

CHAPTER 4-1

NY LESSON 13

LOGICAL REASONING

1. 080120a, P.I. G.G.25
What is the smallest integer greater than 1 that is both the square of an integer and the cube of an integer?
[A] 9 [B] 64 [C] 36 [D] 8
2. 010501a, P.I. G.G.25
Stan was trying to guess Melanie's age. She told him her age was an even number and a multiple of three. What could be Melanie's age?
[A] 15 [B] 10 [C] 16 [D] 12
3. 080701a, P.I. G.G.25
Given the true statements: " t is a multiple of 3" and " t is even." What could be a value of t ?
[A] 24 [B] 8 [C] 15 [D] 9
4. 010803a, P.I. G.G.25
The statement " $a > 2$ and $a < 5$ " is true when a is equal to
[A] 5 [B] 10 [C] 2 [D] 3
5. 010221a, P.I. G.G.25
Seth is thinking of a number between 20 and 30. The number is prime and not more than 2 away from a perfect square. What is the number?
6. 060416a, P.I. G.G.25
The statement " x is *not* the square of an integer and x is a multiple of 3" is true when x is equal to
[A] 36 [B] 18 [C] 9 [D] 32
7. 010706a, P.I. G.G.25
The statement " $x \geq 4$ and $2x - 4 < 6$ " is true when x is equal to
[A] 5 [B] 10 [C] 4 [D] 1
8. 089928a, P.I. G.G.25
Bob and Ray are describing the same number. Bob says, "The number is a positive even integer less than or equal to 20." Ray says, "The number is divisible by 4." If Bob's statement is true and Ray's statement is false, what are all the possible numbers?
9. 080505a, P.I. G.G.25
The statement " x is divisible by 5 or x is divisible by 4" is *false* when x equals
[A] 16 [B] 10 [C] 20 [D] 27
10. 010003a, P.I. G.G.25
Mary says, "The number I am thinking of is divisible by 2 or it is divisible by 3." Mary's statement is false if the number she is thinking of is
[A] 15 [B] 11 [C] 6 [D] 8
11. 060221a
Given the true statement "John is not handsome" and the false statement "John is handsome or smart." Determine the truth value for the statement "John is smart."
12. 010129a, P.I. G.G.25
Mark says, "The number I see is odd." Jan says, "That same number is prime." The teacher says, "Mark is correct or Jan is correct." Some integers would make the teacher's statement true while other integers would make it false. Give and explain one example of when the teacher's statement is true. Give and explain one example of when the teacher's statement is false.

13. 060622a, P.I. G.G.25
If $x = 3$, which statement is *false*?
[A] x is prime and x is odd.
[B] x is odd or x is even.
[C] x is not prime and x is odd.
[D] x is odd and $2x$ is even.
14. 010407a
Given the true statements: "Jason goes shopping or he goes to the movies" and "Jason does not go to the movies." Which statement must also be true?
[A] Jason goes shopping.
[B] Jason does not go shopping and he does not go to the movies.
[C] Jason does not go shopping.
[D] Jason stays home.
15. 069902a, P.I. G.G.25
The statement "If x is divisible by 8, then it is divisible by 6" is false if x equals
[A] 6 [B] 32 [C] 14 [D] 48
16. 060517a, P.I. G.G.25
The statement "If x is prime, then it is odd" is false when x equals
[A] 4 [B] 3 [C] 2 [D] 1
17. 060614a, P.I. G.G.25
Given the statement: "If x is a rational number, then \sqrt{x} is irrational." Which value of x makes the statement *false*?
[A] 2 [B] 3 [C] $\frac{3}{2}$ [D] 4
18. 060110a
At a school costume party, seven girls wore masks and nine boys did not. If there were 15 boys at the party and 20 students did not wear masks, what was the total number of students at the party?
[A] 30 [B] 35 [C] 42 [D] 33

19. 010214a
Frank, George, and Hernando are a plumber, a cabinet maker, and an electrician, though not necessarily in that order. Each can do all work appropriate to his own field, but no work in other fields. Frank was not able to install a new electric line in his home. Hernando was not able to make cabinets. George is also a building contractor who hired one of the other people to do his electrical work. Which statement must be true?
[A] Hernando is an electrician.
[B] Frank is a plumber.
[C] Frank is an electrician.
[D] George is a cabinet maker.
20. 080026a
John, Dan, Karen, and Beth went to a costume ball. They chose to go as Anthony and Cleopatra, and Romeo and Juliet. John got the costumes for Romeo and Cleopatra, but not his own costume. Dan saw the costumes for Juliet and himself. Karen went as Anthony. Beth drove two of her friends, who were dressed as Anthony and Cleopatra, to the ball. What costume did John wear?

CONTRAPOSITIVE

21. 080427a, P.I. G.G.26
What is the contrapositive of the statement "If I study, then I pass the test"?
[A] If I pass the test, then I study.
[B] I pass the test if I study.
[C] If I do not pass the test, then I do not study.
[D] If I do not study, then I do not pass the test.

22. 060308a, P.I. G.G.26
Which statement is logically equivalent to "If it is Saturday, then I am not in school"?
[A] If I am in school, then it is not Saturday.
[B] If I am not in school, then it is Saturday.
[C] If it is Saturday, then I am in school.
[D] If it is not Saturday, then I am in school.
23. 080104a, P.I. G.G.26
Which statement is logically equivalent to "If I did not eat, then I am hungry"?
[A] If I did not eat, then I am not hungry.
[B] If I am not hungry, then I did eat.
[C] If I am not hungry, then I did not eat.
[D] If I am hungry, then I did eat.
24. 060112a, P.I. G.G.26
Which statement is logically equivalent to "If I eat, then I live"?
[A] I live if and only if I eat.
[B] If I eat, then I do not live.
[C] If I live, then I eat.
[D] If I do not live, then I do not eat.
25. 060405a, P.I. G.G.26
Which statement is logically equivalent to "If a triangle is an isosceles triangle, then it has two congruent sides"?
[A] If a triangle is not an isosceles triangle, then it has two congruent sides.
[B] If a triangle does not have two congruent sides, then it is an isosceles triangle.
[C] If a triangle does not have two congruent sides, then it is not an isosceles triangle.
[D] If a triangle is an isosceles triangle, then it does not have two congruent sides.
26. 010220a, P.I. G.G.26
Which statement is logically equivalent to "If the team has a good pitcher, then the team has a good season"?
[A] If the team does not have a good season, then the team does not have a good pitcher.
[B] The team has a good pitcher and the team does not have a good season.
[C] If the team has a good season, then the team has a good pitcher.
[D] If the team does not have a good pitcher, then the team does not have a good season.
27. 010513a, P.I. G.G.26
Which statement is logically equivalent to the statement "If you are an elephant, then you do not forget"?
[A] If you do not forget, then you are an elephant.
[B] If you do not forget, then you are not an elephant.
[C] If you are an elephant, then you forget.
[D] If you forget, then you are not an elephant.
28. 080629a, P.I. G.G.26
Which statement is logically equivalent to the statement "If Corey worked last summer, he buys a car"?
[A] If Corey does not buy a car, he did not work last summer.
[B] If Corey did not work last summer, he does not buy a car.
[C] If Corey buys a car, he worked last summer.
[D] If you are an elephant, then you forget.

29. 010308a, P.I. G.G.26

Given the true statement: "If a person is eligible to vote, then that person is a citizen."
Which statement must also be true?

[A] Kayla is not a citizen; therefore, she is not eligible to vote.

[B] Morgan has never voted; therefore, he is not a citizen.

[C] Juan is a citizen; therefore, he is eligible to vote.

[D] Marie is not eligible to vote; therefore, she is not a citizen.

30. 010837a, P.I. G.G.26

In the spaces provided below, write the converse, the inverse, and the contrapositive of the statement "If I run, then I am tired."

Converse: _____

Inverse: _____

Contrapositive: _____

31. 080739a, P.I. G.G.26

Given the statement: "If I live in Albany, then I am a New Yorker."

In the spaces provided below, write the inverse, the converse, and the contrapositive of this statement.

Inverse: _____

Converse: _____

Contrapositive: _____

Which conditional is logically equivalent to its original statement?

inverse converse contrapositive

CONVERSE AND BICONDITIONAL

32. 010415a, P.I. G.G.26

Which statement is the converse of "If the sum of two angles is 180° , then the angles are supplementary"?

[A] If the sum of two angles is not 180° , then the angles are not supplementary.

[B] If two angles are not supplementary, then their sum is not 180° .

[C] If two angles are supplementary, then their sum is 180° .

[D] If the sum of two angles is not 180° , then the angles are supplementary.

33. 080014a, P.I. G.G.26
What is the converse of the statement "If it is sunny, I will go swimming"?
[A] If I go swimming, it is sunny.
[B] I will go swimming if and only if it is sunny.
[C] If I do not go swimming, then it is not sunny.
[D] If it is not sunny, I will not go swimming.
34. 080116a, P.I. G.G.26
Which statement is the converse of "If it is a 300 ZX, then it is a car"?
[A] If it is not a car, then it is not a 300 ZX.
[B] If it is not a 300 ZX, then it is not a car.
[C] If it is a car, then it is a 300 ZX.
[D] If it is a car, then it is not a 300 ZX.
35. 060520a, P.I. G.G.26
What is the converse of the statement "If it is Sunday, then I do not go to school"?
[A] If it is not Sunday, then I do not go to school.
[B] If it is not Sunday, then I go to school.
[C] If I do not go to school, then it is Sunday.
[D] If I go to school, then it is not Sunday.
36. 080521a, P.I. G.G.26
What is the converse of the statement "If Alicia goes to Albany, then Ben goes to Buffalo"?
[A] If Alicia does not go to Albany, then Ben does not go to Buffalo.
[B] If Ben does not go to Buffalo, then Alicia does not go to Albany.
[C] Alicia goes to Albany if and only if Ben goes to Buffalo.
[D] If Ben goes to Buffalo, then Alicia goes to Albany.
37. 060717a, P.I. G.G.26
What is the converse of the statement "If the Sun rises in the east, then it sets in the west"?
[A] If the Sun does not rise in the east, then it does not set in the west.
[B] If the Sun does not set in the west, then it does not rise in the east.
[C] If the Sun rises in the west, then it sets in the east.
[D] If the Sun sets in the west, then it rises in the east.
38. 089912a, P.I. G.G.26
What is true about the statement "If two angles are right angles, the angles have equal measure " and its converse "If two angles have equal measure then the two angles are right angles"?
[A] The statement is true but its converse is false.
[B] Both the statement and its converse are false.
[C] The statement is false but its converse is true.
[D] Both the statement and its converse are true.
39. 080205a, P.I. G.G.26
Given the statement: "If two lines are cut by a transversal so that the corresponding angles are congruent, then the lines are parallel." What is true about the statement and its converse?
[A] The statement and its converse are both true.
[B] The statement is false, but its converse is true.
[C] The statement is true, but its converse is false.
[D] The statement and its converse are both false.

40. 010112a, P.I. G.G.26

Given the statement: "If two sides of a triangle are congruent, then the angles opposite these sides are congruent."
Given the converse of the statement: "If two angles of a triangle are congruent, then the sides opposite these angles are congruent."
What is true about this statement and its converse?

- [A] Neither the statement nor its converse is true.
- [B] Both the statement and its converse are true.
- [C] The statement is true but its converse is false.
- [D] The statement is false but its converse is true.

41. 060730a, P.I. G.G.26

Given the statement: "A right angle measures 90° ." How is this statement written as a biconditional?

- [A] If an angle does not measure 90° , then it is not a right angle.
- [B] An angle measures 90° and it is a right angle.
- [C] An angle is a right angle if, and only if, it measures 90° .
- [D] If an angle is a right angle, then it measures 90° .

42. 010627a, P.I. G.G.26

Which statement is expressed as a biconditional?

- [A] If two angles are both right angles, then they are congruent.
- [B] If two angles are congruent, then they are both right angles.
- [C] Two angles are congruent if they have the same measure.
- [D] Two angles are congruent if and only if they have the same measure.

INVERSE

43. 010303a, P.I. G.G.26

What is the inverse of the statement "If Mike did his homework, then he will pass this test"?

- [A] If Mike passes this test, then he did his homework.
- [B] If Mike did not do his homework, then he will not pass this test.
- [C] If Mike does not pass this test, then he only did half his homework.
- [D] If Mike does not pass this test, then he did not do his homework.

44. 060317a, P.I. G.G.26

What is the inverse of the statement "If Julie works hard, then she succeeds"?

- [A] If Julie works hard, then she does not succeed.
- [B] If Julie does not work hard, then she does not succeed.
- [C] If Julie succeeds, then she works hard.
- [D] If Julie does not succeed, then she does not work hard.

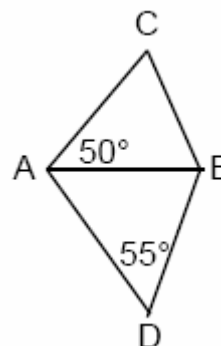
45. 060006a, P.I. G.G.26

What is the inverse of the statement "If it is sunny, I will play baseball"?

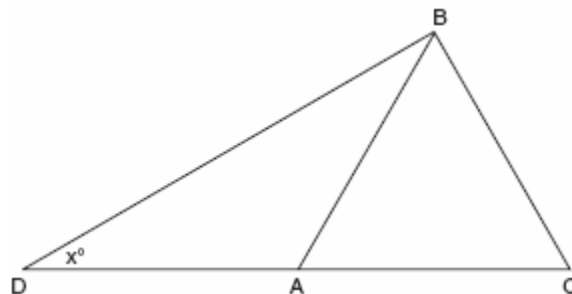
- [A] I will play baseball if and only if it is sunny.
- [B] If I do not play baseball, then it is not sunny.
- [C] If it is not sunny, I will not play baseball.
- [D] If I play baseball, then it is sunny.

46. 080416a, P.I. G.G.26
What is the inverse of the statement "If I do not buy a ticket, then I do not go to the concert"?
- [A] If I buy a ticket, then I do not go to the concert.
- [B] If I do not go to the concert, then I do not buy a ticket.
- [C] If I go to the concert, then I buy a ticket.
- [D] If I buy a ticket, then I go to the concert.
47. 010616a, P.I. G.G.26
Which statement is the inverse of "If the waves are small, I do not go surfing"?
- [A] If the waves are not small, I do not go surfing.
- [B] If the waves are not small, I go surfing.
- [C] If I do not go surfing, the waves are small.
- [D] If I go surfing, the waves are not small.
48. 010715a, P.I. G.G.26
What is the inverse of the statement "If Bob gets hurt, then the team loses the game"?
- [A] If the team does not lose the game, then Bob does not get hurt.
- [B] If Bob does not get hurt, then the team does not lose the game.
- [C] If the team loses the game, then Bob gets hurt.
- [D] Bob gets hurt if the team loses the game.

50. 060107a, P.I. G.G.30
In isosceles triangle DOG , the measure of the vertex angle is three times the measure of one of the base angles. Which statement about $\triangle DOG$ is true?
- [A] $\triangle DOG$ is an obtuse triangle.
- [B] $\triangle DOG$ is a right triangle.
- [C] $\triangle DOG$ is a scalene triangle.
- [D] $\triangle DOG$ is an acute triangle.
51. 069930a, P.I. G.G.31
In the accompanying diagram, $\triangle ABC$ and $\triangle ABD$ are isosceles triangles with $m\angle CAB = 50$ and $m\angle BDA = 55$. If $AB=AC$ and $AB=BD$, what is $m\angle CBD$?



52. 080221a, P.I. G.G.31
In the accompanying diagram of $\triangle BCD$, $\triangle ABC$ is an equilateral triangle and $AD = AB$. What is the value of x , in degrees?



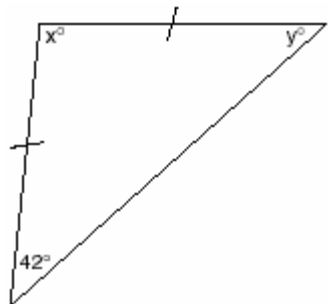
CHAPTER 4-2

SPECIAL TRIANGLES

49. 010223a, P.I. G.G.30
Vertex angle A of isosceles triangle ABC measures 20° more than three times $m\angle B$. Find $m\angle C$.

53. 060510a, P.I. G.G.31

Tina wants to sew a piece of fabric into a scarf in the shape of an isosceles triangle, as shown in the accompanying diagram.



What are the values of x and y ?

[A] $x = 69$ and $y = 69$

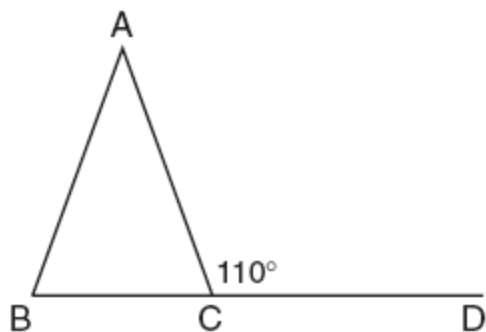
[B] $x = 42$ and $y = 96$

[C] $x = 90$ and $y = 48$

[D] $x = 96$ and $y = 42$

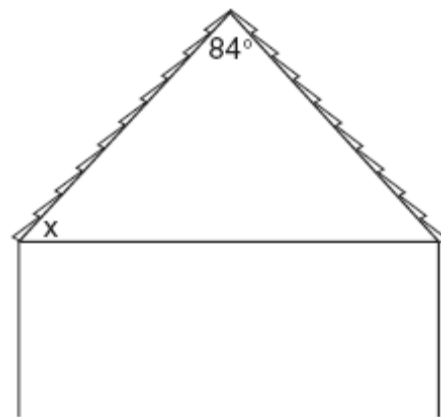
54. 080734a, P.I. G.G.31

In the accompanying diagram of isosceles triangle ABC , $\overline{AB} \cong \overline{AC}$, and exterior angle $ACD = 110^\circ$. What is $m\angle BAC$?



55. 060615a, P.I. G.G.31

The accompanying diagram shows the roof of a house that is in the shape of an isosceles triangle. The vertex angle formed at the peak of the roof is 84° .

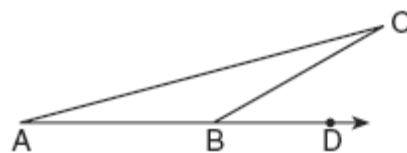


What is the measure of x ?

[A] 84° [B] 138° [C] 48° [D] 96°

56. 010613a, P.I. G.G.31

In the accompanying diagram of $\triangle ABC$, \overline{AB} is extended through D , $m\angle CBD = 30$, and $\overline{AB} \cong \overline{BC}$.



What is the measure of $\angle A$?

[A] 30° [B] 150° [C] 75° [D] 15°

57. 080433a, P.I. G.G.31

Dylan says that all isosceles triangles are acute triangles. Mary Lou wants to prove that Dylan is *not* correct. Sketch an isosceles triangle that Mary Lou could use to show that Dylan's statement is not true. In your sketch, state the measure of *each* angle of the isosceles triangle.

58. 060027a, P.I. G.G.31

Hersch says if a triangle is an obtuse triangle, then it cannot also be an isosceles triangle. Using a diagram, show that Hersch is incorrect, and indicate the measures of all the angles and sides to justify your answer.

CHAPTER 4-4

PROOFS

59. 080608b

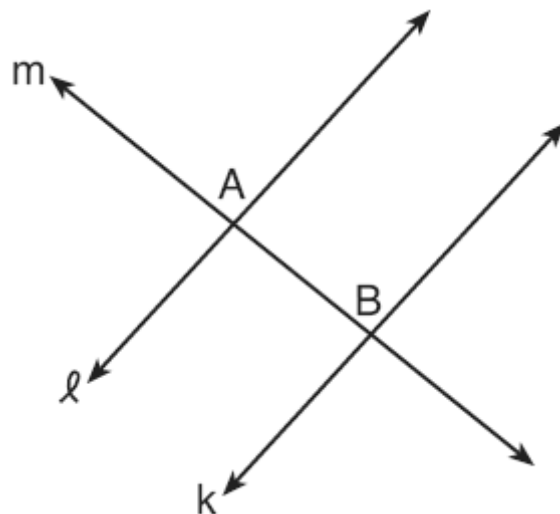
In $\triangle ABC$, D is a point on \overline{AC} such that \overline{BD} is a median. Which statement must be true?

- [A] $\angle ABD \cong \angle CBD$ [B] $\overline{AD} \cong \overline{CD}$
[C] $\overline{BD} \perp \overline{AC}$ [D] $\triangle ABD \cong \triangle CBD$

CHAPTER 4-5

60. 010814b, P.I. G.G.27

In the accompanying diagram, line ℓ is perpendicular to line m at A , line k is perpendicular to line m at B , and lines ℓ , m , and k are in the same plane.

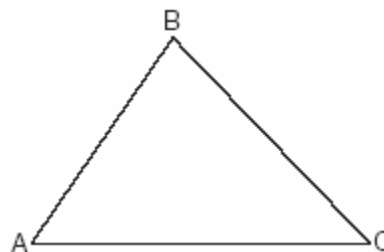


Which statement is the first step in an indirect proof to prove that ℓ is parallel to k ?

- [A] Assume that ℓ is not perpendicular to m .
[B] Assume that ℓ is not parallel to k .
[C] Assume that ℓ , m , and k are not in the same plane.
[D] Assume that ℓ is perpendicular to k .

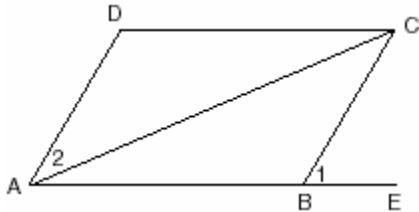
61. 080230b, P.I. G.G.27

In the accompanying diagram, $\triangle ABC$ is *not* isosceles. Prove that if altitude \overline{BD} were drawn, it would *not* bisect \overline{AC} .



62. 060533b, P.I. G.G.27

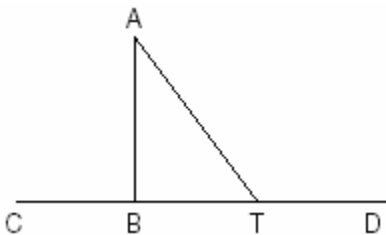
Given: parallelogram $ABCD$, diagonal \overline{AC} ,
and \overline{ABE}



Prove: $m\angle 1 > m\angle 2$

63. 060425b, P.I. G.G.31

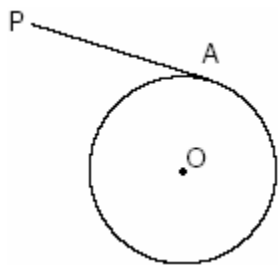
Given: $\triangle ABT$, \overline{CBTD} , and $\overline{AB} \perp \overline{CD}$



Write an indirect proof to show that \overline{AT} is
not perpendicular to \overline{CD} .

64. 010432b, P.I. G.G.27

In the accompanying diagram of circle O , \overline{PA}
is drawn tangent to the circle at A . Place B on
 \overline{PA} anywhere between P and A and draw
 \overline{OA} , \overline{OP} , and \overline{OB} . Prove that \overline{OB} is not
perpendicular to \overline{PA} .



CHAPTER 4-6

TRIANGLE INEQUALITIES

65. 080214a, P.I. G.G.33

If the lengths of two sides of a triangle are 4
and 10, what could be the length of the third
side?

- [A] 6 [B] 8 [C] 16 [D] 14

66. 080018a, P.I. G.G.33

If two sides of a triangle are 1 and 3, the third
side may be

- [A] 4 [B] 3 [C] 2 [D] 5

67. 080520a, P.I. G.G.33

Sara is building a triangular pen for her pet
rabbit. If two of the sides measure 8 feet and
15 feet, the length of the third side could be

- [A] 23 ft [B] 13 ft [C] 3 ft [D] 7 ft

68. 069905a, P.I. G.G.33

The direct distance between city A and city B
is 200 miles. The direct distance between city
 B and city C is 300 miles. Which could be
the direct distance between city C and city A ?

- [A] 650 miles [B] 50 miles
[C] 550 miles [D] 350 miles

69. 080425a, P.I. G.G.33

Which set can *not* represent the lengths of the
sides of a triangle?

- [A] {5,5,11} [B] {7,7,12}
[C] {8,8,8} [D] {4,5,6}

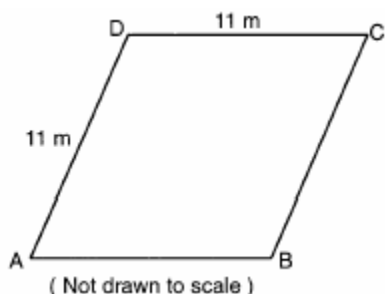
70. 060515a, P.I. G.G.33

Which set could *not* represent the lengths of
the sides of a triangle?

- [A] {2,5,9} [B] {5,10,12}
[C] {7,9,11} [D] {3,4,5}

71. 010010a, P.I. G.G.33

A plot of land is in the shape of rhombus $ABCD$ as shown below.



Which can *not* be the length of diagonal AC ?

[A] 24 m [B] 11 m [C] 18 m [D] 4 m

72. 010534a, P.I. G.G.33

José wants to build a triangular pen for his pet rabbit. He has three lengths of boards already cut that measure 7 feet, 8 feet, and 16 feet. Explain why José cannot construct a pen in the shape of a triangle with sides of 7 feet, 8 feet, and 16 feet.

73. 080120b, P.I. G.G.33

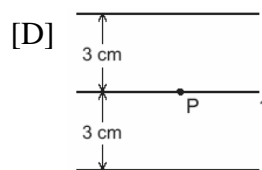
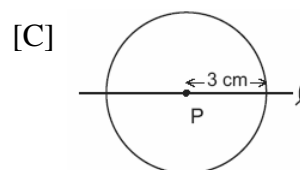
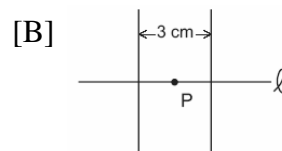
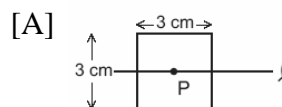
A box contains one 2-inch rod, one 3-inch rod, one 4-inch rod, and one 5-inch rod. What is the maximum number of different triangles that can be made using these rods as sides?

[A] 1 [B] 3 [C] 4 [D] 2

CHAPTER 4-7

LOCUS

74. If point P lies on line ℓ , which diagram represents the locus of points 3 centimeters from point P ?



75. Which equation represents the locus of all points 5 units below the x -axis?

[A] $y = 5$ [B] $x = 5$

[C] $y = -5$ [D] $x = -5$

76. Chantrice is pulling a wagon along a smooth, horizontal street. The path of the center of one of the wagon wheels is best described as

[A] a line perpendicular to the road

[B] a circle [C] two parallel lines

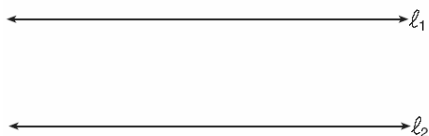
[D] a line parallel to the road

77. 010830a, P.I. G.G.22

The locus of points equidistant from the points (4,-5) and (4,7) is the line whose equation is

- [A] $x = 4$ [B] $y = 1$
[C] $x = 1$ [D] $y = 2$

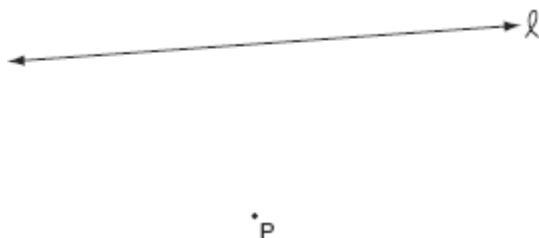
78. In the accompanying diagram, line ℓ_1 is parallel to line ℓ_2 .



Which term describes the locus of all points that are equidistant from line ℓ_1 and ℓ_2 ?

- [A] circle [B] line
[C] rectangle [D] point

79. In the accompanying diagram, point P lies 3 centimeters from line ℓ .



How many points are both 2 centimeters from line ℓ and 1 centimeter from point P ?

- [A] 0 [B] 4 [C] 1 [D] 2

80. The distance between parallel lines ℓ and m is 12 units. Point A is on line ℓ . How many points are equidistant from lines ℓ and m and 8 units from point A .

- [A] 4 [B] 3 [C] 2 [D] 1

81. How many points are equidistant from two parallel lines and also equidistant from two points on one of the lines?

- [A] 2 [B] 4 [C] 1 [D] 3

82. The locus of points equidistant from two sides of an acute scalene triangle is

- [A] a median [B] an angle bisector
[C] the third side [D] an altitude

83. In the coordinate plane, what is the total number of points 5 units from the origin and equidistant from both the x - and y -axes?

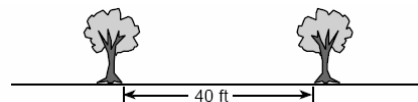
- [A] 1 [B] 0 [C] 4 [D] 2

84. What is the total number of points equidistant from two intersecting straight roads and also 300 feet from the traffic light at the center of the intersection?

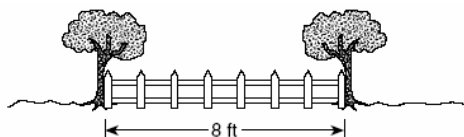
- [A] 2 [B] 0 [C] 4 [D] 1

85. A treasure map shows a treasure hidden in a park near a tree and a statue. The map indicates that the tree and the statue are 10 feet apart. The treasure is buried 7 feet from the base of the tree and also 5 feet from the base of the statue. How many places are possible locations for the treasure to be buried? Draw a diagram of the treasure map, and indicate with an **X** each possible location of the treasure.

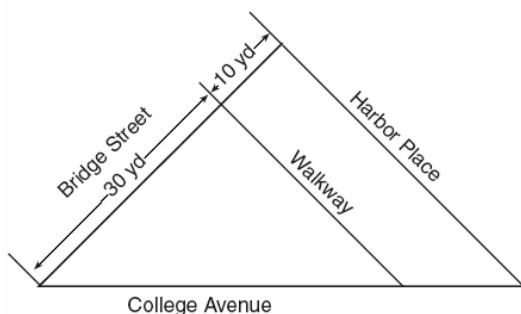
86. Maria's backyard has two trees that are 40 feet apart, as shown in the accompanying diagram. She wants to place lampposts so that the posts are 30 feet from both of the trees. Draw a sketch to show where the lampposts could be placed in relation to the trees. How many locations for the lampposts are possible?



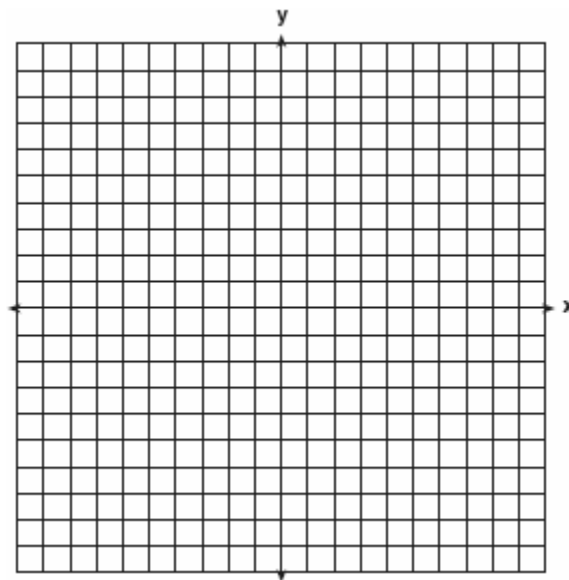
87. Steve has a treasure map, represented in the accompanying diagram, that shows two trees 8 feet apart and a straight fence connecting them. The map states that treasure is buried 3 feet from the fence and equidistant from the two trees.



- a* Sketch a diagram to show all the places where the treasure could be buried. Clearly indicate in your diagram where the treasure could be buried.
- b* What is the distance between the treasure and one of the trees?
88. A triangular park is formed by the intersection of three streets, Bridge Street, Harbor Place, and College Avenue, as shown in the accompanying diagram. A walkway parallel to Harbor Place goes through the park. A time capsule has been buried in the park in a location that is equidistant from Bridge Street and College Avenue and 5 yards from the walkway. Indicate on the diagram with an **X** each possible location where the time capsule could be buried.



89. Dan is sketching a map of the location of his house and his friend Matthew's house on a set of coordinate axes. Dan locates his house at point $D(0,0)$ and locates Matthew's house, which is 6 miles east of Dan's house, at point $M(6,0)$. On the accompanying set of coordinate axes, graph the locus of points equidistant from the two houses. Then write the equation of the locus.



90. In the diagram below, town C lies on straight road p . Sketch the points that are 6 miles from town C . Then sketch the points that are 3 miles from road p . How many points satisfy both conditions?



91. Point P is located on \overline{AB} .
- a* Describe the locus of points that are
- (1) 3 units from \overline{AB}
 - (2) 5 units from point P
- b* How many points satisfy both conditions in part *a*?

[1] B _____

[2] D _____

[3] A _____

[4] D _____

[2] 23, and appropriate work is shown.

[1] Appropriate work is shown, but no answer or an incorrect answer is found.

or [1] 23, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[5] incorrect procedure. _____

[6] B _____

[7] C _____

[3] 2, 6, 10, 14, and 18 and an appropriate method is shown.

[2] One mistake is made with selection, such as including 0.

[1] One of the appropriate sets is found: either 2, 4, 6, 8, 10, 12, 14, 16, 18, 20 or not 4, 8, 12, 16, 20.

or [1] The correct numbers are found, and no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[8] incorrect procedure. _____

[9] D _____

[10] B _____

[2] False, and an appropriate explanation is given.

[1] Appropriate work is shown, but the truth value is missing or is incorrect.

[0] False, but no explanation is given.

or [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an

[11] obviously incorrect procedure. _____

[3] At least one example is shown that makes the statement true, such as 2, 3, 5, 7, 9, or a defined variable; and one example is shown that makes the statement false, such as any even number other than 2, with a correct explanation that shows that the student can recognize odd numbers and prime numbers. The explanation can be in words or as a Venn diagram.

[2] Two correct examples are shown, one that shows the statement is true and one that shows the statement is false, but no explanation or an inappropriate explanation is given.

or [2] Only one correct example is shown, but an appropriate explanation is given.

[1] Only one correct example is shown, and no explanation or an incorrect explanation is given.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[12] incorrect procedure. _____

[13] C _____

[14] A _____

[15] B _____

[16] C _____

[17] D _____

[18] D _____

[19] A _____

[3] Juliet and an explanation is given of how the identification was reached, such as by a narrative or table.

[2] One error is made in the logic statements or the table, but appropriate results are found.

[1] More than one error is made in the logic statements or the table, but appropriate work is shown.

or [1] Juliet but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[20] incorrect procedure.

[21] C

[22] A

[23] B

[24] D

[25] C

[26] A

[27] D

[28] A

[29] A

[3] Three correct statements are written for the converse, the inverse, and the contrapositive.

[2] Two correct statements are written.

[1] One correct statement is written.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[30] incorrect procedure.

[4] Contrapositive, and all three statements are written correctly.

[3] Contrapositive, and only two of the statements are written correctly

or [3] All three statements are written correctly, but the contrapositive is not identified.

[2] Contrapositive, and only one statement is written correctly.

or [2] Only two statements are written correctly, and the contrapositive is not identified.

[1] All three statements are written incorrectly, but the contrapositive is identified.

or [1] Only one statement is written correctly, and the contrapositive is not identified.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[31] incorrect procedure.

[32] C

[33] A

[34] C

[35] C

[36] D

[37] D

[38] A

[39] A

[40] B

[41] C

[42] D

[43] B

[44] B

[45] C

[46] D

[47] B _____

[48] B _____

[2] 32, and appropriate work is shown, such as a diagram or “let” statements and an appropriate equation, such as $5x + 20 = 180$.
or [2] 32, and an appropriate trial-and-error method with at least two trials and appropriate checks are shown.

[1] Appropriate work is shown, but one computational error is made.

or [1] An incorrect equation set equal to 180° is shown, but it is solved appropriately, such as $4x + 20 = 180$; or an incorrect equation set equal to 360° is shown, such as $5x + 20 = 360$.

or [1] 32, and an appropriate trial-and-error method with less than two trials and appropriate checks are shown.

or [1] 32, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[49] incorrect procedure.

[50] A _____

[3] 135 and appropriate work is shown.

[2] The two correct angles of 65° and 70° are found, but their sum is not identified as the answer to the question.

or [2] 65° or 70° and an appropriate sum is found.

[1] Either the 65° or the 70° is correctly identified.

or [1] Two incorrect angle measures are found, but they are added correctly.

or [1] 135 and no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[51] incorrect procedure.

[2] 30, and appropriate work is shown or an appropriate explanation is given.

[1] Angles of the equilateral triangle are shown to be 60° , but x is not determined or is determined incorrectly.

or [1] 30, but no work is shown or no explanation is given.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[52] incorrect procedure.

[53] D _____

[2] 40, and appropriate work is shown, such as $x = 180 - (70 + 70)$ or correctly labeling all the angles in the diagram.

[1] Appropriate work is shown, but one computational error is made.

or [1] Appropriate work is shown, but one conceptual error is made.

or [1] A correct equation is written, but no further correct work is shown.

or [1] The measures of $\angle ACB$ and $\angle ABC$ are both found to be 70° , but no further correct work is shown.

or [1] An incorrect equation of equal difficulty is solved appropriately.

or [1] 40, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[54] incorrect procedure.

[55] C _____

[56] D _____

[2] An isosceles triangle that is not acute is drawn, and its three angles are labeled, such as 20, 20, 140 or 45, 45, 90.

[1] An isosceles triangle is drawn that shows an angle that is not acute, but the base angles are not labeled.

or [1] The three angles are stated correctly, but no triangle is drawn.

[0] The triangle that is drawn and labeled is not isosceles or is acute.

or [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an

[57] obviously incorrect procedure.

[3] The student draws an obtuse triangle and all sides and all angles are correctly calculated, such as by using 120° , 30° , and 30° and sides 4, 4, and 10.

[2] The student has the angles correctly indicated and the two congruent sides marked, but the length of the longest side is incorrect or is missing.

or [2] All sides are correctly marked, but the angles do not add to 180° , but an obtuse angle and two congruent angles are shown.

[1] Only the angles are correctly shown.

or [1] Only the sides are correctly shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[58] incorrect procedure.

[59] B _____

[60] B _____

[4] The proof in column or paragraph form explains clearly, by using contradiction or indirect proof, that altitude \overline{BD} does not bisect side \overline{AC} .

[3] An appropriate conclusion is shown, without specifying that congruent triangles are actually formed only if a triangle is isosceles.

[2] An appropriate diagram is drawn and some evidence that congruence may be an issue is shown, but no further reasoning is given or no conclusion is drawn.

[1] Circular reasoning is used or the statement is said to be true, but no proof by contradiction or indirect proof is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[61] incorrect procedure.

- [6] A complete and correct proof is written.
 [5] A proof is written that demonstrates a thorough understanding of the method of proof and contains no conceptual errors, but one statement and/or reason is missing or is incorrect.
 [4] A proof is written that demonstrates a good understanding of the method of proof and contains no conceptual errors, but two statements and/or reasons are missing or are incorrect.
 [3] A proof is written that demonstrates a good understanding of the method of proof, but one conceptual error is made.
 [2] Some correct relevant statements about the proof are made, but three or four statements and/or reasons are missing or are incorrect.
 [1] Only one correct statement and reason are written.
 [0] The “given” and/or the “prove” statements are rewritten in the style of a formal proof, but no further correct relevant statements are written.
 or [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
- [62] _____
- [2] A correct indirect proof is written with appropriate statements and reasons.
 [1] The assumption that \overline{AT} is perpendicular to \overline{CD} is written, but no further correct work is shown.
 or [1] A method other than an indirect proof is used to show that \overline{AT} is not perpendicular to \overline{CD} .
 [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
- [63] _____

- [4] An appropriate diagram is drawn, and a correct proof is written in statement-reason or paragraph form, such as stating that $\triangle AOB$ cannot have two right angles or that two perpendiculars cannot be drawn to \overline{PA} from point O .
 [3] An appropriate diagram is drawn and an appropriate reason is written to show $\overline{OA} \perp \overline{PA}$, but one statement or one reason is incomplete or is incorrect, but an appropriate conclusion is drawn.
 or [3] The diagram is not drawn, but a complete and correct proof is written.
 [2] An appropriate diagram is drawn, and an appropriate reason is written to show $\overline{OA} \perp \overline{PA}$, but one statement and one reason are incomplete or are incorrect, but an appropriate conclusion is drawn.
 [1] An appropriate diagram is drawn, but the proof contains circular reasoning.
 [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
- [64] _____
- [65] B
- [66] B
- [67] B
- [68] D
- [69] A
- [70] A
- [71] A

[2] The statements $7 + 8 = 15$ and “15 is not greater than 16” are written or the explanation is given that the sum of any two sides of a triangle must be greater than the third side.

[1] An explanation is written that includes a reference to the triangle inequality, but the explanation is not complete or an incorrect conclusion is stated.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[72] incorrect procedure.

[73] B

[74] C

[75] C

[76] D

[77] B

[78] B

[79] C

[80] C

[81] C

[82] B

[83] C

[84] C

[4] A correct diagram is drawn, two X points are marked, a numerical 2 is given for the places to dig, and appropriate work is shown.

[3] The diagram is correct including two X points, but an incorrect answer or no answer is found.

[2] One correct locus situation and one incorrect locus situation are drawn, but the answer is appropriate according to the diagram.

or [2] Each locus situation is correctly drawn, but no X points are marked, and no numerical answer is found.

[1] Only one locus situation is correctly drawn and an incorrect conclusion or no conclusion is shown.

or [1] 2 but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[85] incorrect procedure.

[2] 2 and an appropriate sketch of two circles intersecting in two points is shown.

[1] 2 and no sketch is shown.

or [1] An appropriate sketch is shown, without indicating 2 as the possibilities.

or [1] An appropriate number is found, based on an inappropriate sketch.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[86] incorrect procedure.

a [2] A correct sketch is drawn that shows two possible locations, such as parallel lines and a perpendicular bisector. Students can draw their own sketch or use the diagram given.

[1] A correct sketch is drawn, but with no indication of where the treasure is buried.

or [1] A partial sketch is drawn, showing either the distances from the fence or the distance from the trees.

b [1] 5 feet

or [1] An appropriate answer is found for an incorrect part a.

a and b [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an

[87] obviously incorrect procedure.

[4] Two Xs are indicated at the intersections of the angle bisector and the parallel lines in the correct sketch of the loci.

[3] All loci are drawn correctly, but no Xs are drawn to indicate the locations, or only one X is drawn.

or [3] The angle bisector is drawn correctly, but only one line is drawn parallel to the walkway, but an X is indicated appropriately.

[2] Only one correct locus is drawn, but Xs indicate the two appropriate locations of the intersection of the loci.

[1] Xs are drawn in the correct locations, but no loci are shown.

or [1] Only one correct locus is drawn, and no Xs are indicated.

or [1] Both loci are drawn incorrectly, but Xs are drawn on the appropriate points of intersection.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[88] incorrect procedure.

[2] The points D and M are plotted, the graph of the line $x = 3$ is drawn, and its equation is stated.

[1] One graphing error is made, but an appropriate equation is stated for the locus of points.

or [1] A correct graph is drawn, but the equation is not stated or is stated incorrectly.

or [1] $x = 3$, but no graph is drawn.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[89] incorrect procedure.

[3] 4, and an appropriate sketch is drawn that shows a circle with C as its center and a radius of 6 and two parallel lines, one 3 units above and one 3 units below line p .

[2] An appropriate sketch is drawn, but the answer 4 is not found.

or [2] Only one locus is drawn correctly, but the appropriate number of points of intersection is found.

[1] Only one locus is drawn correctly, and no further correct work is shown.

or [1] Both loci are drawn incorrectly, but the appropriate number of points of intersection is found.

or [1] 4, but no work or sketch is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[90] incorrect procedure.

a [3] Two parallel lines, one 3 units above and one 3 units below \overline{AB} , and a circle with its center at P with a radius of 5 units are described correctly in words or drawn.

[2] Only one parallel line 3 units above or 3 units below \overline{AB} and a correct circle are described in words or drawn.

or [2] Appropriate parallel lines are shown, but the circle is incomplete.

[1] Both parallel lines and the circle have incomplete descriptions or drawings.

[0] Only one incomplete locus is described or drawn.

b [1] 4, and appropriate work is shown.

or [1] An appropriate answer for an incorrect part a is found.

a and b [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct

response that was obtained by an obviously

[91] incorrect procedure.
