Dear Sir

I have to acknowledge the receipt of your favor of May 14. in which you mention that you have finished the 6. first books of Euclid, plane trigonometry, surveying & algebra and ask whether I think a further pursuit of that branch of science would be useful to you. there are some propositions in the latter books of Euclid, & some of Archimedes, which are useful. & I have no doubt you have been made acquainted with them. trigonometry, so far as this, is most valuable to every man, there is scarcely a day in which he will not resort to it for some of the purposes of common life. the science of calculation also is indispensable as far as the extraction of the square & cube roots; Algebra as far as the quadratic equation & the use of logarithms are often of value in ordinary cases: but all beyond these is but a luxury: a delicious luxury indeed; but not to be indulged in by one who is to have a profession to follow for his subsistence. in this light I view the conic sections, curves of the higher orders, perhaps even spherical trigonometry, Algebraical operations beyond the 2d dimension, and fluxions.

Letter from Thomas Jefferson to William G. Munford, Monticello, June 18, 1799.
# TABLE OF CONTENTS

## ADVANCED ALGEBRA

<table>
<thead>
<tr>
<th>PRENTICE HALL CHAPTER</th>
<th>QUESTION NUMBER</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Patterns, Models, and Functions</td>
<td>1-28</td>
</tr>
<tr>
<td>2 Linear Relationships and Functions</td>
<td>29-43</td>
</tr>
<tr>
<td>5 Quadratic Equations and Functions</td>
<td>44-100</td>
</tr>
<tr>
<td>6 Polynomials and Polynomial Functions</td>
<td>101-132</td>
</tr>
<tr>
<td>7 Exponential and Logarithmic Functions</td>
<td>133-186</td>
</tr>
<tr>
<td>8 Rational Functions</td>
<td>187-241</td>
</tr>
<tr>
<td>9 Periodic Functions and Trigonometry</td>
<td>242-319</td>
</tr>
<tr>
<td>10 Quadratic Relations</td>
<td>320-334</td>
</tr>
<tr>
<td>11 More Probability and Statistics</td>
<td>335-360</td>
</tr>
<tr>
<td>12 Sequences and Series</td>
<td>361-373</td>
</tr>
<tr>
<td>NY2 Graphing and Solving Quadratic Inequalities</td>
<td>374-379</td>
</tr>
<tr>
<td>NY8-9 Reciprocal Trigonometric Functions/Identities</td>
<td>380-387</td>
</tr>
<tr>
<td>NY10 Trigonometric Equations</td>
<td>388-399</td>
</tr>
<tr>
<td>NY11 Applying Trigonometric Identities</td>
<td>400-411</td>
</tr>
</tbody>
</table>
CHAPTER 1-3
DEFINING FUNCTIONS

1. If \( f(x) = 4x^0 + (4x)^{-1} \), what is the value of \( f(4) \)?

   [A] \( \frac{1}{16} \)  [B] \( \frac{1}{16} \)  [C] 0  [D] −12

2. Which relation is not a function?

   [A] \( y = x^2 - 4x + 3 \)  [B] \( y = 2x + 4 \)
   [C] \( x = y^2 + 2x - 3 \)  [D] \( x = 3y - 2 \)

3. Which relation is a function?

   [A] \( x = 4 \)  [B] \( x^2 + y^2 = 16 \)
   [C] \( x = y^2 + 1 \)  [D] \( y = \sin x \)

4. Which equation represents a function?

   [A] \( 4y^2 = 36 - 9x^2 \)  [B] \( x^2 + y^2 = 4 \)
   [C] \( y = x^2 - 3x - 4 \)  [D] \( x = y^2 - 6x + 8 \)

5. Which set of ordered pairs is not a function?

   [A] \{ (3,1), (2,1), (1,2), (3,2) \}  [B] \{ (0,0), (1,1), (2,2), (3,3) \}
   [C] \{ (1,2), (3,4), (4,5), (5,6) \}  [D] \{ (4,1), (5,1), (6,1), (7,1) \}

6. Which relation is a function?

   [A] \( x^2 - y^2 = 7 \)  [B] \( x = 7 \)
   [C] \( x^2 + y^2 = 7 \)  [D] \( xy = 7 \)

7. Which equation does not represent a function?

   [A] \( x = \pi \)  [B] \( y = 4 \)
   [C] \( y = |x| \)  [D] \( y = x^2 + 5x \)

8. Which graph is not a function?

   [A]  [B]  [C]  [D]

9. Which graph does not represent a function of \( x \)?

   [A]  [B]  [C]  [D]

10. Which diagram represents a relation in which each member of the domain corresponds to only one member of its range?

    [A]  [B]  [C]  [D]
11. On the accompanying diagram, draw a mapping of a relation from set $A$ to set $B$ that is not a function. Explain why the relationship you drew is not a function.

12. Which diagram represents a one-to-one function?

13. Each graph below represents a possible relationship between temperature and pressure. Which graph does not represent a function?

14. If $f(x) = 5x^2$ and $g(x) = \sqrt{2x}$, what is the value of $(f \circ g)(8)$?
   - [A] 16
   - [B] 80
   - [C] 1,280
   - [D] $8\sqrt{10}$

15. If $f(x) = x^{\frac{2}{3}}$ and $g(x) = 8x^{\frac{1}{2}}$, find $(f \circ g)(x)$ and $(f \circ g)(27)$.

16. If $f(x) = 2^x - 1$ and $g(x) = x^2 - 1$, determine the value of $(f \circ g)(3)$.

17. If $f(x) = -2x + 7$ and $g(x) = x^2 - 2$, then $f(g(3))$ is equal to
   - [A] 7
   - [B] -3
   - [C] -7
   - [D] -1

18. If $f(x) = 5x^2 - 1$ and $g(x) = 3x - 1$, find $g(f(1))$. 
19. If $f$ and $g$ are two functions defined by $f(x) = 3x + 5$ and $g(x) = x^2 + 1$, then $g(f(x))$ is

[A] $3x^2 + 8$  
[B] $9x^2 + 30x + 26$  
[C] $x^2 + 3x + 6$  
[D] $9x^2 + 26$

20. If $f(x) = \frac{2}{x + 3}$ and $g(x) = \frac{1}{x}$, then $(g \circ f)(x)$ is equal to

[A] $\frac{x + 3}{2x}$  
[B] $\frac{2x}{1 + 3x}$  
[C] $\frac{x + 3}{2}$  
[D] $\frac{1 + 3x}{2x}$

21. If $f(x) = x + 1$ and $g(x) = x^2 - 1$, the expression $(g \circ f)(x)$ equals 0 when $x$ is equal to

[A] -2, only  
[B] 0, only  
[C] 0 and -2  
[D] 1 and -1

22. If $f(x) = 2x^2 + 4$ and $g(x) = x - 3$, which number satisfies $f(x) = (f \circ g)(x)$?

[A] 5  
[B] $\frac{3}{2}$  
[C] $\frac{3}{4}$  
[D] 4

23. The accompanying graph is a sketch of the function $y = f(x)$ over the interval $0 \leq x \leq 7$.

What is the value of $(f \circ f)(6)$?

[A] 2  
[B] 0  
[C] -2  
[D] 1

24. A certain drug raises a patient's heart rate, $h(x)$, in beats per minute, according to the function $h(x) = 70 + 0.2x$, where $x$ is the bloodstream drug level, in milligrams. The level of the drug in the patient's bloodstream is a function of time, $t$, in hours, according to the formula $g(t) = 300(0.8)^t$. Find the value of $h(g(4))$, the patient's heart rate in beats per minute, to the nearest whole number.

25. The temperature generated by an electrical circuit is represented by $t = f(m) = 0.3m^2$, where $m$ is the number of moving parts. The resistance of the same circuit is represented by $r = g(t) = 150 + 5t$, where $t$ is the temperature. What is the resistance in a circuit that has four moving parts?

[A] 51  
[B] 174  
[C] 8,670  
[D] 156

**OPERATIONS WITH FUNCTIONS**

26. The revenue, $R(x)$, from selling $x$ units of a product is represented by the equation $R(x) = 35x$, while the total cost, $C(x)$, of making $x$ units of the product is represented by the equation $C(x) = 20x + 500$. The total profit, $P(x)$, is represented by the equation $P(x) = R(x) - C(x)$. For the values of $R(x)$ and $C(x)$ given above, what is $P(x)$?

[A] $15x + 500$  
[B] $15x - 500$  
[C] $10x + 100$  
[D] $15x$

27. A company calculates its profit by finding the difference between revenue and cost. The cost function of producing $x$ hammers is $C(x) = 4x + 170$. If each hammer is sold for $10, the revenue function for selling $x$ hammers is $R(x) = 10x$. How many hammers must be sold to make a profit? How many hammers must be sold to make a profit of $100$?
28. The profit a coat manufacturer makes each day is modeled by the equation
\[ P(x) = -x^3 + 120x - 2000, \]
where \( P \) is the profit and \( x \) is the price for each coat sold. For what values of \( x \) does the company make a profit? [The use of the accompanying grid is optional.]

29. Which equation states that the temperature, \( t \), in a room is less than 3° from 68°?
[A] \(|3 - t| < 68\)  [B] \(|3 + t| < 68\)  [C] \(68 + t < 3\)  [D] \(68 - t < 3\)

30. The solution set of \(|3x + 2| < 1\) contains
[A] only negative real numbers  [B] both positive and negative real numbers  [C] no real numbers  [D] only positive real numbers

31. What is the solution set of the inequality
\[ |3 - 2x| \geq 4? \]
[A] \(\{x|x \leq -\frac{1}{2} \text{ or } x \geq \frac{7}{2}\}\)  [B] \(\{x|\frac{7}{2} \leq x \leq -\frac{1}{2}\}\)  [C] \(\{x|-\frac{1}{2} \leq x \leq \frac{7}{2}\}\)  [D] \(\{x|x \leq \frac{7}{2} \text{ or } x \geq \frac{1}{2}\}\)

32. What is the solution of the inequality
\[ |x + 3| \leq 5? \]
[A] \(x \leq -2 \text{ or } x \geq 8\)  [B] \(-8 \leq x \leq 2\)  [C] \(x \leq -8 \text{ or } x \geq 2\)  [D] \(-2 \leq x \leq 8\)

33. The solution of \(|2x - 3| < 5\) is
[A] \(x > -1\)  [B] \(x < -1 \text{ or } x > 4\)  [C] \(x < 4\)  [D] \(-1 < x < 4\)

34. What is the solution of the inequality
\[ |y + 8| > 3? \]
[A] \(y > -5 \text{ or } y < -11\)  [B] \(-11 < y < -5\)  [C] \(y > -5\)  [D] \(-5 < y < 11\)

35. Which graph represents the solution set of \(|2x - 1| < 7?\)
[A] \[graph\]  [B] \[graph\]  [C] \[graph\]  [D] \[graph\]
36. Which graph represents the solution set for the expression \( |2x + 3| > 7 \)?

[A] \[
\begin{array}{c}
-2 & 0 & 5 \\
-2 & 0 & 5 \\
-5 & 0 & 2 \\
-5 & 0 & 2 \\
\end{array}
\]

[B] \[
\begin{array}{c}
-2 & 0 & 5 \\
-2 & 0 & 5 \\
-5 & 0 & 2 \\
-5 & 0 & 2 \\
\end{array}
\]

[C] \[
\begin{array}{c}
-10 & -9 & -8 & -7 & -6 & -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
-10 & -9 & -8 & -7 & -6 & -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
-10 & -9 & -8 & -7 & -6 & -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
-10 & -9 & -8 & -7 & -6 & -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
\end{array}
\]

[D] \[
\begin{array}{c}
-2 & 0 & 5 \\
-2 & 0 & 5 \\
-5 & 0 & 2 \\
-5 & 0 & 2 \\
\end{array}
\]

37. The solution set of which inequality is represented by the accompanying graph?

[A] \( |2 - x| > -7 \)  
[B] \( |x - 2| < 7 \)  
[C] \( |x - 2| > 7 \)  
[D] \( |2 - x| < -7 \)

38. The inequality \( |1.5C - 24| \leq 30 \) represents the range of monthly average temperatures, \( C \), in degrees Celsius, for Toledo, Ohio. Solve for \( C \).

39. A depth finder shows that the water in a certain place is 620 feet deep. The difference between \( d \), the actual depth of the water, and the reading is \( |d - 620| \) and must be less than or equal to 0.05\( d \). Find the minimum and maximum values of \( d \), to the nearest tenth of a foot.

40. The heights, \( h \), of the students in the chorus at Central Middle School satisfy the inequality \( \left| \frac{h - 57.5}{2} \right| \leq 3.25 \), when \( h \) is measured in inches. Determine the interval in which these heights lie and express your answer to the nearest tenth of a foot. [Only an algebraic solution can receive full credit.]

CHAPTER 2-5

GRAPHING FUNCTIONS

41. Which equation is represented by the accompanying graph?

[A] \( y = |x - 3| \)  
[B] \( y = |x - 3| + 1 \)  
[C] \( y = (x - 3)^2 + 1 \)  
[D] \( y = |x + 3| - 1 \)

42. The graph below represents \( f(x) \).

Which graph best represents \( |f(x)| \)?

[A] \[
\begin{array}{c}
(3) \\
(3) \\
(3) \\
(3) \\
\end{array}
\]

[B] \[
\begin{array}{c}
(4) \\
(4) \\
(4) \\
(4) \\
\end{array}
\]

[C] \[
\begin{array}{c}
(1) \\
(1) \\
(1) \\
(1) \\
\end{array}
\]

[D] \[
\begin{array}{c}
(2) \\
(2) \\
(2) \\
(2) \\
\end{array}
\]

43. Given the function \( y = f(x) \), such that the entire graph of the function lies above the \( x \)-axis. Explain why the equation \( f(x) = 0 \) has no real solutions.
CHAPTER 5-4

INVERSE OF FUNCTIONS

44. If a function is defined by the equation \( y = 3x + 2 \), which equation defines the inverse of this function?

[A] \( x = \frac{1}{3}y + \frac{1}{2} \)  \quad [B] \( y = -3x - 2 \)

[C] \( y = \frac{1}{3}x - \frac{2}{3} \)  \quad [D] \( y = \frac{1}{3}x + \frac{1}{2} \)

45. A function is defined by the equation \( y = 5x - 5 \). Which equation defines the inverse of this function?

[A] \( y = 5x + 5 \)  \quad [B] \( x = \frac{1}{5y - 5} \)

[C] \( y = \frac{1}{5x - 5} \)  \quad [D] \( x = 5y - 5 \)

46. A function is defined by the equation \( y = \frac{1}{2}x - \frac{3}{2} \). Which equation defines the inverse of this function?

[A] \( y = 2x - 3 \)  \quad [B] \( y = 2x + \frac{3}{2} \)

[C] \( y = 2x - \frac{3}{2} \)  \quad [D] \( y = 2x + 3 \)

47. Given: \( f(x) = x^2 \) and \( g(x) = 2^x \)
   a) The inverse of \( g \) is a function, but the inverse of \( f \) is not a function. Explain why this statement is true.
   b) Find \( g^{-1}(f(3)) \) to the nearest tenth.

48. If the point \((a, b)\) lies on the graph \( y = f(x) \), the graph of \( y = f^{-1}(x) \) must contain point

[A] \((-a, -b)\)  \quad [B] \((0, b)\)

[C] \((b, a)\)  \quad [D] \((a, 0)\)

49. Which graph represents the inverse of \( f(x) = \{(0,1), (1,4), (2,3)\} \)?

50. The accompanying diagram shows the graph of the line whose equation is \( y = -\frac{1}{3}x + 2 \).
   On the same set of axes, sketch the graph of the inverse of this function.
   State the coordinates of a point on the inverse function.
51. What is the inverse of the function
\[ y = \log_4 x \]?
[A] \( y^4 = x \)  [B] \( x^4 = y \)
[C] \( 4^y = x \)  [D] \( 4^x = y \)

52. The inverse of a function is a logarithmic function in the form \( y = \log_a x \). Which equation represents the original function?
[A] \( y = b^x \)  [B] \( y = bx \)
[C] \( by = x \)  [D] \( x = b^y \)

53. Draw \( f(x) = 2x^2 \) and \( f^{-1}(x) \) in the interval \( 0 \leq x \leq 2 \) on the accompanying set of axes. State the coordinates of the points of intersection.

54. Expressed in simplest form, \( i^{16} + i^6 - 2i^5 + i^{13} \)
[A] \( i \)  [B] 1  [C] \(-i\)  [D] \(-1\)

55. When simplified, \( i^{27} + i^{34} \) is equal to
[A] \( i \)  [B] \( i-1 \)  [C] \(-i-1 \)  [D] \( i^{61} \)

56. What is the value of \( i^{99} - i^3 \)?
[A] 1  [B] \(-i\)  [C] \(i^{96}\)  [D] 0

57. What is the sum of \( \sqrt{-2} \) and \( \sqrt{-18} \)?
[A] \( 2i\sqrt{5} \)  [B] \( 5i\sqrt{2} \)
[C] \( 4i\sqrt{2} \)  [D] \( 6i \)

58. The expression \( i^0 \cdot i^1 \cdot i^2 \cdot i^3 \cdot i^4 \) is equal to
[A] -1  [B] \(-i\)  [C] \(i\)  [D] \(1\)

59. The expression \( \frac{i^{16}}{i^3} \) is equivalent to
[A] \( 1 \)  [B] \( i \)  [C] \(-1\)  [D] \(-i\)

60. What is the multiplicative inverse of \( 3i \)?
[A] \( \frac{1}{3} \)  [B] \(-\frac{i}{3}\)  [C] \(-3i\)  [D] \(-3\)

61. Mrs. Donahue made up a game to help her class learn about imaginary numbers. The winner will be the student whose expression is equivalent to \(-i\). Which expression will win the game?
[A] \( i^{48} \)  [B] \( i^{46} \)  [C] \( i^{49} \)  [D] \( i^{47} \)

**COMPLEX NUMBERS**

62. Express \( \sqrt{-48} + 3.5 + \sqrt{25} + \sqrt{-27} \) in simplest \( a + bi \) form.

63. What is the sum of \( 2 - \sqrt{-4} \) and \(-3 + \sqrt{-16} \) expressed in simplest \( a + bi \) form?
[A] \(-1 + i\sqrt{20}\)  [B] \(-1 + 12i\)
[C] \(-1 + 2i\)  [D] \(-14 + i\)

64. What is the product of \( 5 + \sqrt{-36} \) and \( 1 - \sqrt{-49} \), expressed in simplest \( a + bi \) form?
[A] \(47 - 29i\)  [B] \(47 + 41i\)
[C] \(-37 + 41i\)  [D] \(5 - 71i\)
65. When expressed as a monomial in terms of \(i\), 
\[
2\sqrt{-32} - 5\sqrt{-8}
\]
is equivalent to 
[A] \(2i\sqrt{2}\)  
[B] \(18i\sqrt{2}\)  
[C] \(2\sqrt{2i}\)  
[D] \(-2i\sqrt{2}\)

66. The expression \((-1+i)^3\) is equivalent to 
[A] \(-3i\)  
[B] \(-2 - 2i\)  
[C] \(-1 - i\)  
[D] \(2 + 2i\)

67. If \(f(x) = x^3 - 2x^2\), then \(f(i)\) is equivalent to 
[A] \(2 - i\)  
[B] \(-2 - i\)  
[C] \(-2 + i\)  
[D] \(2 + i\)

68. The expression \(\frac{2+i}{3+i}\) is equivalent to 
[A] \(\frac{7+i}{10}\)  
[B] \(\frac{7-5i}{10}\)  
[C] \(\frac{6+i}{8}\)  
[D] \(\frac{6+5i}{8}\)

69. What is the value of \(x\) in the equation 
\[
\sqrt{5-2x} = 3i
\]?
[A] \(1\)  
[B] \(-2\)  
[C] \(4\)  
[D] \(7\)

70. Melissa and Joe are playing a game with complex numbers. If Melissa has a score of \(5-4i\) and Joe has a score of \(3+2i\), what is their total score?
[A] \(8 + 6i\)  
[B] \(8 + 2i\)  
[C] \(8 - 2i\)  
[D] \(8 - 6i\)

71. The complex number \(c + di\) is equal to \((2+i)^2\). What is the value of \(c\)?

72. Show that the product of \(a + bi\) and its conjugate is a real number.

73. In an electrical circuit, the voltage, \(E\), in volts, the current, \(I\), in amps, and the opposition to the flow of current, called impedance, \(Z\), in ohms, are related by the equation \(E = IZ\). A circuit has a current of \((3+i)\) amps and an impedance of \((-2+i)\) ohms. Determine the voltage in \(a + bi\) form.

74. The relationship between voltage, \(E\), current, \(I\), and resistance, \(Z\), is given by the equation \(E = IZ\). If a circuit has a current \(I = 3+2i\) and a resistance \(Z = 2-i\), what is the voltage of this circuit?
[A] \(4 + i\)  
[B] \(8 + i\)  
[C] \(8 + 7i\)  
[D] \(4 - i\)

75. Impedance measures the opposition of an electrical circuit to the flow of electricity. The total impedance in a particular circuit is given by the formula \(Z_T = Z_1 + Z_2\). What is the total impedance of a circuit, \(Z_T\), if \(Z_1 = 1+2i\) and \(Z_2 = 1-2i\)?
[A] \(-\frac{3}{2}\)  
[B] \(0\)  
[C] \(\frac{5}{2}\)  
[D] \(1\)

76. Find the sum of \(-2 + 3i\) and \(-1 - 2i\).
Graph the resultant on the accompanying set of axes.
77. Fractal geometry uses the complex number plane to draw diagrams, such as the one shown in the accompanying graph.

Which number is not included in the shaded area?

[A] -0.5 - 0.5i  
[B] -0.9  
[C] -0.9 - 0.9i  
[D] -0.5i

78. Two complex numbers are graphed below.

What is the sum of \( w \) and \( u \), expressed in standard complex number form?

[A] \( 5 + 7i \)  
[B] \( 3 + 7i \)  
[C] \( 7 + 3i \)  
[D] \( -5 + 3i \)

### CHAPTER 5-8

#### QUADRATIC FORMULA

79. If the sum of the roots of \( x^2 + 3x - 5 \) is added to the product of its roots, the result is

[A] -2  
[B] -8  
[C] -15  
[D] 15

80. Express, in simplest \( a + bi \) form, the roots of the equation \( x^2 + 5 = 4x \).

81. Solve for \( x \) in simplest \( a + bi \) form:

\( x^2 + 8x + 25 = 0 \)

82. In physics class, Taras discovers that the behavior of electrical power, \( x \), in a particular circuit can be represented by the function \( f(x) = x^2 + 2x + 7 \). If \( f(x) = 0 \), solve the equation and express your answer in simplest \( a + bi \) form.

83. Barb pulled the plug in her bathtub and it started to drain. The amount of water in the bathtub as it drains is represented by the equation \( L = -5t^2 - 8t + 120 \), where \( L \) represents the number of liters of water in the bathtub and \( t \) represents the amount of time, in minutes, since the plug was pulled. How many liters of water were in the bathtub when Barb pulled the plug? Show your reasoning. Determine, to the nearest tenth of a minute, the amount of time it takes for all the water in the bathtub to drain.

#### USING THE DISCRIMINANT

84. Find all values of \( k \) such that the equation \( 3x^2 - 2x + k = 0 \) has imaginary roots.
85. Jacob is solving a quadratic equation. He executes a program on his graphing calculator and sees that the roots are real, rational, and unequal. This information indicates to Jacob that the discriminant is
[A] negative  [B] zero  
[C] not a perfect square  [D] a perfect square

86. The roots of the equation $x^2 - 3x - 2 = 0$ are
[A] real, rational, and equal  
[B] real, rational, and unequal  
[C] real, irrational, and unequal  [D] imaginary

87. The roots of a quadratic equation are real, rational, and equal when the discriminant is

88. Which equation has imaginary roots?
[A] $x^2 - 1 = 0$  
[B] $x^2 - 2 = 0$  
[C] $x^2 + x + 1 = 0$  [D] $x^2 - x - 1 = 0$

89. The roots of the equation $ax^2 + 4x = -2$ are real, rational, and equal when $a$ has a value of
[A] 2  [B] 4  [C] 3  [D] 1

90. In the equation $ax^2 + 6x - 9 = 0$, imaginary roots will be generated if
[A] $a < 1$, only  
[B] $a < -1$  
[C] $a > -1$, only  [D] $-1 < a < 1$

91. The equation $2x^2 + 8x + n = 0$ has imaginary roots when $n$ is equal to

92. Which equation has imaginary roots?
[A] $(2x + 1)(x - 3) = 7$  
[B] $x(5 + x) = 8$  
[C] $x(x + 6) = -10$  [D] $x(5 - x) = -3$

93. The roots of the equation $2x^2 - 8x - 4 = 0$ are
[A] real, irrational, and unequal  
[B] imaginary  
[C] real, rational, and equal  [D] real, rational, and unequal

94. The roots of the equation $2x^2 - x = 4$ are
[A] real, rational, and unequal  
[B] real and irrational  
[C] real, rational, and equal  [D] imaginary

95. Which statement must be true if a parabola represented by the equation $y = ax^2 + bx + c$ does not intersect the x-axis?
[A] $b^2 - 4ac > 0$, and $b^2 - 4ac$ is not a perfect square.  
[B] $b^2 - 4ac > 0$, and $b^2 - 4ac$ is a perfect square.  
[C] $b^2 - 4ac < 0$  [D] $b^2 - 4ac = 0$

96. For which positive value of $m$ will the equation $4x^2 + mx + 9 = 0$ have roots that are real, equal, and rational?

97. If the roots of $ax^2 + bx + c = 0$ are real, rational, and equal, what is true about the graph of the function $y = ax^2 + bx + c$?
[A] It intersects the x-axis in two distinct points.  
[B] It lies entirely below the x-axis.  
[C] It lies entirely above the x-axis.  [D] It is tangent to the x-axis.

98. If $2 + 3i$ is one root of a quadratic equation with real coefficients, what is the sum of the roots of the equation?
99. The roots of the equation \(2x^2 - 5 = 0\) are
[A] real and irrational  
[B] real, rational, and unequal  
[C] real, rational, and equal  
[D] imaginary

100. Which graph represents a quadratic function with a negative discriminant?

[A]  

[B]  

[C]  

[D]  

101. The expression \(4^2 \cdot 2^3\) is equal to
[A] 16  [B] 4  [C] \(8^3\)  [D] \(4^2\)

102. The value of \(\left(\frac{3^6}{27^5}\right)^{-1}\) is
[A] \(-9\)  [B] \(-\frac{1}{9}\)  [C] 9  [D] \(\frac{1}{9}\)

103. The expression \(\frac{3^3}{3^2}\) is equivalent to
[A] 1  [B] \(\sqrt{3}\)  [C] \(\frac{1}{\sqrt{3}}\)  [D] 3

104. If \(x\) is a positive integer, \(4x^2\) is equivalent to
[A] \(4\sqrt{x}\)  [B] \(2x\)  [C] \(\frac{4\sqrt{x}}{x}\)  [D] \(\frac{2}{x}\)

105. The expression \(b^{\frac{3}{2}}, b > 0\), is equivalent to
[A] \(\frac{1}{(\sqrt{b})^3}\)  [B] \(\frac{1}{(\sqrt{b})^2}\)  [C] \((\sqrt{b})^2\)  [D] \(-(\sqrt{b})^3\)

106. The expression \(\sqrt[4]{16a^6b^4}\) is equivalent to
[A] \(4a^3b\)  [B] \(2a^2b\)  [C] \(2a^3b\)  [D] \(4a^2b\)

107. When simplified, the expression \((\sqrt[3]{m^3})(m^{-\frac{1}{2}})\) is equivalent to
[A] \(\frac{\sqrt[3]{m^5}}{m}\)  [B] \(\sqrt[3]{m^{-4}}\)  [C] \(\sqrt[3]{m^{-2}}\)  [D] \(\frac{4\sqrt[3]{m^3}}{3}\)

108. Find the value of \((x + 2)^0 + (x + 1)^{\frac{2}{3}}\) when \(x = 7\).
109. If \((a^3)^\frac{2}{3} = \frac{1}{a^2}\), what is the value of \(x\)?

[A] 2  [B] -1  [C] -3  [D] 1

110. If \(f(x) = x^{\frac{3}{2}}\), then \(f\left(\frac{1}{4}\right)\) is equal to

[A] -2  [B] -4  [C] 8  [D] -\frac{1}{8}

111. Meteorologists can determine how long a storm lasts by using the function

\[ t(d) = 0.07d^{\frac{3}{2}} \]

where \(d\) is the diameter of the storm, in miles, and \(t\) is the time, in hours. If the storm lasts 4.75 hours, find its diameter, to the nearest tenth of a mile.

**CHAPTER 6-7**

**BINOMIAL EXPANSIONS**

112. What is the last term in the expansion of \((x + 2y)^5\)?

[A] \(2y^5\)  [B] \(32y^5\)  [C] \(y^5\)  [D] \(10y^5\)

113. What is the middle term in the expansion of \((x + y)^4\)?

[A] \(4x^2y^2\)  [B] \(6x^2y^2\)  [C] \(x^2y^2\)  [D] \(2x^2y^2\)

114. What is the fourth term in the expansion of \((y - 1)^7\)?

[A] \(-35y^3\)  [B] \(35y^3\)  [C] \(-35y^4\)  [D] \(35y^4\)

115. What is the third term in the expansion of \((\cos x + 3)^5\)?

[A] \(90\cos^2 x\)  [B] \(60\cos^3 x\)  [C] \(270\cos^2 x\)  [D] \(90\cos^3 x\)

**BINOMIAL PROBABILITY**

116. At a certain intersection, the light for eastbound traffic is red for 15 seconds, yellow for 5 seconds, and green for 30 seconds. Find, to the nearest tenth, the probability that out of the next eight eastbound cars that arrive randomly at the light, exactly three will be stopped by a red light.

117. As shown in the accompanying diagram, a circular target with a radius of 9 inches has a bull's-eye that has a radius of 3 inches. If five arrows randomly hit the target, what is the probability that at least four hit the bull's-eye?

118. Team \(A\) and team \(B\) are playing in a league. They will play each other five times. If the probability that team \(A\) wins a game is \(\frac{1}{3}\), what is the probability that team \(A\) will win at least three of the five games?

119. After studying a couple's family history, a doctor determines that the probability of any child born to this couple having a gene for disease \(X\) is 1 out of 4. If the couple has three children, what is the probability that exactly two of the children have the gene for disease \(X\)?

120. Which fraction represents the probability of obtaining exactly eight heads in ten tosses of a fair coin?

[A] \(\frac{45}{1,024}\)  [B] \(\frac{180}{1,024}\)  [C] \(\frac{90}{1,024}\)  [D] \(\frac{64}{1,024}\)
121. The probability that Kyla will score above a 90 on a mathematics test is \( \frac{4}{5} \). What is the probability that she will score above a 90 on three of the four tests this quarter?

[A] \( \frac{3}{4} \left( \frac{4}{5} \right)^3 \left( \frac{1}{5} \right) \)

[B] \( 4 C_3 \left( \frac{4}{5} \right)^3 \left( \frac{1}{5} \right)^5 \)

[C] \( 4 C_3 \left( \frac{4}{5} \right)^3 \left( \frac{1}{5} \right) \)

[D] \( \frac{3}{4} \left( \frac{4}{5} \right)^3 \left( \frac{1}{5} \right)^3 \)

122. On any given day, the probability that the entire Watson family eats dinner together is \( \frac{2}{5} \). Find the probability that, during any 7-day period, the Watsons eat dinner together at least six times.

123. When Joe bowls, he can get a strike (knock down all the pins) 60% of the time. How many times more likely is it for Joe to bowl at least three strikes out of four tries as it is for him to bowl zero strikes out of four tries? Round your answer to the nearest whole number.

124. A board game has a spinner on a circle that has five equal sectors, numbered 1, 2, 3, 4, and 5, respectively. If a player has four spins, find the probability that the player spins an even number no more than two times on those four spins.

125. The Hiking Club plans to go camping in a State park where the probability of rain on any given day is 0.7. Which expression can be used to find the probability that it will rain on exactly three of the seven days they are there?

[A] \( 7 C_3 \left( 0.3 \right)^3 \left( 0.7 \right)^4 \)

[B] \( 7 C_3 \left( 0.7 \right)^3 \left( 0.3 \right)^4 \)

[C] \( 4 C_3 \left( 0.7 \right)^3 \left( 0.3 \right)^4 \)

[D] \( 4 C_3 \left( 0.4 \right)^3 \left( 0.3 \right)^3 \)

126. Tim Parker, a star baseball player, hits one home run for every ten times he is at bat. If Parker goes to bat five times during tonight's game, what is the probability that he will hit at least four home runs?

127. If the probability that it will rain on any given day this week is 60%, find the probability it will rain exactly 3 out of 7 days this week.

128. The probability that a planted watermelon seed will sprout is \( \frac{3}{4} \). If Peyton plants seven seeds from a slice of watermelon, find, to the nearest ten thousandth, the probability that at least five will sprout.

129. The Coolidge family's favorite television channels are 3, 6, 7, 10, 11, and 13. If the Coolidge family selects a favorite channel at random to view each night, what is the probability that they choose exactly three even-numbered channels in five nights? Express your answer as a fraction or as a decimal rounded to four decimal places.

130. During a recent survey, students at Franconia College were asked if they drink coffee in the morning. The results showed that two-thirds of the students drink coffee in the morning and the remainder do not. What is the probability that of six students selected at random, exactly two of them drink coffee in the morning? Express your answer as a fraction or as a decimal rounded to four decimal places.

131. Ginger and Mary Anne are planning a vacation trip to the island of Capri, where the probability of rain on any day is 0.3. What is the probability that during their five days on the island, they have no rain on exactly three of the five days?
132. On mornings when school is in session in January, Sara notices that her school bus is late one-third of the time. What is the probability that during a 5-day school week in January her bus will be late at least three times?

**MATH TOOLBOX P. 311**

**EXPONENTIAL REGRESSION EQUATIONS**

133. The breaking strength, \( y \), in tons, of steel cable with diameter \( d \), in inches, is given in the table below.

<table>
<thead>
<tr>
<th>( d ) (in)</th>
<th>( y ) (tons)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50</td>
<td>9.85</td>
</tr>
<tr>
<td>0.75</td>
<td>21.80</td>
</tr>
<tr>
<td>1.00</td>
<td>38.30</td>
</tr>
<tr>
<td>1.25</td>
<td>59.20</td>
</tr>
<tr>
<td>1.50</td>
<td>84.40</td>
</tr>
<tr>
<td>1.75</td>
<td>114.00</td>
</tr>
</tbody>
</table>

On the accompanying grid, make a scatter plot of these data. Write the exponential regression equation, expressing the regression coefficients to the nearest tenth.

134. The accompanying table shows the average salary of baseball players since 1984. Using the data in the table, create a scatter plot on the grid and state the exponential regression equation with the coefficient and base rounded to the nearest hundredth. Using your written regression equation, estimate the salary of a baseball player in the year 2005, to the nearest thousand dollars.

<table>
<thead>
<tr>
<th>Numbers of Years Since 1984</th>
<th>Average Salary (thousands of dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>290</td>
</tr>
<tr>
<td>1</td>
<td>320</td>
</tr>
<tr>
<td>2</td>
<td>400</td>
</tr>
<tr>
<td>3</td>
<td>495</td>
</tr>
<tr>
<td>4</td>
<td>600</td>
</tr>
<tr>
<td>5</td>
<td>700</td>
</tr>
<tr>
<td>6</td>
<td>820</td>
</tr>
<tr>
<td>7</td>
<td>1,000</td>
</tr>
<tr>
<td>8</td>
<td>1,250</td>
</tr>
<tr>
<td>9</td>
<td>1,580</td>
</tr>
</tbody>
</table>
135. The table below, created in 1996, shows a history of transit fares from 1955 to 1995. On the accompanying grid, construct a scatter plot where the independent variable is years. State the exponential regression equation with the coefficient and base rounded to the nearest thousandth. Using this equation, determine the prediction that should have been made for the year 1998, to the nearest cent.

<table>
<thead>
<tr>
<th>Year</th>
<th>Fare ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>55</td>
<td>0.10</td>
</tr>
<tr>
<td>60</td>
<td>0.15</td>
</tr>
<tr>
<td>65</td>
<td>0.20</td>
</tr>
<tr>
<td>70</td>
<td>0.30</td>
</tr>
<tr>
<td>75</td>
<td>0.40</td>
</tr>
<tr>
<td>80</td>
<td>0.60</td>
</tr>
<tr>
<td>85</td>
<td>0.80</td>
</tr>
<tr>
<td>90</td>
<td>1.15</td>
</tr>
<tr>
<td>95</td>
<td>1.50</td>
</tr>
</tbody>
</table>

136. A box containing 1,000 coins is shaken, and the coins are emptied onto a table. Only the coins that land heads up are returned to the box, and then the process is repeated. The accompanying table shows the number of trials and the number of coins returned to the box after each trial.

<table>
<thead>
<tr>
<th>Trial</th>
<th>0</th>
<th>1</th>
<th>3</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coins Returned</td>
<td>1,000</td>
<td>610</td>
<td>220</td>
<td>132</td>
<td>45</td>
</tr>
</tbody>
</table>

Write an exponential regression equation, rounding the calculated values to the nearest ten-thousandth. Use the equation to predict how many coins would be returned to the box after the eighth trial.

137. Jean invested $380 in stocks. Over the next 5 years, the value of her investment grew, as shown in the accompanying table.

<table>
<thead>
<tr>
<th>Years Since Investment (x)</th>
<th>Value of Stock, in Dollars (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>380</td>
</tr>
<tr>
<td>1</td>
<td>395</td>
</tr>
<tr>
<td>2</td>
<td>411</td>
</tr>
<tr>
<td>3</td>
<td>427</td>
</tr>
<tr>
<td>4</td>
<td>445</td>
</tr>
<tr>
<td>5</td>
<td>462</td>
</tr>
</tbody>
</table>

Write the exponential regression equation for this set of data, rounding all values to two decimal places. Using this equation, find the value of her stock, to the nearest dollar, 10 years after her initial purchase.
CHAPTER 7-2

EXPONENTIAL FUNCTIONS

138. On the accompanying grid, solve the following system of equations graphically:
   \[ \begin{align*}
   y &= -x^2 + 2x + 1 \\
   y &= 2^x 
   \end{align*} \]

139. On the accompanying grid, sketch the graphs of \( y = 2^x \) and \( 3y = 7x + 3 \) over the interval \(-3 \leq x \leq 4\). Identify and state the coordinates of all points of intersection.

140. The graphs of the equations \( y = 2^x \) and \( y = -2x + a \) intersect in Quadrant I for which values of \( a \)?
   [A] \( a \geq 1 \)  \[ B \] \( a > 1 \)
   [C] \( 0 < a < 1 \)  \[ D \] \( a < 1 \)

141. Which equation models the data in the accompanying table?

<table>
<thead>
<tr>
<th>Time in hours, ( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population, ( y )</td>
<td>5</td>
<td>10</td>
<td>20</td>
<td>40</td>
<td>80</td>
<td>160</td>
<td>320</td>
</tr>
</tbody>
</table>

[A] \( y = 2x \)  \[ B \] \( y = 2^x \)
[C] \( y = 2x + 5 \)  \[ D \] \( y = 5(2^x) \)

142. What is the domain of \( f(x) = 2^x \)?
   [A] \( x \geq 0 \)  \[ B \] all real numbers
   [C] \( x \leq 0 \)  \[ D \] all integers

143. The height, \( f(x) \), of a bouncing ball after \( x \) bounces is represented by \( f(x) = 80(0.5)^x \). How many times higher is the first bounce than the fourth bounce?
   [A] 4  \[ B \] 2  \[ C \] 16  \[ D \] 8

144. The accompanying graph represents the value of a bond over time.

Which type of function does this graph best model?
   [A] trigonometric  \[ B \] exponential
   [C] logarithmic  \[ D \] quadratic
145. The strength of a medication over time is represented by the equation \( y = 200(1.5)^{-x} \), where \( x \) represents the number of hours since the medication was taken and \( y \) represents the number of micrograms per millimeter left in the blood. Which graph best represents this relationship?

[A] [B] [C] [D]

146. On January 1, 1999, the price of gasoline was $1.39 per gallon. If the price of gasoline increased by 0.5% per month, what was the cost of one gallon of gasoline, to the nearest cent, on January 1 one year later?

147. A used car was purchased in July 1999 for $11,900. If the car depreciates 13% of its value each year, what is the value of the car, to the nearest hundred dollars, in July 2002?

148. The Franklins inherited $3,500, which they want to invest for their child's future college expenses. If they invest it at 8.25% with interest compounded monthly, determine the value of the account, in dollars, after 5 years.

Use the formula \( A = P\left(1 + \frac{r}{n}\right)^{nt} \), where \( A \) = value of the investment after \( t \) years, \( P \) = principal invested, \( r \) = annual interest rate, and \( n \) = number of times compounded per year.

CHAPTER 7-3

GRAPHING LOGARITHMIC FUNCTIONS

149. A hotel finds that its total annual revenue and the number of rooms occupied daily by guests can best be modeled by the function \( R = 3\log(n^2 + 10n), \ n > 0 \), where \( R \) is the total annual revenue, in millions of dollars, and \( n \) is the number of rooms occupied daily by guests. The hotel needs an annual revenue of $12 million to be profitable. Graph the function on the accompanying grid over the interval \( 0 < n \leq 100 \).

Calculate the minimum number of rooms that must be occupied daily to be profitable.
150. The cells of a particular organism increase logarithmically. If \( g \) represents cell growth and \( h \) represents time, in hours, which graph best represents the growth pattern of the cells of this organism?

[A]  

[B]  

[C]  

[D]  

151. If \( \log_b xy \), then \( x \) equals  

[A] \( \frac{y}{b} \)  

[B] \( y^b \)  

[C] \( y \cdot b \)  

[D] \( y \cdot b \)  

152. The function \( y = 2^x \) is equivalent to  

[A] \( x = \log_2 y \)  

[B] \( y = x \log 2 \)  

[C] \( x = y \log 2 \)  

[D] \( y = \log_2 x \)  

153. For which value of \( x \) is \( y = \log x \) undefined?

[A] 0  

[B] \( \frac{1}{10} \)  

[C] 1.483  

[D] \( \pi \)  

154. The expression \( \log_3 (8 - x) \) is defined for all values of \( x \) such that  

[A] \( x \geq 8 \)  

[B] \( x > 8 \)  

[C] \( x < 8 \)  

[D] \( x \leq 8 \)  

155. If \( \log 5 = a \), then \( \log 250 \) can be expressed as  

[A] \( 2a + 1 \)  

[B] \( 25a \)  

[C] \( 10 + 2a \)  

[D] \( 50a \)  

156. The speed of sound, \( v \), at temperature \( T \), in degrees Kelvin, is represented by the equation  

\( v = 1087 \sqrt{\frac{T}{273}} \). Which expression is equivalent to \( \log v ? \)

[A] \( \log 1087 + \frac{1}{2} \log T - \frac{1}{2} \log 273 \)  

[B] \( \log 1087 + 2 \log (T + 273) \)  

[C] \( 1087 + \frac{1}{2} \log T - \log 273 \)  

[D] \( 1087(\frac{1}{2} \log T - \frac{1}{2} \log 273) \)  

157. Which expression is not equivalent to \( \log_3 36 \)?

[A] \( \log_3 9 + \log_3 4 \)  

[B] \( 2 \log_3 6 \)  

[C] \( 6 \log_3 2 \)  

[D] \( \log_3 72 - \log_3 2 \)  

158. If \( \log a = 2 \) and \( \log b = 3 \), what is the numerical value of \( \log \frac{\sqrt{a}}{b^3} \)?

[A] -8  

[B] 8  

[C] -25  

[D] 25  

159. If \( \log x = a \), \( \log y = b \), and \( \log z = c \), then \( \log \frac{x^2 y}{\sqrt{z}} \) is equivalent to  

[A] \( 2ab - \frac{1}{2} c \)  

[B] \( 2a + b - \frac{1}{2} c \)  

[C] \( a^2 + b - \frac{1}{2} c \)  

[D] \( 42a + b + \frac{1}{2} c \)
160. The expression \( \log 10^{x^2} - \log 10^x \) is equivalent to

[A] \( \frac{1}{100} \)  \quad [B] -2  \quad [C] 100  \quad [D] 2

161. If \( \log a = x \) and \( \log b = y \), what is \( \log \sqrt{ab} \)?

[A] \( \frac{x+y}{2} \)  \quad [B] \( \frac{x+y}{2} \)  \quad [C] 2x + 2y  \quad [D] x + 2y

CHAPTER 7-4

LOGARITHMIC EQUATIONS

162. If \( \log k = c \log v + \log p \), \( k \) equals

[A] \( v^c + p \)  \quad [B] \( cv + p \)  \quad [C] \( vp^c \)  \quad [D] \( v^c p \)

163. Solve for \( x \): \( \log_4 (x^2 + 3x) - \log_4 (x + 5) = 1 \)

164. In the equation \( \log_x 4 + \log_x 9 = 2 \), \( x \) is equal to

[A] 6  \quad [B] 18  \quad [C] \( \sqrt{13} \)  \quad [D] 6.5

165. If \( \log_3 x = 2 \), what is the value of \( \sqrt{x} \)?

[A] \( \sqrt{5} \)  \quad [B] 25  \quad [C] \( \sqrt{5} \)  \quad [D] 5

166. The relationship between the relative size of an earthquake, \( S \), and the measure of the earthquake on the Richter scale, \( R \), is given by the equation \( \log S = R \). If an earthquake measured 3.2 on the Richter scale, what was its relative size to the nearest hundredth?

167. The magnitude \( (R) \) of an earthquake is related to its intensity \( (I) \) by \( R = \log \left( \frac{I}{T} \right) \), where \( T \) is the threshold below which the earthquake is not noticed. If the intensity is doubled, its magnitude can be represented by

[A] \( 2 \log I - \log T \)  \quad [B] \( \log I - \log T \)
[C] \( 2 \log I - \log T \)  \quad [D] \( \log 2 + \log I - \log T \)

168. The scientists in a laboratory company raise amebas to sell to schools for use in biology classes. They know that one ameba divides into two amebas every hour and that the formula \( t = \log_2 N \) can be used to determine how long it takes to produce a certain number of amebas, \( N \). Determine, to the nearest tenth of an hour, how long it takes to produce 10,000 amebas if they start with one ameba.

169. Solve for \( x \): \( \log_2 (x + 1) = 3 \)

170. Solve for \( x \): \( \log_b 36 - \log_b 2 = \log_b x \)

CHAPTER 7-5

EXPONENTIAL EQUATIONS

171. What is the value of \( x \) in the equation \( 81^{x+2} = 27^{3x+4} \)?

[A] \( -\frac{4}{11} \)  \quad [B] \( \frac{4}{11} \)  \quad [C] \( -\frac{3}{2} \)  \quad [D] \( -\frac{2}{11} \)

172. The solution set of \( 2^{x^2 + 2x} = 2^{-1} \) is

[A] \{1\}  \quad [B] \{-1, 1\}  \quad [C] \{\}  \quad [D] \{-1\}

173. The growth of bacteria in a dish is modeled by the function \( f(t) = 2^t \). For which value of \( t \) is \( f(t) = 32 \)?

[A] 15  \quad [B] 2  \quad [C] 16  \quad [D] 8
174. Solve algebraically for $x$: $27^{2x+1} = 9^x$

175. Determine the value of $x$ and $y$ if $2^y = 8^x$ and $3^y = 3^{x+4}$.

[A] $x = y$  [B] $x = -2, y = -6$
[C] $x = 6, y = 2$  [D] $x = 2, y = 6$

176. Solve for $m$: $3^{m+1} - 5 = 22$

177. Solve algebraically for $x$: $8^{2x} = 4^6$

178. Depreciation (the decline in cash value) on a car can be determined by the formula $V = C(1 - r)^t$, where $V$ is the value of the car after $t$ years, $C$ is the original cost, and $r$ is the rate of depreciation. If a car's cost, when new, is $15,000, the rate of depreciation is 30%, and the value of the car now is $3,000, how old is the car to the nearest tenth of a year?

179. The amount $A$, in milligrams, of a 10-milligram dose of a drug remaining in the body after $t$ hours is given by the formula $A = 10(0.8)^t$. Find, to the nearest tenth of an hour, how long it takes for half of the drug dose to be left in the body.

180. Growth of a certain strain of bacteria is modeled by the equation $G = A(2.7)^{0.584t}$, where:

$G =$ final number of bacteria
$A =$ initial number of bacteria
$t =$ time (in hours)

In approximately how many hours will 4 bacteria first increase to 2,500 bacteria? Round your answer to the nearest hour.

181. The equation for radioactive decay is $p = 0.5^{\frac{t}{H}}$, where $p$ is the part of a substance with half-life $H$ remaining radioactive after a period of time, $t$.

A given substance has a half-life of 6,000 years. After $t$ years, one-fifth of the original sample remains radioactive. Find $t$, to the nearest thousand years.

182. An archaeologist can determine the approximate age of certain ancient specimens by measuring the amount of carbon-14, a radioactive substance, contained in the specimen. The formula used to determine the age of a specimen is $A = A_02^{\frac{-t}{5760}}$, where $A$ is the amount of carbon-14 that a specimen contains, $A_0$ is the original amount of carbon-14, $t$ is time, in years, and 5760 is the half-life of carbon-14.

A specimen that originally contained 120 milligrams of carbon-14 now contains 100 milligrams of this substance. What is the age of the specimen, to the nearest hundred years?
183. An amount of $P$ dollars is deposited in an account paying an annual interest rate $r$ (as a decimal) compounded $n$ times per year. After $t$ years, the amount of money in the account, in dollars, is given by the equation

\[ A = P \left(1 + \frac{r}{n}\right)^{nt}. \]

Rachel deposited $1,000 at 2.8% annual interest, compounded monthly. In how many years, to the nearest tenth of a year, will she have $2,500 in the account? [The use of the grid is optional.]

184. The current population of Little Pond, New York, is 20,000. The population is decreasing, as represented by the formula

\[ P = A(1.3)^{-0.23t}, \]

where $P =$ final population, $t =$ time, in years, and $A =$ initial population. What will the population be 3 years from now? Round your answer to the nearest hundred people.

To the nearest tenth of a year, how many years will it take for the population to reach half the present population? [The use of the grid is optional.]
185. After an oven is turned on, its temperature, $T$, is represented by the equation $T = 400 - 350(3.2)^{-0.1m}$ where $m$ represents the number of minutes after the oven is turned on and $T$ represents the temperature of the oven, in degrees Fahrenheit. How many minutes does it take for the oven's temperature to reach 300°F? Round your answer to the nearest minute. [The use of the grid is optional.]

186. Sean invests $10,000 at an annual rate of 5% compounded continuously, according to the formula $A = Pe^{rt}$, where $A$ is the amount, $P$ is the principal, $e = 2.718$, $r$ is the rate of interest, and $t$ is time, in years. Determine, to the nearest dollar, the amount of money he will have after 2 years. Determine how many years, to the nearest year, it will take for his initial investment to double.

187. Explain how a person can determine if a set of data represents inverse variation and give an example using a table of values.

188. If $R$ varies inversely as $S$, when $S$ is doubled, $R$ is multiplied by

- [A] 2
- [B] $\frac{1}{2}$
- [C] $\frac{1}{4}$
- [D] 4

189. The price per person to rent a limousine for a prom varies inversely as the number of passengers. If five people rent the limousine, the cost is $70 each. How many people are renting the limousine when the cost per couple is $87.50?

190. To balance a seesaw, the distance, in feet, a person is from the fulcrum is inversely proportional to the person's weight, in pounds. Bill, who weighs 150 pounds, is sitting 4 feet away from the fulcrum. If Dan weighs 120 pounds, how far from the fulcrum should he sit to balance the seesaw?

- [A] 3 ft
- [B] 3.5 ft
- [C] 5 ft
- [D] 4.5 ft

191. For a rectangular garden with a fixed area, the length of the garden varies inversely with the width. Which equation represents this situation for an area of 36 square units?

- [A] $xy = 36$
- [B] $y = \frac{36}{x}$
- [C] $x + y = 36$
- [D] $y = 36x$

192. When air is pumped into an automobile tire, the pressure is inversely proportional to the volume. If the pressure is 35 pounds when the volume is 120 cubic inches, what is the pressure, in pounds, when the volume is 140 cubic inches?

193. A pulley that has a diameter of 8 inches is belted to a pulley that has a diameter of 12 inches. The 8-inch-diameter pulley is running at 1,548 revolutions per minute. If the speeds of the pulleys vary inversely to their diameters, how many revolutions per minute does the larger pulley make?
194. Boyle's Law states that the pressure of compressed gas is inversely proportional to its volume. The pressure of a certain sample of a gas is 16 kilopascals when its volume is 1,800 liters. What is the pressure, in kilopascals, when its volume is 900 liters?

195. The time it takes to travel to a location varies inversely to the speed traveled. It takes 4 hours driving at an average speed of 55 miles per hour to reach a location. To the nearest tenth of an hour, how long will it take to reach the same location driving at an average speed of 50 miles per hour?

196. The speed of a laundry truck varies inversely with the time it takes to reach its destination. If the truck takes 3 hours to reach its destination traveling at a constant speed of 50 miles per hour, how long will it take to reach the same location driving at a constant speed of 60 miles per hour?

197. In a given rectangle, the length varies inversely as the width. If the length is doubled, the width will

[A] remain the same
[B] be multiplied by 2
[C] increase by 2
[D] be divided by 2

198. Camisha is paying a band $330 to play at her graduation party. The amount each member earns, \( d \), varies inversely as the number of members who play, \( n \). The graph of the equation that represents the relationship between \( d \) and \( n \) is an example of

[A] a hyperbola
[B] a parabola
[C] a line
[D] an ellipse

199. According to Boyle's Law, the pressure, \( p \), of a compressed gas is inversely proportional to the volume, \( v \). If a pressure of 20 pounds per square inch exists when the volume of the gas is 500 cubic inches, what is the pressure when the gas is compressed to 400 cubic inches?

[A] 25 lb / in²  
[B] 50 lb / in²  
[C] 40 lb / in²  
[D] 16 lb / in²

CHAPTER 8-2

RATIONAL FUNCTIONS

200. The accompanying graph shows the relationship between a person's weight and the distance that the person must sit from the center of a seesaw to make it balanced.

Which equation best represents this graph?

[A] \( y = -120x \)  
[B] \( y = \frac{120}{x} \)  
[C] \( y = 2 \log x \)  
[D] \( y = 12x^2 \)

201. Which function is symmetrical with respect to the origin?

[A] \( y = 5^x \)  
[B] \( y = -\frac{5}{x} \)  
[C] \( y = \sqrt{x + 5} \)  
[D] \( y = |5 - x| \)
202. Which equation represents a hyperbola?
   [A] $y = 16x^2$            [B] $y = 16 - x^2$
   [C] $y^2 = 16 - x^2$        [D] $y = \frac{16}{x}$

203. Which graph represents an inverse variation between stream velocity and the distance from the center of the stream?
   [A] ![Graph A]            [B] ![Graph B]
   [C] ![Graph C]            [D] ![Graph D]

204. Which graph shows that soil permeability varies inversely to runoff?
   [A] ![Graph A]            [B] ![Graph B]
   [C] ![Graph C]            [D] ![Graph D]

**MATH TOOLBOX P. 375**

**RATIONALIZING DENOMINATORS**

205. Which expression is equivalent to $\frac{4}{3 + \sqrt{2}}$?
   [A] $\frac{12 - 4\sqrt{2}}{11}$            [B] $\frac{12 - 4\sqrt{2}}{7}$
   [C] $\frac{12 + 4\sqrt{2}}{11}$          [D] $\frac{12 + 4\sqrt{2}}{7}$

206. Which expression is equal to $\frac{2 + \sqrt{3}}{2 - \sqrt{3}}$?
   [A] $1 - 4\sqrt{3}$            [B] $7 + 4\sqrt{3}$
   [C] $\frac{1 - 4\sqrt{3}}{7}$        [D] $\frac{7 + 4\sqrt{3}}{7}$

207. The expression $\frac{7}{2 - \sqrt{3}}$ is equivalent to
   [A] $\frac{14 + \sqrt{3}}{7}$            [B] $14 + 7\sqrt{3}$
   [C] $\frac{2 + \sqrt{3}}{7}$          [D] $14 - 7\sqrt{3}$

208. The expression $\frac{11}{\sqrt{3} - 5}$ is equivalent to
   [A] $\frac{\sqrt{3} + 5}{2}$            [B] $\frac{-\sqrt{3} + 5}{2}$
   [C] $\frac{-\sqrt{3} - 5}{2}$        [D] $\frac{\sqrt{3} - 5}{2}$

209. The expression $\frac{7}{3 - \sqrt{2}}$ is equivalent to
   [A] $3 + \sqrt{2}$            [B] $\frac{3 + \sqrt{2}}{7}$
   [C] $\frac{21 + \sqrt{2}}{7}$        [D] $3 - \sqrt{2}$

210. The expression $\frac{1}{5 - \sqrt{13}}$ is equivalent to
   [A] $\frac{5 + \sqrt{13}}{-8}$            [B] $\frac{5 + \sqrt{13}}{8}$
   [C] $\frac{5 + \sqrt{13}}{12}$        [D] $\frac{5 + \sqrt{13}}{-12}$
211. Which expression represents the sum of \( \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{2}} \)?

[A] \( \frac{\sqrt{3} + \sqrt{2}}{3} \)  
[B] \( \frac{\sqrt{3} + \sqrt{2}}{2} \)  
[C] \( \frac{2}{\sqrt{5}} \)  
[D] \( \frac{2\sqrt{3} + 3\sqrt{2}}{6} \)

212. The expression \( \frac{5}{\sqrt{5} - 1} \) is equivalent to

[A] \( \frac{5\sqrt{5} - 5}{4} \)  
[B] \( \frac{5}{4} \)  
[C] \( \frac{5\sqrt{5} + 5}{4} \)  
[D] \( \frac{5\sqrt{5} - 5}{6} \)

213. The expression \( \frac{12}{3 + \sqrt{3}} \) is equivalent to

[A] \( 12 - \sqrt{3} \)  
[B] \( 2 + \sqrt{3} \)  
[C] \( 4 - 2\sqrt{3} \)  
[D] \( 6 - 2\sqrt{3} \)

CHAPTER 8-4
RATIONAL EXPRESSIONS

214. Written in simplest form, the expression \( \frac{x^2y^2 - 9}{3 - xy} \) is equivalent to

[A] \( \frac{1}{3 + xy} \)  
[B] \(-1\)  
[C] \(- (3 + xy)\)  
[D] \(3 + xy\)

215. Express the following rational expression in simplest form:
\[
\frac{9 - x^2}{10x^2 - 28x - 6}
\]

216. For all values of \( x \) for which the expression is defined, \( \frac{2x + x^2}{x^2 + 5x + 6} \) is equivalent to

[A] \( \frac{1}{x + 2} \)  
[B] \( \frac{x}{x + 3} \)  
[C] \( \frac{x}{x + 2} \)  
[D] \( \frac{1}{x + 3} \)

217. Written in simplest form, the expression \( \frac{x^2 - 9x}{45x - 5x^2} \) is equivalent to

[A] \(-5\)  
[B] \(-\frac{1}{5}\)  
[C] \(\frac{1}{5}\)  
[D] \(5\)

218. The expression \( \frac{3y^2 - 12y}{4y^2 - y^3} \) is equivalent to

[A] \(\frac{3}{4} - \frac{12}{y^2}\)  
[B] \(\frac{3}{y}\)  
[C] \(-\frac{3}{y}\)  
[D] \(-\frac{9}{4}\)

COMPLEX FRACTIONS

219. The expression \( \frac{a - b}{a + b} \) is equivalent to

[A] \(a + b\)  
[B] \(a - b\)  
[C] \(ab\)  
[D] \(\frac{a - b}{ab}\)

220. The fraction \( \frac{\frac{x}{y} + x}{1 + \frac{1}{y}} \) is equivalent to

[A] \(2x\)  
[B] \(\frac{x^2y}{1 + y}\)  
[C] \(\frac{2xy}{1 + y}\)  
[D] \(x\)
221. In simplest form, \( \frac{x^2 - 1}{x^2 + 1} \) is equal to

[A] \( \frac{x-y}{xy} \)  [B] \( \frac{y-x}{xy} \)  
[C] \( y-x \)  [D] \( x-y \)

222. Express in simplest form:

\[ \frac{x}{1} - \frac{4}{x} \]

223. The expression \( \frac{\frac{1}{x} + \frac{1}{y}}{\frac{1}{x^2} - \frac{1}{y^2}} \) is equivalent to

[A] \( y-x \)  [B] \( \frac{xy}{x-y} \)  
[C] \( \frac{y-x}{xy} \)  [D] \( \frac{xy}{y-x} \)

224. Which expression is equivalent to the complex fraction \( \frac{x}{\frac{x+2}{1}} \)?

[A] \( \frac{x}{2} \)  [B] \( \frac{2x}{x+2} \)  [C] \( \frac{2x}{x^2+4} \)  [D] \( \frac{2}{x} \)

225. When simplified, the complex fraction \( \frac{1}{1 + \frac{1}{\frac{x}{1-x}}} \), \( x \neq 0 \), is equivalent to

[A] \( \frac{1}{1-x} \)  [B] -1  [C] \( \frac{1}{x-1} \)  [D] 1

226. Express in simplest form:

\[ \frac{r-s}{r^2 - s^2} \]

227. Simplify completely:

\[ \frac{1-m}{m-1} \]

228. Simplify for all values of \( a \) for which the expression is defined:

\[ \frac{1-\frac{2}{a}}{\frac{4}{a^2} - 1} \]

229. In a science experiment, when resistor \( A \) and resistor \( B \) are connected in a parallel circuit, the total resistance is \( \frac{1}{\frac{1}{A} + \frac{1}{B}} \). This complex fraction is equivalent to

[A] 1  [B] \( \frac{AB}{A+B} \)  [C] \( AB \)  [D] \( A+B \)

MULTIPLICATION AND DIVISION OF RATIONALS

230. A rectangular prism has a length of \( \frac{2x^2 + 2x - 24}{4x^2 + x} \), a width of \( \frac{x^2 + x - 6}{x + 4} \), and a height of \( \frac{8x^2 + 2x}{x^2 - 9} \). For all values of \( x \) for which it is defined, express, in terms of \( x \), the volume of the prism in simplest form.
231. If the length of a rectangular garden is represented by \( \frac{x^2 + 2x}{x^2 + 2x - 15} \) and its width is represented by \( \frac{2x - 6}{2x + 4} \), which expression represents the area of the garden?

[A] \( \frac{x}{x + 5} \)  [B] \( x \)
[C] \( x + 5 \)  [D] \( \frac{x^2 + 2x}{2(x + 5)} \)

232. Express in simplest form:
\[
\frac{4x + 8}{x + 1} \cdot \frac{2 - x}{3x - 15} \div \frac{x^2 - 4}{2x^2 - 8x - 10}
\]

CHAPTER 8-5

ADDITION AND SUBTRACTION OF RATIONALS

233. What is the sum of \( \frac{3}{x - 3} \) and \( \frac{x}{3 - x} \)?

[A] \( \frac{x + 3}{x - 3} \)  [B] 0  [C] -1  [D] 1

234. What is the sum of \( (y - 5) + \frac{3}{y + 2} \)?

[A] \( \frac{y^2 - 7}{y + 2} \)  [B] \( \frac{y^2 - 3y - 7}{y + 2} \)
[C] \( y - 5 \)  [D] \( \frac{y - 2}{y + 2} \)

235. Express in simplest form:
\[
\frac{1}{x} + \frac{1}{x + 3}
\]

CHAPTER 8-6

SOLVING RATIONALS

236. A rectangle is said to have a golden ratio when \( \frac{w}{h} = \frac{h}{w-h} \), where \( w \) represents width and \( h \) represents height. When \( w = 3 \), between which two consecutive integers will \( h \) lie?

[A] \( 4, -6 \)  [B] \{}  [C] \{4\}  [D] \{-6\}

237. What is the solution set of the equation
\[
\frac{x}{x - 4} - \frac{1}{x + 3} = \frac{28}{x^2 - x - 12}?
\]


238. Solve for \( x \) and express your answer in simplest radical form:
\[
\frac{4}{x} - \frac{3}{x + 1} = 7
\]

239. Solve for all values of \( x \): \( \frac{9}{x} + \frac{9}{x-2} = 12 \)

240. Working by herself, Mary requires 16 minutes more than Antoine to solve a mathematics problem. Working together, Mary and Antoine can solve the problem in 6 minutes. If this situation is represented by the equation \( \frac{6}{t} + \frac{6}{t + 16} = 1 \), where \( t \) represents the number of minutes Antoine works alone to solve the problem, how many minutes will it take Antoine to solve the problem if he works by himself?
241. Electrical circuits can be connected in series, one after another, or in parallel circuits that branch off a main line. If circuits are hooked up in parallel, the reciprocal of the total resistance in the series is found by adding the reciprocals of each resistance, as shown in the accompanying diagram.

\[ \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R_T} \]

If \( R_1 = x \), \( R_2 = x + 3 \), and the total resistance, \( R_T \), is 2.25 ohms, find the positive value of \( R_1 \) to the nearest tenth of an ohm.

246. Which angle is coterminal with an angle of \( 125^\circ \)?


247. In the accompanying diagram, point \( P(0.6, -0.8) \) is on unit circle \( O \). What is the value of \( \theta \), to the nearest degree?

248. If \( \theta \) is an obtuse angle and \( \sin \theta = b \), then it can be concluded that

[A] \( \sin 2\theta < b \)  [B] \( \cos 2\theta > b \)  
[C] \( \tan \theta > b \)  [D] \( \cos \theta > b \)

249. Is \( \frac{1}{2} \sin 2x \) the same expression as \( \sin x \)? Justify your answer.

250. If \( \sin \theta \) is negative and \( \cos \theta \) is negative, in which quadrant does the terminal side of \( \theta \) lie?


251. Expressed as a function of a positive acute angle, \( \sin (-230^\circ) \) is equal to

[A] \( -\cos 50^\circ \)  [B] \( \sin 50^\circ \)  
[C] \( -\sin 50^\circ \)  [D] \( \cos 50^\circ \)

252. Two straight roads intersect at an angle whose measure is \( 125^\circ \). Which expression is equivalent to the cosine of this angle?

[A] \( -\cos 55^\circ \)  [B] \( -\cos 35^\circ \)  
[C] \( \cos 35^\circ \)  [D] \( \cos 55^\circ \)
253. In the accompanying diagram, $PR$ is tangent to circle $O$ at $R$, $QS \perp OR$, and $PR \perp OR$.

Which measure represents $\sin \theta$?

254. In the accompanying diagram of a unit circle, the ordered pair \((-\frac{\sqrt{3}}{2}, -\frac{1}{2})\) represents the point where the terminal side of $\theta$ intersects the unit circle.

What is $m \angle \theta$?

255. If $\theta$ is an angle in standard position and $P(-3,4)$ is a point on the terminal side of $\theta$, what is the value of $\sin \theta$?
[A] $-\frac{3}{5}$  [B] $-\frac{4}{5}$  [C] $\frac{4}{5}$  [D] $\frac{3}{5}$

256. If $\tan \theta = 2.7$ and $\csc \theta < 0$, in which quadrant does $\theta$ lie?

257. If $x$ is a positive acute angle and $\cos x = \frac{\sqrt{3}}{4}$, what is the exact value of $\sin x$?
[A] $\frac{4}{5}$  [B] $\frac{3}{5}$  [C] $\frac{\sqrt{3}}{5}$  [D] $\frac{\sqrt{13}}{4}$

258. The accompanying diagram shows unit circle $O$, with radius $OB = 1$.

Which line segment has a length equivalent to $\cos \theta$?

CHAPTER 9-3
RADIAN MEASURE

259. What is the number of degrees in an angle whose radian measure is $\frac{7\pi}{12}$?
260. A dog has a 20-foot leash attached to the corner where a garage and a fence meet, as shown in the accompanying diagram. When the dog pulls the leash tight and walks from the fence to the garage, the arc the leash makes is 55.8 feet.

What is the measure of angle $\theta$ between the garage and the fence, in radians?

[A] 160  [B] 2.79  [C] 0.36  [D] 3.14

261. Through how many radians does the minute hand of a clock turn in 24 minutes?

[A] $0.6\pi$  [B] $0.4\pi$  [C] $0.8\pi$  [D] $0.2\pi$

262. A wedge-shaped piece is cut from a circular pizza. The radius of the pizza is 6 inches. The rounded edge of the crust of the piece measures 4.2 inches. To the nearest tenth, the angle of the pointed end of the piece of pizza, in radians, is

[A] 1.4  [B] 25.2  [C] 0.7  [D] 7.0

263. An art student wants to make a string collage by connecting six equally spaced points on the circumference of a circle to its center with string. What would be the radian measure of the angle between two adjacent pieces of string, in simplest form?

264. An arc of a circle that is 6 centimeters in length intercepts a central angle of 1.5 radians. Find the number of centimeters in the radius of the circle.

265. What is the radian measure of the angle formed by the hands of a clock at 2:00 p.m.?

[A] $\frac{\pi}{2}$  [B] $\frac{\pi}{3}$  [C] $\frac{\pi}{4}$  [D] $\frac{\pi}{6}$

266. Kristine is riding in car 4 of the Ferris wheel represented in the accompanying diagram. The Ferris wheel is rotating in the direction indicated by the arrows. The eight cars are equally spaced around the circular wheel. Express, in radians, the measure of the smallest angle through which she will travel to reach the bottom of the Ferris wheel.

267. The pendulum of a clock swings through an angle of 2.5 radians as its tip travels through an arc of 50 centimeters. Find the length of the pendulum, in centimeters.

CHAPTER 9-4

TRIGONOMETRIC GRAPHS

268. A modulated laser heats a diamond. Its variable temperature, in degrees Celsius, is given by $f(t) = T \sin at$. What is the period of the curve?

[A] $\frac{1}{a}$  [B] $\frac{2\pi}{a}$  [C] $\frac{2a\pi}{a}$  [D] $|T|$
269. What is the period of the function \( y = 5\sin 3x \)?

[A] 3  [B] \( \frac{2\pi}{3} \)  [C] 5  [D] \( \frac{2\pi}{5} \)

270. The brightness of the star MIRA over time is given by the equation \( y = 2\sin \frac{\pi}{4}x + 6 \), where \( x \) represents time and \( y \) represents brightness. What is the period of this function, in radian measure?

271. A monitor displays the graph \( y = 3\sin 5x \). What will be the amplitude after a dilation of 2?


272. What is the amplitude of the function \( y = \frac{2}{3}\sin 4x \)?

[A] 3\( \pi \)  [B] 4  [C] \( \frac{\pi}{2} \)  [D] \( \frac{2}{3} \)

273. The path traveled by a roller coaster is modeled by the equation \( y = 27\sin 13x + 30 \). What is the maximum altitude of the roller coaster?


274. A certain radio wave travels in a path represented by the equation \( y = 5\sin 2x \). What is the period of this wave?

[A] 2  [B] \( 2\pi \)  [C] 5  [D] \( \pi \)

275. A sound wave is modeled by the curve \( y = 3\sin 4x \). What is the period of this curve?

[A] \( \pi \)  [B] 3  [C] \( \frac{\pi}{2} \)  [D] 4

276. A radio transmitter sends a radio wave from the top of a 50-foot tower. The wave is represented by the accompanying graph.

What is the equation of this radio wave?

[A] \( y = 1.5\sin x \)  [B] \( y = \sin x \)
[C] \( y = \sin 1.5x \)  [D] \( y = 2\sin x \)

277. A student attaches one end of a rope to a wall at a fixed point 3 feet above the ground, as shown in the accompanying diagram, and moves the other end of the rope up and down, producing a wave described by the equation \( y = a\sin bx + c \). The range of the rope's height above the ground is between 1 and 5 feet. The period of the wave is \( 4\pi \). Write the equation that represents this wave.

278. What is the period of the graph of the equation \( y = 2\sin \frac{1}{3}x \)?

[A] \( 6\pi \)  [B] \( \frac{3\pi}{2} \)  [C] \( 2\pi \)  [D] \( \frac{2}{3}\pi \)
279. In physics class, Eva noticed the pattern shown in the accompanying diagram on an oscilloscope.

Which equation best represents the pattern shown on this oscilloscope?

[A] \( y = 2 \sin x + 1 \)    [B] \( y = \sin x + 1 \)

[C] \( y = \sin\left(\frac{1}{2}x\right) + 1 \)

[D] \( y = 2 \sin\left(-\frac{1}{2}x\right) + 1 \)

CHAPTER 9-5

280. The shaded portion of the accompanying map indicates areas of night, and the unshaded portion indicates areas of daylight at a particular moment in time.

Which type of function best represents the curve that divides the area of night from the area of daylight?

[A] logarithmic    [B] quadratic

[C] cosine    [D] tangent

281. The graphs below show the average annual precipitation received at different latitudes on Earth. Which graph is a translated cosine curve?

282. A building's temperature, \( T \), varies with time of day, \( t \), during the course of 1 day, as follows:

\[ T = 8\cos t + 78 \]

The air-conditioning operates when \( T \geq 80^\circ F \). Graph this function for \( 6 \leq t < 17 \) and determine, to the nearest tenth of an hour, the amount of time in 1 day that the air-conditioning is on in the building.
283. A pair of figure skaters graphed part of their routine on a grid. The male skater's path is represented by the equation \( m(x) = 3 \sin \frac{1}{2}x \), and the female skater's path is represented by the equation \( f(x) = -2 \cos x \). On the accompanying grid, sketch both paths and state how many times the paths of the skaters intersect between \( x = 0 \) and \( x = 4\pi \).

284. Which type of symmetry does the equation \( y = \cos x \) have?

[A] line symmetry with respect to the \( x \)-axis

[B] point symmetry with respect to the origin

[C] point symmetry with respect to \( \left( \frac{\pi}{2}, 0 \right) \)

[D] line symmetry with respect to \( y = x \)

285. An object that weighs 2 pounds is suspended in a liquid. When the object is depressed 3 feet from its equilibrium point, it will oscillate according to the formula \( x = 3 \cos(8t) \), where \( t \) is the number of seconds after the object is released. How many seconds are in the period of oscillation?

[A] \( \pi \)  \( [B] 2\pi \)  \( [C] 3 \)  \( [D] \frac{\pi}{4} \)

286. The tide at a boat dock can be modeled by the equation \( y = -2 \cos(\frac{\pi}{6} t) + 8 \), where \( t \) is the number of hours past noon and \( y \) is the height of the tide, in feet. For how many hours between \( t = 0 \) and \( t = 12 \) is the tide at least 7 feet? [The use of the grid is optional.]

287. On the accompanying set of axes, graph the equations \( y = 4 \cos x \) and \( y = 2 \) in the domain \(-\pi \leq x \leq \pi\). Express, in terms of \( \pi \), the interval for which \( 4 \cos x \geq 2 \).
288. The times of average monthly sunrise, as shown in the accompanying diagram, over the course of a 12-month interval can be modeled by the equation \( y = A \cos(Bx) + D \). Determine the values of \( A, B, \) and \( D \), and explain how you arrived at your values.

289. The accompanying diagram shows a section of a sound wave as displayed on an oscilloscope.

Which equation could represent this graph?

[A] \( y = 2 \cos \frac{x}{2} \)  
[B] \( y = \frac{1}{2} \sin \frac{\pi}{2} x \)  
[C] \( y = 2 \sin \frac{x}{2} \)  
[D] \( y = \frac{1}{2} \cos 2x \)

CHAPTER 9-7

LAW OF COSINES

290. A ship at sea is 70 miles from one radio transmitter and 130 miles from another. The angle between the signals sent to the ship by the transmitters is 117.4°. Find the distance between the two transmitters, to the nearest mile.

291. A wooden frame is to be constructed in the form of an isosceles trapezoid, with diagonals acting as braces to strengthen the frame. The sides of the frame each measure 5.30 feet, and the longer base measures 12.70 feet. If the angles between the sides and the longer base each measure 68.4°, find the length of one brace to the nearest tenth of a foot.

292. Two straight roads, Elm Street and Pine Street, intersect creating a 40° angle, as shown in the accompanying diagram. John's house \((J)\) is on Elm Street and is 3.2 miles from the point of intersection. Mary's house \((M)\) is on Pine Street and is 5.6 miles from the intersection. Find, to the nearest tenth of a mile, the direct distance between the two houses.

293. A surveyor is mapping a triangular plot of land. He measures two of the sides and the angle formed by these two sides and finds that the lengths are 400 yards and 200 yards and the included angle is 50°. What is the measure of the third side of the plot of land, to the nearest yard? What is the area of this plot of land, to the nearest square yard?
294. Kieran is traveling from city $A$ to city $B$. As the accompanying map indicates, Kieran could drive directly from $A$ to $B$ along County Route 21 at an average speed of 55 miles per hour or travel on the interstates, 45 miles along I-85 and 20 miles along I-64. The two interstates intersect at an angle of 150° at $C$ and have a speed limit of 65 miles per hour. How much time will Kieran save by traveling along the interstates at an average speed of 65 miles per hour?

![Diagram of County Route 21 and interstates]

295. To measure the distance through a mountain for a proposed tunnel, surveyors chose points $A$ and $B$ at each end of the proposed tunnel and a point $C$ near the mountain. They determined that $AC = 3,800$ meters, $BC = 2,900$ meters, and $\angle ACB = 110°$. Draw a diagram to illustrate this situation and find the length of the tunnel, to the nearest meter.

296. A farmer has determined that a crop of strawberries yields a yearly profit of $1.50 per square yard. If strawberries are planted on a triangular piece of land whose sides are 50 yards, 75 yards, and 100 yards, how much profit, to the nearest hundred dollars, would the farmer expect to make from this piece of land during the next harvest?

297. The Vietnam Veterans Memorial in Washington, D.C., is made up of two walls, each 246.75 feet long, that meet at an angle of 125.2°. Find, to the nearest foot, the distance between the ends of the walls that do not meet.

298. A triangular plot of land has sides that measure 5 meters, 7 meters, and 10 meters. What is the area of this plot of land, to the nearest tenth of a square meter?

299. In the accompanying diagram of $\triangle ABC$, $\angle A = 65°$, $\angle B = 70°$, and the side opposite vertex $B$ is 7. Find the length of the side opposite vertex $A$, and find the area of $\triangle ABC$.

![Diagram of triangle ABC with angles and side lengths]

300. Carmen and Jamal are standing 5,280 feet apart on a straight, horizontal road. They observe a hot-air balloon between them directly above the road. The angle of elevation from Carmen is 60° and from Jamal is 75°. Draw a diagram to illustrate this situation and find the height of the balloon to the nearest foot.

301. A ship captain at sea uses a sextant to sight an angle of elevation of 37° to the top of a lighthouse. After the ship travels 250 feet directly toward the lighthouse, another sighting is made, and the new angle of elevation is 50°. The ship's charts show that there are dangerous rocks 100 feet from the base of the lighthouse. Find, to the nearest foot, how close to the rocks the ship is at the time of the second sighting.

302. While sailing a boat offshore, Donna sees a lighthouse and calculates that the angle of elevation to the top of the lighthouse is 3°, as shown in the accompanying diagram. When she sails her boat 700 feet closer to the lighthouse, she finds that the angle of elevation is now 5°. How tall, to the nearest tenth of a foot, is the lighthouse?

![Diagram of lighthouse and boats with angles of elevation]
303. A sign 46 feet high is placed on top of an office building. From a point on the sidewalk level with the base of the building, the angle of elevation to the top of the sign and the angle of elevation to the bottom of the sign are 40° and 32°, respectively. Sketch a diagram to represent the building, the sign, and the two angles, and find the height of the building to the nearest foot.

304. A ski lift begins at ground level 0.75 mile from the base of a mountain whose face has a 50° angle of elevation, as shown in the accompanying diagram. The ski lift ascends in a straight line at an angle of 20°. Find the length of the ski lift from the beginning of the ski lift to the top of the mountain, to the nearest hundredth of a mile.

305. A ship at sea heads directly toward a cliff on the shoreline. The accompanying diagram shows the top of the cliff, D, sighted from two locations, A and B, separated by distance S. If $m\angle DAC = 30, \ m\angle DBC = 45,$ and $S = 30$ feet, what is the height of the cliff, to the nearest foot?

306. In $\Delta ABC, \ m\angle A = 33, \ a = 12,$ and $b = 15.$ What is $m\angle B$ to the nearest degree?


307. In the accompanying diagram of $\Delta ABC,$ $m\angle A = 30, \ m\angle A = 30,$ and $AC = 13.$

What is the length of side $AB$ to the nearest tenth?


308. In $\Delta ABC, \ a = 19, \ c = 10,$ and $m\angle A = 111.$ Which statement can be used to find the value of $\angle C$?

[A] $\sin C = \frac{10}{19}$ [B] $\sin C = \frac{10 \sin 21^\circ}{19}$
[C] $\sin C = \frac{10 \sin 69^\circ}{19}$
[D] $\sin C = \frac{19 \sin 69^\circ}{10}$

309. As shown in the accompanying diagram, two tracking stations, A and B, are on an east-west line 110 miles apart. A forest fire is located at $F,$ on a bearing 42° northeast of station $A$ and 15° northeast of station $B.$ How far, to the nearest mile, is the fire from station $A$?
310. The accompanying diagram shows the plans for a cell-phone tower that is to be built near a busy highway. Find the height of the tower, to the nearest foot.

![Diagram of a cell-phone tower with angles 32° and 85° and a height of 100 ft.]

311. In \( \triangle ABC \), \( m\angle A = 53 \), \( m\angle B = 14 \), and \( a = 10 \). Find \( b \) to the nearest integer.

MATH TOOLBOX P. 449

TRIANGLE INEQUALITIES

312. An architect commissions a contractor to produce a triangular window. The architect describes the window as \( \triangle ABC \), where \( m\angle A = 50 \), \( BC = 10 \) inches, and \( AB = 12 \) inches. How many distinct triangles can the contractor construct using these dimensions?

[A] 2  [B] 0  [C] 1  [D] more than 2

313. Sam is designing a triangular piece for a metal sculpture. He tells Martha that two of the sides of the piece are 40 inches and 15 inches, and the angle opposite the 40-inch side measures 120°. Martha decides to sketch the piece that Sam described. How many different triangles can she sketch that match Sam's description?

[A] 0  [B] 2  [C] 1  [D] 3

314. How many distinct triangles can be formed if \( m\angle A = 30 \), side \( b = 12 \), and side \( a = 8 \)?

[A] 1  [B] 3  [C] 0  [D] 2

315. A landscape designer is designing a triangular garden with two sides that are 4 feet and 6 feet, respectively. The angle opposite the 4-foot side is 30°. How many distinct triangular gardens can the designer make using these measurements?

316. Main Street and Central Avenue intersect, making an angle measuring 34°. Angela lives at the intersection of the two roads, and Caitlin lives on Central Avenue 10 miles from the intersection. If Leticia lives 7 miles from Caitlin, which conclusion is valid?

[A] Leticia can live at only one location on Main Street.
[B] Leticia cannot live on Main Street.
[C] Leticia can live at one of two locations on Main Street.
[D] Leticia can live at one of three locations on Main Street.

317. In \( \triangle ABC \), if \( AC = 12 \), \( BC = 11 \), and \( m\angle A = 30 \), angle \( C \) could be

[A] an obtuse angle, only
[B] an acute angle, only
[C] either an obtuse angle or an acute angle
[D] a right angle, only

318. What is the total number of distinct triangles that can be constructed if \( AC = 13 \), \( BC = 8 \), and \( m\angle A = 36 \)?

[A] 1  [B] 2  [C] 0  [D] 3

319. Sam needs to cut a triangle out of a sheet of paper. The only requirements that Sam must follow are that one of the angles must be 60°, the side opposite the 60° angle must be 40 centimeters, and one of the other sides must be 15 centimeters. How many different triangles can Sam make?

[A] 2  [B] 0  [C] 1  [D] 3
CHAPTER 10-3

EQUATIONS OF CIRCLES

320. For a carnival game, John is painting two circles, $V$ and $M$, on a square dartboard.
   a On the accompanying grid, draw and label circle $V$, represented by the equation $x^2 + y^2 = 25$, and circle $M$, represented by the equation $(x - 8)^2 + (y + 6)^2 = 4$.

   b A point, $(x,y)$, is randomly selected such that $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$. What is the probability that point $(x,y)$ lies outside both circle $V$ and circle $M$?

321. The center of a circular sunflower with a diameter of 4 centimeters is $(-2,1)$. Which equation represents the sunflower?
   [A] $(x - 2)^2 + (y - 1)^2 = 4$
   [B] $(x + 2)^2 + (y - 1)^2 = 4$
   [C] $(x + 2)^2 + (y - 1)^2 = 2$
   [D] $(x - 2)^2 + (y + 1)^2 = 2$

322. What is the equation of a circle with center $(-3, 1)$ and radius 7?
   [A] $(x + 3)^2 + (y - 1)^2 = 7$
   [B] $(x + 3)^2 + (y - 1)^2 = 49$
   [C] $(x - 3)^2 + (y + 1)^2 = 7$
   [D] $(x - 3)^2 + (y + 1)^2 = 49$

323. A circle has the equation $(x + 1)^2 + (y - 3)^2 = 16$. What are the coordinates of its center and the length of its radius?
   [A] $(1,-3)$ and 4
   [B] $(-1,3)$ and 4
   [C] $(-1,3)$ and 16
   [D] $(1,-3)$ and 16

324. What are the coordinates of the center of the circle represented by the equation $(x + 3)^2 + (y - 4)^2 = 25$?
   [A] $(-3,-4)$
   [B] $(3,-4)$
   [C] $(3,4)$
   [D] $(-3,4)$

325. The center of a circle represented by the equation $(x - 2)^2 + (y + 3)^2 = 100$ is located in Quadrant
   [A] I
   [B] II
   [C] III
   [D] IV

CHAPTER 10-4

EQUATIONS OF ELLIPSES

326. A commercial artist plans to include an ellipse in a design and wants the length of the horizontal axis to equal 10 and the length of the vertical axis to equal 6. Which equation could represent this ellipse?
   [A] $9x^2 + 25y^2 = 225$
   [B] $9x^2 - 25y^2 = 225$
   [C] $3y = 20x^2$
   [D] $x^2 + y^2 = 100$
327. Which equation, when graphed on a Cartesian coordinate plane, would best represent an elliptical racetrack?

[A] $30xy = 288,000$
[B] $3x^2 - 10y^2 = 288,000$
[C] $3x + 10y = 288,000$
[D] $3x^2 + 10y^2 = 288,000$

328. An object orbiting a planet travels in a path represented by the equation $3(y+1)^2 + 5(x+4)^2 = 15$. In which type of pattern does the object travel?

[A] ellipse  [B] circle  
[C] hyperbola  [D] parabola

329. The accompanying diagram represents the elliptical path of a ride at an amusement park.

Which equation represents this path?

[A] $x^2 + y^2 = 300$  
[B] $\frac{x^2}{150^2} - \frac{y^2}{50^2} = 1$

[C] $\frac{x^2}{150^2} + \frac{y^2}{50^2} = 1$  
[D] $y = x^2 + 100x + 300$

330. A designer who is planning to install an elliptical mirror is laying out the design on a coordinate grid. Which equation could represent the elliptical mirror?

[A] $y = 4y^2 + 144$  
[B] $x^2 + y^2 = 144$

[C] $x^2 = 144 + 36y^2$  
[D] $x^2 + 4y^2 = 144$

331. An architect is designing a building to include an arch in the shape of a semi-ellipse (half an ellipse), such that the width of the arch is 20 feet and the height of the arch is 8 feet, as shown in the accompanying diagram.

Which equation models this arch?

[A] $\frac{x^2}{100} + \frac{y^2}{64} = 1$  
[B] $\frac{x^2}{400} + \frac{y^2}{64} = 1$

[C] $\frac{x^2}{64} + \frac{y^2}{400} = 1$  
[D] $\frac{x^2}{64} + \frac{y^2}{100} = 1$

332. The accompanying diagram shows the elliptical orbit of a planet. The foci of the elliptical orbit are $F_1$ and $F_2$.

If $a$, $b$, and $c$ are all positive and $a \neq b \neq c$, which equation could represent the path of the planet?

[A] $y = ax^2 + c^2$  
[B] $ax^2 - by^2 = c^2$

[C] $x^2 + y^2 = c^2$  
[D] $ax^2 + by^2 = c^2$
333. The accompanying diagram shows the construction of a model of an elliptical orbit of a planet traveling around a star. Point $P$ and the center of the star represent the foci of the orbit.

Which equation could represent the relation shown?

\[ [A] \frac{x^2}{81} + \frac{y^2}{225} = 1 \]

\[ [B] \frac{x^2}{225} + \frac{y^2}{81} = 1 \]

\[ [C] \frac{x^2}{15} + \frac{y^2}{9} = 1 \]

\[ [D] \frac{x^2}{15} - \frac{y^2}{9} = 1 \]

334. Solve the following system of equations algebraically:

\[ 9x^2 + y^2 = 9 \]

\[ 3x - y = 3 \]

\[ 335. \text{A linear regression equation of best fit between a student's attendance and the degree of success in school is } h = 0.5x + 68.5. \text{ The correlation coefficient, } r, \text{ for these data would be} \]

\[ [A] r = 0 \]

\[ [B] r = -1 \]

\[ [C] 0 < r < 1 \]

\[ [D] -1 < r < 0 \]

\[ \text{CHAPTER 11-3} \]

\[ \text{CORRELATION COEFFICIENT} \]

336. The relationship of a woman's shoe size and length of a woman's foot, in inches, is given in the accompanying table.

<table>
<thead>
<tr>
<th>Woman's Shoe Size</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foot Length (in)</td>
<td>9.00</td>
<td>9.25</td>
<td>9.50</td>
<td>9.75</td>
</tr>
</tbody>
</table>

The linear correlation coefficient for this relationship is

\[ [A] 0.5 \quad [B] 0 \quad [C] 1 \quad [D] -1 \]

337. Which scatter diagram shows the strongest positive correlation?

\[ [A] \quad [B] \quad [C] \quad [D] \]

338. Which graph represents data used in a linear regression that produces a correlation coefficient closest to $-1$?

\[ [A] \quad [B] \quad [C] \quad [D] \]

\[ \text{CHAPTER 11-4} \]

\[ \text{STANDARD DEVIATION} \]

339. On a standardized test, a score of 86 falls exactly 1.5 standard deviations below the mean. If the standard deviation for the test is 2, what is the mean score for this test?

\[ [A] 84.5 \quad [B] 89 \quad [C] 87.5 \quad [D] 84 \]
340. On a nationwide examination, the Adams School had a mean score of 875 and a standard deviation of 12. The Boswell School had a mean score of 855 and a standard deviation of 20. In which school was there greater consistency in the scores? Explain how you arrived at your answer.

341. An electronics company produces a headphone set that can be adjusted to accommodate different-sized heads. Research into the distance between the top of people's heads and the top of their ears produced the following data, in inches:
4.5, 4.8, 6.2, 5.5, 5.4, 5.8, 6.0, 5.8, 6.2, 4.6, 5.0, 5.4, 5.8
The company decides to design their headphones to accommodate three standard deviations from the mean. Find, to the nearest tenth, the mean, the standard deviation, and the range of distances that must be accommodated.

342. Jean's scores on five mathematics tests were 98, 97, 99, 98, and 96. Her scores on five English tests were 78, 84, 95, 72, and 79. Which statement is true about the standard deviations for the scores?

[A] The standard deviations for both sets of scores are equal.

[B] The standard deviation for the math scores is greater than the standard deviation for the English scores.

[C] The standard deviation for the English scores is greater than the standard deviation for the math scores.

[D] More information is needed to determine the relationship between the standard deviations.

343. Beth's scores on the six Earth science tests she took this semester are 100, 95, 55, 85, 75, and 100. For this population, how many scores are within one standard deviation of the mean?

344. The number of children of each of the first 41 United States presidents is given in the accompanying table. For this population, determine the mean and the standard deviation to the nearest tenth.
How many of these presidents fall within one standard deviation of the mean?

<table>
<thead>
<tr>
<th>Number of Children (x)</th>
<th>Number of Presidents (f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>15</td>
<td>1</td>
</tr>
</tbody>
</table>

345. From 1984 to 1995, the winning scores for a golf tournament were 276, 279, 279, 277, 278, 278, 280, 282, 285, 272, 279, and 278. Using the standard deviation for the sample, \( S_x \), find the percent of these winning scores that fall within one standard deviation of the mean.

CHAPTER 11-7
NORMAL DISTRIBUTIONS

346. In a certain school district, the ages of all new teachers hired during the last 5 years are normally distributed. Within this curve, 95.4% of the ages, centered about the mean, are between 24.6 and 37.4 years. Find the mean age and the standard deviation of the data.
347. Professor Bartrich has 184 students in her mathematics class. The scores on the final examination are normally distributed and have a mean of 72.3 and a standard deviation of 8.9. How many students in the class can be expected to receive a score between 82 and 90?

348. In a New York City high school, a survey revealed the mean amount of cola consumed each week was 12 bottles and the standard deviation was 2.8 bottles. Assuming the survey represents a normal distribution, how many bottles of cola per week will approximately 68.2% of the students drink?

349. The amount of ketchup dispensed from a machine at Hamburger Palace is normally distributed with a mean of 0.9 ounce and a standard deviation of 0.1 ounce. If the machine is used 500 times, approximately how many times will it be expected to dispense 1 or more ounces of ketchup?

350. The amount of juice dispensed from a machine is normally distributed with a mean of 10.50 ounces and a standard deviation of 0.75 ounce. Which interval represents the amount of juice dispensed about 68.2% of the time?

351. The mean of a normally distributed set of data is 56, and the standard deviation is 5. In which interval do approximately 95.4% of all cases lie?

352. The national mean for verbal scores on an exam was 428 and the standard deviation was 113. Approximately what percent of those taking this test had verbal scores between 315 and 541?

353. Mrs. Ramírez is a real estate broker. Last month, the sale prices of homes in her area approximated a normal distribution with a mean of $150,000 and a standard deviation of $25,000. A house had a sale price of $175,000. What is the percentile rank of its sale price, to the nearest whole number? Explain what that percentile means.

354. Twenty high school students took an examination and received the following scores:
70, 60, 75, 68, 85, 86, 78, 72, 82, 88, 88, 73, 74, 79, 86, 82, 90, 92, 93, 73
Determine what percent of the students scored within one standard deviation of the mean. Do the results of the examination approximate a normal distribution? Justify your answer.

355. Battery lifetime is normally distributed for large samples. The mean lifetime is 500 days and the standard deviation is 61 days. Approximately what percent of batteries have lifetimes longer than 561 days?
356. On a standardized test, the distribution of scores is normal, the mean of the scores is 75, and the standard deviation is 5.8. If a student scored 83, the student's score ranks

[A] between the 75th percentile and the 84th percentile
[B] above the 97th percentile
[C] between the 84th percentile and the 97th percentile
[D] below the 75th percentile

357. The mean score on a normally distributed exam is 42 with a standard deviation of 12.1. Which score would be expected to occur less than 5% of the time?


NORMAL PROBABILITY

358. A set of normally distributed student test scores has a mean of 80 and a standard deviation of 4. Determine the probability that a randomly selected score will be between 74 and 82.

359. The amount of time that a teenager plays video games in any given week is normally distributed. If a teenager plays video games an average of 15 hours per week, with a standard deviation of 3 hours, what is the probability of a teenager playing video games between 15 and 18 hours a week?

360. A shoe manufacturer collected data regarding men's shoe sizes and found that the distribution of sizes exactly fits the normal curve. If the mean shoe size is 11 and the standard deviation is 1.5, find:
   a) the probability that a man's shoe size is greater than or equal to 11
   b) the probability that a man's shoe size is greater than or equal to 12.5
   c) \( \frac{P(size \geq 12.5)}{P(size \geq 8)} \)

CHAPTER 12-4

SUMMATIONS

361. What is the value of \( \sum_{m=1}^{3} (2m + 1)^{m-1} \)?


362. What is the value of \( \sum_{m=2}^{5} (m^2 - 1) \)?


363. Evaluate: \( 2 \sum_{n=1}^{5} (2n - 1) \)

364. Evaluate: \( \sum_{n=1}^{5} (n^2 + n) \)

365. The projected total annual profits, in dollars, for the Nutyme Clothing Company from 2002 to 2004 can be approximated by the model \( \sum_{n=0}^{2} (13,567n + 294) \), where \( n \) is the year and \( n = 0 \) represents 2002. Use this model to find the company's projected total annual profits, in dollars, for the period 2002 to 2004.

366. A ball is dropped from a height of 8 feet and allowed to bounce. Each time the ball bounces, it bounces back to half its previous height. The vertical distance the ball travels, \( d \), is given by the formula \( d = 8 + 16 \sum_{k=1}^{n} \left( \frac{1}{2} \right)^k \), where \( n \) is the number of bounces. Based on this formula, what is the total vertical distance that the ball has traveled after four bounces?

[A] 8.9 ft [B] 22.0 ft [C] 15.0 ft [D] 23.0 ft
367. If \( \binom{n}{r} \) represents the number of combinations of \( n \) items taken \( r \) at a time, what is the value of \( \sum_{r=1}^{4} \binom{4}{r} \)?


368. The value of \( \sum_{r=2}^{4} \binom{5}{r} \) is


369. Evaluate: \( \sum_{k=0}^{3} (3\cos k\pi + 1) \)

370. Evaluate: \( \sum_{k=1}^{2} \frac{(-1)^{k-1}}{(2k-1)!} \)

371. What is the value of \( \sum_{b=0}^{3} (2 - (b)i) \)?

[A] 8-5i  [B] 2-6i  [C] 8-6i  [D] 2-5i

372. What is the value of \( \sum_{n=1}^{5} (-2n + 100) \)?


373. Jonathan's teacher required him to express the sum \( \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6} + \frac{6}{7} \) using sigma notation. Jonathan proposed four possible answers. Which of these four answers is not correct?

[A] \( \sum_{k=3}^{7} \frac{k-1}{k} \)  [B] \( \sum_{k=1}^{5} \frac{k}{k+1} \)

[C] \( \sum_{k=2}^{6} \frac{k}{k+1} \)  [D] \( \sum_{k=1}^{5} \frac{k+1}{k+2} \)

NY LESSON 2

QUADRATIC INEQUALITIES

374. Which graph represents the solution set of the inequality \( x^2 - 4x - 5 < 0 \)?

[A]  [B]  [C]  [D]

375. Which graph represents the solution set of \( x^2 - x - 12 < 0 \)?

[A]  [B]  [C]  [D]
376. The height of a projectile is modeled by the equation $y = -2x^2 + 38x + 10$, where $x$ is time, in seconds, and $y$ is height, in feet. During what interval of time, to the nearest tenth of a second, is the projectile at least 125 feet above ground? [The use of the accompanying grid is optional.]

377. When a baseball is hit by a batter, the height of the ball, $h(t)$, at time $t$, $t \geq 0$, is determined by the equation $h(t) = -16t^2 + 64t + 4$. For which interval of time is the height of the ball greater than or equal to 52 feet?

378. The cost ($C$) of selling $x$ calculators in a store is modeled by the equation $C = \frac{3,200,000}{x} + 60,000$. The store profit ($P$) for these sales is modeled by the equation $P = 500x$. What is the minimum number of calculators that have to be sold for profit to be greater than cost?

379. The profit, $P$, for manufacturing a wireless device is given by the equation $P = -10x^2 + 750x - 9,000$, where $x$ is the selling price, in dollars, for each wireless device. What range of selling prices allows the manufacturer to make a profit on this wireless device? [The use of the grid is optional.]

380. The expression $(1 + \cos x)(1 - \cos x)$ is equivalent to

[A] $\sec^2 x$  
[B] $\sin^2 x$  
[C] $\csc^2 x$  
[D] 1

381. If $\theta$ is a positive acute angle and $\sin \theta = a$, which expression represents $\cos \theta$ in terms of $a$?

[A] $\frac{1}{\sqrt{1-a^2}}$  
[B] $\sqrt{a}$  
[C] $\sqrt{1-a^2}$  
[D] $\frac{1}{\sqrt{a}}$
382. The expression $\frac{\tan \theta}{\sec \theta}$ is equivalent to

[A] $\sin \theta$  [B] $\cos \theta$

[C] $\frac{\sin \theta}{\cos^2 \theta}$  [D] $\frac{\cos^2 \theta}{\sin \theta}$

383. The expression $\frac{\sec \theta}{\csc \theta}$ is equivalent to

[A] $\sin \theta$  [B] $\cot \theta$

[C] $\cos \theta$  [D] $\sec \theta$

384. The expression $\frac{2 \cos \theta}{\sin 2\theta}$ is equivalent to

[A] $\sin \theta$  [B] $\csc \theta$

[C] $\sec \theta$  [D] $\cot \theta$

385. The expression $\frac{1 - \cos^2 x}{\sin^2 x}$ is equivalent to

[A] $\cos x$  [B] $\sin x$  [C] $-1$  [D] $1$

386. A crate weighing $w$ pounds sits on a ramp positioned at an angle of $\theta$ with the horizontal. The forces acting on this crate are modeled by the equation $Mw \cos \theta = w \sin \theta$, where $M$ is the coefficient of friction. What is an expression for $M$ in terms of $\theta$?

[A] $M = \cot \theta$  [B] $M = \tan \theta$

[C] $M = \sec \theta$  [D] $M = \csc \theta$

387. Express in simplest terms: $\frac{2 - 2 \sin^2 x}{\cos x}$

NY LESSON 10

SOLVING TRIGONOMETRIC EQUATIONS

388. On a monitor, the graphs of two impulses are recorded on the same screen, where $0^\circ \leq x < 360^\circ$. The impulses are given by the following equations:

$y = 2 \sin^2 x$

$y = 1 - \sin x$

Find all values of $x$, in degrees, for which the two impulses meet in the interval $0^\circ \leq x < 360^\circ$. [Only an algebraic solution will be accepted.]

389. If $(\sec x - 2)(2 \sec x - 1) = 0$, then $x$ terminates in

[A] Quadrants I and II, only

[B] Quadrant I, only

[C] Quadrants I, II, III, and IV

[D] Quadrants I and IV, only

390. If $\sin 6A = \cos 9A$, then $m\angle$ is equal to

[A] $54^\circ$  [B] $\frac{1}{2}$  [C] $36^\circ$  [D] $6^\circ$

391. What value of $x$ in the interval $0^\circ \leq x \leq 180^\circ$ satisfies the equation $\sqrt{3} \tan x + 1 = 0$?

[A] $30^\circ$  [B] $150^\circ$  [C] $60^\circ$  [D] $-30^\circ$

392. What is a positive value of $x$ for which $9 - \cos x = \frac{1}{3}$?

[A] $90^\circ$  [B] $60^\circ$  [C] $30^\circ$  [D] $45^\circ$

393. A solution set of the equation $5 \sin \theta + 3 = 3$ contains all multiples of

[A] $135^\circ$  [B] $90^\circ$  [C] $45^\circ$  [D] $180^\circ$
394. Navigators aboard ships and airplanes use nautical miles to measure distance. The length of a nautical mile varies with latitude. The length of a nautical mile, \( L \), in feet, on the latitude line \( \theta \) is given by the formula
\[
L = 6077 - 31 \cos \theta.
\]
Find, to the nearest degree, the angle \( \theta \), \( 0 \leq \theta \leq 90^\circ \), at which the length of a nautical mile is approximately 6,076 feet.

395. Solve algebraically for all values of \( \theta \) in the interval \( 0 \leq \theta \leq 360^\circ \) that satisfy the equation
\[
\sin \theta = \frac{1}{2}.
\]

396. Solve the following equation algebraically for all values of \( \theta \) in the interval \( 0 \leq \theta \leq 180^\circ \).
\[
2 \sin \theta - 1 = 0
\]

397. In the interval \( 0 \leq A \leq 360^\circ \), solve for all values of \( A \) in the equation
\[
\cos 2A = -3 \sin A - 1.
\]

398. Find, to the nearest degree, all values of \( \theta \) in the interval \( 0^\circ < \theta < 360^\circ \) that satisfy the equation
\[
3 \cos 2\theta + \sin \theta - 1 = 0.
\]

399. An architect is using a computer program to design the entrance of a railroad tunnel. The outline of the opening is modeled by the function \( f(x) = 8 \sin x + 2 \), in the interval \( 0 \leq x \leq \pi \), where \( x \) is expressed in radians. Solve algebraically for all values of \( x \) in the interval \( 0 \leq x \leq \pi \), where the height of the opening, \( f(x) \), is 6. Express your answer in terms of \( \pi \).
If the \( x \)-axis represents the base of the tunnel, what is the maximum height of the entrance of the tunnel?

### NY LESSON 11

#### DOUBLE ANGLE AND ANGLE SUM AND DIFFERENCE IDENTITIES

400. If \( \sin x = \frac{4}{5} \), where \( 0^\circ < x < 90^\circ \), find the value of \( \cos (x + 180^\circ) \).

401. If \( \sin A = \frac{4}{5} \), \( \tan B = \frac{5}{12} \), and angles \( A \) and \( B \) are in Quadrant I, what is the value of \( \sin(A + B) \)?

\[
[A] \quad \frac{63}{65} \quad [B] \quad \frac{33}{65} \quad [C] \quad \frac{33}{65} \quad [D] \quad \frac{63}{65}
\]

402. If \( A \) is a positive acute angle and \( \sin A = \frac{\sqrt{5}}{3} \), what is \( \cos 2A \)?

\[
[A] \quad \frac{1}{3} \quad [B] \quad \frac{1}{9} \quad [C] \quad \frac{1}{9} \quad [D] \quad -\frac{1}{3}
\]

403. If \( \sin x = \frac{12}{13} \), \( \cos y = \frac{3}{5} \), and \( x \) and \( y \) are acute angles, the value of \( \cos(x - y) \) is

\[
[A] \quad -\frac{33}{65} \quad [B] \quad -\frac{14}{65} \quad [C] \quad \frac{21}{65} \quad [D] \quad \frac{63}{65}
\]

404. If \( x \) is an acute angle and \( \sin x = \frac{12}{13} \), then \( \cos 2x \) equals

\[
[A] \quad \frac{25}{169} \quad [B] \quad \frac{119}{169} \quad [C] \quad -\frac{25}{169} \quad [D] \quad -\frac{119}{169}
\]

405. If \( \theta \) is an acute angle such that \( \sin \theta = \frac{5}{13} \), what is the value of \( \sin 2\theta \)?

\[
[A] \quad \frac{120}{169} \quad [B] \quad \frac{10}{26} \quad [C] \quad \frac{12}{13} \quad [D] \quad \frac{60}{169}
\]
406. If \( \sin \theta = \frac{\sqrt{5}}{3} \), then \( \cos 2\theta \) equals

[A] \(-\frac{1}{9}\) [B] \(\frac{1}{3}\) [C] \(-\frac{1}{3}\) [D] \(\frac{1}{9}\)

407. If \( A \) and \( B \) are positive acute angles,
\( \sin A = \frac{5}{13} \), and \( \cos B = \frac{4}{5} \), what is the value of \( \sin (A + B) \)?

[A] \(\frac{33}{65}\) [B] \(\frac{56}{65}\) [C] \(-\frac{16}{65}\) [D] \(\frac{63}{65}\)

408. The expression \( \cos 40^\circ \cos 10^\circ + \sin 40^\circ \sin 10^\circ \) is equivalent to

[A] \(\sin 50^\circ\) [B] \(\cos 50^\circ\) [C] \(\cos 30^\circ\) [D] \(\sin 30^\circ\)

409. If \( \theta \) is a positive acute angle and
\( \sin 2\theta = \frac{\sqrt{3}}{2} \), then \( (\cos \theta + \sin \theta)^2 \) equals

[A] 1 [B] \(1 + \frac{\sqrt{3}}{2}\) [C] \(60^\circ\) [D] \(30^\circ\)

410. If \( x \) is a positive acute angle and \( \sin x = \frac{1}{2} \), what is \( \sin 2x \)?

[A] \(-\frac{\sqrt{3}}{2}\) [B] \(\frac{\sqrt{3}}{2}\) [C] \(\frac{1}{2}\) [D] \(-\frac{1}{2}\)

411. The expression \( \frac{\sin 2\theta}{\sin^2 \theta} \) is equivalent to

[A] \(2 \tan \theta\) [B] \(2 \cot \theta\)
[C] \(2 \cos \theta\) [D] \(\frac{2}{\sin \theta}\)