw score points necessary to sustain all of the Mathematics A examinations as well as the Integrated Algebra examinations administered through January 2010 is shown as Figure 5-4.

The Conversion Charts for the Regents Mathematics A Examinations			
Math A Test Date	Raw Score Needed for Passing Score of 65%	Math A Test Date	Raw Score Needed for Passing Score of 65%
June 99	43 of 85 points (51%)	August 03	No Test Given
August 99	47 of 85 points (55%)	January 04	37 of 84 points (44%)
January 00	44 of 85 points (52%)	June 04	37 of 84 points (44%)
June 00	41 of 85 points (48%)	August 04	36 of 84 points (43%)
August 00	41 of 85 points (48%)	January 05	34 of 84 points (40%)
January 01	46 of 85 points (54%)	June 05	36 of 84 points (43%)
June 01	46 of 85 points (54%)	August 05	34 of 84 points (40%)
August 01	47 of 85 points (55%)	January 06	33 of 84 points (39%)
January 02	48 of 85 points (56%)	June 06	35 of 84 points (42%)
June 02	52 of 85 points (61%)	August 06	34 of 84 points (40%)
August 02	53 of 85 points (62%)	January 07	35 of 84 points (42%)
January 03	52 of 85 points (61%)	August 07	34 of 84 points (40%)
June 03	51 of 85 points (60%)	January 08	34 of 84 points (40%)
		June 08	36 of 84 points (43%)
		August 08	36 of 84 points (43%)
		January 09	35 of 84 points (42%)

The Conversion Charts for the Regents Integrated Algebra Examinations	
IA Test Date	Raw Score Needed for Passing Score of 65%
June 08	30 of 87 points (34%)
August 08	30 of 87 points (34%)
January 09	31 of 87 points (36%)
June 09	30 of 87 points (34%)
August 09	30 of 87 points (34%)
January 10	30 of 87 points (34%)

Plummeting Scores Necessary to Sustain an Examination Figure 5-4

The above table is divided into two columns to highlight the position and importance of the June 2003 Regents Mathematics A examination in the historical record. The June 2003 Regents Mathematics A examination appears in the above table as the bottom entry in the left column. The information in the right column is taken from Regents mathematics examinations that were administered in the aftermath of the June 2003 Regents Mathematics A examination, including the first six Regents mathematics examinations of the Integrated Algebra curriculum . The right hand column shows a significant and continuing decline in minimum standards necessary to sustain an examination during the Mathematics A curriculum, and this decline in minimum standards continues into the successor curriculum, Integrated Algebra. The question posed by these data is simply, "What happened with the June 2003 Regents Mathematics A examination?" What follows is one plausible explanation.

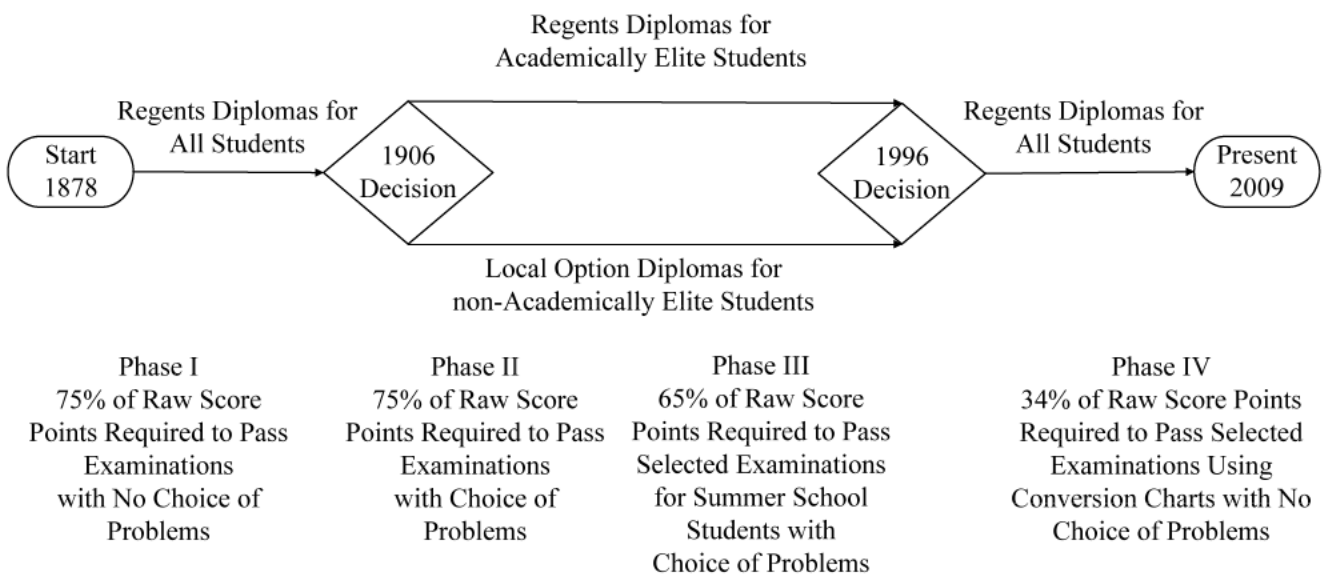
From June 1999 to January 2003, schools throughout the state of New York were beginning to implement new graduation standards. These new graduation standards required that schools make incremental progress toward eliminating local option diplomas and expand the Regents examination system to assess the achievement of all secondary school students who were not exempt from the Regents examination system because of individual education plans. Anecdotal evidence from interviews collected during the course of this research suggest that the profile of past academic abilities of large numbers of students who took the June 2003 Regents Mathematics A examination were lower than the academic abilities of students who had historically been tested. Echoing the ideas of Lawrence A. Cremin, the Regents examination system, once a hallmark of elitism in secondary education, was becoming popularized. (See page 191.) The Regents process of examination no longer segregated elite students from common students. Everyone was being tested. And in June 2003, when everyone was tested, approximately two-thirds of the examinees failed. The public outcry from parents and politicians across the state caused the State Education Department to blink. The SED cancelled the August 2003 administration of the Regents Mathematics A examination, and immediately set upon a plan to prevent another repeat of the examination that morphed into a political spectacle.

Immediately after the June 2003 administration, Regents Mathematics A testing was suspended throughout the state of New York and schools were given flexibility in graduating students, even if they had not achieved the requisite 51 raw score points necessary to achieve a 65% scaled score on the “underperforming” examination. A “blue ribbon” panel was created to review the causes of the failure of the examination and what had gone wrong. No Regents Mathematics A examination was administered in August 2003, while the committee was doing its work. After the blue ribbon panel completed its work and reported to the Board of Regents in October, 2003, the Regents Mathematics A examination was redesigned to have more multiple choice problems and fewer open response problems (Mills, 2003).

When Regents Mathematics A testing resumed in January 2004 with newly redesigned examinations, the number of raw points required for a passing grade of 65% was significantly lowered. Prior and up to June 2003, all Regents Mathematics A examinations had 85 raw score points, and a student needed approximately 55% of the raw score points to obtain a passing scaled score of 65% or higher. The redesigned examinations beginning in January 2004 had a maximum raw score of 84 points and the number of raw score points needed to obtain a passing scaled score of 65% was reduced to approximately 42% of the available raw points. This standard of approximately 42% of available raw score points held constant for the remaining administrations of the Regents examinations for the Mathematics A curriculum. When the Integrated Algebra was implemented to replace the Mathematics A curriculum, the standard of

42% was dropped to approximately 34%, where it has remained through January 2010. There have been no repeats of the June 2003 debacle involving the Regents Mathematics A examination, and there is little if any evidence to suggest any conclusion other than minimum standards have been lowered as Regents mathematics examinations have become popularized.

With this analysis of what happened during Phase IV complete, the four scoring phases described in the preceding paragraphs can be loosely correlated with the three eras of diplomas as shown in Figure 5-5.



Four Phases of Academic Rigor Figure 5-5

Summary

Based on the historical record and as evidenced by the research sample, subjecting non-academically elite students to the Regents examination system has resulted in a lowering of the minimum thresholds for sustaining examinations and qualifying for the Regents academic diploma credential. This lowering of scoring standards does not necessarily mean that desired levels of educational achievement have been lowered. Rather, it reflects a reality that academically elite students typically have had higher achievement levels on Regents examinations than has the general population. When an examination system tests only the elite, higher standards are to be expected. Under this interpretation of the historical record, by making the Regents examination mandatory for all students, the state changed the demographics of examinees, which eventually resulted in a dilemma – either lower the standards associated with passing a Regents examination, or fail large numbers of students. The state appears to have anticipated that some adjustments might be necessary, and changed to a system of "grading on a curve" as the transition from a dual diploma system to a single diploma system began. This allowed the 65% standard for passing a Regents examination to be manipulated to fit a statistical curve. When this statistical curve is stripped from the analysis, and a more consistent metric applied to the historical record, the deterioration in scoring standards for Regents mathematics examinations during the first decade of the 21st Century looks like the curve of a hockey stick, and the curve is pointed down.

CHAPTER VI - SUMMARY AND RECOMMENDATIONS

Overview

This chapter summarizes the research findings in relation to the two research questions, critiques the overall research effort, and suggests new pathways for continuing research.

A Summary of Changes Observed in Regents Mathematics Assessments

The history of the Regents examination system as a control system over mathematics education in the state of New York can be divided into numerous different eras, some of which overlap one another. Each of these eras has some impact on the instrumental and expressive orders of public schools. Some of the more significant eras are:

1. The era of the preliminary examinations (1866 - 1959), in which the state regulated admission passage from elementary school into a class of privileged academic scholars in secondary schools.
2. The era of the academic examinations (1878 - 2010), in which the state used the Regents examination system to control the curricula of publically funded secondary schools.
3. The first era of a single diploma system (1878 - 1906), in which the state recognized no secondary school diplomas other that the Regents academic diploma.
4. The era of the dual diploma system (1906 - after 1996), in which the state recognized both local option diplomas and Regents academic diplomas in publically funded secondary schools.

5. The second era of a single diploma system (circa1996 - present), in which the state once again recognizes no secondary school diplomas other that the Regents academic diploma for general education students.
6. The four general scoring eras of: a) 75% minimums with no choice (c.1866 - circa1906); b) 75% minimums with choice (circa1906 - circa1950); c) 65% minimums with choice (circa1960 - 2004); and d) 34% minimums with no choice (c.2008 - Present).
7. The era when Regents examinations were targeted at academically elite students (1866 - circa 2002).
8. The era when Regents examinations were targeted at all general education students (circa 2003 - present).
9. The era of slide rules and reference tables (1866 - circa1990).
10. The era of electronic calculators (circa 1994 - present)

Throughout each of this incomplete list of eras, the research sample suggests that assessed Regents mathematics curricula at the secondary school level have remained grounded in a classical humanist agenda. The research sample also reflects a general decade-to-decade stability in the topics that are assessed, with incremental change during each decade. The examinations of long ago and the examinations of today are quite similar, as are the examinations administered during the intervening years. This observation suggests that the ongoing struggle between progressive and traditional forces for control of mathematics education in the state of New York has not penetrated the curriculum and assessment practices of the Regents examination system. Rather, the Regents examination system has historically insulated the classical humanist agenda from the agendas of more progressive educators.

The history of the Regents examination system also reflects important decisions and accommodations between competing interest groups for control of mathematics curricula and assessment practices in the public schools of the state of New York. Prior to the Civil War, the state of New York did not exercise effective control over curricula and assessment practices in publicly funded schools. When the old academies of New York were perceived as abusing state funding by lowering academic standards during the midst of the Civil War, the State moved to establish more rigorous regulatory control over both curricula and assessment practices. Thus were born the Regents preliminary examinations. Since the advent of the Regents academic examinations in 1878, the state of New York has mandated a traditional humanist agenda in mathematics education, and given it preference over more progressive agenda by associating the traditional humanist agenda with state sponsored academic credentials known as Regents diplomas.

When progressive voices called for alternative approaches in public education at the beginning of the 20th Century, the state of New York ceded control over progressive education practices to local schools and school districts by creating local option diplomas. Thus the era of the dual diploma system came into being and would last for approximately 100 years. During the last decade of the 20th century and the first decade of the 21st Century, as conservative and progressive voices both argued for more and better mathematics education for all students, the state eliminated the local option diploma that was associated with progressive education practices and restored the old “one size fits all” standard, thus requiring once again that all students study a Regents based traditional humanist mathematics curriculum in order to graduate from secondary school.

The second era of a single diploma system for secondary education in the state of New York is significantly different than the first era, which ended in 1906, because the very nature of public schools and the students who attend them has changed during the intervening century of the dual diploma system, which featured shared state and local control over curricula and assessment practices in public education. As the first decade of the 21st Century ends, with local control almost gone from the curricula and assessment practices of New York's public schools, the Regents examination system continues to assess classical humanist mathematics topics that are not significantly different than assessed topics of decades long past. What is being assessed has changed little. Who is being assessed has changed much.

Research Question # 1:

How has the classification and framing of assessed knowledge in the core subject area of mathematics changed in Regents level examinations administered in the public schools of New York since 1866?

The classification and framing of assessed knowledge in the core subject area of mathematics in the secondary schools of New York has changed little since 1890, when the first academic examinations appear in the research sample. Approximately two-thirds of the topics assessed in modern Regents mathematics curricula in calendar year 2009 were also assessed in the mathematics curricula of 1890 and before. Analysis of data from the research sample suggests that curricular reform in mathematics education in the public schools of New York since the Civil War, as measured by changes in assessment practices in the Regents examination system, is metaphorically comparable to pouring old wine into new glasses. Despite numerous changes in curricula over a span of 14 decades, and despite much public discourse and rhetoric suggesting otherwise, there has been relatively little change from decade to decade in assessment practices and the mathematical topics that are assessed. The cumulative total of all incremental changes over the decades has never moved the Regents mathematics curricula away from its early fundamental grounding in classical humanism. Accordingly, the recent movement toward egalitarian education in the public schools of New York, with one curriculum for all students, has restricted progressive education opportunities for students and placed all general education students in the predicament of having to sustain a mathematics class that is grounded in classical humanism in order to meet high school graduation requirements.

Elements of Basil Bernstein’s theory of educational transmissions, status attainment theory, and control theory are well supported by the findings of this research study. The Regents examination system features the necessary characteristics of an effective control system. In exercising control, the Regents examination system relies on valued educational credentials to shape and define the instrumental and expressive orders of public schools, which Bernstein posits as important factors in mediating the different experiences of students from different classes in schools. The elaborated codes of secondary school mathematics are seen as relatively independent from their evoking contexts in mathematics assessments, thus lending support to Bernstein’s theories concerning class codes. In defining standards associated with what will be taught and assessed in Regents curricula in the secondary schools of the state of New York, the Regents examination system exemplifies Bernstein’s theories concerning knowledge production, knowledge recontextualization and knowledge reproduction in schools.

Research Question #2:

How has popularization influenced the contents, structure and academic rigor of Regents mathematics examinations?

The historical record supports the view that Regents curricula and Regents academic examinations were optional for most secondary school students between 1878 and the first decade of the 21st Century. Prior to 1906, the Regents curricula can be considered optional because secondary schooling itself was optional for most students. As effective compulsory school attendance and child labor laws were enacted, and as the general demand for secondary education increased, class diversity of students also increased. However, the Regents academic curricula and Regents diplomas were made optional in 1906, and local option diplomas facilitated the demand for more progressive secondary school curricula. This dual option diploma system was effectively dismantled between 2000 and 2010, resulting in significant growth in the numbers of students taking Regents curricula and Regents examinations. What was once an optional secondary school curriculum for elite academic students is now a mandatory curriculum for all general education students.

This research effort provides empirical evidence that the academic rigor of Regents mathematics examinations has decreased in the past decade, concurrent with the elimination of the dual option diploma system and significant increases in the number of students participating in the Regents examination system. When minimum passing standards for Regents mathematics examinations are analyzed over a long period of time using a consistent metric, which is defined in this study as the percentage of raw score points necessary to sustain a Regents mathematics examination and earn credit toward a Regents academic diploma, the popularization of the Regents examination system is accompanied by deterioration in academic rigor.

There are four scoring phases associated with the Regents examination system over a span of 144 years. During the first three of these scoring phases, Regents mathematics examinations were optional and Regents curricula were taken primarily by academically elite students. During these first three scoring phases, the minimum percentage of total raw score points necessary to sustain a Regents mathematics examination is believed to have never dropped below 60%. During the fourth of these scoring phases, all students are required to take Regents mathematics examinations and the minimum percentage of total raw score points necessary to sustain a Regents mathematics examination is approximately 34%. These facts support the idea that the academic rigor necessary to sustain a Regents mathematics examination has been lowered due to the influence of non-academically elite students being exposed to the Regents examination system.

The Importance of this Research Effort

This research is important for several reasons. First, a total of 1,534 Regents mathematics examinations have been located and preserved in digital format during the course of this research, constituting what is believed to be the largest single repository of any state's historical mathematics assessment practices. Second, a robust and representative sample of these Regents mathematics examinations has been converted into a primary source database of historical Regents mathematics assessment practices. This database has been placed in the public domain and has potential value not only for future academic research, but also as a collection of free, high quality mathematics education resources for classroom instruction. Also of importance is the idea that a new pathway has been pioneered for understanding long term trends in micro-level assessment practices in public schools. Said differently, the research sample developed in this research effort has potential applicability to a wide variety of research interests.

This study is also important because the Regents examination system is an excellent exemplar for research purposes of the control paradigm involving high stakes testing that is embedded in the federal No Child Left Behind (NCLB) Act, which is currently scheduled for congressional debate and renewal during calendar year 2010. This research suggests that high stakes testing is associated with more academic rigor when all general education students are not exposed to the same curricula and evaluative standards. On this view, educators have failed in their attempts to raise all students to the level of academic achievement that was previously associated with academically elite students, and different standards and different assessment paradigms for students of different interests and abilities might lead to higher academic standards for all students. In short, when “one size fits all” is used as a guiding principle in curriculum design, as it has been in New York since 1996, high stakes testing is not sufficient to ensure high levels of academic rigor.

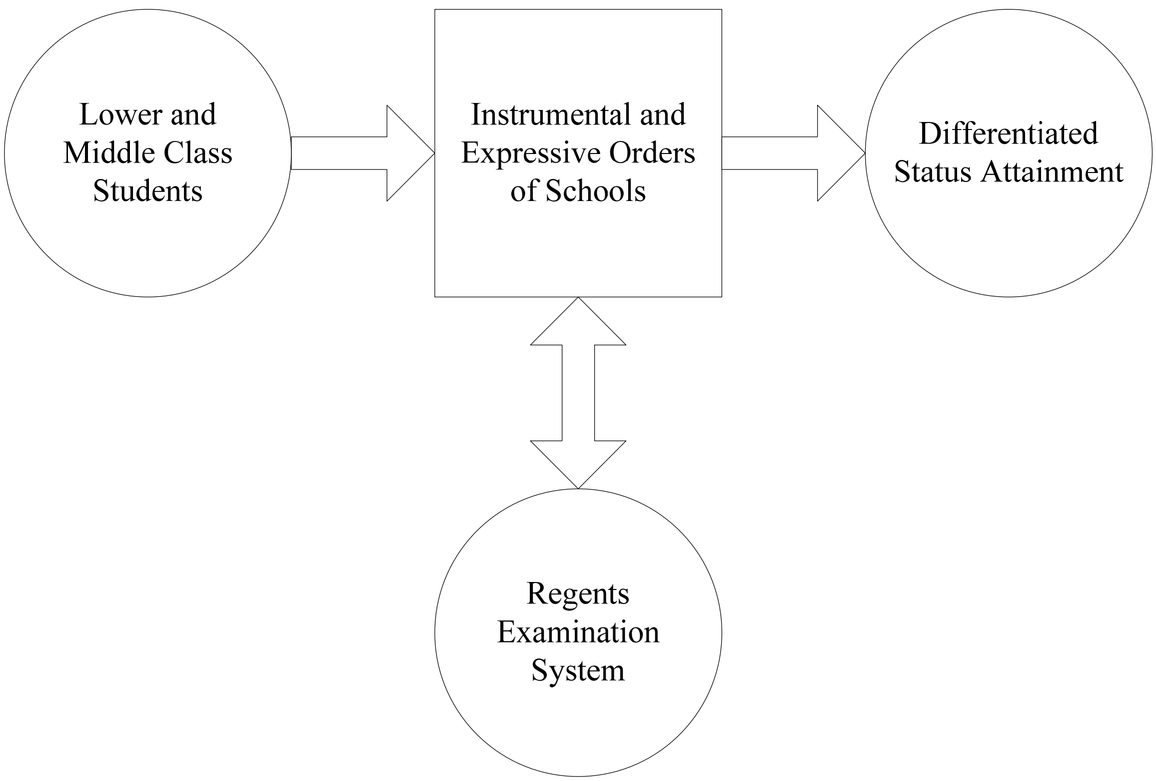
The findings of this research effort have numerous educational policy implications not only for high stakes testing, but also for the design and implementation of new schools and new curricula. In the absence of new and improved teaching methods that effectively and efficiently raise all students to the levels of achievement previously associated with the Regents examination system and Regents academic diplomas before popularization, policymakers may wish to reconsider the appropriateness of curricula based on the assumption that college is the appropriate outcome of all secondary schooling. This leads to questions about school and curricula design that involve tracking and differentiated outcomes of schooling that may reasonably be expected considering the differentiated efficacy of educators when dealing with students of differentiated academic abilities, differentiated social status, differentiated educational interests, and differentiated vocational goals.

What This Study is Lacking - A General Critique of the Current Research Effort

The positives of the preceding paragraphs notwithstanding, the current research effort has several shortcomings. These shortcomings are mostly due to the narrow focus of the original research questions and the specific research methodologies used in answering them. The two research questions have been answered. The first question concerns stability and change in Regents assessment practices. The second question concerns the impact of popularization on the Regents examination system. In answering these two research questions, a larger question is illuminated, and that question is whether schools reproduce social stratification. The evidence left by the Regents examination system suggests that the experience of schooling has rarely been the same for all students and that current efforts to make the experience of schooling more egalitarian are associated with lowered standards for academically elite students. What is not known is: 1) the extent to which non-academically elite students are learning more mathematics as a result of their exposure to the Regents examination system; and 2) whether the life skills and opportunities of non-academically elite students are improved as a result of their exposure to the Regents examination system. On this view, questions remain as to the differential impact of the

Regents examination system in mediating status attainment for different social classes of students.

The following diagram generally represents the idea, which is grounded in both Basil Bernstein's theory of educational transmissions and credentials theory, that students of various social classes are inputs into the instrumental and expressive orders of schools, whereupon, their interactions with these instrumental and expressive orders influence their status attainment and station in life after leaving school.



Social Stratification and the Regents Examination System Figure 6-1

The current research illuminates much about the vertical dimensions of this model, which is to say that this study illuminates how the Regents examination system has influenced the instrumental and expressive orders of public schools in the state of New York. The current research also illuminates the political reality in New York that the instrumental and expressive orders of public schools cannot fail large numbers of students. Hence, the relationship between the Regents examination system and the instrumental and expressive orders of schools is bidirectional, which explains why the academic standards of rigor associated with Regents mathematics examinations were decreased by the state when more and more non-academically elite students began taking the examinations.

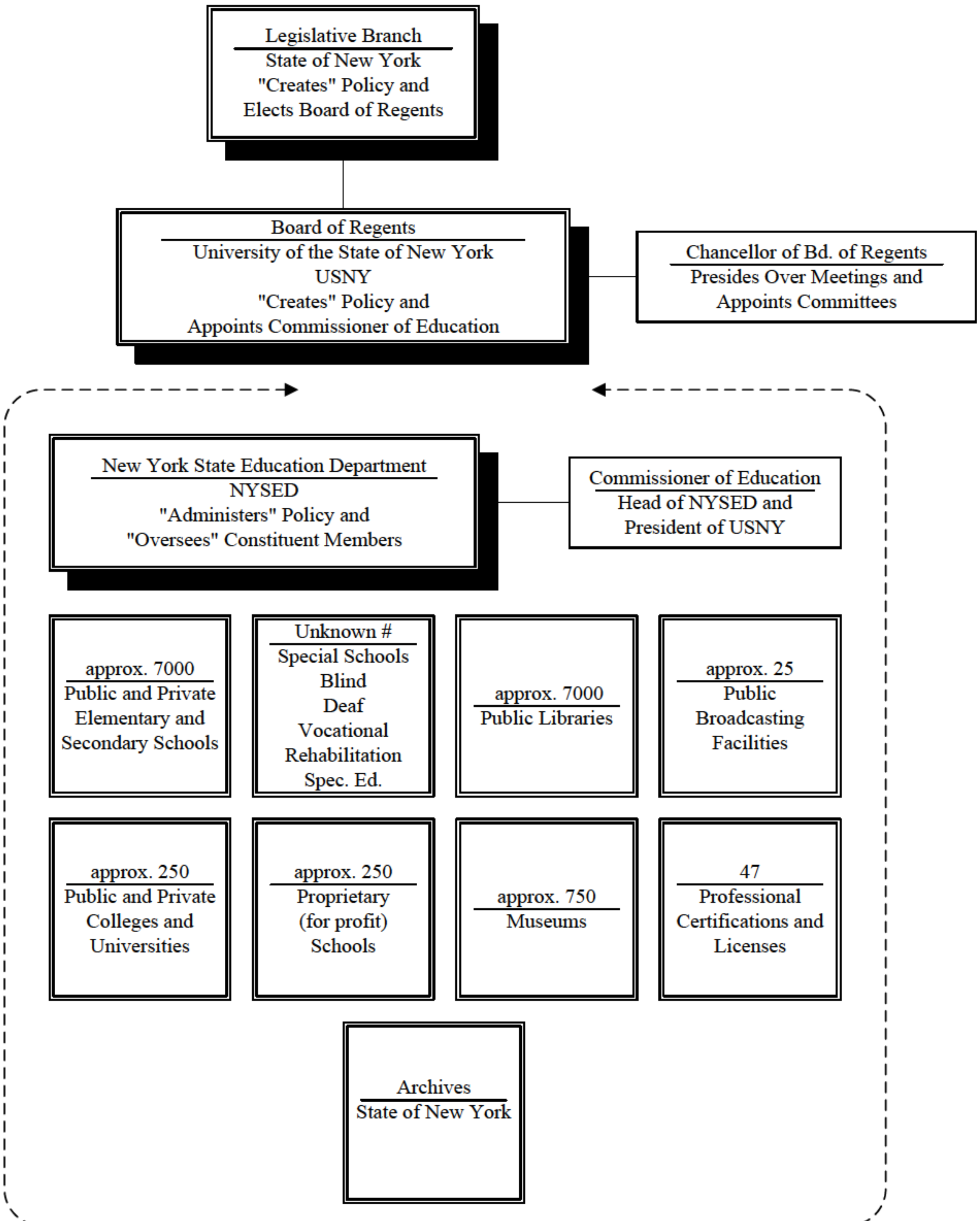
What this study does not do is link the horizontal components of Figure 6-1, which is to say that this study does not, except in very general terms, link differentiated social classes of students input into the instrumental and expressive orders of schools with differentiated status attainment outcomes. The current study focuses on the micro-level practices of schools and not on the outcomes of schooling. Accordingly, the current research effort sheds only a small light on the social stratification effects of public schooling, other than to note in very general terms that, historically: 1) tracking decisions have often been associated with class bias; and 2) students from middle class backgrounds have been more likely to be exposed to the Regents examination system and thus to receive credentials of greater value. These findings are generally consistent with the theories of social stratification embedded within Randall Collins' credentialism and Basil Bernstein's educational transmissions. The following paragraph suggests how the horizontal axis of Figure 6-1 might be examined in a future research effort.

A Suggestion for Further Research

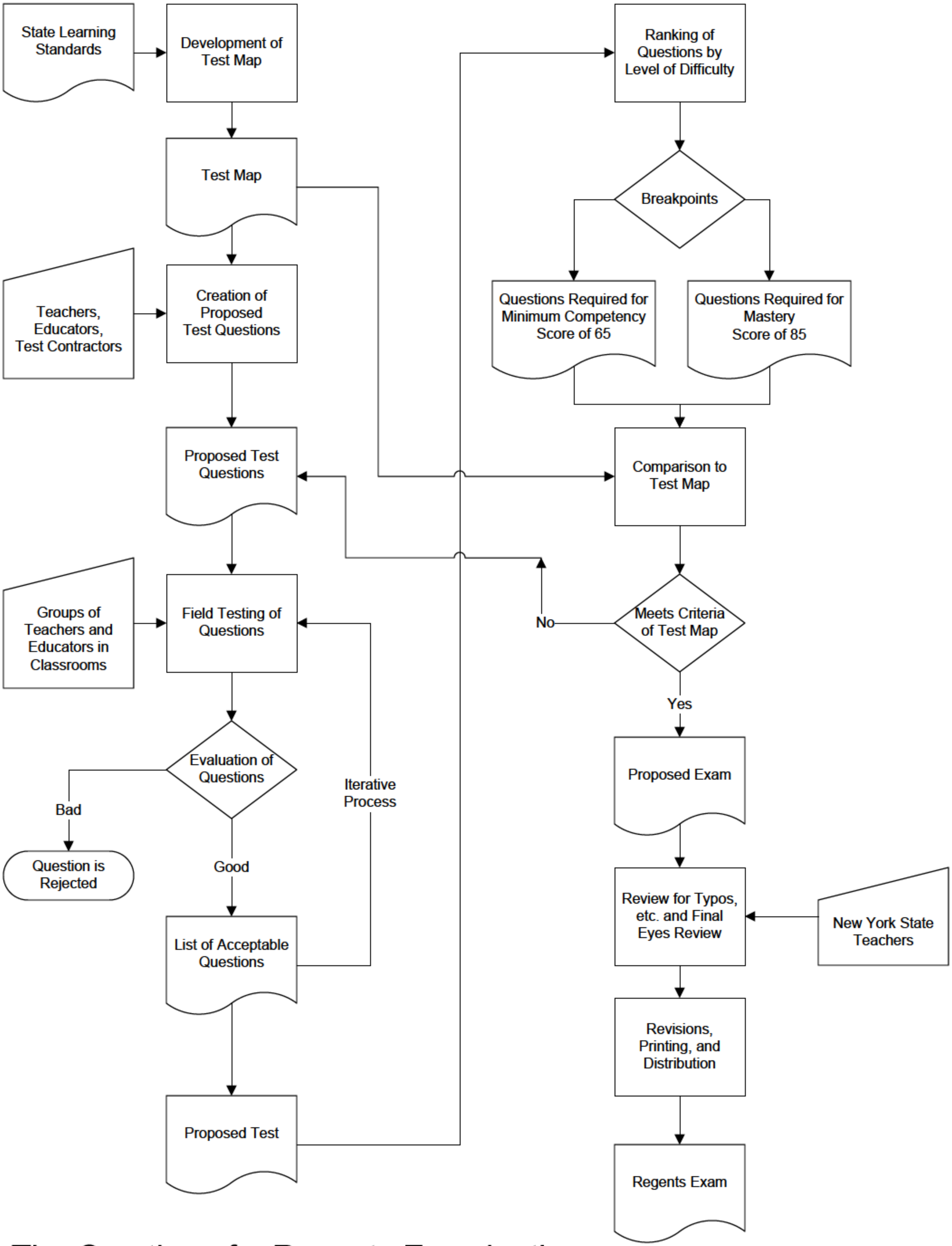
With regards to recommendations for future research, it would be desirable to focus on the horizontal arrows of Figure 6.1. These arrows represent two new research questions, neither of which can be answered using the methodologies associated with the current research effort.

These two new research questions are: 1) What are the relationships between student socio-economic status and assignment to the instrumental and expressive orders of schools that are associated with the Regents examination system? and 2) What are the relationships between Regents academic diplomas and status attainment following high school? The first question relates to the left arrow and the second question relates to the right arrow. In designing a future research effort to address these questions, one might gather and analyze data from three sources: 1) identification of students as low income under federal school funding guidelines; 2) achievement scores on Regents examinations; and 3) status attainment following high school. Such a study could involve either quantitative or qualitative methods, or both, and would be a natural extension of the current research study, which sheds light primarily on the vertical dimensions of Figure 6-1. Ideally, such a study would look at cohorts of students during the second and third eras of diplomas. Specific avenues of inquiry could be directed toward: 1) understanding the input relationships between student eligibility for federal Title 1 funding and Regents examination scores during the decades preceding and following the 1996 decision to move to a single diploma system; and 2) understanding the output relationships between Regents examination scores and high school graduation rates for students of different socio-economic status. Such a study could significantly increase our understanding of the social stratification effects of public schooling and would be an appropriate follow-up study to the current research effort.

THE SCOPE AND AUTHORITY OF THE NEW YORK STATE BOARD OF REGENTS IN THE 21ST CENTURY



Constituent Members of the University of the State of New York (USNY)



The Creation of a Regents Examination

The Known Population In 2009 of Extant Regents Mathematical Examinations including The Sample of Examinations Selected for Analysis

Legend	Assessed Curricula	Number of Extant Examinations Collected During Research Effort
A2	Algebra 2 & Trigonometry	1
AA	Advanced Algebra	125
AAR	Advanced Arithmetic	51
AG	Analytical Geometry	1
AL	Algebra	48
AR	Arithmetic	165
BA	Business Arithmetic	3
BK	Bookkeeping II	8
CA	Commercial Arithmetic	13
CS	Conic Sections	1
EA	Elementary Algebra	66
EY	11th Year Math	54
GE	Geometry	3
HA	Higher Algebra	2
IA	Integrated Algebra	6
IN	Intermediate Algebra	128
MA	Math A	30
MB	Math B	27
MP	Mathematics (Preliminary)	21
NY	9th Year Math	30
PG	Plane Geometry	166
PT	Plane Trigonometry	86
S1	Sequential Course I	55
S2	Sequential Course II	64
S3	Sequential Course III	53
SG	Solid Geometry	128
SP	Special Geometry	3
ST	Spheric Trigonometry	14
TH	Third Year Mathematics	14
TR	Trigonometry	65

TW	12th Year Math	26
TY	10th Year Math	77
	Total Number of Extant Examinations	1534

All Examinations from Shaded Years are Included in the Research Sample Selected for Analysis

The Known Population In 2009 of Extant Regents Mathematical Examinations including The Sample of Examinations Selected for Analysis

Months During Which Examinations Were Administered

	January-April	June	August-December
2009	MA MB IA	MB IA GE	MB IA GE
2008	MA MB	MA MB IA	MA MB IA
2007	MA MB	MA MB	MA MB
2006	MA MB	MA MB	MA MB
2005	MA MB	MA MB	MA MB
2004	S3 MA MB	MA MB	MA MB
2003	S2 S3 MA MB	S3 MA MB	S3 MB
2002	S1 S2 S3 MA MB	S2 S3 MA MB	S2 S3 MA MB
2001	S1 S2 S3 MA	S1 S2 S3 MA MB	S1 S2 S3 MA MB
2000	S1 S2 S3 MA	S1 S2 S3 MA	S1 S2 S3 MA
1999	S1 S2 S3	S1 S2 S3 MA	S1 S2 S3 MA
1998	S1 S2 S3	S1 S2 S3	S1 S2 S3
1997	S1 S2 S3	S1 S2 S3	S1 S2 S3
1996	S1 S2 S3	S1 S2 S3	S1 S2 S3
1995	S1 S2 S3	S1 S2 S3	S2 S3
1994	S1 S2 S3	S1 S2 S3	S1 S2 S3
1993			S1 S2 S3
1992			
1991			
1990	EY S2 S3	S1 S2 S3	S1 S2 S3
1989	S1 S2 S3	S1 S2 S3	S1 S2 S3
1988	NY S1 S2	S1 S2 S3	
1987	NY S1 S2 S3	NY S1 S2 S3	NY S1 S2 S3
1986	NY TY EY S1 S2 S3	NY TY EY S1 S2 S3	NY TY S1 S2 S3
1985	EY S2 S3	EY S2 S3	EY S2 S3
1984	EY S2 S3	EY S2 S3	TY EY S2 S3
1983	TY S1 S2	TY S1 S2	TY S1 S2
1982	TY S1 S2	TY S1 S2	TY S1 S2
1980	TY S1	TY EY S1 S3	TY S1 S2
198	NY TY EY S1 S2	NY TY EY S1 S2 S3	NY TY EY S1 S2
1979	NY S1 S2	TY S1 S2	TY S1 S2
1978	TY	TY S1 S2	TY EY
1977	NY TY	NY TY S1 S2	TY
1976	TY	TY SP	NY
1975	TY	NY TY EY SP	NY TY
1974	NY NY		NY

The Known Population In 2009 of Extant Regents Mathematical Examinations including The Sample of Examinations Selected for Analysis

Months During Which Examinations Were Administered

	January-April	June	August-December
1973	NY	NY	NY
1972	TY	NY	TY
1971	TY	TY	TY
1970	NY TY EY	NY TY EY SP	NY TY EY
1969	TY EY	TY EY	TY EY
1968	TY	TY	TY
1967	TY EY	TY	NY
1966	EY	NY TY EY TWA TWB	
1965	TY TWA TWB	NY TY EY TWA TWB TWE	NY TY EY
1964	IN TR TY TWA TWB	TY EY TWA TWB	
1963	IN TY TWB	IN TR TY EY TWA TWB	IN TR TY EY
1962	IN TR TY TWA TWB	IN TR TY EY TWB	IN TR TY EY
1961	IN TR TY TWA TWB	IN TR TY EY TWA TWB	BA IN TR TY EY
1960	IN AA TR TY EY TWA SG	IN TR TY EY TWA TWB	IN TR TY EY
1959	IN AA TR TY EY TWA SG		
1958	IN AA TR TY EY TWA PG SG	IN EY	IN
1957	PG IN EY	PG IN TY EY	IN
1956	IN EY	PG TY EY	IN
1955	IN TY EY	IN EY	IN
1954	EY	IN EY	IN
1953	IN AA TR PG SG MP TY EY BK	IN AA TR PG SG MP TY EY BK	
1952	IN AA TR PG SG MP TY EY BK	IN AA TR PG SG MP TY EY BK	IN TR PG SG
1951	IN AA TR PG SG MP BK	IN AA TR PG SG MP TY EY BK	IN TR PG SG
1950	IN AA TR PG SG MP BK	IN AA TR PG SG MP TY EY BK	IN TR PG SG
1949	MP IN AA TR PG	MP IN AA TR PG SG TY EY	IN TR PG SG
1948	MP IN TR PG SG	MP TR PG SG TY	EY
1947	MP IN AA TR PG SG	MP IN AA PG SG TR TY	
1946	MP IN AA TR PG SG	MP IN AA TR PG SG	
1945	MP IN AA PG	MP IN AA PG TR	
1944	MP IN AA TR PG	MP IN AA PG TR	
1943	AR IN AA TR PG SG	MP IN AA PG SG TR	
1942	AR IN AA PT PG SG	AR IN AA PG SG PT	
1941	AR IN AA PT PG SG	AR IN AA PG SG PT	
1940	AR IN AA PT PG SG	BA AR IN AA PG SG PT	BA IN PG SG PT
1939	AR IN AA TH PT SG	AR IN AA SG PT	
1938	AR IN TH PG SG	AR TH IN AA PG SG	

The Known Population In 2009 of Extant Regents Mathematical Examinations including The Sample of Examinations Selected for Analysis

	Months During Which Examinations Were Administered		
	January-April	June	August-December
1937	AR IN AA TH PT PG SG	AR TH IN AA PG PT	
1936	AR IN AA TH PT SG	AR TH IN AA PG SG PT	
1935	AR IN AA PG TH PT SG CA	AR TH AA PG SG PT	
1934	AR IN AA TH PG SG CA	AR TH IN AA PG SG PT CA	CA
1933	AR EA IN AA TH PT PG SG CA	AR EA IN AA TH PG SG PT CA	CA
1932	CA	CA	CA
1931	AR EA IN AA PT PG SG CA	AR EA IN AA TH PG SG PT CA	
1930	AR EA IN AA PT PG SG CA	AR EA IN AA PG SG PT	EA IN AA PG SG PT
1929	AR EA IN AA PT PG SG	AR EA IN AA PG SG PT	
1928	AR EA PT PG SG	AR EA AA PG PT	
1927	AR EA IN AA PT PG SG	AR EA IN AA PG SG PT	EA AA PG SG PT
1926	AR EA IN AA PT PG SG	AR EA IN AA PG SG PT	EA IN AA PG SG PT
1925	AR IN AA PT PG SG	AR EA IN AA PG SG PT	EA IN AA PG SG PT
1924	AR ST EA IN AA PT PG SG	AR EA IN AA PG SG ST PT	IN AA PG SG PT
1923	AR ST EA IN AA PT PG	EA IN AA PG SG ST PT	EA IN AA PG SG PT
1922	AR ST EA IN AA PT PG SG	AR EA IN AA PG SG ST PT	EA IN AA PG SG PT
1921	ST EA IN AA PT PG SG	AR EA IN AA PG SG SG ST PT	EA IN AA PG SG PT
1920	AR EA IN AA TR PT PG SG	AR EA IN AA TR PG SG PT	EA IN AA SG PT
1919	AR EA IN AA TR PG SG	AR EA IN AA TR PG SG PT	EA IN AA PG SG PT
1918	AR EA IN AA TR PT PG SG	AR EA IN AA TR PG SG PT	EA IN AA PG SG PT
1917	AR EA IN AA TR PG SG	AR IN AA EA PG SG	EA IN AA PG SG PT
1916	AR AAR EA IN AA TR PG SG	AR AAR EA IN TR PG SG	AAR EA AA PG
1915	AR AAR EA AA IN TR PG SG	AR AAR EA IN AA TR PG SG	
1914	AR AAR EA IN AA TR PG SG	AAR EA IN AA TR PG SG	AAR EA AA PG
1913	AR AAR EA IN AA TR PG SG	AR AAR EA IN TR PG SG	EA AA PG
1912			AAR EA AA PG
1911	AR	AR AAR EA IN AA TR PG SG	
1910			
1909	AR AAR EA IN AA TR PG SG	AR AAR EA IN AA TR PG SG	
1908	AR AAR EA IN AA TR PG SG	AR AAR EA IN AA TR PG SG	AR AAR EA IN PG
1907	AR AAR EA IN AA TR PG PT SG	AR AAR EA IN AA TR PG SG	AR AAR EA AA PG
1906	AR AAR AL AA TR PG SG	AR AAR EA IN AA TR PG SG	AR AAR EA PG
1905	AR AR AAR AAR AL AL AA TR PG PG SG	AR AAR AL AA TR PG SG	AR AAR AL PG
1904			AR AL PG
1903	AR AAR		
1902			AR

The Known Population In 2009 of Extant Regents Mathematical Examinations including The Sample of Examinations Selected for Analysis

Months During Which Examinations Were Administered

	January-April	June	August-December
1901			
1900	AR AR AAR AL AL AA PT PG PG SG	AR AAR AL AA PT ST PG SG	
1899	AR AR AAR AL AL AA PT PG PG SG	AR AAR AL AA PT ST PG SG	AR AL PG PG
1898	AR AAR AL AL PG PG SG PT	AAR ST AL AA PG PT	AR AL AL PG PG
1897	AR AR AAR AAR AL AL AA AA PG PG SG SG PT PT	AR AAR ST AL AA PG SG PT	AR AL PG
1896	AR AR AAR AAR AL AL AA PG PG SG PT PT	AR AAR ST AL AA PG SG PT	AR AR AL AL AA PG PG PT
1895	AR AAR AAR AL AL PG PG SG SG AA PT PT	AR AAR AL AA PG SG PT	AR AL AA PG
1894	AR AAR AAR PT AL AL PG PG SG SG PT PT	AR AAR AL AA PG SG PT	AR AL AL AL AA PG PG PG PT
1893	AR AR AAR AAR AL AL PG PG SG SG PT PT	AR AAR AL AA PG SG PT	AR AR AR AL AL PG PG PG
1892			AR AR AR AL PG PG PG
1891	AR AAR AL AL AA AG PG PG SG PT	AR AAR AL AA CS ST PG SG PT	
1890	AR AR AR AL AL HA HA PG PG PG SG SG PT PT	AR EA AA PG SG PT	
1889			
1888			
1887			
1886			
1885			
1884			
1883			
1882	AR	AR	
1881	AR AR	AR	AR
1880	AR	AR AR	AR
1879	AR	AR	AR
1878	AR	AR	AR
1877	AR	AR	AR
1876	AR	AR	AR
1875	AR	AR	AR
1874	AR	AR	AR
1873	AR	AR	AR
1872	AR	AR	AR
1871	AR	AR	AR
1870	AR	AR	AR
1869	AR	AR	AR
1868	AR	AR	AR
1867	AR	AR	AR
1866			AR

A COMPARISON BY CURRICULUM OF THE EXTANT POPULATION OF REGENTS MATHEMATICS EXAMINATIONS TO THE DATABASE

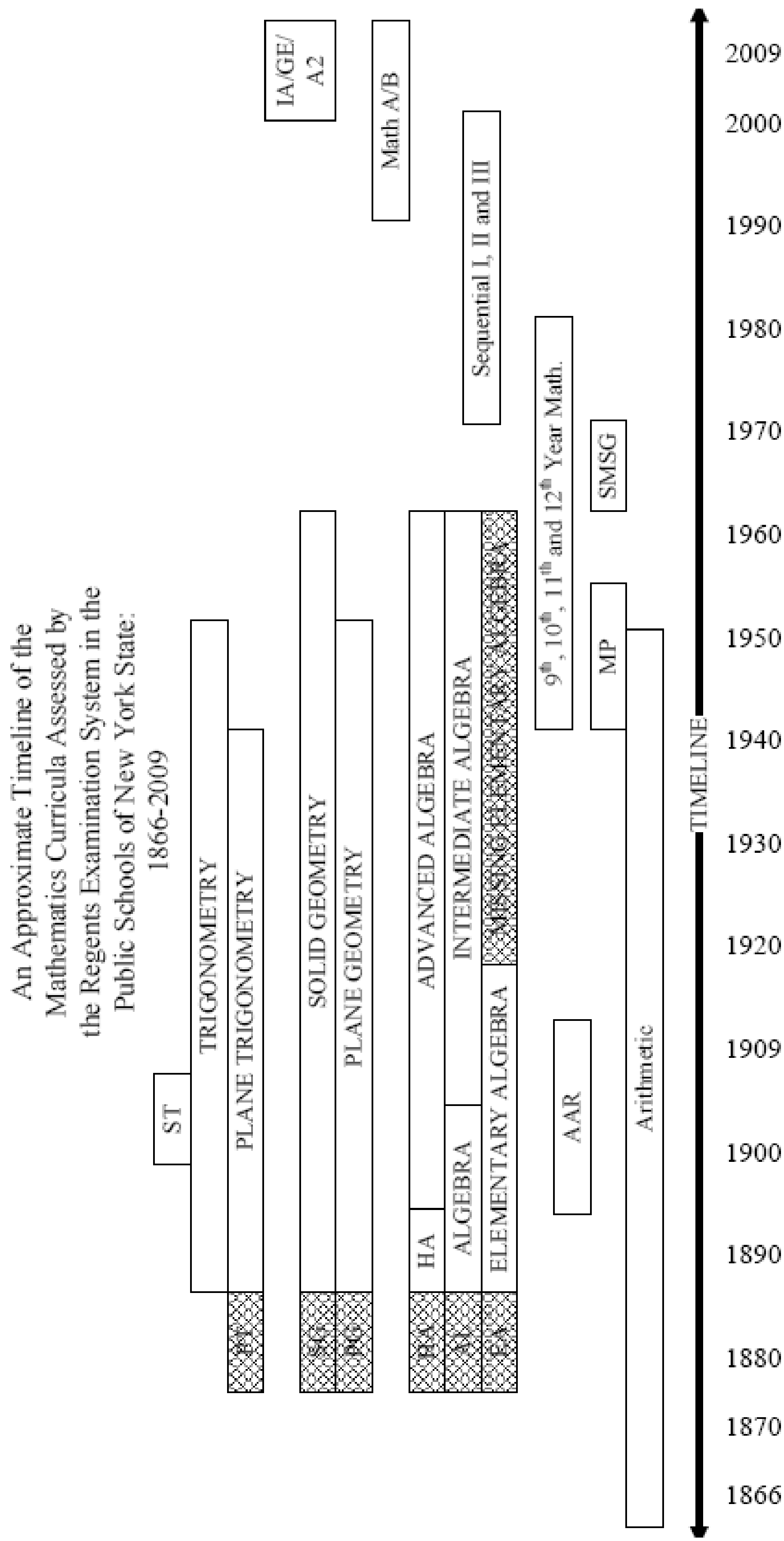
(as of December 31, 2009)

	INTEGRATED CURRICULA			
	# EXTANT EXAMS	# EXAMS IN DATABASE	FIRST DECADE	LAST DECADE
Algebra 2 & Trigonometry (A2)	0	0		
Geometry (GE)	3	2	2009	2009
Integrated Algebra (IA)	6	3	2009	2009
Math B (MB)	27	4	2009	2009
Math A (MA)	30	3	2000	2009
Sequential III (S3)	53	7	1980	2000
Sequential II (S2)	64	8	1980	2000
Sequential I (S1)	55	7	1980	2000
12th Year Math (TWA)	26	3	1960	1960
11th Year Math (EY)	54	12	1950	1990
10th Year Math (TY)	77	11	1950	1970
9th Year Math (NY)	30	6	1970	1980
Third Year Mathematics	14	0		

	DIFFERENTIATED CURRICULA			
	# EXTANT EXAMS	# EXAMS IN DATABASE	FIRST DECADE	LAST DECADE
Higher Algebra (HA)	2	2	1890	1890
Algebra (AL)	48	5	1890	1950
Advanced Algebra (AA)	125	17	1890	1890
Intermediate Algebra (IN)	128	16	1909	1960
Elementary Algebra (EA)	66	9	1890	1909
Solid Geometry (SG)	128	19	1890	1960
Plane Geometry (PG)	166	19	1890	1950
Analytical Geometry (AG)	1	0		
Special Geometry (SG)	3	1	1970	1970
Conic Sections	1	0		
Spheric Trigonometry (ST)	14	1	1900	1900
Plane Trigonometry (PT)	86	14	1890	1940
Trigonometry (TR)	65	10	1909	1960
Advanced Arithmetic (AA)	51	4	1900	1909

	PRELIMINARY CURRICULA			
	# of EXAMS IN COLLECTION	# of EXAMS IN DISSERTATION	START YEAR	STOP YEAR
Mathematics (Preliminary) (PM)	21	2	1950	1950
Arithmetic (AR)	165	22	1866	1940

	BUSINESS CURRICULA			
	# of EXAMS IN COLLECTION	# of EXAMS IN DISSERTATION	START YEAR	STOP YEAR
Bookkeeping II	8	0		
Business Arithmetic (BA)	3	1	1940	1940
Commercial Arithmetic	13	0		


An Approximate Timeline of the
Mathematics Curricula Assessed by
the Regents Examination System in the
Public Schools of New York State:
1866-2009
ST
TRIGONOMETRY
PLANE TRIGONOMETRY
SOLID GEOMETRY
PLANE GEOMETRY
HA
ADVANCED ALGEBRA
ALGEBRA
INTERMEDIATE ALGEBRA
ELEMENTARY ALGEBRA
MISSING ELEMENTARY ALGEBRA
IA/GE/
A2
Math A/B
Sequential I, II and III
9th, 10th, 11th and 12th Year Math.
AAR
MP
SMSG
Arithmetic
TIMELINE
2009
2000
1990
1980
1970
1960
1950
1940
1930
1920
1909
1900
1890
1880
1870
1866

Taxonomy of Knowledge Reproduced in New York Schools	A LISTING OF CURRICULA IN WHICH ASSESSED TOPICS ARE INCLUDED																								
	AA	AAR	AL	AR	BA	EA	EY	GE	HA	IA	IN	MA	MB	MP	NY	PG	PT	S1	S2	S3	SG	SMSG	ST	TR	TW
Absolute Value										IA					NY										
Alligation	AA					EA					IN														TW
Analysis of Data										IA															
Area and the Coordinate Plane																		S1	S2						
Arithmetic Operations		AAR		AR																					
Arithmetic: Addition				AR	BA									MP											
Arithmetic: Division		AAR		AR										MP											
Arithmetic: Multiplication			AL	AR										MP											
Arithmetic: Numeration				AR																					
Arithmetic: Place Value				AR											NY										
Arithmetic: Subtraction				AR										MP											
Bills and Receipts				AR										MP											
Binomial Expansions	AA		AL			EA			HA		IN		MB							S3					TW
Binomial Expansions: Undetermined Coefficients	AA																								
Brokerage and Commission		AAR		AR										MP											
Calculus: Differential	AA																								TW
Calculus: Integral	AA																								TW
Central Tendency										IA		MA						S1		S3					
Central Tendency: Average Known with Missing Data												MA						S1							
Central Tendency: Averages	AA			AR	BA						IN	MA		MP	NY			S1							
Central Tendency: Dispersion													MB							S3					
Central Tendency: Normal Distributions	AA												MB							S3					
Circles: Arc Measure							EY									PG	PT								
Circles: Area of				AR							IN	MA		MP		PG		S1							
Circles: Center, Radius and Circumference				AR			EY	GE		IA		MA	MB			PG	PT			S3		SMSG		TR	TW
Circles: Chords								GE					MB			PG				S3		SMSG			
Circles: Chords, Secants and Tangents								GE					MB			PG	PT			S3		SMSG			
Circles: Equations of	AA						EY	GE		IA		MA	MB						S2						TW
Circles: Radian Measure							EY						MB				PT			S3				TR	
Circles: Tangents								GE					MB			PG				S3		SMSG			
Combinatorics: Combinations	AA								HA			MA						S1	S2						TW
Combinatorics: Multiplication Counting Principle	AA								HA									S1	S2						TW
Combinatorics: Permutations	AA									IA		MA						S1	S2						
Complementary, Supplementary and Vertical Angles												MA		MP	NY	PG		S1	S2						
Conics	AA						EY				IN		MB												
Consecutive Integers						EA	EY			IA	IN	MA			NY			S1							
Constructions								GE				MA				PG	PT		S2						
Continued Fractions (or 1 1 8 2 Special Numbers – Pi)	AA																								
Conversions	AA	AAR		AR		EA				IA	IN	MA			NY			S1							TW
Cost				AR	BA									MP											
Decimals				AR										MP											
Definitions: Advanced Arithmetic		AAR																							
Definitions: Algebra			AL			EA				IA	IN														
Definitions: Arithmetic		AAR		AR																					
Definitions: Geometry																PG									
Definitions: Solid Geometry																					SG				
Definitions: Trigonometry																	PT								
Distance								GE				MA							S2			SMSG		TR	
Equations and Expressions: Modeling	AA		AL	AR		EA	EY			IA	IN	MA		MP	NY			S1							
Equations and Expressions: Using Substitution in	AA			AR		EA	EY				IN	MA		MP	NY		PT	S1	S2						TW
Equations: Absolute Value							EY						MB							S3					
Equations: Degrees of	AA										IN														
Equations: Forming from Imaginary Roots	AA																								

Taxonomy of Knowledge Reproduced in New York Schools	A LISTING OF CURRICULA IN WHICH ASSESSED TOPICS ARE INCLUDED																								
	AA	AAR	AL	AR	BA	EA	EY	GE	HA	IA	IN	MA	MB	MP	NY	PG	PT	S1	S2	S3	SG	SMSG	ST	TR	TW
Equations: Forming Higher Order from Roots	AA								HA																
Equations: Forming New from Modified Roots	AA																								
Equations: Forming Quadratics from Roots	AA		AL				EY				IN								S2						TW
Equations: Graphing						EA					IN							S1	S2						
Equations: Higher Order	AA								HA		IN														
Equations: Literal	AA					EA	EY			IA	IN	MA	MB		NY			S1	S2						TW
Equations: Logarithmic	AA						EY		HA		IN		MB				PT			S3				TR	TW
Equations: Modeling from a Table						EA				IA	IN														
Equations: Roots of Higher Order	AA								HA		IN														TW
Equations: Simple				AR		EA					IN	MA		MP	NY			S1							
Equations: Simple with Decimals												MA			NY			S1							
Equations: Simple with Fractional Expressions	AA		AL			EA				IA	IN			MP	NY										
Equations: Writing Linear	AA						EY			IA	IN	MA							S2			SMSG			
Error										IA															
Estimating and Rounding															NY										
Exponential Functions and Equations	AA						EY			IA	IN		MB					S1		S3				TR	TW
Exponential Growth													MB												
Exponents	AA						EY			IA	IN	MA	MB					S1		S3					TW
Exponents: Operations with			AL	AR					HA	IA	IN	MA			NY			S1							
Factors: Greatest Common		AAR	AL	AR		EA									NY			S1							
Factors: Least Common Multiples				AR																					
Factors: Prime			AL	AR		EA	EY				IN														
Fraction Madness				AR								MA		MP											
Fractions		AAR		AR										MP											
Fractions: Complex	AA		AL	AR		EA	EY			IA	IN	MA	MB							S3					TW
Fractions: Partial	AA								HA																
Functional Notation	AA						EY													S3					
Functions: Compositions of							EY						MB												
Functions: Defining							EY			IA			MB							S3					
Functions: Domain and Range							EY						MB							S3					
Functions: Inverses of													MB				PT			S3					
Graphic Representation																									
Graphic Representation of Data						EA				IA		MA						S1	S2						
Graphic Representation: Histograms and Tables						EA				IA		MA		MP				S1							
Graphing Functions and Relations													MB												
Graphing Higher Order Equations	AA																								
Graphs: Identifying Equations of						EA	EY			IA		MA	MB		NY				S2						
Inequalities: Absolute Value							EY						MB							S3		SMSG			
Inequalities: Graphing Systems of										IA		MA			NY			S1							
Inequalities: Linear							EY			IA		MA			NY			S1				SMSG			TW
Inequalities: Systems of							EY																		
Inequalities: Writing Systems of										IA								S1							
Locus								GE				MA			NY	PG			S2		SG	SMSG			TW
Locus with Equations																									
Logarithms	AA						EY		HA		IN		MB				PT			S3				TR	TW
Logical Reasoning								GE				MA							S2						
Logical Reasoning: Biconditional												MA										SMSG			
Logical Reasoning: Contrapositive								GE				MA						S1	S2						
Logical Reasoning: Converse												MA				PG			S2						
Logical Reasoning: Inverse												MA													
Logical Reasoning: Symbolic Logic																		S1	S2						
Logical Reasoning: Venn Diagrams												MA													
Longitude		AAR		AR																					

Taxonomy of Knowledge Reproduced in New York Schools	A LISTING OF CURRICULA IN WHICH ASSESSED TOPICS ARE INCLUDED																							
	AA	AAR	AL	AR	BA	EA	EY	GE	HA	IA	IN	MA	MB	MP	NY	PG	PT	S1	S2	S3	SG	SMSG	ST	TR	TW
Matrices	AA																								TW
Medians, Altitudes, Bisectors and Midsegments								GE								PG	PT		S2			SMSG		TR	
Mensuration		AAR		AR										MP											
Midpoint								GE		IA		MA							S2			SMSG			
Notes and Interest	AA	AAR	AL	AR	BA	EA				IA	IN			MP	NY										
Numbers: Comparing Reals				AR								MA		MP											
Numbers: Complex	AA						EY						MB				PT			S3					TW
Numbers: Imaginary	AA						EY				IN		MB							S3					
Numbers: Prime and Composite				AR																					
Numbers: Properties of Real	AA						EY			IA	IN	MA			NY			S1	S2	S3					
Order of Operations		AAR	AL	AR		EA												S1							
Parallel and Perpendicular Lines	AA						EY	GE		IA	IN	MA				PG			S2						TW
Parallel Lines: Angles Involving								GE										S1	S2						
Percent		AAR		AR	BA					IA		MA		MP	NY			S1							
Perimeter				AR				GE			IN		MB		NY			S1							
Points on a Line: Identification of	AA					EA					IN	MA			NY			S1	S2						TW
Polygons and Circles: Compositions of										IA															
Polygons and Circles: Inscribed				AR						IA						PG	PT			S3		SMSG		TR	
Polygons: Area of				AR	BA						IN	MA	MB			PG		S1	S2		SG	SMSG			
Polygons: Interior and Exterior Angles of																PG			S2			SMSG			
Polynomials: Addition and Subtraction of			AL			EA				IA		MA			NY			S1	S2						
Polynomials: Factoring	AA		AL			EA	EY			IA	IN	MA						S1		S3					TW
Polynomials: Multiplication and Division of	AA		AL			EA	EY				IN	MA						S1	S2	S3					TW
Probability: Binomial with "At Least or At Most"													MB							S3					
Probability: Binomial with "Exactly"													MB							S3					
Probability: Conditional										IA															
Probability: Dependent Events																		S1	S2						TW
Probability: Experimental										IA		MA						S1							
Probability: Geometric																			S2						
Probability: Independent Events	AA									IA									S2						
Probability: Mutually Exclusive Events	AA																	S1							TW
Probability: Sample Space										IA								S1							
Probability: Theoretical	AA									IA		MA						S1	S2	S3					TW
Profit and Loss		AAR		AR	BA	EA								MP											
Progressions: Arithmetic	AA	AAR		AR			EY		HA		IN														TW
Progressions: Arithmetic and Geometric	AA								HA		IN														TW
Progressions: Geometric	AA	AAR					EY		HA		IN														
Proofs: Algebraic	AA	AAR																							
Proofs: Circle													MB			PG						SMSG			
Proofs: Coordinate												MA	MB					S1	S2			SMSG			
Proofs: Dihedral and Polyhedral Angles																					SG				
Proofs: General Polyhedrons																					SG				
Proofs: Geometry																PG						SMSG			
Proofs: Lines and Planes in Space																					SG				
Proofs: Polygon								GE					MB			PG			S2			SMSG			
Proofs: Prisms and Cylinders																					SG				
Proofs: Pyramids and Cones																					SG				
Proofs: Pythagoras																PG									
Proofs: Solid Geometry																					SG				TW
Proofs: Spheres																					SG	SMSG			
Proofs: Spherical Polygons																					SG		ST	TR	
Proofs: Triangle								GE					MB			PG			S2			SMSG			
Proofs: Trigonometric							EY										PT			S3				TR	

Taxonomy of Knowledge Reproduced in New York Schools	A LISTING OF CURRICULA IN WHICH ASSESSED TOPICS ARE INCLUDED																								
	AA	AAR	AL	AR	BA	EA	EY	GE	HA	IA	IN	MA	MB	MP	NY	PG	PT	S1	S2	S3	SG	SMSG	ST	TR	TW
Proportions	AA	AAR		AR		EA								MP	NY			S1							
Quadratics: a > 1	AA		AL				EY				IN		MB						S2						
Quadratics: Axis of Symmetry	AA						EY			IA	IN		MB						S2						TW
Quadratics: Completing the Square									HA		IN														
Quadratics: Difference of Perfect Squares							EY			IA	IN				NY			S1		S3					
Quadratics: Find Vertex Given Equation	AA									IA	IN		MB						S2						TW
Quadratics: Graphing							EY			IA	IN	MA							S2						
Quadratics: Imaginary Solutions							EY						MB							S3					TW
Quadratics: Inequalities							EY						MB						S2	S3					TW
Quadratics: Noninteger Solutions						EA	EY				IN								S2						TW
Quadratics: Solving	AA					EA				IA	IN	MA			NY				S2						TW
Quadratics: Solving by Factoring							EY			IA	IN	MA			NY			S1	S2						
Quadratics: Sum and Product of Roots	AA						EY				IN		MB							S3					TW
Quadratics: Using the Discriminant	AA						EY				IN		MB		NY				S2	S3					TW
Quadratics: Writing	AA					EA					IN				NY			S1							
Radicals: N-Roots	AA	AAR	AL	AR		EA					IN							S1						TR	TW
Radicals: Operations with	AA		AL			EA	EY		HA		IN	MA			NY			S1							
Radicals: Rationalizing Denominators	AA								HA		IN		MB							S3					
Radicals: Simplifying	AA		AL			EA				IA	IN				NY			S1							
Radicals: Solving	AA		AL			EA	EY				IN		MB							S3					TW
Radicals: Square Roots	AA	AAR	AL	AR		EA			HA		IN	MA			NY										
Rate	AA			AR	BA						IN				NY			S1							TW
Rate, Time and Distance	AA		AL	AR		EA				IA	IN	MA			NY										TW
Ratio		AAR		AR	BA						IN	MA		MP											
Rationals: Addition and Subtraction of						EA				IA	IN	MA	MB		NY			S1	S2						
Rationals: Solving	AA		AL			EA				IA	IN	MA			NY			S1	S2						
Rationals: Undefined							EY			IA		MA			NY			S1		S3					
Regression: Linear													MB												
Regression: Logarithmic													MB												
Regression: Power													MB												
Scientific Notation							EY			IA	IN	MA						S1		S3					
Series	AA								HA																
Series: Infinite																									
Set Theory										IA					NY				S2						
Sets: Replacement							EY			IA					NY			S1							
Similarity								GE				MA			NY	PG		S1	S2						
Similarity: Right Triangles													MB		NY	PG		S1	S2						
Slope	AA					EA	EY			IA	IN	MA			NY			S1	S2			SMSG			TW
Slope Intercept Form of a Line	AA						EY				IN				NY			S1							
Solid Geometry: Dihedral and Polyhedral Angles																					SG	SMSG			TW
Solid Geometry: General Polyhedrons																					SG	SMSG			
Solid Geometry: Lines and Planes in Space				AR				GE													SG	SMSG			TW
Solid Geometry: Prisms and Cylinders		AAR		AR				GE						MP				S1			SG				TW
Solid Geometry: Pyramids and Cones								GE		IA											SG	SMSG			TW
Solid Geometry: Spheres		AAR																			SG	SMSG			TW
Solid Geometry: Spherical Polygons																					SG		ST	TR	TW
Special Quadrilaterals														MP			PT		S2						
Special Quadrilaterals: Parallelograms								GE				MA				PG		S1	S2			SMSG			
Special Quadrilaterals: Rectangles and Squares				AR								MA			NY	PG		S1	S2			SMSG			
Special Quadrilaterals: Rhombuses								GE								PG			S2			SMSG			
Special Quadrilaterals: Trapezoids								GE								PG		S1	S2						
Summations							EY				IN		MB							S3					
Symmetry											IN	MA						S1		S3					

Taxonomy of Knowledge Reproduced in New York Schools	A LISTING OF CURRICULA IN WHICH ASSESSED TOPICS ARE INCLUDED																								
	AA	AAR	AL	AR	BA	EA	EY	GE	HA	IA	IN	MA	MB	MP	NY	PG	PT	S1	S2	S3	SG	SMSG	ST	TR	TW
Systems: Linear			AL			EA	EY			IA	IN	MA			NY			S1	S2						
Systems: Other Nonlinear	AA		AL			EA	EY		HA		IN	MA	MB				PT		S2	S3					TW
Systems: Quadratic Linear	AA					EA	EY	GE		IA	IN	MA						S1	S2						
Systems: Three Variables	AA		AL				EY				IN														
Systems: Writing	AA	AAR	AL	AR		EA	EY			IA	IN	MA			NY			S1	S2						TW
Systems: Writing Quadratic	AA										IN				NY										
Transformations: Classifications of								GE					MB					S1							
Transformations: Compositions of								GE				MA	MB						S2	S3					
Transformations: Dilations												MA	MB						S2	S3					
Transformations: Isometries												MA	MB							S3					
Transformations: Reflections								GE				MA	MB					S1	S2	S3		SMSG			
Transformations: Rotations								GE												S3					
Transformations: Translations													MB						S2	S3					
Triangle Inequalities								GE				MA				PG			S2			SMSG		TR	
Triangles: Equilateral				AR									MB		NY	PG			S2						
Triangles: Interior and Exterior Angles of								GE						MP	NY	PG		S1	S2						
Triangles: Isosceles								GE				MA				PG	PT	S1	S2						
Triangles: Mean Proportionals	AA							GE					MB			PG			S2			SMSG			
Triangles: Pythagoras	AA			AR		EA				IA	IN	MA			NY	PG		S1	S2						
Triangles: Special Right							EY									PG			S2						
Triangles: Vectors							EY										PT			S3				TR	
Trigonometric Equations													MB				PT			S3				TR	
Trigonometric Expressions: Factoring							EY																		TW
Trigonometric Formulas: Derivations of							EY										PT						ST	TR	
Trigonometric Functions: Evaluating						EA	EY										PT			S3				TR	
Trigonometric Functions: Inverses of						EA	EY						MB		NY		PT			S3				TR	
Trigonometric Functions: Logarithms of							EY				IN						PT							TR	
Trigonometric Functions: Properties of						EA	EY						MB				PT			S3				TR	
Trigonometric Graphs							EY										PT			S3				TR	
Trigonometric Identities							EY						MB				PT			S3				TR	
Trigonometric Identities: Angle Sum or Difference							EY										PT			S3				TR	
Trigonometric Identities: Double and Half Angle							EY						MB				PT			S3				TR	
Trigonometric Ratios: Basic										IA					NY		PT		S2					TR	
Trigonometric Ratios: Cofunction & Reciprocal													MB				PT			S3				TR	
Trigonometry: Finding Angles											IN				NY	PG	PT		S2			SMSG		TR	
Trigonometry: Finding Area							EY						MB				PT			S3		SMSG		TR	
Trigonometry: Finding Sides						EA	EY			IA	IN	MA			NY	PG	PT		S2					TR	
Trigonometry: Finding Sides Using Two Triangles							EY									PG	PT							TR	
Trigonometry: Law of Cosines							EY						MB			PG	PT			S3				TR	
Trigonometry: Law of Sines							EY						MB			PG	PT		S2	S3				TR	
Trigonometry: Law of Sines - The Ambiguous Case							EY										PT			S3				TR	
Trigonometry: Law of Tangents							EY										PT								
Trigonometry: Polar Coordinates																									TW
Trigonometry: Polar Form	AA																PT								TW
Trigonometry: Reference Angles							EY										PT			S3				TR	
Trigonometry: Terminal Sides of Angles																				S3				TR	
Trigonometry: Unit Circles																				S3					
Valuation				AR																					
Variation: Direct							EY				IN	MA						S1							TW
Variation: Inverse	AA					EA	EY				IN		MB							S3					TW
Volume	AA	AAR		AR		EA				IA	IN	MA		MP				S1			SG				
# of Different Topics Assessed Each Year	80	25	27	52	9	49	80	33	19		80	78	69	29	56	39	40	76	76	66	19	35	3	35	58

Taxonomy of Mathematical Knowledge		Taxonomy of Knowledge Reproduced in New York Schools: 1866-2009
1	Numbers and Computation	
1.1	Number Concepts	
1.1		Arithmetic: Numeration
1.1		Arithmetic: Place Value
1.1		Numbers: Prime and Composite
1.1		Absolute Value
1.1.1	Natural	
1.1.2	Integers	
1.1.2		Consecutive Integers
1.1.3	Rational	
1.1.3		Rationals: Undefined
1.1.4	Irrational	
1.1.5	Algebraic	
1.1.6	Real	
1.1.6		Numbers: Properties of Real
1.1.7	Complex	
1.1.7		Numbers: Imaginary
1.1.7		Numbers: Complex
1.1.8	FamousNumbers	
1.1.8.1	0	
1.1.8.2	pi	
1.1.8.3	e	
1.1.8.4	i	
1.1.8.5	Golden Mean	
1.2	Arithmetic	
1.2		Definitions: Arithmetic
1.2		Definitions: Advanced Arithmetic
1.2.1	Operations	
1.2.1		Arithmetic Operations
1.2.1		Order of Operations
1.2.1.1	Addition	
1.2.1.1		Arithmetic: Addition
1.2.1.2	Subtraction	
1.2.1.2		Arithmetic: Subtraction
1.2.1.3	Multiplication	
1.2.1.3		Arithmetic: Multiplication
1.2.1.4	Division	
1.2.1.4		Arithmetic: Division
1.2.1.5	Roots	
1.2.1.5		Radicals: Square Roots
1.2.1.5		Radicals: N-Roots
1.2.1.6	Factorials	
1.2.1.7	Factoring	
1.2.1.7		Factors: Prime
1.2.1.7		Factors: Least Common Multiples
1.2.1.7		Factors: Greatest Common
1.2.1.8	Properties of Operations	

Taxonomy of Mathematical Knowledge		Taxonomy of Knowledge Reproduced in New York Schools: 1866-2009
1.2.1.9	Estimation	
1.2.2	Fractions	
1.2.2		Fractions
1.2.2		Fraction Madness
1.2.2		Fractions: Complex
1.2.2.1	Addition	
1.2.2.1		Rationals: Addition and Subtraction of
1.2.2.2	Subtraction	
1.2.2.3	Multiplication	
1.2.2.4	Division	
1.2.2.5	Ratio and Proportion	
1.2.2.5		Ratio
1.2.2.5		Proportions
1.2.2.5		Rate
1.2.2.6	Equivalent Fractions	
1.2.3		Decimals
1.2.3	Decimals	
1.2.3.1	Addition	
1.2.3.2	Subtraction	
1.2.3.3	Multiplication	
1.2.3.4	Division	
1.2.3.5	Percents	
1.2.3.5		Percent
1.2.4	Comparison of numbers	
1.2.4		Numbers: Comparing Reals
1.2.5	Exponents	
3.1.7		Exponents
1.2.5.1	Multiplication	
1.2.5.2	Division	
1.2.5.3	Powers	
1.2.5.4	Integer Exponents	
1.2.5.5	Rational Exponents	
1.3		Binomial Expansions: Undetermined Coefficients
1.3	Patterns and Sequences	
1.3.1	Number Patterns	
1.3.1		Binomial Expansions
1.3.1		Summations
1.3.1		Continued Fractions (or 1.1.8.2 Special Numbers – Pi)
1.3.1		Fractions: Partial
1.3.2	Fibonacci Sequence	
1.3.3	Arithmetic Sequence	
1.3.3		Progressions: Arithmetic
1.3.3		Progressions: Arithmetic and Geometric
1.3.4	Geometric Sequence	
1.3.4		Progressions: Geometric
1.4	Measurement	
1.4.1	Units of Measurement	

	Taxonomy of Mathematical Knowledge	Taxonomy of Knowledge Reproduced in New York Schools: 1866-2009
1.4.1		Mensuration
1.4.1		Conversions
1.4.1.9		Estimating and Rounding
1.4.1.1	Metric System	
1.4.1.2	Standard Units	
1.4.1.3	Nonstandard Units	
1.4.2	Linear Measure	
1.4.2.1	Distance	
1.4.2.2	Circumference	
1.4.2.3	Perimeter	
1.4.2.3		Perimeter
1.4.3	Area	
1.4.3.1	Area of Polygons	
1.4.3.1		Polygons: Area of
1.4.3.2	Area of Circles	
1.4.3.3	Surface Area	
1.4.3.4	Nonstandard Shapes	
1.4.4	Volume	
1.4.4		Volume
1.4.5	Weight and Mass	
1.4.6	Temperature	
1.4.7	Time	
1.4.8	Speed	
1.4.8		Rate, Time and Distance
1.4.9	Money	
1.4.10	Scale	
2	Logic and Foundations	
2.1	Logic	
2.1.1	Venn Diagrams	
2.1.1		Logical Reasoning: Venn Diagrams
2.1.2	Propositional and Predicate Logic	
2.1.2		Logical Reasoning
2.1.2		Logical Reasoning: Biconditional
2.1.2		Logical Reasoning: Contrapositive
2.1.2		Logical Reasoning: Converse
2.1.2		Logical Reasoning: Inverse
2.1.2		Logical Reasoning: Symbolic Logic
2.1.3	Methods of Proof	
2.1.3		Constructions
2.2	Set Theory	
2.2		Set Theory
2.2.1	Sets and Set Operations	
2.2.1		Sets: Replacement
2.2.2	Relations and Functions	
2.2.3	Cardinality	
2.2.4	Axiom of Choice	
2.3	Computability and Decidability	

	Taxonomy of Mathematical Knowledge	Taxonomy of Knowledge Reproduced in New York Schools: 1866-2009
2.4	Model Theory	
3	Algebra and Number Theory	
3.1	Algebra	
3.1		Definitions: Algebra
3.1.1	Graphing Techniques	
3.1.1		Graphing Functions and Relations
3.1.1		Graphs: Identifying Equations of
3.1.1		Graphing Higher Order Equations
3.1.2	Algebraic Manipulation	
3.1.2		Equations and Expressions: Modeling
3.1.2		Equations: Modeling from a Table
3.1.2		Equations: Literal
3.1.2		Equations and Expressions: Using Substitution in
3.1.3	Functions	
3.1.3		Functions: Defining
3.1.3		Functions: Domain and Range
3.1.3		Functional Notation
3.1.3		Functions: Inverses of
3.1.3		Functions: Compositions of
3.1.3.1	Linear	
3.1.3.1		Equations: Absolute Value
3.1.3.2	Quadratic	
3.1.3.2		Quadratics: Axis of Symmetry
3.1.3.3	Polynomial	
3.1.3.4	Rational	
3.1.3.5	Exponential	
3.1.3.6	Logarithmic	
3.1.3.7	Piece-wise	
3.1.3.8	Step	
3.1.4	Equations	
3.1.4		Polynomials: Multiplication and Division of
3.1.4.1	Linear	
3.1.4.1		Slope
3.1.4.1		Slope Intercept Form of a Line
3.1.4.1		Equations: Graphing
3.1.4.1		Points on a Line: Identification of
3.1.4.1		Variation: Direct
3.1.4.1		Equations: Writing Linear
3.1.4.2	Quadratic	
3.1.4.2		Equations: Degrees of
3.1.4.2		Quadratics: Graphing
3.1.4.2		Quadratics: Find Vertex Given Equation
3.1.4.2		Quadratics: Solving
3.1.4.2		Quadratics: Solving by Factoring
3.1.4.2		Quadratics: Using the Discriminant
3.1.4.2		Quadratics: Sum and Product of Roots
3.1.4.2		Quadratics: Noninteger Solutions

	Taxonomy of Mathematical Knowledge	Taxonomy of Knowledge Reproduced in New York Schools: 1866-2009
3.1.4.2		Quadratics: Imaginary Solutions
3.1.4.2		Quadratics: Difference of Perfect Squares
3.1.4.2		Quadratics: Completing the Square
3.1.4.2		Equations: Roots of Higher Order
3.1.4.2		Equations: Forming from Imaginary Roots
3.1.4.2		Equations: Forming New from Modified Roots
3.1.4.2		Equations: Forming Higher Order from Roots
3.1.4.2		Quadratics: Writing
3.1.4.2		Equations: Forming Quadratics from Roots
3.1.4.2		Quadratics: $a > 1$
3.1.4.2		Systems: Writing Quadratic
3.1.4.3	Polynomial	
3.1.4.3		Polynomials: Addition and Subtraction of
3.1.4.3		Polynomials: Factoring
3.1.4.4	Rational	
3.1.4.4		Variation: Inverse
3.1.4.4		Rationals: Solving
3.1.4.5	Exponential	
3.1.4.5		Scientific Notation
3.1.4.5		Exponents: Operations with
3.1.4.5		Exponential Functions and Equations
3.1.4.5		Exponential Growth
3.1.4.5		Radicals: Operations with
3.1.4.5		Radicals: Rationalizing Denominators
3.1.4.5		Radicals: Simplifying
3.1.4.5		Radicals: Solving
3.1.4.6	Logarithmic	
3.1.4.6		Logarithms
3.1.4.6		Equations: Logarithmic
3.1.4.7	Systems	
3.1.4.7		Systems: Writing
3.1.5	Inequalities	
3.1.5		Inequalities: Linear
3.1.5		Inequalities: Absolute Value
3.1.5		Quadratics: Inequalities
3.1.5		Inequalities: Writing Systems of
3.1.5		Inequalities: Systems of
3.1.5		Inequalities: Graphing Systems of
3.1.6	Matrices	
3.1.6		Matrices
3.1.7	Sequences and Series	
3.1.7		Series
3.1.7		Series: Infinite
3.1.8	Algebraic Proof	
3.1.8		Proofs: Algebraic
3.2	Linear Algebra	
3.2.1	Systems of Linear Equations	

	Taxonomy of Mathematical Knowledge	Taxonomy of Knowledge Reproduced in New York Schools: 1866-2009
3.2.1		Systems: Linear
3.2.2	Matrix algebra	
3.2.3	Vectors in R3	
3.2.4	Vector Spaces	
3.2.4		Triangles: Vectors
3.2.5	Linear Transformations	
3.2.6	Eigenvalues and Eigenvectors	
3.2.7	Inner Product Spaces	
3.3	Abstract Algebra	
3.3.1	Groups	
3.3.2	Rings and Ideals	
3.3.3	Fields	
3.3.4	Galois Theory	
3.3.5	Multilinear Algebra	
3.4	Number Theory	
3.4.1	Integers	
3.4.2	Primes	
3.4.2.1	Divisibility	
3.4.2.2	Factorization	
3.4.2.3	Distributions of Primes	
3.4.3	Congruences	
3.4.4	Diophantine Equations	
3.4.5	Irrational Numbers	
3.4.6	Famous Problems	
3.4.7	Coding Theory	
3.4.8	Cryptography	
3.5	Category Theory	
3.6	K-Theory	
3.7	Homological Algebra	
3.8	Modular Arithmetic	
4	Discrete Mathematics	
4.1	Cellular Automata	
4.1	Chaos	
4.2	Combinatorics	
4.2.1	Combinations	
4.2.2	Permutations	
4.3	Game Theory	
4.4	Algorithms	
4.5	Recursion	
4.6	Graph Theory	
4.7	Linear Programming	
4.8	Order and Lattices	
4.9	Theory of Computation	
5	Geometry and Topology	
5		Definitions: Geometry
5.1	Geometric Proof	
5.1		Proofs: Circle

	Taxonomy of Mathematical Knowledge	Taxonomy of Knowledge Reproduced in New York Schools: 1866-2009
5.1		Proofs: Coordinate
5.1		Proofs: Dihedral and Polyhedral Angles
5.1		Proofs: General Polyhedrons
5.1		Proofs: Geometry
5.1		Proofs: Lines and Planes in Space
5.1		Proofs: Polygon
5.1		Proofs: Prisms and Cylinders
5.1		Proofs: Pyramids and Cones
5.1		Proofs: Pythagoras
5.1		Proofs: Solid Geometry
5.1		Proofs: Spheres
5.1		Proofs: Spherical Polygons
5.1		Proofs: Triangle
5.1		Proofs: Trigonometric
5.2	Plane Geometry	
5.2.1	Measurement	
5.2.1		Polygons and Circles: Compositions of
5.2.2	Lines and Planes	
5.2.2		Parallel Lines: Angles Involving
5.2.3	Angles	
5.2.3		Complementary, Supplementary and Vertical Angles
5.2.4	Triangles	
5.2.4		Triangles: Special Right
5.2.4.1		Triangles: Interior and Exterior Angles of
5.2.4.1	Properties	
5.2.4.1		Triangle Inequalities
5.2.4.1		Triangles: Isosceles
5.2.4.1		Triangles: Equilateral
5.2.4.1		Trigonometry: Law of Cosines
5.2.4.1		Trigonometry: Law of Sines
5.2.4.1		Trigonometry: Law of Sines - The Ambiguous Case
5.2.4.1		Trigonometry: Law of Tangents
5.2.4.2	Congruence	
5.2.4.3	Similarity	
5.2.4.3		Similarity
5.2.4.3		Similarity: Right Triangles
5.2.4.3		Triangles: Mean Proportionals
5.2.4.4	Pythagorean Theorem	
5.2.4.4		Triangles: Pythagoras
5.2.5.1		Polygons: Interior and Exterior Angles of
5.2.5.1		Special Quadrilaterals
5.2.5.1		Special Quadrilaterals: Parallelograms
5.2.5.1		Special Quadrilaterals: Rectangles and Squares
5.2.5.1		Special Quadrilaterals: Rhombuses
5.2.5.1		Special Quadrilaterals: Trapezoids
5.2.5	Polygons	
5.2.5.1	Properties	

Taxonomy of Mathematical Knowledge		Taxonomy of Knowledge Reproduced in New York Schools: 1866-2009
5.2.4.1		Medians, Altitudes, Bisectors and Midsegments
5.2.5.2	Regular	
5.2.5.3	Irregular	
5.2.5.4	Congruence	
5.2.5.5	Similarity	
5.2.6	Circles	
5.2.6		Circles: Area of
5.2.6		Circles: Center, Radius and Circumference
5.2.6		Circles: Equations of
5.2.6		Circles: Radian Measure
5.2.6		Polygons and Circles: Inscribed
5.2.6		Circles: Arc Measure
5.2.6		Circles: Chords
5.2.6		Circles: Chords, Secants and Tangents
5.2.6		Circles: Tangents
5.2.7	Patterns	
5.2.7		Locus
5.2.7		Locus with Equations
5.2.7.1	Geometric Patterns	
5.2.7.2	Tilings and Tessellations	
5.2.7.3	Symmetry	
5.2.7.3		Symmetry
5.2.7.4	Golden Ratio	
5.2.8	Transformations	
5.2.8		Transformations: Classifications of
5.2.8		Transformations: Isometries
5.2.8		Transformations: Compositions of
5.2.8.1	Translation	
5.2.8.1		Transformations: Translations
5.2.8.2	Rotation	
5.2.8.2		Transformations: Rotations
5.2.8.3	Reflection	
5.2.8.3		Transformations: Reflections
5.2.8.4	Scaling	
5.2.8.4		Transformations: Dilations
5.3	Solid Geometry	
5.3		Solid Geometry: Lines and Planes in Space
5.3		Definitions: Solid Geometry
5.3.1	Dihedral Angles	
5.3.1		Solid Geometry: Dihedral and Polyhedral Angles
5.3.2	Spheres	
5.3.2		Solid Geometry: Spheres
5.3.2		Longitude
5.3.7		Solid Geometry: Spherical Polygons
5.3.3	Cones	
5.3.4	Cylinders	
5.3.5	Pyramids	

	Taxonomy of Mathematical Knowledge	Taxonomy of Knowledge Reproduced in New York Schools: 1866-2009
5.5.5		Solid Geometry: Pyramids and Cones
5.3.6	Prisms	
5.3.6		Solid Geometry: Prisms and Cylinders
5.3.7	Polyhedra	
5.3.7		Solid Geometry: General Polyhedrons
5.4	Analytic Geometry	
5.4		Area and the Coordinate Plane
5.4.1	Cartesian Coordinates	
5.4.2	Lines	
5.4.2		Parallel and Perpendicular Lines
5.4.2		Midpoint
5.4.3	Circles	
5.4.4	Planes	
5.4.5	Conics	
5.4.5		Conics
5.4.6	Polar Coordinates	
5.4.6		Trigonometry: Polar Coordinates
5.4.6		Trigonometry: Polar Form
5.4.7	Parametric Curves	
5.4.8	Surfaces	
5.4.9	Distance Formula	
5.4.9		Distance
5.5	Projective Geometry	
5.6	Differential Geometry	
5.7	Algebraic Geometry	
5.8	Topology	
5.8.1	Point Set Topology	
5.8.2	General Topology	
5.8.3	Differential Topology	
5.8.4	Algebraic Topology	
5.9	Trigonometry	
5.9		Definitions: Trigonometry
5.9.1		Trigonometry: Terminal Sides of Angles
5.9.1	Angles	
5.9.1		Trigonometry: Reference Angles
5.9.2	Trigonometric Functions	
5.9.4		Trigonometry: Unit Circles
5.9.2		Trigonometric Ratios: Basic
5.9.3		Trigonometric Ratios: Cofunction & Reciprocal
5.9.2		Trigonometric Functions: Evaluating
5.2.1		Trigonometry: Finding Sides
5.2.1		Trigonometry: Finding Sides Using Two Triangles
5.2.1		Trigonometry: Finding Angles
5.2.1		Trigonometry: Finding Area
5.9.2		Trigonometric Graphs
5.9.2		Trigonometric Functions: Properties of
5.9.2		Trigonometric Functions: Logarithms of

	Taxonomy of Mathematical Knowledge	Taxonomy of Knowledge Reproduced in New York Schools: 1866-2009
5.9.3	Inverse Trigonometric Functions	
5.9.3		Trigonometric Functions: Inverses of
5.9.4	Trigonometric Identities	
5.9.4		Trigonometric Identities
5.9.4		Trigonometric Identities: Angle Sum or Difference
5.9.4		Trigonometric Identities: Double and Half Angle
5.9.2		Trigonometric Formulas: Derivations of
5.9.5	Trigonometric Equations	
5.9.5		Trigonometric Equations
5.9.5		Trigonometric Expressions: Factoring
5.9.6	Roots of Unity	
5.9.7	Spherical Trigonometry	
5.1	Fractal Geometry	
6	Calculus	
6.1	Single Variable	
6.1.1	Functions	
6.1.2	Limits	
6.1.3	Continuity	
6.1.4	Differentiation	
6.1.4		Calculus: Differential
6.1.5	Integration	
6.1.5		Calculus: Integral
6.1.6	Series	
6.2	Several Variables	
6.2.1	Functions of Several Variables	
6.2.2	Limits	
6.2.3	Continuity	
6.2.4	Partial Derivatives	
6.2.5	Multiple integrals	
6.2.6	Taylor Series	
6.3	Advanced Calculus	
6.3.1	Vector Valued Functions	
6.3.2	Line Integrals	
6.3.3	Surface Integrals	
6.3.4	Stokes Theorem	
6.3.5	Curvilinear Coordinates	
6.3.6	Linear spaces	
6.3.7	Fourier Series	
6.3.8	Orthogonal Functions	
6.4	Tensor Calculus	
6.5	Calculus of Variations	
6.6	Operational Calculus	
7	Analysis	
7.1	Real Analysis	
7.1.1	Metric Spaces	
7.1.2	Convergence	
7.1.3	Continuity	

	Taxonomy of Mathematical Knowledge	Taxonomy of Knowledge Reproduced in New York Schools: 1866-2009
7.1.4	Differentiation	
7.1.5	Integration	
7.1.6	Measure Theory	
7.2	Complex Analysis	
7.2.1	Convergence	
7.2.2	Infinite Series	
7.2.3	Analytic Functions	
7.2.4	Integration	
7.2.5	Contour Integrals	
7.2.6	Conformal Mappings	
7.2.7	Several Complex Variables	
7.3	Numerical Analysis	
7.3.1	Computer Arithmetic	
7.3.2	Solutions of Equations	
7.3.2		Equations: Simple
7.3.2		Equations: Simple with Decimals
7.3.2		Equations: Simple with Fractional Expressions
7.3.2		Equations: Higher Order
7.3.2		Alligation
7.3.3		Systems: Quadratic Linear
7.3.3	Solutions of Systems	
7.3.3		Systems: Other Nonlinear
7.3.3		Systems: Three Variables
7.3.4	Interpolation	
7.3.5	Numerical Differentiation	
7.3.6	Numerical Integration	
7.3.7	Numerical Solutions of ODEs	
7.3.8	Numerical Solutions of PDEs	
7.4	Integral Transforms	
7.4.1	Fourier Transforms	
7.4.2	Laplace Transforms	
7.4.3	Hankel Transforms	
7.4.4	Wavelets	
7.4.5	Other Transforms	
7.5	Signal Analysis	
7.5.1	Sampling Theory	
7.5.2	Filters	
7.5.3	Noise	
7.5.4	Data Compression	
7.5.5	Image Processing	
7.6.1	Hilbert Spaces	
7.6.2	Banach Spaces	
7.6.3	Topological Spaces	
7.6.4	Locally Convex Spaces	
7.6.5	Bounded Operators	
7.6	Functional Analysis	
7.6.6	Spectral Theorem	

Taxonomy of Mathematical Knowledge		Taxonomy of Knowledge Reproduced in New York Schools: 1866-2009
7.6.7	Unbounded Operators	
7.7	Harmonic Analysis	
7.8	Global Analysis	
8	Differential and Difference Equations	
8.1	Ordinary Differential Equations	
8.1.1	First Order	
8.1.2	Second Order	
8.1.3	Linear Oscillations	
8.1.4	Nonlinear Oscillations	
8.1.5	Systems of Differential Equations	
8.1.6	Sturm Liouville Problems	
8.1.7	Special Functions	
8.1.8	Power Series Methods	
8.1.9	Laplace Transforms	
8.2	Partial Differential Equations	
8.2.1	First Order	
8.2.2	Elliptic	
8.2.3	Parabolic	
8.2.4	Hyperbolic	
8.2.5	Integral Transforms	
8.2.6	Integral Equations	
8.2.7	Potential Theory	
8.2.8	Nonlinear Equations	
8.2.9	Symmetries and Integrability	
8.3	Difference Equations	
8.3.1	First Order	
8.3.2	Second Order	
8.3.3	Linear Systems	
8.3.4	Z Transforms	
8.3.5	Orthogonal Polynomials	
8.4	Dynamical Systems	
8.4.1	1D Maps	
8.4.2	2D Maps	
8.4.3	Lyapunov Exponents	
8.4.4	Bifurcations	
8.4.5	Fractals	
8.4.6	Differentiable Dynamics	
8.4.7	Conservative Dynamics	
8.4.8	Chaos	
8.4.9	Complex Dynamical Systems	
9	Statistics and Probability	
9.1	Data Collection	
9.1		Analysis of Data
9.1.1	Experimental Design	
9.1.2	Sampling and Surveys	
9.1.3	Data and Measurement Issues	
9.2	Data Summary and Presentation	

	Taxonomy of Mathematical Knowledge	Taxonomy of Knowledge Reproduced in New York Schools: 1866-2009
9.2.1	Summary Statistics	
9.2.1.1	Measures of Central Tendencies	
9.2.1.1		Central Tendency: Averages
9.2.1.1		Central Tendency: Average Known with Missing Data
9.2.1.1		Central Tendency
9.2.1.2	Measures of Spread	
9.2.1.2		Central Tendency: Dispersion
9.2.2	Data Representation	
9.2.2		Graphic Representation
9.2.2.1	Graphs and Plots	
9.2.2.1		Graphic Representation of Data
9.2.2.1		Graphic Representation: Histograms and Tables
9.2.2.2	Tables	
9.3	Statistical Inference and Techniques	
9.3.1	Sampling Distributions	
9.3.2	Regression and Correlation	
9.3.2		Regression: Linear
9.3.2		Regression: Logarithmic
9.3.2		Regression: Power
9.3.3	Confidence Intervals	
9.3.3		Error
9.3.4	Hypothesis Tests	
9.3.5	Statistical Quality Control	
9.3.6	Non-parametric Techniques	
9.3.7	Multivariate Techniques	
9.3.8	Survival Analysis	
9.3.9	Bayesian Statistics	
9.4	Probability	
9.4.1	Elementary Probability	
9.4.1.1		Probability: Independent Events
9.4.1.1		Probability: Mutually Exclusive Events
9.4.1.1		Probability: Dependent Events
9.4.1.1		Probability: Theoretical
9.4.1.1		Probability: Conditional
9.4.1.1		Probability: Experimental
9.4.1.1		Probability: Geometric
9.4.1.1	Sample Space and Sets	
9.4.1.1		Probability: Sample Space
9.4.1.2	General Rules	
9.4.1.3	Combinations and Permutations	
9.4.1.3		Combinatorics: Multiplication Counting Principle
9.4.1.3		Combinatorics: Permutations
9.4.1.3		Combinatorics: Combinations
9.4.1.4	Random Variables	
9.4.2	Univariate Distributions	
9.4.2		Central Tendency: Normal Distributions
9.4.2		Probability: Binomial with "Exactly"

Taxonomy of Mathematical Knowledge		Taxonomy of Knowledge Reproduced in New York Schools: 1866-2009
9.4.2		Probability: Binomial with "At Least or At Most"
9.4.2.1	Discrete Distributions	
9.4.2.2	Continuous Distributions	
9.4.2.3	Expected Value	
9.4.3	Limit Theorems	
9.4.3.1	Central Limit Theorem	
9.4.3.2	Law of Large Numbers	
9.4.4	Multivariate Distributions	
9.4.4.1	Joint	
9.4.4.2	Conditional	
9.4.4.3	Expectations	
9.4.5	Stochastic Processes	
9.4.5.1	Brownian Motion	
9.4.5.2	Markov Chains	
9.4.5.3	Queuing Theory	
9.4.6	Probability Measures	
9.4.7	Simulation	
10	Applied Mathematics	
10.1	Consumer Mathematics	
10.1		Cost
10.1		Bills and Receipts
10.1		Valuation
10.1	Mathematical Physics	
10.2	Mathematical Economics	
10.3	Mathematical Biology	
10.4	Mathematics for Business	
10.4		Profit and Loss
10.4		Notes and Interest
10.4		Brokerage and Commission
10.5	Engineering Mathematics	
10.6	Mathematical Sociology	
10.7	Mathematics for Social Sciences	
10.8	Mathematics for Computer Science	
10.9	Mathematics for Humanities	
11	Mathematics History	
11.1	General	
11.2	Famous Problems	
11.3	Biographies of Mathematicians	

A Longitudinal Census of Observed Topics in the Mathematics Curricula of the Public Schools of New York State: 1866-2009

Taxonomy of Knowledge Assessed on Regents Mathematics Examinations	1866	1870	1880	1890	1900	1909	1920	1930	1940	1950	1960	1970	1980	1990	2000	2009	Current Status
	Preliminary Exams Only			Preliminary and Academic Exams							Academic Exams Only						
Absolute Value												+	+			+	Common
Alligation							+	+		+	+						Threatened
Analysis of Data																+	Common
Area and the Coordinate Plane													+	+	+		Common
Arithmetic Operations			+		+	+		+									Extinct
Arithmetic: Addition		+	+	+		+	+	+	+	+							Extinct
Arithmetic: Division	+	+	+	+		+		+	+	+							Extinct
Arithmetic: Multiplication	+	+	+	+					+	+							Extinct
Arithmetic: Numeration	+	+	+			+											Extinct
Arithmetic: Place Value				+								+					Threatened
Arithmetic: Subtraction		+	+					+	+	+							Extinct
Bills and Receipts		+	+	+	+	+	+			+							Extinct
Binomial Expansions				+	+		+	+	+	+	+		+	+	+	+	Very Common
Binomial Expansions: Undetermined Coefficients				+	+	+											Extinct
Brokerage and Commission			+	+	+	+	+		+	+							Extinct
Calculus: Differential										+	+						Threatened
Calculus: Integral											+						Threatened
Central Tendency													+	+	+	+	Very Common
Central Tendency: Average Known with Missing Data														+	+	+	Very Common
Central Tendency: Averages				+			+	+	+	+	+		+	+	+		Common
Central Tendency: Dispersion													+	+	+	+	Very Common
Central Tendency: Normal Distributions									+	+			+	+	+	+	Very Common
Circles: Arc Measure						+		+	+	+	+	+	+				Common
Circles: Area of					+			+	+	+	+	+	+		+		Common
Circles: Center, Radius and Circumference				+	+			+	+	+	+	+	+	+	+	+	Very Common
Circles: Chords				+	+	+		+	+	+	+	+	+	+	+	+	Very Common
Circles: Chords, Secants and Tangents				+	+		+	+	+	+	+	+	+	+	+	+	Very Common
Circles: Equations of									+	+	+		+	+	+	+	Very Common
Circles: Radian Measure								+		+	+	+	+	+	+	+	Very Common
Circles: Tangents							+	+	+	+	+	+	+	+	+	+	Very Common
Combinatorics: Combinations				+	+	+	+	+	+	+	+		+	+	+	+	Very Common
Combinatorics: Multiplication Counting Principle				+					+	+	+		+		+		Common
Combinatorics: Permutations								+		+			+	+	+	+	Very Common
Complementary, Supplementary and Vertical Angles					+			+	+	+	+	+	+	+	+	+	Very Common
Conics									+	+	+	+	+			+	Common
Consecutive Integers							+	+			+	+	+		+	+	Common
Constructions				+	+	+	+	+	+	+	+	+	+	+	+	+	Very Common
Continued Fractions					+												Extinct

A Longitudinal Census of Observed Topics in the Mathematics Curricula of the Public Schools of New York State: 1866-2009

Taxonomy of Knowledge Assessed on Regents Mathematics Examinations	Calendar Years in Which Topic Appears in Research Database																
	Preliminary Exams Only			Preliminary and Academic Exams							Academic Exams Only						
	1866	1870	1880	1890	1900	1909	1920	1930	1940	1950	1960	1970	1980	1990	2000	2009	Current Status
Conversions	+	+	+	+	+	+	+	+	+	+	+		+	+	+	+	Very Common
Cost	+	+	+	+	+	+	+	+	+	+							Extinct
Decimals	+	+	+					+	+	+							Very Common
Definitions: Advanced Arithmetic					+												Extinct
Definitions: Algebra				+	+		+	+								+	Common
Definitions: Arithmetic	+		+	+	+	+			+								Extinct
Definitions: Geometry				+	+		+			+	+						Threatened
Definitions: Solid Geometry				+	+		+										Extinct
Definitions: Trigonometry				+	+												Extinct
Distance										+	+	+	+	+	+	+	Very Common
Equations and Expressions: Modeling				+		+	+	+	+	+	+	+	+	+	+	+	Very Common
Equations and Expressions: Using Substitution in						+	+	+	+	+	+	+	+	+	+	+	Very Common
Equations: Absolute Value												+		+	+	+	Very Common
Equations: Degrees of								+		+							Extinct
Equations: Forming from Imaginary Roots							+	+									Extinct
Equations: Forming Higher Order from Roots				+		+		+			+						Threatened
Equations: Forming New from Modified Roots								+	+	+							Extinct
Equations: Forming Quadratics from Roots				+			+	+	+		+		+	+	+		Common
Equations: Graphing							+	+	+					+	+		Common
Equations: Higher Order				+				+	+	+	+						Threatened
Equations: Literal					+	+	+	+	+	+	+	+	+	+	+	+	Very Common
Equations: Logarithmic				+	+	+	+	+	+	+	+		+	+	+	+	Very Common
Equations: Modeling from a Table								+	+	+	+					+	Common
Equations: Roots of Higher Order				+		+	+	+	+	+	+						Threatened
Equations: Simple								+	+	+		+	+	+	+	+	Very Common
Equations: Simple with Decimals												+	+	+	+	+	Very Common
Equations: Simple with Fractional Expressions				+			+	+	+	+			+			+	Common
Equations: Writing Linear									+	+	+	+	+	+	+	+	Very Common
Error																+	Common
Estimating and Rounding													+				Common
Exponential Functions and Equations							+	+	+	+	+	+	+	+	+	+	Very Common
Exponential Growth																+	Common
Exponents								+	+	+	+		+		+	+	Common
Exponents: Operations with	+	+	+	+	+	+	+	+	+	+		+	+	+	+	+	Very Common
Factors: Greatest Common		+	+	+	+								+		+		Common
Factors: Least Common Multiples		+	+														Extinct
Factors: Prime		+	+	+		+	+						+				Threatened
Fraction Madness			+		+	+				+					+		Common

A Longitudinal Census of Observed Topics in the Mathematics Curricula of the Public Schools of New York State: 1866-2009

Taxonomy of Knowledge Assessed on Regents Mathematics Examinations	Calendar Years in Which Topic Appears in Research Database																
	1866	1870	1880	1890	1900	1909	1920	1930	1940	1950	1960	1970	1980	1990	2000	2009	
	Preliminary Exams Only			Preliminary and Academic Exams							Academic Exams Only						Current Status
Fractions	+	+	+			+	+	+		+							Extinct
Fractions: Complex			+	+	+		+	+	+	+	+	+	+	+	+	+	Very Common
Fractions: Partial				+	+												Extinct
Functional Notation										+	+		+	+	+		Common
Functions: Compositions of												+				+	Common
Functions: Defining												+			+	+	Common
Functions: Domain and Range												+	+	+	+	+	Very Common
Functions: Inverses of									+					+	+	+	Very Common
Graphic Representation							+	+					+				Common
Graphic Representation of Data															+	+	Common
Graphic Representation: Histograms and Tables										+				+	+	+	Very Common
Graphing Functions and Relations																+	Common
Graphing Higher Order Equations						+	+	+	+								Extinct
Graphs: Identifying Equations of								+					+		+	+	Common
Inequalities: Absolute Value												+	+	+		+	Very Common
Inequalities: Graphing Systems of												+	+	+		+	Very Common
Inequalities: Linear											+	+	+	+	+	+	Very Common
Inequalities: Systems of													+				Common
Inequalities: Writing Systems of															+	+	Common
Locus					+	+		+	+	+	+	+	+	+	+	+	Very Common
Locus with Equations												+	+				Common
Logarithms				+	+	+	+	+	+	+	+	+	+	+	+	+	Very Common
Logical Reasoning												+	+	+	+	+	Very Common
Logical Reasoning: Biconditional												+				+	Common
Logical Reasoning: Contrapositive											+		+	+	+	+	Very Common
Logical Reasoning: Converse									+	+	+	+	+		+		Common
Logical Reasoning: Inverse															+		Common
Logical Reasoning: Symbolic Logic													+	+	+		Common
Logical Reasoning: Venn Diagrams															+		Common
Longitude	+	+	+		+	+											Extinct
Matrices						+					+						Threatened
Medians, Altitudes, Bisectors and Midsegments				+				+	+	+	+	+	+	+	+	+	Very Common
Mensuration	+	+	+	+	+	+			+	+							Very Common
Midpoint										+	+	+	+	+	+	+	Very Common
Notes and Interest	+	+	+	+	+	+	+	+	+	+		+				+	Common
Numbers: Comparing Reals									+	+					+		Common
Numbers: Complex						+	+	+	+	+	+	+	+	+	+	+	Very Common
Numbers: Imaginary								+	+	+	+		+		+	+	Common

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Taxonomy of Knowledge Assessed on Regents Mathematics Examinations	Calendar Years in Which Topic Appears in Research Database																
	1866	1870	1880	1890	1900	1909	1920	1930	1940	1950	1960	1970	1980	1990	2000	2009	
	Preliminary Exams Only			Preliminary and Academic Exams							Academic Exams Only						Current Status
Numbers: Prime and Composite	+	+		+													Extinct
Numbers: Properties of Real								+		+	+	+	+	+	+	+	Very Common
Order of Operations					+	+	+								+		Common
Parallel and Perpendicular Lines				+				+	+	+	+	+	+	+	+	+	Very Common
Parallel Lines: Angles Involving										+		+	+	+	+	+	Very Common
Percent		+	+	+	+	+	+	+	+	+			+		+	+	Very Common
Perimeter									+	+	+		+		+	+	Common
Points on a Line: Identification of								+	+		+	+	+	+	+		Common
Polygons and Circles: Compositions of																+	Common
Polygons and Circles: Inscribed				+	+	+	+	+	+	+	+	+	+	+	+	+	Very Common
Polygons: Area of		+	+	+	+		+	+	+	+	+	+	+		+	+	Very Common
Polygons: Interior and Exterior Angles of					+	+	+	+	+	+	+	+	+	+	+		Very Common
Polynomials: Addition and Subtraction of				+			+					+	+	+	+	+	Very Common
Polynomials: Factoring					+	+	+	+	+	+	+	+	+	+	+	+	Very Common
Polynomials: Multiplication and Division of				+	+	+	+	+	+	+	+	+	+	+	+		Very Common
Probability: Binomial with "At Least or At Most"														+	+	+	Very Common
Probability: Binomial with "Exactly"														+	+	+	Very Common
Probability: Conditional																+	Common
Probability: Dependent Events											+		+	+	+		Common
Probability: Experimental													+		+	+	Very Common
Probability: Geometric														+			Common
Probability: Independent Events									+		+				+	+	Common
Probability: Mutually Exclusive Events										+	+		+	+	+		Common
Probability: Sample Space															+	+	Common
Probability: Theoretical									+	+	+		+	+	+	+	Very Common
Profit and Loss		+	+	+	+	+	+	+	+	+							Extinct
Progressions: Arithmetic		+		+		+	+	+	+	+	+						Threatened
Progressions: Arithmetic and Geometric				+	+	+	+		+	+	+						Threatened
Progressions: Geometric				+	+	+		+	+	+	+						Threatened
Proofs: Algebraic					+	+		+	+	+							Extinct
Proofs: Circle				+	+	+	+	+	+	+	+	+	+			+	Very Common
Proofs: Coordinate											+	+	+	+	+	+	Very Common
Proofs: Dihedral and Polyhedral Angles				+		+	+			+	+						Threatened
Proofs: General Polyhedrons				+			+	+									Extinct
Proofs: Geometry					+							+					Threatened
Proofs: Lines and Planes in Space				+	+	+	+	+	+	+	+						Threatened
Proofs: Polygon				+	+	+	+	+	+	+		+	+	+		+	Very Common
Proofs: Prisms and Cylinders				+	+	+	+	+	+								Extinct

A Longitudinal Census of Observed Topics in the Mathematics Curricula of the Public Schools of New York State: 1866-2009

Taxonomy of Knowledge Assessed on Regents Mathematics Examinations	1866	1870	1880	1890	1900	1909	1920	1930	1940	1950	1960	1970	1980	1990	2000	2009	Current Status
	Preliminary Exams Only			Preliminary and Academic Exams							Academic Exams Only						
Proofs: Pyramids and Cones				+		+	+		+	+							Extinct
Proofs: Pythagoras				+													Extinct
Proofs: Solid Geometry				+							+						Threatened
Proofs: Spheres				+	+	+	+		+			+					Threatened
Proofs: Spherical Polygons					+	+	+	+	+	+	+						Threatened
Proofs: Triangle				+	+	+	+	+	+	+	+	+	+	+	+	+	Very Common
Proofs: Trigonometric				+	+	+	+	+	+	+	+		+	+	+		Very Common
Proportions		+	+	+	+	+	+	+	+	+		+	+	+	+		Very Common
Quadratics: a > 1					+	+			+		+	+	+		+	+	Common
Quadratics: Axis of Symmetry										+	+		+	+	+	+	Very Common
Quadratics: Completing the Square				+					+								Extinct
Quadratics: Difference of Perfect Squares										+	+	+	+		+	+	Common
Quadratics: Find Vertex Given Equation								+		+	+			+	+	+	Common
Quadratics: Graphing								+	+	+	+		+	+	+	+	Very Common
Quadratics: Imaginary Solutions											+	+	+	+	+	+	Very Common
Quadratics: Inequalities											+	+	+	+	+	+	Very Common
Quadratics: Noninteger Solutions						+	+	+	+	+	+		+	+	+		Common
Quadratics: Solving							+	+	+	+	+	+	+	+	+	+	Very Common
Quadratics: Solving by Factoring						+		+			+	+	+	+	+	+	Very Common
Quadratics: Sum and Product of Roots							+	+	+	+	+	+		+	+	+	Common
Quadratics: Using the Discriminant					+	+	+	+	+	+	+	+	+	+	+	+	Very Common
Quadratics: Writing						+		+	+	+			+	+	+		Common
Radicals: N-Roots	+	+	+	+	+	+	+	+	+	+	+			+			Common
Radicals: Operations with				+	+	+	+	+	+	+		+	+	+	+	+	Very Common
Radicals: Rationalizing Denominators				+	+		+	+	+	+	+	+	+	+		+	Very Common
Radicals: Simplifying					+	+	+	+	+				+	+	+	+	Common
Radicals: Solving					+	+		+	+	+	+	+		+	+	+	Very Common
Radicals: Square Roots	+	+	+	+	+	+	+	+	+	+		+	+		+		Very Common
Rate				+		+		+	+		+	+	+				Common
Rate, Time and Distance			+	+	+	+	+	+	+	+	+	+	+		+	+	Very Common
Ratio	+	+	+	+	+	+		+	+	+	+				+		Common
Rationals: Addition and Subtraction of				+			+	+	+	+	+	+	+	+	+	+	Very Common
Rationals: Solving				+			+	+			+	+	+	+	+	+	Very Common
Rationals: Undefined												+	+	+	+	+	Very Common
Regression: Linear																+	Common
Regression: Logarithmic																+	Common
Regression: Power																+	Common
Scientific Notation											+	+	+	+	+	+	Very Common

Calendar Years in Which Topic Appears in Research Database

A Longitudinal Census of Observed Topics in the Mathematics Curricula of the Public Schools of New York State: 1866-2009

Calendar Years in Which Topic Appears in Research Database

Taxonomy of Knowledge Assessed on Regents Mathematics Examinations	1866	1870	1880	1890	1900	1909	1920	1930	1940	1950	1960	1970	1980	1990	2000	2009	
	Preliminary Exams Only			Preliminary and Academic Exams							Academic Exams Only						Current Status
Series					+	+											Extinct
Series: Infinite				+													Extinct
Set Theory												+	+			+	Common
Sets: Replacement												+	+	+		+	Common
Similarity								+	+	+	+	+	+	+	+	+	Very Common
Similarity: Right Triangles								+			+	+	+	+	+	+	Very Common
Slope								+	+	+	+	+	+	+	+	+	Very Common
Slope Intercept Form of a Line									+	+	+	+	+		+		Common
Solid Geometry: Dihedral and Polyhedral Angles								+	+	+	+	+					Threatened
Solid Geometry: General Polyhedrons				+		+	+	+	+	+	+	+					Threatened
Solid Geometry: Lines and Planes in Space				+				+	+	+	+	+				+	Common
Solid Geometry: Prisms and Cylinders				+	+	+	+	+	+	+	+				+	+	Common
Solid Geometry: Pyramids and Cones				+	+	+	+	+	+	+	+	+				+	Common
Solid Geometry: Spheres				+	+	+	+	+	+	+	+	+					Threatened
Solid Geometry: Spherical Polygons					+	+	+	+	+	+	+						Threatened
Special Quadrilaterals								+		+		+		+			Common
Special Quadrilaterals: Parallelograms								+	+	+	+	+	+	+	+	+	Very Common
Special Quadrilaterals: Rectangles and Squares								+	+	+	+	+	+	+	+	+	Very Common
Special Quadrilaterals: Rhombuses							+	+	+	+	+	+	+	+	+	+	Very Common
Special Quadrilaterals: Trapezoids								+	+		+	+	+	+	+	+	Very Common
Summations								+		+			+	+	+	+	Very Common
Symmetry													+	+	+		Common
Systems: Linear				+		+	+	+		+	+	+	+	+	+	+	Very Common
Systems: Other Nonlinear				+	+	+	+	+	+	+	+	+	+	+	+	+	Very Common
Systems: Quadratic Linear				+		+	+	+	+	+	+	+	+	+	+	+	Very Common
Systems: Three Variables					+	+			+	+	+		+				Common
Systems: Writing					+	+	+	+	+	+	+	+	+	+	+	+	Very Common
Systems: Writing Quadratic							+		+		+		+				Common
Transformations: Classifications of															+	+	Common
Transformations: Compositions of													+	+	+	+	Very Common
Transformations: Dilations														+	+	+	Very Common
Transformations: Isometries														+	+	+	Very Common
Transformations: Reflections												+	+	+	+	+	Very Common
Transformations: Rotations													+	+		+	Very Common
Transformations: Translations													+	+	+	+	Very Common
Triangle Inequalities						+		+	+	+		+	+	+	+	+	Very Common
Triangles: Equilateral					+			+	+	+	+	+	+	+		+	Common
Triangles: Interior and Exterior Angles of						+		+	+	+	+	+	+	+	+	+	Very Common

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Taxonomy of Knowledge Assessed on Regents Mathematics Examinations	1866	1870	1880	1890	1900	1909	1920	1930	1940	1950	1960	1970	1980	1990	2000	2009	
	Preliminary Exams Only			Preliminary and Academic Exams							Academic Exams Only						Current Status
Triangles: Isosceles								+	+	+		+	+	+	+	+	Very Common
Triangles: Mean Proportionals				+	+			+	+	+	+	+	+	+	+	+	Very Common
Triangles: Pythagoras			+	+		+	+	+	+		+	+	+	+	+	+	Very Common
Triangles: Special Right								+	+			+	+		+		Common
Triangles: Vectors							+		+	+	+	+		+	+		Common
Trigonometric Equations					+	+	+	+	+	+	+	+	+	+	+	+	Very Common
Trigonometric Expressions: Factoring											+						Threatened
Trigonometric Formulas: Derivations of					+	+		+	+	+	+	+	+	+			Common
Trigonometric Functions: Evaluating				+			+	+	+	+	+	+	+	+	+		Common
Trigonometric Functions: Inverses of								+	+	+	+	+	+	+	+	+	Very Common
Trigonometric Functions: Logarithms of								+	+	+	+	+	+				Common
Trigonometric Functions: Properties of				+	+		+	+	+	+	+	+	+	+	+	+	Very Common
Trigonometric Graphs				+	+			+	+	+	+	+	+	+	+		Common
Trigonometric Identities					+		+	+	+	+	+	+	+	+	+	+	Very Common
Trigonometric Identities: Angle Sum or Difference				+			+	+	+	+	+		+	+	+		Common
Trigonometric Identities: Double and Half Angle					+		+	+	+	+	+		+	+	+	+	Very Common
Trigonometric Ratios: Basic				+	+	+		+			+	+	+		+	+	Common
Trigonometric Ratios: Cofunction & Reciprocal								+		+			+	+		+	Common
Trigonometry: Finding Angles					+			+		+	+	+	+	+			Common
Trigonometry: Finding Area					+	+	+	+	+	+	+	+	+	+	+	+	Very Common
Trigonometry: Finding Sides				+	+	+	+	+	+	+	+	+	+	+	+	+	Very Common
Trigonometry: Finding Sides Using Two Triangles					+	+	+	+	+	+	+	+					Threatened
Trigonometry: Law of Cosines				+	+	+	+	+	+	+	+	+	+	+	+	+	Very Common
Trigonometry: Law of Sines				+	+		+	+	+	+	+	+	+	+	+	+	Very Common
Trigonometry: Law of Sines - The Ambiguous Case							+	+	+	+	+	+		+	+		Common
Trigonometry: Law of Tangents								+		+							Extinct
Trigonometry: Polar Coordinates											+						Threatened
Trigonometry: Polar Form									+	+	+						Threatened
Trigonometry: Reference Angles									+	+	+	+	+	+			Common
Trigonometry: Terminal Sides of Angles											+		+	+			Common
Trigonometry: Unit Circles															+		Common
Valuation	+	+	+	+	+	+		+	+								Extinct
Variation: Direct									+	+	+	+	+	+	+	+	Very Common
Variation: Inverse								+	+	+	+	+		+	+	+	Common
Volume			+	+	+	+	+	+	+	+	+			+	+	+	Very Common

YEAR	TOPICS ASSESSED	TOPICS OBSERVED FOR THE FIRST TIME	TOPICS OBSERVED FOR THE LAST TIME
	1866		
1866	Arithmetic: Division	Arithmetic: Division (1866-1950)	
1866	Arithmetic: Multiplication	Arithmetic: Multiplication (1866-1950)	
1866	Arithmetic: Numeration	Arithmetic: Numeration (1866-1909)	
1866	Conversions	Conversions (1866-2009)	
1866	Cost	Cost (1866-1950)	
1866	Decimals	Decimals (1866-1950)	
1866	Definitions: Arithmetic	Definitions: Arithmetic (1866-1940)	
1866	Exponents: Operations with	Exponents: Operations with (1866-2009)	
1866	Fractions	Fractions (1866-1950)	
1866	Longitude	Longitude (1866-1909)	
1866	Mensuration	Mensuration (1866-1950)	
1866	Notes and Interest	Notes and Interest (1866-2009)	
1866	Numbers: Prime and Composite	Numbers: Prime and Composite (1866-1890)	
1866	Radicals: N-Roots	Radicals: N-Roots (1866-1990)	
1866	Radicals: Square Roots	Radicals: Square Roots (1866-2000)	
1866	Ratio	Ratio (1866-2000)	
1866	Valuation	Valuation (1866-1940)	
	1870		
1870	Arithmetic: Addition	Arithmetic: Addition (1870-1950)	
1870	Arithmetic: Division	Arithmetic: Subtraction (1870-1950)	
1870	Arithmetic: Multiplication	Bills and Receipts (1870-1950)	
1870	Arithmetic: Numeration	Factors: Greatest Common (1870-2000)	
1870	Arithmetic: Subtraction	Factors: Least Common Multiples (1870-1880)	
1870	Bills and Receipts	Factors: Prime (1870-1880)	
1870	Conversions	Percent (1870-2009)	
1870	Cost	Polygons: Area of (1870-2009)	
1870	Decimals	Profit and Loss (1870-1950)	
1870	Exponents: Operations with	Progressions: Arithmetic (1870-1960)	
1870	Factors: Greatest Common	Proportions (1870-2000)	
1870	Factors: Least Common Multiples		
1870	Factors: Prime		
1870	Fractions		
1870	Longitude		
1870	Mensuration		
1870	Notes and Interest		
1870	Numbers: Prime and Composite		
1870	Percent		
1870	Polygons: Area of		
1870	Profit and Loss		
1870	Progressions: Arithmetic		
1870	Proportions		
1870	Radicals: N-Roots		
1870	Radicals: Square Roots		
1870	Ratio		
1870	Valuation		

YEAR	TOPICS ASSESSED	TOPICS OBSERVED FOR THE FIRST TIME	TOPICS OBSERVED FOR THE LAST TIME
	1880		
1880	Arithmetic Operations	Arithmetic Operations (1880-1930)	Factors: Least Common Multiples (1870-1880)
1880	Arithmetic: Addition	Brokerage and Commission (1880-1950)	
1880	Arithmetic: Division	Fraction Madness (1880-2000)	
1880	Arithmetic: Multiplication	Fractions: Complex (1880-2009)	
1880	Arithmetic: Numeration	Rate, Time and Distance (1880-2009)	
1880	Arithmetic: Subtraction	Triangles: Pythagoras (1880-2009)	
1880	Bills and Receipts	Volume (1880-2009)	
1880	Brokerage and Commission		
1880	Conversions		
1880	Cost		
1880	Decimals		
1880	Definitions: Arithmetic		
1880	Exponents: Operations with		
1880	Factors: Greatest Common		
1880	Factors: Least Common Multiples		
1880	Factors: Prime		
1880	Fraction Madness		
1880	Fractions		
1880	Fractions: Complex		
1880	Longitude		
1880	Mensuration		
1880	Notes and Interest		
1880	Percent		
1880	Polygons: Area of		
1880	Profit and Loss		
1880	Proportions		
1880	Radicals: N-Roots		
1880	Radicals: Square Roots		
1880	Rate, Time and Distance		
1880	Ratio		
1880	Triangles: Pythagoras		
1880	Valuation		
1880	Volume		
	1890		
1890	Arithmetic: Addition	Arithmetic: Place Value (1890-1970)	Numbers: Prime and Composite (1866-1890)
1890	Arithmetic: Division	Binomial Expansions (1890-2009)	Proofs: Pythagoras (1890-1890)
1890	Arithmetic: Multiplication	Binomial Expansions: Undetermined Coefficients (1890-1909)	Series: Infinite (1890-1890)
1890	Arithmetic: Place Value	Central Tendency: Averages (1890-2000)	
1890	Bills and Receipts	Circles: Center, Radius and Circumference (1890-2009)	
1890	Binomial Expansions	Circles: Chords (1890-2009)	
1890	Binomial Expansions: Undetermined Coefficients	Circles: Chords, Secants and Tangents (1890-2009)	
1890	Brokerage and Commission	Combinatorics: Combinations (1890-2009)	
1890	Central Tendency: Averages	Combinatorics: Multiplication Counting Principle (1890-2000)	
1890	Circles: Center, Radius and Circumference	Constructions (1890-2009)	
1890	Circles: Chords	Definitions: Algebra (1890-2009)	
1890	Circles: Chords, Secants and Tangents	Definitions: Geometry (1890-1960)	

YEAR	TOPICS ASSESSED	TOPICS OBSERVED FOR THE FIRST TIME	TOPICS OBSERVED FOR THE LAST TIME
1890	Combinatorics: Combinations	Definitions: Solid Geometry (1890-1920)	
1890	Combinatorics: Multiplication Counting Principle	Definitions: Trigonometry (1890-1900)	
1890	Constructions	Equations and Expressions: Modeling (1890-2009)	
1890	Conversions	Equations: Forming Higher Order from Roots (1890-1960)	
1890	Cost	Equations: Forming Quadratics from Roots (1890-2000)	
1890	Definitions: Algebra	Equations: Higher Order (1890-1960)	
1890	Definitions: Arithmetic	Equations: Logarithmic (1890-2009)	
1890	Definitions: Geometry	Equations: Roots of Higher Order (1890-1960)	
1890	Definitions: Solid Geometry	Equations: Simple with Fractional Expressions (1890-2009)	
1890	Definitions: Trigonometry	Fractions: Partial (1890-1900)	
1890	Equations and Expressions: Modeling	Logarithms (1890-2009)	
1890	Equations: Forming Higher Order from Roots	Medians, Altitudes, Bisectors and Midsegments (1890-2009)	
1890	Equations: Forming Quadratics from Roots	Parallel and Perpendicular Lines (1890-2009)	
1890	Equations: Higher Order	Polygons and Circles: Inscribed (1890-2009)	
1890	Equations: Logarithmic	Polynomials: Addition and Subtraction of (1890-2009)	
1890	Equations: Roots of Higher Order	Polynomials: Multiplication and Division of (1890-2000)	
1890	Equations: Simple with Fractional Expressions	Progressions: Arithmetic and Geometric (1890-1960)	
1890	Exponents: Operations with	Progressions: Geometric (1890-1960)	
1890	Factors: Greatest Common	Proofs: Circle (1890-2009)	
1890	Factors: Prime	Proofs: Dihedral and Polyhedral Angles (1890-1960)	
1890	Fractions: Complex	Proofs: General Polyhedrons (1890-1930)	
1890	Fractions: Partial	Proofs: Lines and Planes in Space (1890-1960)	
1890	Logarithms	Proofs: Polygon (1890-2009)	
1890	Medians, Altitudes, Bisectors and Midsegments	Proofs: Prisms and Cylinders (1890-1940)	
1890	Mensuration	Proofs: Pyramids and Cones (1890-1950)	
1890	Notes and Interest	Proofs: Pythagoras (1890-1890)	
1890	Numbers: Prime and Composite	Proofs: Solid Geometry (1890-1960)	
1890	Parallel and Perpendicular Lines	Proofs: Spheres (1890-1970)	
1890	Percent	Proofs: Triangle (1890-2009)	
1890	Polygons and Circles: Inscribed	Proofs: Trigonometric (1890-2000)	
1890	Polygons: Area of	Quadratics: Completing the Square (1890-1940)	
1890	Polynomials: Addition and Subtraction of	Radicals: Operations with (1890-2009)	
1890	Polynomials: Multiplication and Division of	Radicals: Rationalizing Denominators (1890-2009)	
1890	Profit and Loss	Rate (1890-1980)	
1890	Progressions: Arithmetic	Rationals: Addition and Subtraction of (1890-2009)	
1890	Progressions: Arithmetic and Geometric	Rationals: Solving (1890-2009)	
1890	Progressions: Geometric	Series: Infinite (1890-1890)	
1890	Proofs: Circle	Solid Geometry: General Polyhedrons (1890-1970)	
1890	Proofs: Dihedral and Polyhedral Angles	Solid Geometry: Lines and Planes in Space (1890-2009)	
1890	Proofs: General Polyhedrons	Solid Geometry: Prisms and Cylinders (1890-2009)	
1890	Proofs: Lines and Planes in Space	Solid Geometry: Pyramids and Cones (1890-2009)	
1890	Proofs: Polygon	Solid Geometry: Spheres (1890-1970)	
1890	Proofs: Prisms and Cylinders	Systems: Linear (1890-2009)	
1890	Proofs: Pyramids and Cones	Systems: Other Nonlinear (1890-2009)	
1890	Proofs: Pythagoras	Systems: Quadratic Linear (1890-2009)	
1890	Proofs: Solid Geometry	Triangles: Mean Proportionals (1890-2009)	
1890	Proofs: Spheres	Trigonometric Functions: Evaluating (1890-2000)	
1890	Proofs: Triangle	Trigonometric Functions: Properties of (1890-2009)	
1890	Proofs: Trigonometric	Trigonometric Graphs (1890-2000)	

YEAR	TOPICS ASSESSED	TOPICS OBSERVED FOR THE FIRST TIME	TOPICS OBSERVED FOR THE LAST TIME
1890	Proportions	Trigonometric Identities: Angle Sum or Difference (1890-2000)	
1890	Quadratics: Completing the Square	Trigonometric Ratios: Basic (1890-2009)	
1890	Radicals: N-Roots	Trigonometry: Finding Sides (1890-2009)	
1890	Radicals: Operations with	Trigonometry: Law of Cosines (1890-2009)	
1890	Radicals: Rationalizing Denominators	Trigonometry: Law of Sines (1890-2009)	
1890	Radicals: Square Roots		
1890	Rate		
1890	Rate, Time and Distance		
1890	Ratio		
1890	Rationals: Addition and Subtraction of		
1890	Rationals: Solving		
1890	Series: Infinite		
1890	Solid Geometry: General Polyhedrons		
1890	Solid Geometry: Lines and Planes in Space		
1890	Solid Geometry: Prisms and Cylinders		
1890	Solid Geometry: Pyramids and Cones		
1890	Solid Geometry: Spheres		
1890	Systems: Linear		
1890	Systems: Other Nonlinear		
1890	Systems: Quadratic Linear		
1890	Triangles: Mean Proportionals		
1890	Triangles: Pythagoras		
1890	Trigonometric Functions: Evaluating		
1890	Trigonometric Functions: Properties of		
1890	Trigonometric Graphs		
1890	Trigonometric Identities: Angle Sum or Difference		
1890	Trigonometric Ratios: Basic		
1890	Trigonometry: Finding Sides		
1890	Trigonometry: Law of Cosines		
1890	Trigonometry: Law of Sines		
1890	Valuation		
1890	Volume		
	1900		
1900	Arithmetic Operations	Circles: Area of (1900-2000)	Continued Fractions (1900-1900)
1900	Bills and Receipts	Complementary, Supplementary and Vertical Angles (1900-2009)	Definitions: Advanced Arithmetic (1900-1900)
1900	Binomial Expansions	Continued Fractions (1900-1900)	Definitions: Trigonometry (1890-1900)
1900	Binomial Expansions: Undetermined Coefficients	Definitions: Advanced Arithmetic (1900-1900)	Fractions: Partial (1890-1900)
1900	Brokerage and Commission	Equations: Literal (1900-2009)	Series (1900-1900)
1900	Circles: Area of	Locus (1900-2009)	
1900	Circles: Center, Radius and Circumference	Order of Operations (1900-2000)	
1900	Circles: Chords	Polygons: Interior and Exterior Angles of (1900-2000)	
1900	Circles: Chords, Secants and Tangents	Polynomials: Factoring (1900-2009)	
1900	Combinatorics: Combinations	Proofs: Algebraic (1900-1950)	
1900	Complementary, Supplementary and Vertical Angles	Proofs: Geometry (1900-1970)	
1900	Constructions	Proofs: Spherical Polygons (1900-1960)	
1900	Continued Fractions (or 1.1.8.2 Special Numbers – Pi)	Quadratics: $a > 1$ (1900-2009)	
1900	Conversions	Quadratics: Using the Discriminant (1900-2009)	

YEAR	TOPICS ASSESSED	TOPICS OBSERVED FOR THE FIRST TIME	TOPICS OBSERVED FOR THE LAST TIME
1900	Cost	Radicals: Simplifying (1900-2009)	
1900	Definitions: Advanced Arithmetic	Radicals: Solving (1900-2009)	
1900	Definitions: Algebra	Series (1900-1900)	
1900	Definitions: Arithmetic	Solid Geometry: Spherical Polygons (1900-1960)	
1900	Definitions: Geometry	Systems: Three Variables (1900-1980)	
1900	Definitions: Solid Geometry	Systems: Writing (1900-2009)	
1900	Definitions: Trigonometry	Triangles: Equilateral (1900-2009)	
1900	Equations: Literal	Trigonometric Equations (1900-2009)	
1900	Equations: Logarithmic	Trigonometric Formulas: Derivations of (1900-1990)	
1900	Exponents: Operations with	Trigonometric Identities (1900-2009)	
1900	Factors: Greatest Common	Trigonometric Identities: Double and Half Angle (1900-2009)	
1900	Fraction Madness	Trigonometry: Finding Angles (1900-1990)	
1900	Fractions: Complex	Trigonometry: Finding Area (1900-2009)	
1900	Fractions: Partial	Trigonometry: Finding Sides Using Two Triangles (1900-1970)	
1900	Locus		
1900	Logarithms		
1900	Longitude		
1900	Mensuration		
1900	Notes and Interest		
1900	Order of Operations		
1900	Percent		
1900	Polygons and Circles: Inscribed		
1900	Polygons: Area of		
1900	Polygons: Interior and Exterior Angles of		
1900	Polynomials: Factoring		
1900	Polynomials: Multiplication and Division of		
1900	Profit and Loss		
1900	Progressions: Arithmetic and Geometric		
1900	Progressions: Geometric		
1900	Proofs: Algebraic		
1900	Proofs: Circle		
1900	Proofs: Geometry		
1900	Proofs: Lines and Planes in Space		
1900	Proofs: Polygon		
1900	Proofs: Prisms and Cylinders		
1900	Proofs: Spheres		
1900	Proofs: Spherical Polygons		
1900	Proofs: Triangle		
1900	Proofs: Trigonometric		
1900	Proportions		
1900	Quadratics: a > 1		
1900	Quadratics: Using the Discriminant		
1900	Radicals: N-Roots		
1900	Radicals: Operations with		
1900	Radicals: Rationalizing Denominators		
1900	Radicals: Simplifying		
1900	Radicals: Solving		
1900	Radicals: Square Roots		
1900	Rate, Time and Distance		

YEAR	TOPICS ASSESSED	TOPICS OBSERVED FOR THE FIRST TIME	TOPICS OBSERVED FOR THE LAST TIME
1900	Ratio		
1900	Series		
1900	Solid Geometry: Prisms and Cylinders		
1900	Solid Geometry: Pyramids and Cones		
1900	Solid Geometry: Spheres		
1900	Solid Geometry: Spherical Polygons		
1900	Systems: Other Nonlinear		
1900	Systems: Three Variables		
1900	Systems: Writing		
1900	Triangles: Equilateral		
1900	Triangles: Mean Proportionals		
1900	Trigonometric Equations		
1900	Trigonometric Formulas: Derivations of		
1900	Trigonometric Functions: Properties of		
1900	Trigonometric Graphs		
1900	Trigonometric Identities		
1900	Trigonometric Identities: Double and Half Angle		
1900	Trigonometric Ratios: Basic		
1900	Trigonometry: Finding Angles		
1900	Trigonometry: Finding Area		
1900	Trigonometry: Finding Sides		
1900	Trigonometry: Finding Sides Using Two Triangles		
1900	Trigonometry: Law of Cosines		
1900	Trigonometry: Law of Sines		
1900	Valuation		
1900	Volume		
	1909		
1909	Arithmetic Operations	Circles: Arc Measure (1909-1909)	Arithmetic: Numeration (1866-1909)
1909	Arithmetic: Addition	Equations and Expressions: Using Substitution in (1909-2009)	Binomial Expansions: Undetermined Coefficients (1890-1909)
1909	Arithmetic: Division	Graphing Higher Order Equations (1909-1940)	Longitude (1866-1909)
1909	Arithmetic: Numeration	Matrices (1909-1960)	
1909	Bills and Receipts	Numbers: Complex (1909-2009)	
1909	Binomial Expansions: Undetermined Coefficients	Quadratics: Noninteger Solutions (1909-2000)	
1909	Brokerage and Commission	Quadratics: Solving by Factoring (1909-2009)	
1909	Circles: Arc Measure	Quadratics: Writing (1909-2000)	
1909	Circles: Chords	Triangle Inequalities (1909-2009)	
1909	Combinatorics: Combinations	Triangles: Interior and Exterior Angles of (1909-2009)	
1909	Constructions		
1909	Conversions		
1909	Cost		
1909	Definitions: Arithmetic		
1909	Equations and Expressions: Modeling		
1909	Equations and Expressions: Using Substitution in		
1909	Equations: Forming Higher Order from Roots		
1909	Equations: Literal		
1909	Equations: Logarithmic		
1909	Equations: Roots of Higher Order		

YEAR	TOPICS ASSESSED	TOPICS OBSERVED FOR THE FIRST TIME	TOPICS OBSERVED FOR THE LAST TIME
1909	Exponents: Operations with		
1909	Factors: Prime		
1909	Fraction Madness		
1909	Fractions		
1909	Graphing Higher Order Equations		
1909	Locus		
1909	Logarithms		
1909	Longitude		
1909	Matrices		
1909	Mensuration		
1909	Notes and Interest		
1909	Numbers: Complex		
1909	Order of Operations		
1909	Percent		
1909	Polygons and Circles: Inscribed		
1909	Polygons: Interior and Exterior Angles of		
1909	Polynomials: Factoring		
1909	Polynomials: Multiplication and Division of		
1909	Profit and Loss		
1909	Progressions: Arithmetic		
1909	Progressions: Arithmetic and Geometric		
1909	Progressions: Geometric		
1909	Proofs: Algebraic		
1909	Proofs: Circle		
1909	Proofs: Dihedral and Polyhedral Angles		
1909	Proofs: Lines and Planes in Space		
1909	Proofs: Polygon		
1909	Proofs: Prisms and Cylinders		
1909	Proofs: Pyramids and Cones		
1909	Proofs: Spheres		
1909	Proofs: Spherical Polygons		
1909	Proofs: Triangle		
1909	Proofs: Trigonometric		
1909	Proportions		
1909	Quadratics: a > 1		
1909	Quadratics: Noninteger Solutions		
1909	Quadratics: Solving by Factoring		
1909	Quadratics: Using the Discriminant		
1909	Quadratics: Writing		
1909	Radicals: N-Roots		
1909	Radicals: Operations with		
1909	Radicals: Simplifying		
1909	Radicals: Solving		
1909	Radicals: Square Roots		
1909	Rate		
1909	Rate, Time and Distance		
1909	Ratio		
1909	Series		
1909	Solid Geometry: General Polyhedrons		

YEAR	TOPICS ASSESSED	TOPICS OBSERVED FOR THE FIRST TIME	TOPICS OBSERVED FOR THE LAST TIME
1909	Solid Geometry: Prisms and Cylinders		
1909	Solid Geometry: Pyramids and Cones		
1909	Solid Geometry: Spheres		
1909	Solid Geometry: Spherical Polygons		
1909	Systems: Linear		
1909	Systems: Other Nonlinear		
1909	Systems: Quadratic Linear		
1909	Systems: Three Variables		
1909	Systems: Writing		
1909	Triangle Inequalities		
1909	Triangles: Interior and Exterior Angles of		
1909	Triangles: Pythagoras		
1909	Trigonometric Equations		
1909	Trigonometric Formulas: Derivations of		
1909	Trigonometric Ratios: Basic		
1909	Trigonometry: Finding Area		
1909	Trigonometry: Finding Sides		
1909	Trigonometry: Finding Sides Using Two Triangles		
1909	Trigonometry: Law of Cosines		
1909	Valuation		
1909	Volume		
	1920		
1920	Alligation	Alligation (1920-1960)	Definitions: Solid Geometry (1890-1920)
1920	Arithmetic: Addition	Circles: Tangents (1920-2009)	
1920	Bills and Receipts	Consecutive Integers (1920-2009)	
1920	Binomial Expansions	Equations: Forming from Imaginary Roots (1920-1930)	
1920	Brokerage and Commission	Equations: Graphing (1920-2000)	
1920	Central Tendency: Averages	Exponential Functions and Equations (1920-2009)	
1920	Circles: Chords, Secants and Tangents	Graphic Representation (1920-1980)	
1920	Circles: Tangents	Quadratics: Solving (1920-2009)	
1920	Combinatorics: Combinations	Quadratics: Sum and Product of Roots (1920-2009)	
1920	Consecutive Integers	Special Quadrilaterals: Rhombuses (1920-2009)	
1920	Constructions	Systems: Writing Quadratic (1920-1980)	
1920	Conversions	Triangles: Vectors (1920-2000)	
1920	Cost	Trigonometry: Law of Sines - The Ambiguous Case (1920-2000)	
1920	Definitions: Algebra		
1920	Definitions: Geometry		
1920	Definitions: Solid Geometry		
1920	Equations and Expressions: Modeling		
1920	Equations and Expressions: Using Substitution in		
1920	Equations: Forming from Imaginary Roots		
1920	Equations: Forming Quadratics from Roots		
1920	Equations: Graphing		
1920	Equations: Literal		
1920	Equations: Logarithmic		
1920	Equations: Roots of Higher Order		
1920	Equations: Simple with Fractional Expressions		

YEAR	TOPICS ASSESSED	TOPICS OBSERVED FOR THE FIRST TIME	TOPICS OBSERVED FOR THE LAST TIME
1920	Exponential Functions and Equations		
1920	Exponents: Operations with		
1920	Factors: Prime		
1920	Fractions		
1920	Fractions: Complex		
1920	Graphic Representation		
1920	Graphing Higher Order Equations		
1920	Logarithms		
1920	Notes and Interest		
1920	Numbers: Complex		
1920	Order of Operations		
1920	Percent		
1920	Polygons and Circles: Inscribed		
1920	Polygons: Area of		
1920	Polygons: Interior and Exterior Angles of		
1920	Polynomials: Addition and Subtraction of		
1920	Polynomials: Factoring		
1920	Polynomials: Multiplication and Division of		
1920	Profit and Loss		
1920	Progressions: Arithmetic		
1920	Progressions: Arithmetic and Geometric		
1920	Proofs: Circle		
1920	Proofs: Dihedral and Polyhedral Angles		
1920	Proofs: General Polyhedrons		
1920	Proofs: Lines and Planes in Space		
1920	Proofs: Polygon		
1920	Proofs: Prisms and Cylinders		
1920	Proofs: Pyramids and Cones		
1920	Proofs: Spheres		
1920	Proofs: Spherical Polygons		
1920	Proofs: Triangle		
1920	Proofs: Trigonometric		
1920	Proportions		
1920	Quadratics: Noninteger Solutions		
1920	Quadratics: Solving		
1920	Quadratics: Sum and Product of Roots		
1920	Quadratics: Using the Discriminant		
1920	Radicals: N-Roots		
1920	Radicals: Operations with		
1920	Radicals: Rationalizing Denominators		
1920	Radicals: Simplifying		
1920	Radicals: Square Roots		
1920	Rate, Time and Distance		
1920	Rationals: Addition and Subtraction of		
1920	Rationals: Solving		
1920	Solid Geometry: General Polyhedrons		
1920	Solid Geometry: Prisms and Cylinders		
1920	Solid Geometry: Pyramids and Cones		
1920	Solid Geometry: Spheres		

YEAR	TOPICS ASSESSED	TOPICS OBSERVED FOR THE FIRST TIME	TOPICS OBSERVED FOR THE LAST TIME
1920	Solid Geometry: Spherical Polygons		
1920	Special Quadrilaterals: Rhombuses		
1920	Systems: Linear		
1920	Systems: Other Nonlinear		
1920	Systems: Quadratic Linear		
1920	Systems: Writing		
1920	Systems: Writing Quadratic		
1920	Triangles: Pythagoras		
1920	Triangles: Vectors		
1920	Trigonometric Equations		
1920	Trigonometric Functions: Evaluating		
1920	Trigonometric Functions: Properties of		
1920	Trigonometric Identities		
1920	Trigonometric Identities: Angle Sum or Difference		
1920	Trigonometric Identities: Double and Half Angle		
1920	Trigonometry: Finding Area		
1920	Trigonometry: Finding Sides		
1920	Trigonometry: Finding Sides Using Two Triangles		
1920	Trigonometry: Law of Cosines		
1920	Trigonometry: Law of Sines		
1920	Trigonometry: Law of Sines - The Ambiguous Case		
1920	Volume		
	1930		
1930	Alligation	Circles: Radian Measure (1930-2009)	Arithmetic Operations (1880-1930)
1930	Arithmetic Operations	Combinatorics: Permutations (1930-2009)	Equations: Forming from Imaginary Roots (1920-1930)
1930	Arithmetic: Addition	Equations: Degrees of (1930-1950)	Proofs: General Polyhedrons (1890-1930)
1930	Arithmetic: Division	Equations: Forming New from Modified Roots (1930-1950)	
1930	Arithmetic: Subtraction	Equations: Modeling from a Table (1930-2009)	
1930	Binomial Expansions	Equations: Simple (1930-2009)	
1930	Central Tendency: Averages	Exponents (1930-2009)	
1930	Circles: Arc Measure	Graphs: Identifying Equations of (1930-2009)	
1930	Circles: Area of	Numbers: Imaginary (1930-2009)	
1930	Circles: Center, Radius and Circumference	Numbers: Properties of Real (1930-2009)	
1930	Circles: Chords	Points on a Line: Identification of (1930-2000)	
1930	Circles: Chords, Secants and Tangents	Quadratics: Find Vertex Given Equation (1930-2009)	
1930	Circles: Radian Measure	Quadratics: Graphing (1930-2009)	
1930	Circles: Tangents	Similarity (1930-2009)	
1930	Combinatorics: Combinations	Similarity: Right Triangles (1930-2009)	
1930	Combinatorics: Permutations	Slope (1930-2009)	
1930	Complementary, Supplementary and Vertical Angles	Solid Geometry: Dihedral and Polyhedral Angles (1930-1970)	
1930	Consecutive Integers	Special Quadrilaterals (1930-1990)	
1930	Constructions	Special Quadrilaterals: Parallelograms (1930-2009)	
1930	Conversions	Special Quadrilaterals: Rectangles and Squares (1930-2009)	
1930	Cost	Special Quadrilaterals: Trapezoids (1930-2009)	
1930	Decimals	Summations (1930-2009)	
1930	Definitions: Algebra	Triangles: Isosceles (1930-)	
1930	Equations and Expressions: Modeling	Triangles: Special Right (1930-2000)	

YEAR	TOPICS ASSESSED	TOPICS OBSERVED FOR THE FIRST TIME	TOPICS OBSERVED FOR THE LAST TIME
1930	Equations and Expressions: Using Substitution in	Trigonometric Functions: Inverses of (1930-2009)	
1930	Equations: Degrees of	Trigonometric Functions: Logarithms of (1930-1980)	
1930	Equations: Forming from Imaginary Roots	Trigonometric Ratios: Cofunction & Reciprocal (1930-2009)	
1930	Equations: Forming Higher Order from Roots	Trigonometry: Law of Tangents (1930-1950)	
1930	Equations: Forming New from Modified Roots	Variation: Inverse (1930-2009)	
1930	Equations: Forming Quadratics from Roots		
1930	Equations: Graphing		
1930	Equations: Higher Order		
1930	Equations: Literal		
1930	Equations: Logarithmic		
1930	Equations: Modeling from a Table		
1930	Equations: Roots of Higher Order		
1930	Equations: Simple		
1930	Equations: Simple with Fractional Expressions		
1930	Exponential Functions and Equations		
1930	Exponents		
1930	Exponents: Operations with		
1930	Fractions		
1930	Fractions: Complex		
1930	Graphic Representation		
1930	Graphing Higher Order Equations		
1930	Graphs: Identifying Equations of		
1930	Locus		
1930	Logarithms		
1930	Medians, Altitudes, Bisectors and Midsegments		
1930	Notes and Interest		
1930	Numbers: Complex		
1930	Numbers: Imaginary		
1930	Numbers: Properties of Real		
1930	Parallel and Perpendicular Lines		
1930	Percent		
1930	Points on a Line: Identification of		
1930	Polygons and Circles: Inscribed		
1930	Polygons: Area of		
1930	Polygons: Interior and Exterior Angles of		
1930	Polynomials: Factoring		
1930	Polynomials: Multiplication and Division of		
1930	Profit and Loss		
1930	Progressions: Arithmetic		
1930	Progressions: Geometric		
1930	Proofs: Algebraic		
1930	Proofs: Circle		
1930	Proofs: General Polyhedrons		
1930	Proofs: Lines and Planes in Space		
1930	Proofs: Polygon		
1930	Proofs: Prisms and Cylinders		
1930	Proofs: Spherical Polygons		
1930	Proofs: Triangle		
1930	Proofs: Trigonometric		

YEAR	TOPICS ASSESSED	TOPICS OBSERVED FOR THE FIRST TIME	TOPICS OBSERVED FOR THE LAST TIME
1930	Proportions		
1930	Quadratics: Find Vertex Given Equation		
1930	Quadratics: Graphing		
1930	Quadratics: Noninteger Solutions		
1930	Quadratics: Solving		
1930	Quadratics: Solving by Factoring		
1930	Quadratics: Sum and Product of Roots		
1930	Quadratics: Using the Discriminant		
1930	Quadratics: Writing		
1930	Radicals: N-Roots		
1930	Radicals: Operations with		
1930	Radicals: Rationalizing Denominators		
1930	Radicals: Simplifying		
1930	Radicals: Solving		
1930	Radicals: Square Roots		
1930	Rate		
1930	Rate, Time and Distance		
1930	Ratio		
1930	Rationals: Addition and Subtraction of		
1930	Rationals: Solving		
1930	Similarity		
1930	Similarity: Right Triangles		
1930	Slope		
1930	Solid Geometry: Dihedral and Polyhedral Angles		
1930	Solid Geometry: General Polyhedrons		
1930	Solid Geometry: Lines and Planes in Space		
1930	Solid Geometry: Prisms and Cylinders		
1930	Solid Geometry: Pyramids and Cones		
1930	Solid Geometry: Spheres		
1930	Solid Geometry: Spherical Polygons		
1930	Special Quadrilaterals		
1930	Special Quadrilaterals: Parallelograms		
1930	Special Quadrilaterals: Rectangles and Squares		
1930	Special Quadrilaterals: Rhombuses		
1930	Special Quadrilaterals: Trapezoids		
1930	Summations		
1930	Systems: Linear		
1930	Systems: Other Nonlinear		
1930	Systems: Quadratic Linear		
1930	Systems: Writing		
1930	Triangle Inequalities		
1930	Triangles: Equilateral		
1930	Triangles: Interior and Exterior Angles of		
1930	Triangles: Isosceles		
1930	Triangles: Mean Proportionals		
1930	Triangles: Pythagoras		
1930	Triangles: Special Right		
1930	Trigonometric Equations		
1930	Trigonometric Formulas: Derivations of		

YEAR	TOPICS ASSESSED	TOPICS OBSERVED FOR THE FIRST TIME	TOPICS OBSERVED FOR THE LAST TIME
1930	Trigonometric Functions: Evaluating		
1930	Trigonometric Functions: Inverses of		
1930	Trigonometric Functions: Logarithms of		
1930	Trigonometric Functions: Properties of		
1930	Trigonometric Graphs		
1930	Trigonometric Identities		
1930	Trigonometric Identities: Angle Sum or Difference		
1930	Trigonometric Identities: Double and Half Angle		
1930	Trigonometric Ratios: Basic		
1930	Trigonometric Ratios: Cofunction & Reciprocal		
1930	Trigonometry: Finding Angles		
1930	Trigonometry: Finding Area		
1930	Trigonometry: Finding Sides		
1930	Trigonometry: Finding Sides Using Two Triangles		
1930	Trigonometry: Law of Cosines		
1930	Trigonometry: Law of Sines		
1930	Trigonometry: Law of Sines - The Ambiguous Case		
1930	Trigonometry: Law of Tangents		
1930	Valuation		
1930	Variation: Inverse		
1930	Volume		
	1940		
1940	Arithmetic: Addition	Central Tendency: Normal Distributions (1940-2009)	Definitions: Arithmetic (1866-1940)
1940	Arithmetic: Division	Circles: Equations of (1940-2009)	Graphing Higher Order Equations (1909-1940)
1940	Arithmetic: Multiplication	Conics (1940-2009)	Proofs: Prisms and Cylinders (1890-1940)
1940	Arithmetic: Subtraction	Equations: Writing Linear (1940-2009)	Quadratics: Completing the Square (1890-1940)
1940	Binomial Expansions	Functions: Inverses of (1940-2009)	Valuation (1866-1940)
1940	Brokerage and Commission	Logical Reasoning: Converse (1940-2000)	
1940	Central Tendency: Averages	Numbers: Comparing Reals (1940-2000)	
1940	Central Tendency: Normal Distributions	Perimeter (1940-2009)	
1940	Circles: Arc Measure	Probability: Independent Events (1940-2009)	
1940	Circles: Area of	Probability: Theoretical (1940-2009)	
1940	Circles: Center, Radius and Circumference	Slope Intercept Form of a Line (1940-2000)	
1940	Circles: Chords	Trigonometry: Polar Form (1940-1960)	
1940	Circles: Chords, Secants and Tangents	Trigonometry: Reference Angles (1940-1990)	
1940	Circles: Equations of	Variation: Direct (1940-2009)	
1940	Circles: Tangents		
1940	Combinatorics: Combinations		
1940	Combinatorics: Multiplication Counting Principle		
1940	Complementary, Supplementary and Vertical Angles		
1940	Conics		
1940	Constructions		
1940	Conversions		
1940	Cost		
1940	Decimals		
1940	Definitions: Arithmetic		
1940	Equations and Expressions: Modeling		

YEAR	TOPICS ASSESSED	TOPICS OBSERVED FOR THE FIRST TIME	TOPICS OBSERVED FOR THE LAST TIME
1940	Equations and Expressions: Using Substitution in		
1940	Equations: Forming New from Modified Roots		
1940	Equations: Forming Quadratics from Roots		
1940	Equations: Graphing		
1940	Equations: Higher Order		
1940	Equations: Literal		
1940	Equations: Logarithmic		
1940	Equations: Modeling from a Table		
1940	Equations: Roots of Higher Order		
1940	Equations: Simple		
1940	Equations: Simple with Fractional Expressions		
1940	Equations: Writing Linear		
1940	Exponential Functions and Equations		
1940	Exponents		
1940	Exponents: Operations with		
1940	Fractions: Complex		
1940	Functions: Inverses of		
1940	Graphing Higher Order Equations		
1940	Locus		
1940	Logarithms		
1940	Logical Reasoning: Converse		
1940	Medians, Altitudes, Bisectors and Midsegments		
1940	Mensuration		
1940	Notes and Interest		
1940	Numbers: Comparing Reals		
1940	Numbers: Complex		
1940	Numbers: Imaginary		
1940	Parallel and Perpendicular Lines		
1940	Percent		
1940	Perimeter		
1940	Points on a Line: Identification of		
1940	Polygons and Circles: Inscribed		
1940	Polygons: Area of		
1940	Polygons: Interior and Exterior Angles of		
1940	Polynomials: Factoring		
1940	Polynomials: Multiplication and Division of		
1940	Probability: Independent Events		
1940	Probability: Theoretical		
1940	Profit and Loss		
1940	Progressions: Arithmetic		
1940	Progressions: Arithmetic and Geometric		
1940	Progressions: Geometric		
1940	Proofs: Algebraic		
1940	Proofs: Circle		
1940	Proofs: Lines and Planes in Space		
1940	Proofs: Polygon		
1940	Proofs: Prisms and Cylinders		
1940	Proofs: Pyramids and Cones		
1940	Proofs: Spheres		

YEAR	TOPICS ASSESSED	TOPICS OBSERVED FOR THE FIRST TIME	TOPICS OBSERVED FOR THE LAST TIME
1940	Proofs: Spherical Polygons		
1940	Proofs: Triangle		
1940	Proofs: Trigonometric		
1940	Proportions		
1940	Quadratics: a > 1		
1940	Quadratics: Completing the Square		
1940	Quadratics: Graphing		
1940	Quadratics: Noninteger Solutions		
1940	Quadratics: Solving		
1940	Quadratics: Sum and Product of Roots		
1940	Quadratics: Using the Discriminant		
1940	Quadratics: Writing		
1940	Radicals: N-Roots		
1940	Radicals: Operations with		
1940	Radicals: Rationalizing Denominators		
1940	Radicals: Simplifying		
1940	Radicals: Solving		
1940	Radicals: Square Roots		
1940	Rate		
1940	Rate, Time and Distance		
1940	Ratio		
1940	Rationals: Addition and Subtraction of		
1940	Similarity		
1940	Slope		
1940	Slope Intercept Form of a Line		
1940	Solid Geometry: Dihedral and Polyhedral Angles		
1940	Solid Geometry: General Polyhedrons		
1940	Solid Geometry: Lines and Planes in Space		
1940	Solid Geometry: Prisms and Cylinders		
1940	Solid Geometry: Pyramids and Cones		
1940	Solid Geometry: Spheres		
1940	Solid Geometry: Spherical Polygons		
1940	Special Quadrilaterals: Parallelograms		
1940	Special Quadrilaterals: Rectangles and Squares		
1940	Special Quadrilaterals: Rhombuses		
1940	Special Quadrilaterals: Trapezoids		
1940	Systems: Other Nonlinear		
1940	Systems: Quadratic Linear		
1940	Systems: Three Variables		
1940	Systems: Writing		
1940	Systems: Writing Quadratic		
1940	Triangle Inequalities		
1940	Triangles: Equilateral		
1940	Triangles: Interior and Exterior Angles of		
1940	Triangles: Isosceles		
1940	Triangles: Mean Proportionals		
1940	Triangles: Pythagoras		
1940	Triangles: Special Right		
1940	Triangles: Vectors		

YEAR	TOPICS ASSESSED	TOPICS OBSERVED FOR THE FIRST TIME	TOPICS OBSERVED FOR THE LAST TIME
1940	Trigonometric Equations		
1940	Trigonometric Formulas: Derivations of		
1940	Trigonometric Functions: Evaluating		
1940	Trigonometric Functions: Inverses of		
1940	Trigonometric Functions: Logarithms of		
1940	Trigonometric Functions: Properties of		
1940	Trigonometric Graphs		
1940	Trigonometric Identities		
1940	Trigonometric Identities: Angle Sum or Difference		
1940	Trigonometric Identities: Double and Half Angle		
1940	Trigonometry: Finding Area		
1940	Trigonometry: Finding Sides		
1940	Trigonometry: Finding Sides Using Two Triangles		
1940	Trigonometry: Law of Cosines		
1940	Trigonometry: Law of Sines		
1940	Trigonometry: Law of Sines - The Ambiguous Case		
1940	Trigonometry: Polar Form		
1940	Trigonometry: Reference Angles		
1940	Valuation		
1940	Variation: Direct		
1940	Variation: Inverse		
1940	Volume		
	1950		
1950	Alligation	Calculus: Differential (1950-1960)	Arithmetic: Addition (1870-1950)
1950	Arithmetic: Addition	Distance (1950-2009)	Arithmetic: Division (1866-1950)
1950	Arithmetic: Division	Functional Notation (1950-2000)	Arithmetic: Multiplication (1866-1950)
1950	Arithmetic: Multiplication	Graphic Representation: Histograms and Tables (1950-2009)	Arithmetic: Subtraction (1870-1950)
1950	Arithmetic: Subtraction	Midpoint (1950-2009)	Bills and Receipts (1870-1950)
1950	Bills and Receipts	Parallel Lines: Angles Involving (1950-2009)	Brokerage and Commission (1880-1950)
1950	Binomial Expansions	Probability: Mutually Exclusive Events (1950-2000)	Cost (1866-1950)
1950	Brokerage and Commission	Quadratics: Axis of Symmetry (1950-2009)	Decimals (1866-1950)
1950	Calculus: Differential	Quadratics: Difference of Perfect Squares (1950-2009)	Equations: Degrees of (1930-1950)
1950	Central Tendency: Averages		Equations: Forming New from Modified Roots (1930-1950)
1950	Central Tendency: Normal Distributions		Fractions (1866-1950)
1950	Circles: Arc Measure		Mensuration (1866-1950)
1950	Circles: Area of		Profit and Loss (1870-1950)
1950	Circles: Center, Radius and Circumference		Proofs: Algebraic (1900-1950)
1950	Circles: Chords		Proofs: Pyramids and Cones (1890-1950)
1950	Circles: Chords, Secants and Tangents		Trigonometry: Law of Tangents (1930-1950)
1950	Circles: Equations of		
1950	Circles: Radian Measure		
1950	Circles: Tangents		
1950	Combinatorics: Combinations		
1950	Combinatorics: Multiplication Counting Principle		
1950	Combinatorics: Permutations		
1950	Complementary, Supplementary and Vertical Angles		
1950	Conics		

YEAR	TOPICS ASSESSED	TOPICS OBSERVED FOR THE FIRST TIME	TOPICS OBSERVED FOR THE LAST TIME
1950	Constructions		
1950	Conversions		
1950	Cost		
1950	Decimals		
1950	Definitions: Geometry		
1950	Distance		
1950	Equations and Expressions: Modeling		
1950	Equations and Expressions: Using Substitution in		
1950	Equations: Degrees of		
1950	Equations: Forming New from Modified Roots		
1950	Equations: Higher Order		
1950	Equations: Literal		
1950	Equations: Logarithmic		
1950	Equations: Modeling from a Table		
1950	Equations: Roots of Higher Order		
1950	Equations: Simple		
1950	Equations: Simple with Fractional Expressions		
1950	Equations: Writing Linear		
1950	Exponential Functions and Equations		
1950	Exponents		
1950	Exponents: Operations with		
1950	Fraction Madness		
1950	Fractions		
1950	Fractions: Complex		
1950	Functional Notation		
1950	Graphic Representation: Histograms and Tables		
1950	Locus		
1950	Logarithms		
1950	Logical Reasoning: Converse		
1950	Medians, Altitudes, Bisectors and Midsegments		
1950	Mensuration		
1950	Midpoint		
1950	Notes and Interest		
1950	Numbers: Comparing Reals		
1950	Numbers: Complex		
1950	Numbers: Imaginary		
1950	Numbers: Properties of Real		
1950	Parallel and Perpendicular Lines		
1950	Parallel Lines: Angles Involving		
1950	Percent		
1950	Perimeter		
1950	Polygons and Circles: Inscribed		
1950	Polygons: Area of		
1950	Polygons: Interior and Exterior Angles of		
1950	Polynomials: Factoring		
1950	Polynomials: Multiplication and Division of		
1950	Probability: Mutually Exclusive Events		
1950	Probability: Theoretical		
1950	Profit and Loss		

YEAR	TOPICS ASSESSED	TOPICS OBSERVED FOR THE FIRST TIME	TOPICS OBSERVED FOR THE LAST TIME
1950	Progressions: Arithmetic		
1950	Progressions: Arithmetic and Geometric		
1950	Progressions: Geometric		
1950	Proofs: Algebraic		
1950	Proofs: Circle		
1950	Proofs: Dihedral and Polyhedral Angles		
1950	Proofs: Lines and Planes in Space		
1950	Proofs: Polygon		
1950	Proofs: Pyramids and Cones		
1950	Proofs: Spherical Polygons		
1950	Proofs: Triangle		
1950	Proofs: Trigonometric		
1950	Proportions		
1950	Quadratics: Axis of Symmetry		
1950	Quadratics: Difference of Perfect Squares		
1950	Quadratics: Find Vertex Given Equation		
1950	Quadratics: Graphing		
1950	Quadratics: Noninteger Solutions		
1950	Quadratics: Solving		
1950	Quadratics: Sum and Product of Roots		
1950	Quadratics: Using the Discriminant		
1950	Quadratics: Writing		
1950	Radicals: N-Roots		
1950	Radicals: Operations with		
1950	Radicals: Rationalizing Denominators		
1950	Radicals: Solving		
1950	Radicals: Square Roots		
1950	Rate, Time and Distance		
1950	Ratio		
1950	Rationals: Addition and Subtraction of		
1950	Similarity		
1950	Slope		
1950	Slope Intercept Form of a Line		
1950	Solid Geometry: Dihedral and Polyhedral Angles		
1950	Solid Geometry: General Polyhedrons		
1950	Solid Geometry: Lines and Planes in Space		
1950	Solid Geometry: Prisms and Cylinders		
1950	Solid Geometry: Pyramids and Cones		
1950	Solid Geometry: Spheres		
1950	Solid Geometry: Spherical Polygons		
1950	Special Quadrilaterals		
1950	Special Quadrilaterals: Parallelograms		
1950	Special Quadrilaterals: Rectangles and Squares		
1950	Special Quadrilaterals: Rhombuses		
1950	Summations		
1950	Systems: Linear		
1950	Systems: Other Nonlinear		
1950	Systems: Quadratic Linear		
1950	Systems: Three Variables		

YEAR	TOPICS ASSESSED	TOPICS OBSERVED FOR THE FIRST TIME	TOPICS OBSERVED FOR THE LAST TIME
1950	Systems: Writing		
1950	Triangle Inequalities		
1950	Triangles: Equilateral		
1950	Triangles: Interior and Exterior Angles of		
1950	Triangles: Isosceles		
1950	Triangles: Mean Proportionals		
1950	Triangles: Vectors		
1950	Trigonometric Equations		
1950	Trigonometric Formulas: Derivations of		
1950	Trigonometric Functions: Evaluating		
1950	Trigonometric Functions: Inverses of		
1950	Trigonometric Functions: Logarithms of		
1950	Trigonometric Functions: Properties of		
1950	Trigonometric Graphs		
1950	Trigonometric Identities		
1950	Trigonometric Identities: Angle Sum or Difference		
1950	Trigonometric Identities: Double and Half Angle		
1950	Trigonometric Ratios: Cofunction & Reciprocal		
1950	Trigonometry: Finding Angles		
1950	Trigonometry: Finding Area		
1950	Trigonometry: Finding Sides		
1950	Trigonometry: Finding Sides Using Two Triangles		
1950	Trigonometry: Law of Cosines		
1950	Trigonometry: Law of Sines		
1950	Trigonometry: Law of Sines - The Ambiguous Case		
1950	Trigonometry: Law of Tangents		
1950	Trigonometry: Polar Form		
1950	Trigonometry: Reference Angles		
1950	Variation: Direct		
1950	Variation: Inverse		
1950	Volume		
	1960		
1960	Alligation	Calculus: Integral (1960-1960)	Alligation (1920-1960)
1960	Binomial Expansions	Inequalities: Linear (1960-2009)	Calculus: Differential (1950-1960)
1960	Calculus: Differential	Logical Reasoning: Contrapositive (1960-2009)	Calculus: Integral (1960-1960)
1960	Calculus: Integral	Probability: Dependent Events (1960-2000)	Definitions: Geometry (1890-1960)
1960	Central Tendency: Averages	Proofs: Coordinate (1960-2009)	Equations: Forming Higher Order from Roots (1890-1960)
1960	Circles: Arc Measure	Quadratics: Imaginary Solutions (1960-2009)	Equations: Higher Order (1890-1960)
1960	Circles: Area of	Quadratics: Inequalities (1960-2009)	Equations: Roots of Higher Order (1890-1960)
1960	Circles: Center, Radius and Circumference	Scientific Notation (1960-2009)	Matrices (1909-1960)
1960	Circles: Chords	Trigonometric Expressions: Factoring (1960-1960)	Progressions: Arithmetic (1870-1960)
1960	Circles: Chords, Secants and Tangents	Trigonometry: Polar Coordinates (1960-1960)	Progressions: Arithmetic and Geometric (1890-1960)
1960	Circles: Equations of	Trigonometry: Terminal Sides of Angles (1960-1990)	Progressions: Geometric (1890-1960)
1960	Circles: Radian Measure		Proofs: Dihedral and Polyhedral Angles (1890-1960)
1960	Circles: Tangents		Proofs: Lines and Planes in Space (1890-1960)
1960	Combinatorics: Combinations		Proofs: Solid Geometry (1890-1960)
1960	Combinatorics: Multiplication Counting Principle		Proofs: Spherical Polygons (1900-1960)

YEAR	TOPICS ASSESSED	TOPICS OBSERVED FOR THE FIRST TIME	TOPICS OBSERVED FOR THE LAST TIME
1960	Complementary, Supplementary and Vertical Angles		Solid Geometry: Spherical Polygons (1900-1960)
1960	Conics		Trigonometric Expressions: Factoring (1960-1960)
1960	Consecutive Integers		Trigonometry: Polar Coordinates (1960-1960)
1960	Constructions		Trigonometry: Polar Form (1940-1960)
1960	Conversions		
1960	Definitions: Geometry		
1960	Distance		
1960	Equations and Expressions: Modeling		
1960	Equations and Expressions: Using Substitution in		
1960	Equations: Forming Higher Order from Roots		
1960	Equations: Forming Quadratics from Roots		
1960	Equations: Higher Order		
1960	Equations: Literal		
1960	Equations: Logarithmic		
1960	Equations: Modeling from a Table		
1960	Equations: Roots of Higher Order		
1960	Equations: Writing Linear		
1960	Exponential Functions and Equations		
1960	Exponents		
1960	Fractions: Complex		
1960	Functional Notation		
1960	Inequalities: Linear		
1960	Locus		
1960	Logarithms		
1960	Logical Reasoning: Contrapositive		
1960	Logical Reasoning: Converse		
1960	Matrices		
1960	Medians, Altitudes, Bisectors and Midsegments		
1960	Midpoint		
1960	Numbers: Complex		
1960	Numbers: Imaginary		
1960	Numbers: Properties of Real		
1960	Parallel and Perpendicular Lines		
1960	Perimeter		
1960	Points on a Line: Identification of		
1960	Polygons and Circles: Inscribed		
1960	Polygons: Area of		
1960	Polygons: Interior and Exterior Angles of		
1960	Polynomials: Factoring		
1960	Polynomials: Multiplication and Division of		
1960	Probability: Dependent Events		
1960	Probability: Independent Events		
1960	Probability: Mutually Exclusive Events		
1960	Probability: Theoretical		
1960	Progressions: Arithmetic		
1960	Progressions: Arithmetic and Geometric		
1960	Progressions: Geometric		
1960	Proofs: Circle		
1960	Proofs: Coordinate		

YEAR	TOPICS ASSESSED	TOPICS OBSERVED FOR THE FIRST TIME	TOPICS OBSERVED FOR THE LAST TIME
1960	Proofs: Dihedral and Polyhedral Angles		
1960	Proofs: Lines and Planes in Space		
1960	Proofs: Solid Geometry		
1960	Proofs: Spherical Polygons		
1960	Proofs: Triangle		
1960	Proofs: Trigonometric		
1960	Quadratics: a > 1		
1960	Quadratics: Axis of Symmetry		
1960	Quadratics: Difference of Perfect Squares		
1960	Quadratics: Find Vertex Given Equation		
1960	Quadratics: Graphing		
1960	Quadratics: Imaginary Solutions		
1960	Quadratics: Inequalities		
1960	Quadratics: Noninteger Solutions		
1960	Quadratics: Solving		
1960	Quadratics: Solving by Factoring		
1960	Quadratics: Sum and Product of Roots		
1960	Quadratics: Using the Discriminant		
1960	Radicals: N-Roots		
1960	Radicals: Rationalizing Denominators		
1960	Radicals: Solving		
1960	Rate		
1960	Rate, Time and Distance		
1960	Ratio		
1960	Rationals: Addition and Subtraction of		
1960	Rationals: Solving		
1960	Scientific Notation		
1960	Similarity		
1960	Similarity: Right Triangles		
1960	Slope		
1960	Slope Intercept Form of a Line		
1960	Solid Geometry: Dihedral and Polyhedral Angles		
1960	Solid Geometry: General Polyhedrons		
1960	Solid Geometry: Lines and Planes in Space		
1960	Solid Geometry: Prisms and Cylinders		
1960	Solid Geometry: Pyramids and Cones		
1960	Solid Geometry: Spheres		
1960	Solid Geometry: Spherical Polygons		
1960	Special Quadrilaterals: Parallelograms		
1960	Special Quadrilaterals: Rectangles and Squares		
1960	Special Quadrilaterals: Rhombuses		
1960	Special Quadrilaterals: Trapezoids		
1960	Systems: Linear		
1960	Systems: Other Nonlinear		
1960	Systems: Quadratic Linear		
1960	Systems: Three Variables		
1960	Systems: Writing		
1960	Systems: Writing Quadratic		
1960	Trian_les: E_uilateral		

YEAR	TOPICS ASSESSED	TOPICS OBSERVED FOR THE FIRST TIME	TOPICS OBSERVED FOR THE LAST TIME
1960	Triangles: Interior and Exterior Angles of		
1960	Triangles: Mean Proportionals		
1960	Triangles: Pythagoras		
1960	Triangles: Vectors		
1960	Trigonometric Equations		
1960	Trigonometric Expressions: Factoring		
1960	Trigonometric Formulas: Derivations of		
1960	Trigonometric Functions: Evaluating		
1960	Trigonometric Functions: Inverses of		
1960	Trigonometric Functions: Logarithms of		
1960	Trigonometric Functions: Properties of		
1960	Trigonometric Graphs		
1960	Trigonometric Identities		
1960	Trigonometric Identities: Angle Sum or Difference		
1960	Trigonometric Identities: Double and Half Angle		
1960	Trigonometric Ratios: Basic		
1960	Trigonometry: Finding Angles		
1960	Trigonometry: Finding Area		
1960	Trigonometry: Finding Sides		
1960	Trigonometry: Finding Sides Using Two Triangles		
1960	Trigonometry: Law of Cosines		
1960	Trigonometry: Law of Sines		
1960	Trigonometry: Law of Sines - The Ambiguous Case		
1960	Trigonometry: Polar Coordinates		
1960	Trigonometry: Polar Form		
1960	Trigonometry: Reference Angles		
1960	Trigonometry: Terminal Sides of Angles		
1960	Variation: Direct		
1960	Variation: Inverse		
1960	Volume		
	1970		
1970	Absolute Value	Absolute Value (1970-2009)	Arithmetic: Place Value (1890-1970)
1970	Arithmetic: Place Value	Equations: Absolute Value (1970-2009)	Proofs: Geometry (1900-1970)
1970	Circles: Arc Measure	Equations: Simple with Decimals (1970-2009)	Proofs: Spheres (1890-1970)
1970	Circles: Area of	Functions: Compositions of (1970-2009)	Solid Geometry: Dihedral and Polyhedral Angles (1930-1970)
1970	Circles: Center, Radius and Circumference	Functions: Defining (1970-2009)	Solid Geometry: General Polyhedrons (1890-1970)
1970	Circles: Chords	Functions: Domain and Range (1970-2009)	Solid Geometry: Spheres (1890-1970)
1970	Circles: Chords, Secants and Tangents	Inequalities: Absolute Value (1970-2009)	Trigonometry: Finding Sides Using Two Triangles (1900-1970)
1970	Circles: Radian Measure	Inequalities: Graphing Systems of (1970-2009)	
1970	Circles: Tangents	Locus with Equations (1970-1980)	
1970	Complementary, Supplementary and Vertical Angles	Logical Reasoning (1970-2009)	
1970	Conics	Logical Reasoning: Biconditional (1970-2009)	
1970	Consecutive Integers	Rationals: Undefined (1970-2009)	
1970	Constructions	Set Theory (1970-2009)	
1970	Distance	Sets: Replacement (1970-2009)	
1970	Equations and Expressions: Modeling	Transformations: Reflections (1970-2009)	
1970	Equations and Expressions: Using Substitution in		

YEAR	TOPICS ASSESSED	TOPICS OBSERVED FOR THE FIRST TIME	TOPICS OBSERVED FOR THE LAST TIME
1970	Equations: Absolute Value		
1970	Equations: Literal		
1970	Equations: Simple		
1970	Equations: Simple with Decimals		
1970	Equations: Writing Linear		
1970	Exponential Functions and Equations		
1970	Exponents: Operations with		
1970	Fractions: Complex		
1970	Functions: Compositions of		
1970	Functions: Defining		
1970	Functions: Domain and Range		
1970	Inequalities: Absolute Value		
1970	Inequalities: Graphing Systems of		
1970	Inequalities: Linear		
1970	Locus		
1970	Locus with Equations		
1970	Logarithms		
1970	Logical Reasoning		
1970	Logical Reasoning: Biconditional		
1970	Logical Reasoning: Converse		
1970	Medians, Altitudes, Bisectors and Midsegments		
1970	Midpoint		
1970	Notes and Interest		
1970	Numbers: Complex		
1970	Numbers: Properties of Real		
1970	Parallel and Perpendicular Lines		
1970	Parallel Lines: Angles Involving		
1970	Points on a Line: Identification of		
1970	Polygons and Circles: Inscribed		
1970	Polygons: Area of		
1970	Polygons: Interior and Exterior Angles of		
1970	Polynomials: Addition and Subtraction of		
1970	Polynomials: Factoring		
1970	Polynomials: Multiplication and Division of		
1970	Proofs: Circle		
1970	Proofs: Coordinate		
1970	Proofs: Geometry		
1970	Proofs: Polygon		
1970	Proofs: Spheres		
1970	Proofs: Triangle		
1970	Proportions		
1970	Quadratics: a > 1		
1970	Quadratics: Difference of Perfect Squares		
1970	Quadratics: Imaginary Solutions		
1970	Quadratics: Inequalities		
1970	Quadratics: Solving		
1970	Quadratics: Solving by Factoring		
1970	Quadratics: Sum and Product of Roots		
1970	uadratics: Usin the Discriminant		

YEAR	TOPICS ASSESSED	TOPICS OBSERVED FOR THE FIRST TIME	TOPICS OBSERVED FOR THE LAST TIME
1970	Radicals: Operations with		
1970	Radicals: Rationalizing Denominators		
1970	Radicals: Solving		
1970	Radicals: Square Roots		
1970	Rate		
1970	Rate, Time and Distance		
1970	Rationals: Addition and Subtraction of		
1970	Rationals: Solving		
1970	Rationals: Undefined		
1970	Scientific Notation		
1970	Set Theory		
1970	Sets: Replacement		
1970	Similarity		
1970	Similarity: Right Triangles		
1970	Slope		
1970	Slope Intercept Form of a Line		
1970	Solid Geometry: Dihedral and Polyhedral Angles		
1970	Solid Geometry: General Polyhedrons		
1970	Solid Geometry: Lines and Planes in Space		
1970	Solid Geometry: Pyramids and Cones		
1970	Solid Geometry: Spheres		
1970	Special Quadrilaterals		
1970	Special Quadrilaterals: Parallelograms		
1970	Special Quadrilaterals: Rectangles and Squares		
1970	Special Quadrilaterals: Rhombuses		
1970	Special Quadrilaterals: Trapezoids		
1970	Systems: Linear		
1970	Systems: Other Nonlinear		
1970	Systems: Quadratic Linear		
1970	Systems: Writing		
1970	Transformations: Reflections		
1970	Triangle Inequalities		
1970	Triangles: Equilateral		
1970	Triangles: Interior and Exterior Angles of		
1970	Triangles: Isosceles		
1970	Triangles: Mean Proportionals		
1970	Triangles: Pythagoras		
1970	Triangles: Special Right		
1970	Triangles: Vectors		
1970	Trigonometric Equations		
1970	Trigonometric Formulas: Derivations of		
1970	Trigonometric Functions: Evaluating		
1970	Trigonometric Functions: Inverses of		
1970	Trigonometric Functions: Logarithms of		
1970	Trigonometric Functions: Properties of		
1970	Trigonometric Graphs		
1970	Trigonometric Identities		
1970	Trigonometric Ratios: Basic		
1970	Tri_onometr_: Findin_ An_les		

YEAR	TOPICS ASSESSED	TOPICS OBSERVED FOR THE FIRST TIME	TOPICS OBSERVED FOR THE LAST TIME
1970	Trigonometry: Finding Area		
1970	Trigonometry: Finding Sides		
1970	Trigonometry: Finding Sides Using Two Triangles		
1970	Trigonometry: Law of Cosines		
1970	Trigonometry: Law of Sines		
1970	Trigonometry: Law of Sines - The Ambiguous Case		
1970	Trigonometry: Reference Angles		
1970	Variation: Direct		
1970	Variation: Inverse		
	1980		
1980	Absolute Value	Area and the Coordinate Plane (1980-2000)	Circles: Arc Measure (1909-1909)
1980	Area and the Coordinate Plane	Central Tendenc¸ ¸1980-2009¸	Estimatin¸ and Roundin¸ ¸1980-1980¸
1980	Binomial Expansions	Central Tendency: Dispersion (1980-2009)	Factors: Prime (1870-1880)
1980	Central Tendency	Estimating and Rounding (1980-1980)	Graphic Representation (1920-1980)
1980	Central Tendency: Averages	Inequalities: Systems of (1980-1980	Inequalities: Systems of (1980-1980
1980	Central Tendency: Dispersion	Logical Reasoning: Symbolic Logic (1980-2000)	Locus with Equations (1970-1980)
1980	Central Tendency: Normal Distributions	Probability: Experimental (1980-2009)	Rate (1890-1980)
1980	Circles: Arc Measure	Symmetry (1980-2000)	Systems: Three Variables (1900-1980)
1980	Circles: Area of	Transformations: Compositions of (1980-2009)	Systems: Writing Quadratic (1920-1980)
1980	Circles: Center, Radius and Circumference	Transformations: Rotations (1980-2009)	Trigonometric Functions: Logarithms of (1930-1980)
1980	Circles: Chords	Transformations: Translations (1980-2009)	
1980	Circles: Chords, Secants and Tangents		
1980	Circles: Equations of		
1980	Circles: Radian Measure		
1980	Circles: Tangents		
1980	Combinatorics: Combinations		
1980	Combinatorics: Multiplication Counting Principle		
1980	Combinatorics: Permutations		
1980	Complementary, Supplementary and Vertical Angles		
1980	Conics		
1980	Consecutive Integers		
1980	Constructions		
1980	Conversions		
1980	Distance		
1980	Equations and Expressions: Modeling		
1980	Equations and Expressions: Using Substitution in		
1980	Equations: Forming Quadratics from Roots		
1980	Equations: Literal		
1980	Equations: Logarithmic		
1980	Equations: Simple		
1980	Equations: Simple with Decimals		
1980	Equations: Simple with Fractional Expressions		
1980	Equations: Writing Linear		
1980	Estimating and Rounding		
1980	Exponential Functions and Equations		
1980	Exponents		
1980	Exponents: Operations with		

YEAR	TOPICS ASSESSED	TOPICS OBSERVED FOR THE FIRST TIME	TOPICS OBSERVED FOR THE LAST TIME
1980	Factors: Greatest Common		
1980	Factors: Prime		
1980	Fractions: Complex		
1980	Functional Notation		
1980	Functions: Domain and Range		
1980	Graphic Representation		
1980	Graphs: Identifying Equations of		
1980	Inequalities: Absolute Value		
1980	Inequalities: Graphing Systems of		
1980	Inequalities: Linear		
1980	Inequalities: Systems of		
1980	Locus		
1980	Locus with Equations		
1980	Logarithms		
1980	Logical Reasoning		
1980	Logical Reasoning: Contrapositive		
1980	Logical Reasoning: Converse		
1980	Logical Reasoning: Symbolic Logic		
1980	Medians, Altitudes, Bisectors and Midsegments		
1980	Midpoint		
1980	Numbers: Complex		
1980	Numbers: Imaginary		
1980	Numbers: Properties of Real		
1980	Parallel and Perpendicular Lines		
1980	Parallel Lines: Angles Involving		
1980	Percent		
1980	Perimeter		
1980	Points on a Line: Identification of		
1980	Polygons and Circles: Inscribed		
1980	Polygons: Area of		
1980	Polygons: Interior and Exterior Angles of		
1980	Polynomials: Addition and Subtraction of		
1980	Polynomials: Factoring		
1980	Polynomials: Multiplication and Division of		
1980	Probability: Dependent Events		
1980	Probability: Experimental		
1980	Probability: Mutually Exclusive Events		
1980	Probability: Theoretical		
1980	Proofs: Circle		
1980	Proofs: Coordinate		
1980	Proofs: Polygon		
1980	Proofs: Triangle		
1980	Proofs: Trigonometric		
1980	Proportions		
1980	Quadratics: $a > 1$		
1980	Quadratics: Axis of Symmetry		
1980	Quadratics: Difference of Perfect Squares		
1980	Quadratics: Graphing		
1980	Quadratics: Imaginary Solutions		

YEAR	TOPICS ASSESSED	TOPICS OBSERVED FOR THE FIRST TIME	TOPICS OBSERVED FOR THE LAST TIME
1980	Quadratics: Inequalities		
1980	Quadratics: Noninteger Solutions		
1980	Quadratics: Solving		
1980	Quadratics: Solving by Factoring		
1980	Quadratics: Using the Discriminant		
1980	Quadratics: Writing		
1980	Radicals: Operations with		
1980	Radicals: Rationalizing Denominators		
1980	Radicals: Simplifying		
1980	Radicals: Square Roots		
1980	Rate		
1980	Rate, Time and Distance		
1980	Rationals: Addition and Subtraction of		
1980	Rationals: Solving		
1980	Rationals: Undefined		
1980	Scientific Notation		
1980	Set Theory		
1980	Sets: Replacement		
1980	Similarity		
1980	Similarity: Right Triangles		
1980	Slope		
1980	Slope Intercept Form of a Line		
1980	Special Quadrilaterals: Parallelograms		
1980	Special Quadrilaterals: Rectangles and Squares		
1980	Special Quadrilaterals: Rhombuses		
1980	Special Quadrilaterals: Trapezoids		
1980	Summations		
1980	Symmetry		
1980	Systems: Linear		
1980	Systems: Other Nonlinear		
1980	Systems: Quadratic Linear		
1980	Systems: Three Variables		
1980	Systems: Writing		
1980	Systems: Writing Quadratic		
1980	Transformations: Compositions of		
1980	Transformations: Reflections		
1980	Transformations: Rotations		
1980	Transformations: Translations		
1980	Triangle Inequalities		
1980	Triangles: Equilateral		
1980	Triangles: Interior and Exterior Angles of		
1980	Triangles: Isosceles		
1980	Triangles: Mean Proportionals		
1980	Triangles: Pythagoras		
1980	Triangles: Special Right		
1980	Trigonometric Equations		
1980	Trigonometric Formulas: Derivations of		
1980	Trigonometric Functions: Evaluating		
1980	Trigonometric Functions: Inverses of		

YEAR	TOPICS ASSESSED	TOPICS OBSERVED FOR THE FIRST TIME	TOPICS OBSERVED FOR THE LAST TIME
1980	Trigonometric Functions: Logarithms of		
1980	Trigonometric Functions: Properties of		
1980	Trigonometric Graphs		
1980	Trigonometric Identities		
1980	Trigonometric Identities: Angle Sum or Difference		
1980	Trigonometric Identities: Double and Half Angle		
1980	Trigonometric Ratios: Basic		
1980	Trigonometric Ratios: Cofunction & Reciprocal		
1980	Trigonometry: Finding Angles		
1980	Trigonometry: Finding Area		
1980	Trigonometry: Finding Sides		
1980	Trigonometry: Law of Cosines		
1980	Trigonometry: Law of Sines		
1980	Trigonometry: Reference Angles		
1980	Trigonometry: Terminal Sides of Angles		
1980	Variation: Direct		
	1990		
1990	Area and the Coordinate Plane	Central Tendency: Average Known with Missing Data (1990-	Probability: Geometric (1990-1990)
1990	Binomial Expansions	Probability: Binomial with "At Least or At Most" (1990-2009)	Radicals: N-Roots (1866-1990)
1990	Central Tendency	Probability: Binomial with "Exactly" (1990-2009)	Special Quadrilaterals (1930-1990)
1990	Central Tendency: Average Known with Missing Data	Probability: Geometric (1990-1990)	Trigonometric Formulas: Derivations of (1900-1990)
1990	Central Tendency: Averages	Transformations: Dilations (1990-2009)	Trigonometry: Finding Angles (1900-1990)
1990	Central Tendency: Dispersion	Transformations: Isometries (1990-2009)	Trigonometry: Reference Angles (1940-1990)
1990	Central Tendency: Normal Distributions		Trigonometry: Terminal Sides of Angles (1960-1990)
1990	Circles: Center, Radius and Circumference		
1990	Circles: Chords		
1990	Circles: Chords, Secants and Tangents		
1990	Circles: Equations of		
1990	Circles: Radian Measure		
1990	Circles: Tangents		
1990	Combinatorics: Combinations		
1990	Combinatorics: Permutations		
1990	Complementary, Supplementary and Vertical Angles		
1990	Constructions		
1990	Conversions		
1990	Distance		
1990	Equations and Expressions: Modeling		
1990	Equations and Expressions: Using Substitution in		
1990	Equations: Absolute Value		
1990	Equations: Forming Quadratics from Roots		
1990	Equations: Graphing		
1990	Equations: Literal		
1990	Equations: Logarithmic		
1990	Equations: Simple		
1990	Equations: Simple with Decimals		
1990	Equations: Writing Linear		
1990	Exponential Functions and Equations		

YEAR	TOPICS ASSESSED	TOPICS OBSERVED FOR THE FIRST TIME	TOPICS OBSERVED FOR THE LAST TIME
1990	Exponents: Operations with		
1990	Fractions: Complex		
1990	Functional Notation		
1990	Functions: Domain and Range		
1990	Functions: Inverses of		
1990	Graphic Representation: Histograms and Tables		
1990	Inequalities: Absolute Value		
1990	Inequalities: Graphing Systems of		
1990	Inequalities: Linear		
1990	Locus		
1990	Logarithms		
1990	Logical Reasoning		
1990	Logical Reasoning: Contrapositive		
1990	Logical Reasoning: Symbolic Logic		
1990	Medians, Altitudes, Bisectors and Midsegments		
1990	Midpoint		
1990	Numbers: Complex		
1990	Numbers: Properties of Real		
1990	Parallel and Perpendicular Lines		
1990	Parallel Lines: Angles Involving		
1990	Points on a Line: Identification of		
1990	Polygons and Circles: Inscribed		
1990	Polygons: Interior and Exterior Angles of		
1990	Polynomials: Addition and Subtraction of		
1990	Polynomials: Factoring		
1990	Polynomials: Multiplication and Division of		
1990	Probability: Binomial with "At Least or At Most"		
1990	Probability: Binomial with "Exactly"		
1990	Probability: Dependent Events		
1990	Probability: Geometric		
1990	Probability: Mutually Exclusive Events		
1990	Probability: Theoretical		
1990	Proofs: Coordinate		
1990	Proofs: Polygon		
1990	Proofs: Triangle		
1990	Proofs: Trigonometric		
1990	Proportions		
1990	Quadratics: Axis of Symmetry		
1990	Quadratics: Find Vertex Given Equation		
1990	Quadratics: Graphing		
1990	Quadratics: Imaginary Solutions		
1990	Quadratics: Inequalities		
1990	Quadratics: Noninteger Solutions		
1990	Quadratics: Solving		
1990	Quadratics: Solving by Factoring		
1990	Quadratics: Sum and Product of Roots		
1990	Quadratics: Using the Discriminant		
1990	Quadratics: Writing		
1990	Radicals: N-Roots		

YEAR	TOPICS ASSESSED	TOPICS OBSERVED FOR THE FIRST TIME	TOPICS OBSERVED FOR THE LAST TIME
1990	Radicals: Operations with		
1990	Radicals: Rationalizing Denominators		
1990	Radicals: Simplifying		
1990	Radicals: Solving		
1990	Rationals: Addition and Subtraction of		
1990	Rationals: Solving		
1990	Rationals: Undefined		
1990	Scientific Notation		
1990	Sets: Replacement		
1990	Similarity		
1990	Similarity: Right Triangles		
1990	Slope		
1990	Special Quadrilaterals		
1990	Special Quadrilaterals: Parallelograms		
1990	Special Quadrilaterals: Rectangles and Squares		
1990	Special Quadrilaterals: Rhombuses		
1990	Special Quadrilaterals: Trapezoids		
1990	Summations		
1990	Symmetry		
1990	Systems: Linear		
1990	Systems: Other Nonlinear		
1990	Systems: Quadratic Linear		
1990	Systems: Writing		
1990	Transformations: Compositions of		
1990	Transformations: Dilations		
1990	Transformations: Isometries		
1990	Transformations: Reflections		
1990	Transformations: Rotations		
1990	Transformations: Translations		
1990	Triangle Inequalities		
1990	Triangles: Equilateral		
1990	Triangles: Interior and Exterior Angles of		
1990	Triangles: Isosceles		
1990	Triangles: Mean Proportionals		
1990	Triangles: Pythagoras		
1990	Triangles: Vectors		
1990	Trigonometric Equations		
1990	Trigonometric Formulas: Derivations of		
1990	Trigonometric Functions: Evaluating		
1990	Trigonometric Functions: Inverses of		
1990	Trigonometric Functions: Properties of		
1990	Trigonometric Graphs		
1990	Trigonometric Identities		
1990	Trigonometric Identities: Angle Sum or Difference		
1990	Trigonometric Identities: Double and Half Angle		
1990	Trigonometric Ratios: Cofunction & Reciprocal		
1990	Trigonometry: Finding Angles		
1990	Trigonometry: Finding Area		
1990	Tri_onometr_: Findin_ Sides		

YEAR	TOPICS ASSESSED	TOPICS OBSERVED FOR THE FIRST TIME	TOPICS OBSERVED FOR THE LAST TIME
1990	Trigonometry: Law of Cosines		
1990	Trigonometry: Law of Sines		
1990	Trigonometry: Law of Sines - The Ambiguous Case		
1990	Trigonometry: Reference Angles		
1990	Trigonometry: Terminal Sides of Angles		
1990	Variation: Direct		
1990	Variation: Inverse		
1990	Volume		
	2000		
2000	Area and the Coordinate Plane	Graphic Representation of Data (2000-2009)	Area and the Coordinate Plane (1980-2000)
2000	Binomial Expansions	Inequalities: Writing Systems of (2000-2009)	Central Tendency: Averages (1890-2000)
2000	Central Tendency	Lo_ical Reasonin_: Inverse _2000-2000_	Circles: Area of _1900-2000_
2000	Central Tendency: Average Known with Missing Data	Logical Reasoning: Venn Diagrams (2000-2000)	Combinatorics: Multiplication Counting Principle (1890-2000)
2000	Central Tendency: Averages	Probability: Sample Space (2000-2009)	Equations: Forming Quadratics from Roots (1890-2000)
2000	Central Tendency: Dispersion	Transformations: Classifications of (2000-2009)	Equations: Graphing (1920-2000)
2000	Central Tendency: Normal Distributions	Trigonometry: Unit Circles (2000-2000)	Factors: Greatest Common (1870-2000)
2000	Circles: Area of		Fraction Madness (1880-2000)
2000	Circles: Center, Radius and Circumference		Functional Notation (1950-2000)
2000	Circles: Chords		Logical Reasoning: Converse (1940-2000)
2000	Circles: Chords, Secants and Tangents		Logical Reasoning: Inverse (2000-2000)
2000	Circles: Equations of		Logical Reasoning: Symbolic Logic (1980-2000)
2000	Circles: Radian Measure		Logical Reasoning: Venn Diagrams (2000-2000)
2000	Circles: Tangents		Numbers: Comparing Reals (1940-2000)
2000	Combinatorics: Combinations		Order of Operations (1900-2000)
2000	Combinatorics: Multiplication Counting Principle		Points on a Line: Identification of (1930-2000)
2000	Combinatorics: Permutations		Polygons: Interior and Exterior Angles of (1900-2000)
2000	Complementary, Supplementary and Vertical Angles		Polynomials: Multiplication and Division of (1890-2000)
2000	Consecutive Integers		Probability: Dependent Events (1960-2000)
2000	Constructions		Probability: Mutually Exclusive Events (1950-2000)
2000	Conversions		Proofs: Trigonometric (1890-2000)
2000	Distance		Proportions (1870-2000)
2000	Equations and Expressions: Modeling		Quadratics: Noninteger Solutions (1909-2000)
2000	Equations and Expressions: Using Substitution in		Quadratics: Writing (1909-2000)
2000	Equations: Absolute Value		Radicals: Square Roots (1866-2000)
2000	Equations: Forming Quadratics from Roots		Ratio (1866-2000)
2000	Equations: Graphing		Slope Intercept Form of a Line (1940-2000)
2000	Equations: Literal		Symmetry (1980-2000)
2000	Equations: Logarithmic		Triangles: Special Right (1930-2000)
2000	Equations: Simple		Triangles: Vectors (1920-2000)
2000	Equations: Simple with Decimals		Trigonometric Functions: Evaluating (1890-2000)
2000	Equations: Writing Linear		Trigonometric Graphs (1890-2000)
2000	Exponential Functions and Equations		Trigonometric Identities: Angle Sum or Difference (1890-2000)
2000	Exponents		Trigonometry: Law of Sines - The Ambiguous Case (1920-2000)
2000	Exponents: Operations with		Trigonometry: Unit Circles (2000-2000)
2000	Factors: Greatest Common		
2000	Fraction Madness		
2000	Fractions: Complex		

YEAR	TOPICS ASSESSED	TOPICS OBSERVED FOR THE FIRST TIME	TOPICS OBSERVED FOR THE LAST TIME
2000	Functional Notation		
2000	Functions: Defining		
2000	Functions: Domain and Range		
2000	Functions: Inverses of		
2000	Graphic Representation of Data		
2000	Graphic Representation: Histograms and Tables		
2000	Graphs: Identifying Equations of		
2000	Inequalities: Linear		
2000	Inequalities: Writing Systems of		
2000	Locus		
2000	Logarithms		
2000	Logical Reasoning		
2000	Logical Reasoning: Contrapositive		
2000	Logical Reasoning: Converse		
2000	Logical Reasoning: Inverse		
2000	Logical Reasoning: Symbolic Logic		
2000	Logical Reasoning: Venn Diagrams		
2000	Medians, Altitudes, Bisectors and Midsegments		
2000	Midpoint		
2000	Numbers: Comparing Reals		
2000	Numbers: Complex		
2000	Numbers: Imaginary		
2000	Numbers: Properties of Real		
2000	Order of Operations		
2000	Parallel and Perpendicular Lines		
2000	Parallel Lines: Angles Involving		
2000	Percent		
2000	Perimeter		
2000	Points on a Line: Identification of		
2000	Polygons and Circles: Inscribed		
2000	Polygons: Area of		
2000	Polygons: Interior and Exterior Angles of		
2000	Polynomials: Addition and Subtraction of		
2000	Polynomials: Factoring		
2000	Polynomials: Multiplication and Division of		
2000	Probability: Binomial with "At Least or At Most"		
2000	Probability: Binomial with "Exactly"		
2000	Probability: Dependent Events		
2000	Probability: Experimental		
2000	Probability: Independent Events		
2000	Probability: Mutually Exclusive Events		
2000	Probability: Sample Space		
2000	Probability: Theoretical		
2000	Proofs: Coordinate		
2000	Proofs: Triangle		
2000	Proofs: Trigonometric		
2000	Proportions		
2000	Quadratics: a > 1		
2000	Quadratics: Axis of Symmetry		

YEAR	TOPICS ASSESSED	TOPICS OBSERVED FOR THE FIRST TIME	TOPICS OBSERVED FOR THE LAST TIME
2000	Quadratics: Difference of Perfect Squares		
2000	Quadratics: Find Vertex Given Equation		
2000	Quadratics: Graphing		
2000	Quadratics: Imaginary Solutions		
2000	Quadratics: Inequalities		
2000	Quadratics: Noninteger Solutions		
2000	Quadratics: Solving		
2000	Quadratics: Solving by Factoring		
2000	Quadratics: Sum and Product of Roots		
2000	Quadratics: Using the Discriminant		
2000	Quadratics: Writing		
2000	Radicals: Operations with		
2000	Radicals: Simplifying		
2000	Radicals: Solving		
2000	Radicals: Square Roots		
2000	Rate, Time and Distance		
2000	Ratio		
2000	Rationals: Addition and Subtraction of		
2000	Rationals: Solving		
2000	Rationals: Undefined		
2000	Scientific Notation		
2000	Similarity		
2000	Similarity: Right Triangles		
2000	Slope		
2000	Slope Intercept Form of a Line		
2000	Solid Geometry: Prisms and Cylinders		
2000	Special Quadrilaterals: Parallelograms		
2000	Special Quadrilaterals: Rectangles and Squares		
2000	Special Quadrilaterals: Rhombuses		
2000	Special Quadrilaterals: Trapezoids		
2000	Summations		
2000	Symmetry		
2000	Systems: Linear		
2000	Systems: Other Nonlinear		
2000	Systems: Quadratic Linear		
2000	Systems: Writing		
2000	Transformations: Classifications of		
2000	Transformations: Compositions of		
2000	Transformations: Dilations		
2000	Transformations: Isometries		
2000	Transformations: Reflections		
2000	Transformations: Translations		
2000	Triangle Inequalities		
2000	Triangles: Interior and Exterior Angles of		
2000	Triangles: Isosceles		
2000	Triangles: Mean Proportionals		
2000	Triangles: Pythagoras		
2000	Triangles: Special Right		
2000	Triangles: Vectors		

YEAR	TOPICS ASSESSED	TOPICS OBSERVED FOR THE FIRST TIME	TOPICS OBSERVED FOR THE LAST TIME
2000	Trigonometric Equations		
2000	Trigonometric Functions: Evaluating		
2000	Trigonometric Functions: Inverses of		
2000	Trigonometric Functions: Properties of		
2000	Trigonometric Graphs		
2000	Trigonometric Identities		
2000	Trigonometric Identities: Angle Sum or Difference		
2000	Trigonometric Identities: Double and Half Angle		
2000	Trigonometric Ratios: Basic		
2000	Trigonometry: Finding Area		
2000	Trigonometry: Finding Sides		
2000	Trigonometry: Law of Cosines		
2000	Trigonometry: Law of Sines		
2000	Trigonometry: Law of Sines - The Ambiguous Case		
2000	Trigonometry: Unit Circles		
2000	Variation: Direct		
2000	Variation: Inverse		
2000	Volume		

2009

2009	Absolute Value	Analysis of Data (2009-)	Absolute Value (1970-2009)
2009	Analysis of Data	Error (2009-)	Analysis of Data (2009-)
2009	Binomial Expansions	Exponential Growth (2009-)	Binomial Expansions (1890-2009)
2009	Central Tendency	Graphing Functions and Relations (2009-)	Central Tendency (1980-2009)
2009	Central Tendency: Average Known with Missing Data	Polygons and Circles: Compositions of (2009-)	Central Tendency: Average Known with Missing Data (1990-2009)
2009	Central Tendency: Dispersion	Probability: Conditional (2009-)	Central Tendency: Dispersion (1980-2009)
2009	Central Tendency: Normal Distributions	Regression: Linear (2009-)	Central Tendency: Normal Distributions (1940-2009)
2009	Circles: Center, Radius and Circumference	Regression: Logarithmic (2009-)	Circles: Center, Radius and Circumference (1890-2009)
2009	Circles: Chords	Regression: Power (2009-)	Circles: Chords (1890-2009)
2009	Circles: Chords, Secants and Tangents		Circles: Chords, Secants and Tangents (1890-2009)
2009	Circles: Equations of		Circles: Equations of (1940-2009)
2009	Circles: Radian Measure		Circles: Radian Measure (1930-2009)
2009	Circles: Tangents		Circles: Tangents (1920-2009)
2009	Combinatorics: Combinations		Combinatorics: Combinations (1890-2009)
2009	Combinatorics: Permutations		Combinatorics: Permutations (1930-2009)
2009	Complementary, Supplementary and Vertical Angles		Complementary, Supplementary and Vertical Angles (1900-2009)
2009	Conics		Conics (1940-2009)
2009	Consecutive Integers		Consecutive Integers (1920-2009)
2009	Constructions		Constructions (1890-2009)
2009	Conversions		Conversions (1866-2009)
2009	Definitions: Algebra		Definitions: Algebra (1890-2009)
2009	Distance		Distance (1950-2009)
2009	Equations and Expressions: Modeling		Equations and Expressions: Modeling (1890-2009)
2009	Equations and Expressions: Using Substitution in		Equations and Expressions: Using Substitution in (1909-2009)
2009	Equations: Absolute Value		Equations: Absolute Value (1970-2009)
2009	Equations: Literal		Equations: Literal (1900-2009)
2009	Equations: Logarithmic		Equations: Logarithmic (1890-2009)
2009	Equations: Modeling from a Table		Equations: Modeling from a Table (1930-2009)

YEAR	TOPICS ASSESSED	TOPICS OBSERVED FOR THE FIRST TIME	TOPICS OBSERVED FOR THE LAST TIME
2009	Equations: Simple		Equations: Simple (1930-2009)
2009	Equations: Simple with Decimals		Equations: Simple with Decimals (1970-2009)
2009	Equations: Simple with Fractional Expressions		Equations: Simple with Fractional Expressions (1890-2009)
2009	Equations: Writing Linear		Equations: Writing Linear (1940-2009)
2009	Error		Error (2009-)
2009	Exponential Functions and Equations		Exponential Functions and Equations (1920-2009)
2009	Exponential Growth		Exponential Growth (2009-)
2009	Exponents		Exponents (1930-2009)
2009	Exponents: Operations with		Exponents: Operations with (1866-2009)
2009	Fractions: Complex		Fractions: Complex (1880-2009)
2009	Functions: Compositions of		Functions: Compositions of (1970-2009)
2009	Functions: Defining		Functions: Defining (1970-2009)
2009	Functions: Domain and Range		Functions: Domain and Range (1970-2009)
2009	Functions: Inverses of		Functions: Inverses of (1940-2009)
2009	Graphic Representation of Data		Graphic Representation of Data (2000-2009)
2009	Graphic Representation: Histograms and Tables		Graphic Representation: Histograms and Tables (1950-2009)
2009	Graphing Functions and Relations		Graphing Functions and Relations (2009-)
2009	Graphs: Identifying Equations of		Graphs: Identifying Equations of (1930-2009)
2009	Inequalities: Absolute Value		Inequalities: Absolute Value (1970-2009)
2009	Inequalities: Graphing Systems of		Inequalities: Graphing Systems of (1970-2009)
2009	Inequalities: Linear		Inequalities: Linear (1960-2009)
2009	Inequalities: Writing Systems of		Inequalities: Writing Systems of (2000-2009)
2009	Locus		Locus (1900-2009)
2009	Logarithms		Logarithms (1890-2009)
2009	Logical Reasoning		Logical Reasoning (1970-2009)
2009	Logical Reasoning: Biconditional		Logical Reasoning: Biconditional (1970-2009)
2009	Logical Reasoning: Contrapositive		Logical Reasoning: Contrapositive (1960-2009)
2009	Medians, Altitudes, Bisectors and Midsegments		Medians, Altitudes, Bisectors and Midsegments (1890-2009)
2009	Midpoint		Midpoint (1950-2009)
2009	Notes and Interest		Notes and Interest (1866-2009)
2009	Numbers: Complex		Numbers: Complex (1909-2009)
2009	Numbers: Imaginary		Numbers: Imaginary (1930-2009)
2009	Numbers: Properties of Real		Numbers: Properties of Real (1930-2009)
2009	Parallel and Perpendicular Lines		Parallel and Perpendicular Lines (1890-2009)
2009	Parallel Lines: Angles Involving		Parallel Lines: Angles Involving (1950-2009)
2009	Percent		Percent (1870-2009)
2009	Perimeter		Perimeter (1940-2009)
2009	Polygons and Circles: Compositions of		Polygons and Circles: Compositions of (2009-)
2009	Polygons and Circles: Inscribed		Polygons and Circles: Inscribed (1890-2009)
2009	Polygons: Area of		Polygons: Area of (1870-2009)
2009	Polynomials: Addition and Subtraction of		Polynomials: Addition and Subtraction of (1890-2009)
2009	Polynomials: Factoring		Polynomials: Factoring (1900-2009)
2009	Probability: Binomial with "At Least or At Most"		Probability: Binomial with "At Least or At Most" (1990-2009)
2009	Probability: Binomial with "Exactly"		Probability: Binomial with "Exactly" (1990-2009)
2009	Probability: Conditional		Probability: Conditional (2009-)
2009	Probability: Experimental		Probability: Experimental (1980-2009)
2009	Probability: Independent Events		Probability: Independent Events (1940-2009)
2009	Probability: Sample Space		Probability: Sample Space (2000-2009)
2009	Probability: Theoretical		Probability: Theoretical (1940-2009)

YEAR	TOPICS ASSESSED	TOPICS OBSERVED FOR THE FIRST TIME	TOPICS OBSERVED FOR THE LAST TIME
2009	Proofs: Circle		Proofs: Circle (1890-2009)
2009	Proofs: Coordinate		Proofs: Coordinate (1960-2009)
2009	Proofs: Polygon		Proofs: Polygon (1890-2009)
2009	Proofs: Triangle		Proofs: Triangle (1890-2009)
2009	Quadratics: a > 1		Quadratics: a > 1 (1900-2009)
2009	Quadratics: Axis of Symmetry		Quadratics: Axis of Symmetry (1950-2009)
2009	Quadratics: Difference of Perfect Squares		Quadratics: Difference of Perfect Squares (1950-2009)
2009	Quadratics: Find Vertex Given Equation		Quadratics: Find Vertex Given Equation (1930-2009)
2009	Quadratics: Graphing		Quadratics: Graphing (1930-2009)
2009	Quadratics: Imaginary Solutions		Quadratics: Imaginary Solutions (1960-2009)
2009	Quadratics: Inequalities		Quadratics: Inequalities (1960-2009)
2009	Quadratics: Solving		Quadratics: Solving (1920-2009)
2009	Quadratics: Solving by Factoring		Quadratics: Solving by Factoring (1909-2009)
2009	Quadratics: Sum and Product of Roots		Quadratics: Sum and Product of Roots (1920-2009)
2009	Quadratics: Using the Discriminant		Quadratics: Using the Discriminant (1900-2009)
2009	Radicals: Operations with		Radicals: Operations with (1890-2009)
2009	Radicals: Rationalizing Denominators		Radicals: Rationalizing Denominators (1890-2009)
2009	Radicals: Simplifying		Radicals: Simplifying (1900-2009)
2009	Radicals: Solving		Radicals: Solving (1900-2009)
2009	Rate, Time and Distance		Rate, Time and Distance (1880-2009)
2009	Rationals: Addition and Subtraction of		Rationals: Addition and Subtraction of (1890-2009)
2009	Rationals: Solving		Rationals: Solving (1890-2009)
2009	Rationals: Undefined		Rationals: Undefined (1970-2009)
2009	Regression: Linear		Regression: Linear (2009-)
2009	Regression: Logarithmic		Regression: Logarithmic (2009-)
2009	Regression: Power		Regression: Power (2009-)
2009	Scientific Notation		Scientific Notation (1960-2009)
2009	Set Theory		Set Theory (1970-2009)
2009	Sets: Replacement		Sets: Replacement (1970-2009)
2009	Similarity		Similarity (1930-2009)
2009	Similarity: Right Triangles		Similarity: Right Triangles (1930-2009)
2009	Slope		Slope (1930-2009)
2009	Solid Geometry: Lines and Planes in Space		Solid Geometry: Lines and Planes in Space (1890-2009)
2009	Solid Geometry: Prisms and Cylinders		Solid Geometry: Prisms and Cylinders (1890-2009)
2009	Solid Geometry: Pyramids and Cones		Solid Geometry: Pyramids and Cones (1890-2009)
2009	Special Quadrilaterals: Parallelograms		Special Quadrilaterals: Parallelograms (1930-2009)
2009	Special Quadrilaterals: Rectangles and Squares		Special Quadrilaterals: Rectangles and Squares (1930-2009)
2009	Special Quadrilaterals: Rhombuses		Special Quadrilaterals: Rhombuses (1920-2009)
2009	Special Quadrilaterals: Trapezoids		Special Quadrilaterals: Trapezoids (1930-2009)
2009	Summations		Summations (1930-2009)
2009	Systems: Linear		Systems: Linear (1890-2009)
2009	Systems: Other Nonlinear		Systems: Other Nonlinear (1890-2009)
2009	Systems: Quadratic Linear		Systems: Quadratic Linear (1890-2009)
2009	Systems: Writing		Systems: Writing (1900-2009)
2009	Transformations: Classifications of		Transformations: Classifications of (2000-2009)
2009	Transformations: Compositions of		Transformations: Compositions of (1980-2009)
2009	Transformations: Dilations		Transformations: Dilations (1990-2009)
2009	Transformations: Isometries		Transformations: Isometries (1990-2009)
2009	Transformations: Reflections		Transformations: Reflections (1970-2009)

YEAR	TOPICS ASSESSED	TOPICS OBSERVED FOR THE FIRST TIME	TOPICS OBSERVED FOR THE LAST TIME
2009	Transformations: Rotations		Transformations: Rotations (1980-2009)
2009	Transformations: Translations		Transformations: Translations (1980-2009)
2009	Triangle Inequalities		Triangle Inequalities (1909-2009)
2009	Triangles: Equilateral		Triangles: Equilateral (1900-2009)
2009	Triangles: Interior and Exterior Angles of		Triangles: Interior and Exterior Angles of (1909-2009)
2009	Triangles: Isosceles		Triangles: Isosceles (1930-)
2009	Triangles: Mean Proportionals		Triangles: Mean Proportionals (1890-2009)
2009	Triangles: Pythagoras		Triangles: Pythagoras (1880-2009)
2009	Trigonometric Equations		Trigonometric Equations (1900-2009)
2009	Trigonometric Functions: Inverses of		Trigonometric Functions: Inverses of (1930-2009)
2009	Trigonometric Functions: Properties of		Trigonometric Functions: Properties of (1890-2009)
2009	Trigonometric Identities		Trigonometric Identities (1900-2009)
2009	Trigonometric Identities: Double and Half Angle		Trigonometric Identities: Double and Half Angle (1900-2009)
2009	Trigonometric Ratios: Basic		Trigonometric Ratios: Basic (1890-2009)
2009	Trigonometric Ratios: Cofunction & Reciprocal		Trigonometric Ratios: Cofunction & Reciprocal (1930-2009)
2009	Trigonometry: Finding Area		Trigonometry: Finding Area (1900-2009)
2009	Trigonometry: Finding Sides		Trigonometry: Finding Sides (1890-2009)
2009	Trigonometry: Law of Cosines		Trigonometry: Law of Cosines (1890-2009)
2009	Trigonometry: Law of Sines		Trigonometry: Law of Sines (1890-2009)
2009	Variation: Direct		Variation: Direct (1940-2009)
2009	Variation: Inverse		Variation: Inverse (1930-2009)
2009	Volume		Volume (1880-2009)

The Regents Go to War: A Case Study of Rare Societal Events Being Used as Evoking Contexts for Mathematics Assessment

1 1918_01_PT_06 Law of Sines
Two ships are 4 miles apart. The angular distance of the first ship from a hostile warship, as observed by the second ship, is 52°, 20'; the angular distance of the second ship from the first ship is 63°, 10'. Find the distance of each ship from the hostile warship.

2 1918_06_AR_06 Conversions
Suppose that an average family wastes a slice of bread every day and that there are 20 slices in a 12 ounce loaf. If a soldier's ration of bread is 10 ounces a day. how many day's rations are wasted weekly in a village containing 500 families? [10]

3 1918_06_AR_10 Central Tendency: Averages
From February 18 to March 16, the 165 pupils in a public school in the village of Edison, Ohio, sold $11,296.25 worth of thrift stamps. Four pupils sold stamps valued as follows: $2892; $2717.50; $1973; $1547.75. State (a) the total number of 25 cent stamps sold [2], (b) the average number of 25 cent stamps sold by each pupil in the school [4], (c) the total number of 25 cent stamps sold by the four pupils [4].

4 1918_06_AR_06 Notes and Interest
If a $500 Liberty Bond bearing $4\frac{1}{4}$% interest and purchased at par should be held for 10 years, how much interest on the bond would the owner receive during that time? [10]

5 1918_06_AA_11 Combinatorics: Combinations
From 16 soldiers in how many ways can a guard of 5 be chosen? In how many ways can a guard of 7 be arranged in line? In how many ways can the 16 be divided into two equal groups?

6 1918_06_AA_12 Combinatorics: Permuations
A signal corps has six different flags; by using one, or two, or three flags at a time, how many different signals can be formed with these flags?

7 1918_06_AA_02 Combinatorics
a) How many parties, each consisting of 1 sergeant, 2 corporals and 5 privates can be formed from 3 sergeants, 8 corporals and 16 privates?
b) How many permutations can be made of the letters in the word New York, each one beginning with N?

8 1918_09_AA_08 Rate, Time and Distance
An army truck going from Buffalo to New York travels at the rate of 12 miles an hour. After traveling $2\frac{1}{2}$ hours it is delayed $1\frac{1}{2}$ hours by an obstruction on the road; it then proceeds at its former rate. Three hours after the first truck starts a second one follows at the rate of 15 miles an hour. How far will they travel before the second overtakes the first? [Solve graphically.]

9 1919_01_AR_07 Notes and Interest
How many $1000 Liberty Bonds bearing interest at $4\frac{1}{2}$% must a man buy to receive $850 in yearly interest? [10]

10 1919_01_AR_08 Percent
During the week of Dec. 9, 1918, the children of the grammar schools of a certain city invested $6168.50 in Thrift Stamps; during the week of Dec. 16, 1918, they inversted an amount equal to 120% of the amount invested during the week of Dec. 9. How much did they invest during both weeks? [10]

11 1919_01_IA_13 Equations and Expressions: Using Substitution in
A projectile weighing W pounds, whose diameter is d inches, strikes a wrought iron plat when moving at the rate of v feet per second. The depth of penetration p (in inches) is given by the formula

$$p = \frac{v}{608.3}\sqrt{\frac{w}{d}} - 0.14d$$

Find p when $d = 12.5$, $w = 1250$ and $v = 2016$

12 1919_01_AA_03 Equations and Expressions: Using Substitution in

A bomb dropped from a point H feet above the earth by an airplane moving s feet per second, will fall D feet ahead of the perpendicular on which it was dropped, D being found by the formula

$$D = \left(\frac{\sqrt{H}}{4} + \frac{H}{8000}\right)s - \frac{H}{40}$$

If it is known that $s = 100$ and $D = 2000$ feet, find the height of the airplane to the nearest 100 feet.

13 1919_06_AA_13 Quadratics: Solving

The time of the fall of a bomb from an airplane is given by the formula $T = \dfrac{\sqrt{H}}{4} + \dfrac{H}{9000}$ where T is the number of seconds and H is the height in feet; find the value of H in terms of T and thus obtain a formula for the height of the airplane when the time of descent of the bomb is known.

14 1919_06_TR_05 Trigonometry: Finding Sides

An observer in a war balloon observes the angle of depression of an enemy battery to be 27.5°; an instrument registers the height as 3250 feet. At what distance from a point on the ground directly below the observer is the battery located, if the point and the battery are on the same horizontal plane?

15 1919_06_TR_06 Trigonometry: Finding Sides Using Two Triangles

An observer in a war balloon at a definite height locates two distant forts; if the forts and the point on the ground directly below the observer are on the same horizontal plane, what further observations should be made and how should these observations be made use of to determine the distance between the forts?

16 1943_01_AR_26 Central Tendency: Averages

Pupils of four classes invested during September in War Saving Stamps as follows:

In the first class 35 pupils each invested an average of 40 cents per month.

In the second class 36 pupils each invested an average of 50 cents per month.

In the third class 32 pupils each invested an average of 35 cents per month.

In the fourth class 36 pupils each invested an average of 37 cents per month.

a How much was invested by each class during the month? [4]

b What was the total amount invested in War Savings Stamps by these four classes? [2]

c How much would these pupils invest in War Savings Stamps at the same rate during a 10 month school year? [4]

17 1943_01_AR_28 Percent

A man worked 55 hours one week, in a defense factory, at the rate of $.80 per hour for the first 40 hours. For every hour over 40 hours he received $1\frac{1}{2}$ times as much per hour.

a How much was his week's pay? [5]

b If he used 10% of his wages to buy war stamps, how much did he invest in stamps? [5]

18 1943_01_AA_25 Circles: Center, Radius and Circumference

An airplane has just enough gasoline to travel from its base B to a point P and return. The distance r from B to P is known as the Radius of Action.

a If the speed of the plane on its outward trip is v_1 miles per hour, the speed returning over the course v_2 miles per hour and the total time of the round trip is t hours, derive a formula for r in terms of v_1, v_2 and t. [7]

b Find correct to the *nearest mile*, the radius of action of a plane if $v_1 = 150$ m.p.h., $v_2 = 125$ m.p.h. and $t = 3$ hours 20 minutes. [3]

19 1943_01_AA_29 Quaratics: Find Vertex Given Equation

If a gun is fired at an angle of 45° to the horizontal and with a muzzle velocity of 1600 feet per second, the path of the projectile is given by the equation $y = x - \frac{x^2}{80,000}$ where y represents the height in feet of the projectile above the ground and x representes the horizontal distance in feet traveled by the projectile.

a Determine the maximum height attained by the projectile. [7]

b Find, correct to the *nearest mile*, the distance from the gun to the point where the projectile strikes the ground. [3]

Note: This problem is based on an optional topic in the syllabus.

20 1943_01_TR_28 Law of Cosines

A gun fired at A was heard at B and at C two seconds and three seconds respectively after it was fired. If angle BAC = 110°30’ and the sound traveled 1150 feet per second, compute, correct to the nearest foot, the distance between B and C. [10]

21 1943_06_PM_26 Notes and Interest

From a certain community 225 men entered the armed forces. The people of the community decided they would buy enough additional war bonds to provide each man with teh following articles: a steel helmet at $2.50; a gas mask at $9; a mess kit at $2; a blanket at $6.50.

a How much did the articles for one man cost? [2]

b What was the total cost of the articles for the 225 men in the armed forces? [2]

c How many citizens of that community wouold each have to buy an $18.75 bond to provide the men with these articles? [4]

d What will be the total maturity value of these bonds? [2]

22 1943_06_PM_27 Notes and Interest

For $75 it is possible to buy a war savings bond that will be worth $100 in 10 years. Using the formula $A = P(1+r)^n$, find, correct to the *nearest tenth of a per cent*, the rate of interest oin this investment if interest is compounded annually. [10]

23 1943_06_TR_23 Solid Geometry: Spherical Polygons

War maneuvers in the vicinity of the Solomon Islands occupy a portion of the earth’s surface bounded by the equator, the parallel of latitude 15° S. and the meridians of longitude 150° E. and 165° E.

a If a represents the radius of the earth, show the area K of this protion is given by the formula $K = \frac{\pi r^2 \sin 15°}{12}$ [Suggestion: The altitude of the zone whose bases are the equator and the parallel of latitude 15° S is $r \sin 15°$] [7]

b Which of the following is correct: The area of this portion of the earth;s surface is (1) less than 250,000 square miles, (2) approximately 500,000 square miles or (3) more than 1,000,000 square miles? [Use 4,000 miles as the radiius of the earth.] [3]

24 1943_06_TR_23 Trigonometry: Finding Sides Using Two Triangles

From two points due west of a captive balloon, the angles of elevation of the balloon are x and y ($x > y$). The distance between the two points is d. Show that the distance s from the point on the ground directly beneath the balloon to the nearer point of observation is given by the formula $s = \frac{d \cos x \sin y}{\sin(x - y)}$ [10]

25 1943_06_TR_26 Trigonometry: Law of Cosines

A merchant vessel sails from a certain port directly east at 12 knots. A submarine is 10 nautical miles S. W. from this point. At what rate must the submarine proceed in order to overtake the vessel in 2 hours? [Express answer to the *nearest knot*.] [1 knot = 1 nauticl miles per hour]. [10]

26 1944_01_PM_10 Notes and Interest

If a boy purchases five 25-cent war stamps each week, how many weeks will it take him to fill a stamp book for a war bond costing $18.75?

27 1944_01_PM_10 Profit and Loss

A boy sold the produce from his Victory garden as follows:

80 lb. of snap beans at 12¢ a lb.
36 lb. of peas at 15¢ a lb.
200 cucumbers at 3¢ each
37 bunches of beets at 10¢ a bunch
400 ears of sweet corn at 2¢ an ear

The only expenses he had were $1.20 for seed and $1.50 for fertilizer.

a How much did he receive for his produce? [5]
b What was his net profit?
c If he worked a total of 60 hours, how much did he receive per hour for his work? [2]

28 1944_01_PM_28 Rate

A bomber supplied with 2000 gallons of gasoline carried a bomb load of eight long tons. It completed a mission over a target 625 miles from its airbase. [1 long ton = 2240 lb.]

a How many miles did the bonber fly from the time it left until it again reached its air base? [1]
b Find the number of pounds of bombs carried for each gallon of gasoline provided. [4]
c How many miles to the gallon, correct to the *nearest hundredth*, did the gasonline supply allow for? [5]

29 1944_01_IA_12 Trignometry: Finding Sides

On a certain night, to determine the celing over an airport, a celing light projector threw a spotlight vertically on the underside of a cloud. At a distance of 500 feet from the projector, the angle of elevation of the spot of light on the cloud was found to be 66°. What was the ceiling (height of the cloud)?

30 1944_01_TR_25 Trigonometry: Law of Sines

Two observers, A and B, at the ends of a level base line 1000 yards long, measure angles from the base line to a gun emplacement *G*. If angle $BAG = 37°20'$ and angle $ABG = 62°30'$, find *BG*.

31 1944_06_PM_26 Percent

In a recent newspaper collection campaign held throughout the entire nation, one Junior high school of 175 students collected 28 tons of paper.

a What was the average number of pounds of paper collected per student? [5]
b How much did the school receive by selling the paper for $60 a hundred pounds? [1]
c Of the amount collected, $168 was given to the local servicemen’s organization. What per cent did it receive? [2]

32 1944_06_TR_23 Trigonometry: Finding Sides Using Two Triangles

An artillery range spotter is flying at an altitude of *h* feet. He observes that a gun *G* and its target *T*, both in the same horizontal plane, are due west of his position, the target being at the greater distance. The angles of depression of the gun and the target are *x* and *y* respectively. Derive a formula for the range *r*, that is, the distance *GT*. [10]

33 1945_01_PG_32 Circles: Chords

A straight road crosses a straight railroad at an angle of 60°. On the road, 40 miles from the crossing, a gun with a range of 37 miles is located. A train moving along the railroad track has just passed the crossing.

a Find, correct to the nearest mile, the distance of the gun from the railroad. [3]
b Show that the train will come within range of the gun. [2]
c How far from the crossing will the train first come within range of the gun? [4]
d At what point will the train pass out of range of the gun? [1]

34 1945_06_PM_26 Profit and Loss

A young man receives a salary of \$275 a month. During the year he plans to spend \$480 for rent, \$820 for food, \$375 for clothing, \$425 for fuel and household expenses and \$300 for other expenses. He also plans to buy as many war bonds at \$375 each as possible with the balance.

a How much is his annual salarty? [3]

b If he spends his mopney according to his plans, how much will his total expenses be? [3]

c How mnany war bonds at \$375 each will he be able to buy? [2]

d How much will he have left after pauying his expenses and buying the war bonds? [2]

35 1945_06_IA_32 Notes and Interest

A man has \$5000 invested in a mortgage that pays 5% annually. He buys Series G War Bonds paying $2\frac{1}{2}$% and now his total investment pays him 3% annually. How much has he invested in Seriies G War Bonds? [10]

36 1945_06_PG_33 Trigonometry: Finding Sides Using Two Triangles

R is a camp situated 240 rods from a straight road. On this road a second camp *S* is located, 400 rods from *R*. It is desired to build a supply depot at a point *P* on the road, which shall be the same distance from the two camps.

a Explain how point *P* can be located geometrically. [4]

b Find the distance from the supply depot to each of the two camps. [6]

37 1946_01_PM_11 Percent

A certain type of gun fired 300 shots per minute. After improvement, the firing speed of the gun was increased by 20%. What is the new firing rate per minute?

38 1946_01_TR_27 Trigonometry: Law of Cosines

A railroad runs from point *A* directly north to point *B*, a distance of 60 miles. An enemy gun is located east of the railroad, 30 miles from *A* and 40 miles from *B* and has a range of 19 miles. Is the railroad within range of the gun? [All computation in this problems must be shown.] [10]

39 1946_01_TR_29 Longitude

The great circle arc betwwen Tokyo and Wake Island is 28°45’ and the bearing of Tokyo from Wake Island is N 49° 43’ W. The longitude of Tokyo is 139° 45’ E and the longitude of Wake Island is 166°35’ E. Find the latitude of Tokyo. [10]

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