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ENRICHMENT ACTIVITY 2-1A

Logic Using Tables

A table is often the most efficient way to organize information and list the possibilities that lead to a logical conclusion. In this activity, you will use tables to solve the following problem.

Category	A	B	C	D	E
1	low	high			
2	low	low	high		high
3	high	low	low	low	high
4			low	high	
5	high	high	high	low	low

Mrs. Cabellero divides the food and household products for which she must shop into five categories: (1) meat, (2) fresh vegetables, (3) diary products, (4) nonperishable foods, and (5) paper products. There are five stores at which she usually shops: A, B, C, D, and E. She has made the table above to keep track of which stores are the best places to shop for particular items, noting which stores have high prices in a particular category and which have low. Where she has made no notation, the prices vary.

Mrs. Cabellero usually shops for two categories of products at one time and wants to choose the one store at which she will get the best prices. The stores that have the best (that is, low) prices for each category are listed in the table. For each pair of products, Mrs. Cabellero's choice will be the store with a low price for one type of product and prices that vary for the other.

Complete the table below based on the information given above, so that each store occurs exactly twice as the store with the best price for a particular pair of products.

Categories		Store with Best Prices
Meat and fresh vegetables	1, 2	A
Meat and dairy products	1, 3	
Meat and nonperishable foods	1, 4	
Meat and paper products	1, 5	
Fresh vegetables and dairy products	2, 3	B
Fresh vegetables and nonperishable foods	2, 4	
Fresh vegetables and paper products	2, 5	
Dairy products and nonperishable foods	3, 4	C
Dairy products and paper products	3, 5	D
Nonperishable foods and paper products	4, 5	

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ENRICHMENT ACTIVITY 2-1B

Negating Quantified Statements

Consider this statement:

p: All grapes are purple.

A common error is to negate a statement such as p by saying that "No grapes are purple." However, remember that a statement and its negation must be opposite in truth value. Therefore, if we had a bowl of grapes, some of which were purple and some of which were green, then the two statements "All grapes are purple" and "No grapes are purple" would both be false. They cannot be negations of each other. Moreover, the statement "All grapes are not purple" is also improper since it could be interpreted to mean there are no purple grapes. To avoid this type of confusion, we can negate statements of the form "All are" by stating that "Some are not." In logic, we use "some" to mean "at least one," so another proper negation for "All grapes are purple" would be "There is at least one grape that is not purple." (Compare this to the use of a single counterexample to disprove a general assertion.) The following examples illustrate how to correctly negate quantified statements.

Examples

1. p: All months have 30 days. (False)
$\sim p$: Some months do not have 30 days. (True)

2. p: For every real number x, $x \cdot 0 = 0$. (True)
$\sim p$: There is at least one real number x for which $x \cdot 0 \neq 0$. (False)

Next, consider negating the statement

q: Some students study Spanish.

Since "some" means "at least one," it seems reasonable that the negation of a "some" statement is a "no" statement. Therefore, the negation of statement q above is "No students study Spanish."

Examples

3. q: Some numbers are prime. (True)
$\sim q$: No numbers are prime. (False)

4. q: Some rectangles are trapezoids. (False)
$\sim q$: No rectangles are trapezoids. (True)

5. q: Some polygons are not octagons. (True)
$\sim q$: No polygons are not octagons. (False)
or, equivalently
All polygons are octagons. (False)

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Exercises

In 1–8, form the negation of each statement. Then determine the truth value for the given statement and its negation.

1. All dogs are mammals.

2. Some square are pentagons.

3. No books are biographies.

4. Every U.S. President was elected after the age of 60.

5. No integers are irrational numbers.

6. Some chords are not diameters.

7. Not all triangles are right triangles.

8. All lions are not horses.

9. Find at least three examples of "all," "some," and "no" statements in newspapers or magazines, or online. Write the negation of each statement, then determine the truth value of the original statement and its negation. Discuss with classmates how not knowing how to form correct negations could lead to misinterpretation or controversy.

10. Jodi says that since the statement "All circles are congruent" is false, then "All circles are not congruent" must be true. Do you agree with Jodi? Explain.