

## Integrated Algebra

In implementing the Algebra process and content performance indicators, it is expected that students will identify and justify mathematical relationships. The intent of both the process and content performance indicators is to provide a variety of ways for students to acquire and demonstrate mathematical reasoning ability when solving problems. Local curriculum and local/state assessments must support and allow students to use any mathematically correct method when solving a problem.

Throughout this document the performance indicators use the words *investigate*, *explore*, *discover*, *conjecture*, *reasoning*, *argument*, *justify*, *explain*, *proof*, and *apply*. Each of these terms is an important component in developing a student's mathematical reasoning ability. It is therefore important that a clear and common definition of these terms be understood. The order of these terms reflects different stages of the reasoning process.

**Investigate/Explore** - Students will be given situations in which they will be asked to look for patterns or relationships between elements within the setting.

**Discover** - Students will make note of possible patterns and generalizations that result from investigation/exploration.

**Conjecture** - Students will make an overall statement, thought to be true, about the new discovery.

**Reasoning** - Students will engage in a process that leads to knowing something to be true or false.

**Argument** - Students will communicate, in verbal or written form, the reasoning process that leads to a conclusion. A valid argument is the end result of the conjecture/reasoning process.

**Justify/Explain** - Students will provide an argument for a mathematical conjecture. It may be an intuitive argument or a set of examples that support the conjecture. The argument may include, but is not limited to, a written paragraph, measurement using appropriate tools, the use of dynamic software, or a written proof.

**Proof** - Students will present a valid argument, expressed in written form, justified by axioms, definitions, and theorems.

**Apply** - Students will use a theorem or concept to solve an algebraic or numerical problem.

## **Problem Solving Strand**

*Students will build new mathematical knowledge through problem solving.*

- A.PS.1 Use a variety of problem solving strategies to understand new mathematical content
- A.PS.2 Recognize and understand equivalent representations of a problem situation or a mathematical concept

*Students will solve problems that arise in mathematics and in other contexts.*

- A.PS.3 Observe and explain patterns to formulate generalizations and conjectures
- A.PS.4 Use multiple representations to represent and explain problem situations (e.g., verbally, numerically, algebraically, graphically)

*Students will apply and adapt a variety of appropriate strategies to solve problems.*

- A.PS.5 Choose an effective approach to solve a problem from a variety of strategies (numeric, graphic, algebraic)
- A.PS.6 Use a variety of strategies to extend solution methods to other problems
- A.PS.7 Work in collaboration with others to propose, critique, evaluate, and value alternative approaches to problem solving

*Students will monitor and reflect on the process of mathematical problem solving.*

- A.PS.8 Determine information required to solve a problem, choose methods for obtaining the information, and define parameters for acceptable solutions
- A.PS.9 Interpret solutions within the given constraints of a problem
- A.PS.10 Evaluate the relative efficiency of different representations and solution methods of a problem

## **Reasoning and Proof Strand**

*Students will recognize reasoning and proof as fundamental aspects of mathematics.*

- A.RP.1 Recognize that mathematical ideas can be supported by a variety of strategies

*Students will make and investigate mathematical conjectures.*

- A.RP.2 Use mathematical strategies to reach a conclusion and provide supportive arguments for a conjecture
- A.RP.3 Recognize when an approximation is more appropriate than an exact answer

*Students will develop and evaluate mathematical arguments and proofs.*

- A.RP.4 Develop, verify, and explain an argument, using appropriate mathematical ideas and language
- A.RP.5 Construct logical arguments that verify claims or counterexamples that refute them
- A.RP.6 Present correct mathematical arguments in a variety of forms
- A.RP.7 Evaluate written arguments for validity

*Students will select and use various types of reasoning and methods of proof.*

- A.RP.8 Support an argument by using a systematic approach to test more than one case
- A.RP.9 Devise ways to verify results or use counterexamples to refute incorrect statements
- A.RP.10 Extend specific results to more general cases
- A.RP.11 Use a Venn diagram to support a logical argument
- A.RP.12 Apply inductive reasoning in making and supporting mathematical conjectures

### **Communication Strand**

*Students will organize and consolidate their mathematical thinking through communication.*

- A.CM.1 Communicate verbally and in writing a correct, complete, coherent, and clear design (outline) and explanation for the steps used in solving a problem

- A.CM.2 Use mathematical representations to communicate with appropriate accuracy, including numerical tables, formulas, functions, equations, charts, graphs, Venn diagrams, and other diagrams

*Students will communicate their mathematical thinking coherently and clearly to peers, teachers, and others.*

- A.CM.3 Present organized mathematical ideas with the use of appropriate standard notations, including the use of symbols and other representations when sharing an idea in verbal and written form
- A.CM.4 Explain relationships among different representations of a problem
- A.CM.5 Communicate logical arguments clearly, showing why a result makes sense and why the reasoning is valid
- A.CM.6 Support or reject arguments or questions raised by others about the correctness of mathematical work

*Students will analyze and evaluate the mathematical thinking and strategies of others.*

- A.CM.7 Read and listen for logical understanding of mathematical thinking shared by other students
- A.CM.8 Reflect on strategies of others in relation to one's own strategy
- A.CM.9 Formulate mathematical questions that elicit, extend, or challenge strategies, solutions, and/or conjectures of others

*Students will use the language of mathematics to express mathematical ideas precisely.*

- A.CM.10 Use correct mathematical language in developing mathematical questions that elicit, extend, or challenge other students' conjectures
- A.CM.11 Represent word problems using standard mathematical notation
- A.CM.12 Understand and use appropriate language, representations, and terminology when describing objects, relationships, mathematical solutions, and rationale

- A.CM.13 Draw conclusions about mathematical ideas through decoding, comprehension, and interpretation of mathematical visuals, symbols, and technical writing

### **Connections Strand**

*Students will recognize and use connections among mathematical ideas.*

- A.CN.1 Understand and make connections among multiple representations of the same mathematical idea
- A.CN.2 Understand the corresponding procedures for similar problems or mathematical concepts

*Students will understand how mathematical ideas interconnect and build on one another to produce a coherent whole.*

- A.CN.3 Model situations mathematically, using representations to draw conclusions and formulate new situations
- A.CN.4 Understand how concepts, procedures, and mathematical results in one area of mathematics can be used to solve problems in other areas of mathematics
- A.CN.5 Understand how quantitative models connect to various physical models and representations

*Students will recognize and apply mathematics in contexts outside of mathematics.*

- A.CN.6 Recognize and apply mathematics to situations in the outside world
- A.CN.7 Recognize and apply mathematical ideas to problem situations that develop outside of mathematics
- A.CN.8 Develop an appreciation for the historical development of mathematics

### **Representation Strand**

*Students will create and use representations to organize, record, and communicate mathematical ideas.*

- A.R.1 Use physical objects, diagrams, charts, tables, graphs, symbols, equations, or objects created using technology as representations of mathematical concepts
- A.R.2 Recognize, compare, and use an array of representational forms
- A.R.3 Use representation as a tool for exploring and understanding mathematical ideas

*Students will select, apply, and translate among mathematical representations to solve problems.*

- A.R.4 Select appropriate representations to solve problem situations
- A.R.5 Investigate relationships between different representations and their impact on a given problem

*Students will use representations to model and interpret physical, social, and mathematical phenomena.*

- A.R.6 Use mathematics to show and understand physical phenomena (e.g., find the height of a building if a ladder of a given length forms a given angle of elevation with the ground)
- A.R.7 Use mathematics to show and understand social phenomena (e.g., determine profit from student and adult ticket sales)
- A.R.8 Use mathematics to show and understand mathematical phenomena (e.g., compare the graphs of the functions represented by the equations  $y = x^2$  and  $y = -x^2$ )

## Number Sense and Operations Strand

*Students will understand numbers, multiple ways of representing numbers, relationships among numbers, and number systems.*

- Number Theory*
- A.N.1 Identify and apply the properties of real numbers (closure, commutative, associative, distributive, identity, inverse) *Note: Students do not need to identify groups and fields, but students should be engaged in the ideas.*

*Students will understand meanings of operations and procedures, and how they relate to one another.*

<i>Operations</i>	A.N.2	Simplify radical terms (no variable in the radicand)
	A.N.3	Perform the four arithmetic operations using like and unlike radical terms and express the result in simplest form
	A.N.4	Understand and use scientific notation to compute products and quotients of numbers
	A.N.5	Solve algebraic problems arising from situations that involve fractions, decimals, percents (decrease/increase and discount), and proportionality/direct variation
	A.N.6	Evaluate expressions involving factorial(s), absolute value(s), and exponential expression(s)
	A.N.7	Determine the number of possible events, using counting techniques or the Fundamental Principle of Counting
	A.N.8	Determine the number of possible arrangements (permutations) of a list of items

### **Algebra Strand**

*Students will represent and analyze algebraically a wide variety of problem solving situations.*

<i>Variables and Expressions</i>	A.A.1	Translate a quantitative verbal phrase into an algebraic expression
	A.A.2	Write a verbal expression that matches a given mathematical expression
<i>Equations and Inequalities</i>	A.A.3	Distinguish the difference between an algebraic expression and an algebraic equation
	A.A.4	Translate verbal sentences into mathematical equations or inequalities
	A.A.5	Write algebraic equations or inequalities that represent a situation
	A.A.6	Analyze and solve verbal problems whose solution requires solving a linear equation in one variable or linear inequality in one variable

- A.A.7 Analyze and solve verbal problems whose solution requires solving systems of linear equations in two variables
- A.A.8 Analyze and solve verbal problems that involve quadratic equations
- A.A.9 Analyze and solve verbal problems that involve exponential growth and decay
- A.A.10 Solve systems of two linear equations in two variables algebraically (See A.G.7)
- A.A.11 Solve a system of one linear and one quadratic equation in two variables, where only factoring is required *Note: The quadratic equation should represent a parabola and the solution(s) should be integers.*

***Students will perform algebraic procedures accurately.***

*Variables and Expressions*

- A.A.12 Multiply and divide monomial expressions with a common base, using the properties of exponents *Note: Use integral exponents only.*
- A.A.13 Add, subtract, and multiply monomials and polynomials
- A.A.14 Divide a polynomial by a monomial or binomial, where the quotient has no remainder
- A.A.15 Find values of a variable for which an algebraic fraction is undefined
- A.A.16 Simplify fractions with polynomials in the numerator and denominator by factoring both and renaming them to lowest terms
- A.A.17 Add or subtract fractional expressions with monomial or like binomial denominators
- A.A.18 Multiply and divide algebraic fractions and express the product or quotient in simplest form
- A.A.19 Identify and factor the difference of two perfect squares
- A.A.20 Factor algebraic expressions completely, including trinomials with a lead coefficient of one (after factoring a GCF)

*Equations and  
Inequalities*

- A.A.21 Determine whether a given value is a solution to a given linear equation in one variable or linear inequality in one variable
- A.A.22 Solve all types of linear equations in one variable
- A.A.23 Solve literal equations for a given variable
- A.A.24 Solve linear inequalities in one variable
- A.A.25 Solve equations involving fractional expressions *Note: Expressions which result in linear equations in one variable.*
- A.A.26 Solve algebraic proportions in one variable which result in linear or quadratic equations
- A.A.27 Understand and apply the multiplication property of zero to solve quadratic equations with integral coefficients and integral roots
- A.A.28 Understand the difference and connection between roots of a quadratic equation and factors of a quadratic expression

***Students will recognize, use, and represent algebraically patterns, relations, and functions.***

*Patterns,  
Relations,  
and Functions*

- A.A.29 Use set-builder notation and/or interval notation to illustrate the elements of a set, given the elements in roster form
- A.A.30 Find the complement of a subset of a given set, within a given universe
- A.A.31 Find the intersection of sets (no more than three sets) and/or union of sets (no more than three sets)

*Coordinate  
Geometry*

- A.A.32 Explain slope as a rate of change between dependent and independent variables
- A.A.33 Determine the slope of a line, given the coordinates of two points on the line
- A.A.34 Write the equation of a line, given its slope and the coordinates of a point on the line
- A.A.35 Write the equation of a line, given the coordinates of two points on the line

- A.A.36 Write the equation of a line parallel to the x- or y-axis
- A.A.37 Determine the slope of a line, given its equation in any form
- A.A.38 Determine if two lines are parallel, given their equations in any form
- A.A.39 Determine whether a given point is on a line, given the equation of the line
- A.A.40 Determine whether a given point is in the solution set of a system of linear inequalities
- A.A.41 Determine the vertex and axis of symmetry of a parabola, given its equation (See A.G.10 )
- Trigonometric Functions*
- A.A.42 Find the sine, cosine, and tangent ratios of an angle of a right triangle, given the lengths of the sides
- A.A.43 Determine the measure of an angle of a right triangle, given the length of any two sides of the triangle
- A.A.44 Find the measure of a side of a right triangle, given an acute angle and the length of another side
- A.A.45 Determine the measure of a third side of a right triangle using the Pythagorean theorem, given the lengths of any two sides

## Geometry Strand

*Students will use visualization and spatial reasoning to analyze characteristics and properties of geometric shapes.*

- Shapes*
- A.G.1 Find the area and/or perimeter of figures composed of polygons and circles or sectors of a circle *Note: Figures may include triangles, rectangles, squares, parallelograms, rhombuses, trapezoids, circles, semi-circles, quarter-circles, and regular polygons (perimeter only).*
- A.G.2 Use formulas to calculate volume and surface area of rectangular solids and cylinders

*Students will apply coordinate geometry to analyze problem solving situations.*

<i>Coordinate Geometry</i>	A.G.3	Determine when a relation is a function, by examining ordered pairs and inspecting graphs of relations
	A.G.4	Identify and graph linear, quadratic (parabolic), absolute value, and exponential functions
	A.G.5	Investigate and generalize how changing the coefficients of a function affects its graph
	A.G.6	Graph linear inequalities
	A.G.7	Graph and solve systems of linear equations and inequalities with rational coefficients in two variables (See A.A.10)
	A.G.8	Find the roots of a parabolic function graphically <i>Note: Only quadratic equations with integral solutions.</i>
	A.G.9	Solve systems of linear and quadratic equations graphically <i>Note: Only use systems of linear and quadratic equations that lead to solutions whose coordinates are integers.</i>
	A.G.10	Determine the vertex and axis of symmetry of a parabola, given its graph (See A.A.41) <i>Note: The vertex will have an ordered pair of integers and the axis of symmetry will have an integral value.</i>

### Measurement Strand

*Students will determine what can be measured and how, using appropriate methods and formulas.*

<i>Units of Measurement</i>	A.M.1	Calculate rates using appropriate units (e.g., rate of a space ship versus the rate of a snail)
	A.M.2	Solve problems involving conversions within measurement systems, given the relationship between the units

*Students will understand that all measurement contains error and be able to determine its significance.*

<i>Error and Magnitude</i>	A.M.3	Calculate the relative error in measuring square and cubic units, when there is an error in the linear measure
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## Statistics and Probability Strand

*Students will collect, organize, display, and analyze data.*

*Organization and  
Display of Data*

- A.S.1 Categorize data as qualitative or quantitative
- A.S.2 Determine whether the data to be analyzed is univariate or bivariate
- A.S.3 Determine when collected data or display of data may be biased
- A.S.4 Compare and contrast the appropriateness of different measures of central tendency for a given data set
- A.S.5 Construct a histogram, cumulative frequency histogram, and a box-and-whisker plot, given a set of data
- A.S.6 Understand how the five statistical summary (minimum, maximum, and the three quartiles) is used to construct a box-and-whisker plot
- A.S.7 Create a scatter plot of bivariate data
- A.S.8 Construct manually a reasonable line of best fit for a scatter plot and determine the equation of that line

*Analysis of Data*

- A.S.9 Analyze and interpret a frequency distribution table or histogram, a cumulative frequency distribution table or histogram, or a box-and-whisker plot
- A.S.10 Evaluate published reports and graphs that are based on data by considering: experimental design, appropriateness of the data analysis, and the soundness of the conclusions
- A.S.11 Find the percentile rank of an item in a data set and identify the point values for first, second, and third quartiles
- A.S.12 Identify the relationship between the independent and dependent variables from a scatter plot (positive, negative, or none)
- A.S.13 Understand the difference between correlation and causation
- A.S.14 Identify variables that might have a correlation but not a causal relationship

*Students will make predictions that are based upon data analysis.*

<i>Predictions from Data</i>	A.S.15	Identify and describe sources of bias and its effect, drawing conclusions from data
	A.S.16	Recognize how linear transformations of one-variable data affect the data's mean, median, mode, and range
	A.S.17	Use a reasonable line of best fit to make a prediction involving interpolation or extrapolation

*Students will understand and apply concepts of probability.*

<i>Probability</i>	A.S.18	Know the definition of conditional probability and use it to solve for probabilities in finite sample spaces
	A.S.19	Determine the number of elements in a sample space and the number of favorable events
	A.S.20	Calculate the probability of an event and its complement
	A.S.21	Determine empirical probabilities based on specific sample data
	A.S.22	Determine, based on calculated probability of a set of events, if: <ul style="list-style-type: none"><li>○ some or all are equally likely to occur</li><li>○ one is more likely to occur than another</li><li>○ whether or not an event is certain to happen or not to happen</li></ul>
	A.S.23	Calculate the probability of: <ul style="list-style-type: none"><li>○ a series of independent events</li><li>○ a series of dependent events</li><li>○ two mutually exclusive events</li><li>○ two events that are not mutually exclusive</li></ul>