## JMAP REGENTS BY DATE

NY Algebra II Regents Exam Questions from Spring 2015 to January 2018 Sorted by Date www.jmap.org

## 2015 Algebra II Common Core State Standards Sample Items

1 If $a, b$, and $c$ are all positive real numbers, which graph could represent the sketch of the graph of $p(x)=-a(x+b)\left(x^{2}-2 c x+c^{2}\right)$ ?
1)

2)

3)

4)


2 Which equation represents a parabola with a focus of $(0,4)$ and a directrix of $y=2$ ?

1) $y=x^{2}+3$
2) $y=-x^{2}+1$
3) $y=\frac{x^{2}}{2}+3$
4) $y=\frac{x^{2}}{4}+3$

3 If the terminal side of angle $\theta$, in standard position, passes through point $(-4,3)$, what is the numerical value of $\sin \theta$ ?

1) $\frac{3}{5}$
2) $\frac{4}{5}$
3) $-\frac{3}{5}$
4) $-\frac{4}{5}$

4 A study of the annual population of the red-winged blackbird in Ft. Mill, South Carolina, shows the population, $B(t)$, can be represented by the function $B(t)=750(1.16)^{t}$, where the $t$ represents the number of years since the study began. In terms of the monthly rate of growth, the population of red-winged blackbirds can be best approximated by the function

1) $B(t)=750(1.012)^{t}$
2) $B(t)=750(1.012)^{12 t}$
3) $B(t)=750(1.16)^{12 t}$
4) $B(t)=750(1.16)^{\frac{t}{12}}$

5 Use the properties of rational exponents to determine the value of $y$ for the equation:

$$
\frac{\sqrt[3]{x^{8}}}{\left(x^{4}\right)^{\frac{1}{3}}}=x^{y}, x>1
$$

6 Write $(5+2 y i)(4-3 i)-(5-2 y i)(4-3 i)$ in $a+b i$ form, where $y$ is a real number.

7 Use an appropriate procedure to show that $x-4$ is a factor of the function $f(x)=2 x^{3}-5 x^{2}-11 x-4$. Explain your answer.

8 Solve algebraically for all values of $x: \sqrt{x-5}+x=7$
9 Monthly mortgage payments can be found using the formula below:

$$
\begin{gathered}
M=\frac{P\left(\frac{r}{12}\right)\left(1+\frac{r}{12}\right)^{n}}{\left(1+\frac{r}{12}\right)^{n}-1} \\
M=\text { monthly payment } \\
P=\text { amount borrowed } \\
r=\text { annual interest rate } \\
n=\text { number of monthly payments }
\end{gathered}
$$

The Banks family would like to borrow $\$ 120,000$ to purchase a home. They qualified for an annual interest rate of $4.8 \%$. Algebraically determine the fewest number of whole years the Banks family would need to include in the mortgage agreement in order to have a monthly payment of no more than $\$ 720$.

10 Solve the following system of equations algebraically for all values of $x, y$, and $z$ :

$$
\begin{gathered}
x+3 y+5 z=45 \\
6 x-3 y+2 z=-10 \\
-2 x+3 y+8 z=72
\end{gathered}
$$

11 Write an explicit formula for $a_{n}$, the $n$th term of the recursively defined sequence below.

$$
\begin{aligned}
& a_{1}=x+1 \\
& a_{n}=x\left(a_{n-1}\right)
\end{aligned}
$$

For what values of $x$ would $a_{n}=0$ when $n>1$ ?

12 Stephen's Beverage Company is considering whether to produce a new brand of cola. The company will launch the product if at least $25 \%$ of cola drinkers will buy the product. Fifty cola drinkers are randomly selected to take a blind taste-test of products $A, B$, and the new product. Nine out of fifty participants preferred Stephen's new cola to products $A$ and $B$. The company then devised a simulation based on the requirement that $25 \%$ of cola drinkers will buy the product. Each dot in the graph shown below represents the proportion of people who preferred Stephen's new product, each of sample size 50, simulated 100 times.


Assume the set of data is approximately normal and the company wants to be $95 \%$ confident of its results. Does the sample proportion obtained from the blind taste-test, nine out of fifty, fall within the margin of error developed from the simulation? Justify your answer. The company decides to continue developing the product even though only nine out of fifty participants preferred its brand of cola in the taste-test. Describe how the simulation data could be used to support this decision.

13 In contract negotiations between a local government agency and its workers, it is estimated that there is a $50 \%$ chance that an agreement will be reached on the salaries of the workers. It is estimated that there is a $70 \%$ chance that there will be an agreement on the insurance benefits. There is a $20 \%$ chance that no agreement will be reached on either issue. Find the probability that an agreement will be reached on both issues. Based on this answer, determine whether the agreement on salaries and the agreement on insurance are independent events. Justify your answer.

14 The ocean tides near Carter Beach follow a repeating pattern over time, with the amount of time between each low and high tide remaining relatively constant. On a certain day, low tide occurred at 8:30 a.m. and high tide occurred at 3:00 p.m. At high tide, the water level was 12 inches above the average local sea level; at low tide it was 12 inches below the average local sea level. Assume that high tide and low tide are the maximum and minimum water levels each day, respectively. Write a cosine function of the form $f(t)=A \cos (B t)$, where $A$ and $B$ are real numbers, that models the water level, $f(t)$, in inches above or below the average Carter Beach sea level, as a function of the time measured in $t$ hours since 8:30 a.m. On the grid below, graph one cycle of this function.


People who fish in Carter Beach know that a certain species of fish is most plentiful when the water level is increasing. Explain whether you would recommend fishing for this species at 7:30 p.m. or 10:30 p.m. using evidence from the given context.

15 What is the solution set of the equation $\frac{3 x+25}{x+7}-5=\frac{3}{x}$ ?

1) $\left\{\frac{3}{2}, 7\right\}$
2) $\left\{\frac{7}{2},-3\right\}$
3) $\left\{-\frac{3}{2}, 7\right\}$
4) $\left\{-\frac{7}{2},-3\right\}$

Algebra II Sample Items 2015
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16 Functions $f, g$, and $h$ are given below.

$$
\begin{aligned}
& f(x)=\sin (2 x) \\
& g(x)=f(x)+1
\end{aligned}
$$



Which statement is true about functions $f, g$, and $h$ ?

1) $f(x)$ and $g(x)$ are odd, $h(x)$ is even.
2) $f(x)$ and $g(x)$ are even, $h(x)$ is odd.
3) $f(x)$ is odd, $g(x)$ is neither, $h(x)$ is even.
4) $f(x)$ is even, $g(x)$ is neither, $h(x)$ is odd.

17 The expression $\frac{6 x^{3}+17 x^{2}+10 x+2}{2 x+3}$ equals

1) $3 x^{2}+4 x-1+\frac{5}{2 x+3}$
2) $6 x^{2}+8 x-2+\frac{5}{2 x+3}$
3) $6 x^{2}-x+13-\frac{37}{2 x+3}$
4) $3 x^{2}+13 x+\frac{49}{2}+\frac{151}{2 x+3}$

18 The solutions to the equation $-\frac{1}{2} x^{2}=-6 x+20$ are

1) $-6 \pm 2 i$
2) $-6 \pm 2 \sqrt{19}$
3) $6 \pm 2 i$
4) $6 \pm 2 \sqrt{19}$

19 What is the completely factored form of $k^{4}-4 k^{2}+8 k^{3}-32 k+12 k^{2}-48$ ?

1) $(k-2)(k-2)(k+3)(k+4)$
2) $(k-2)(k-2)(k+6)(k+2)$
3) $(k+2)(k-2)(k+3)(k+4)$
4) $(k+2)(k-2)(k+6)(k+2)$

20 Which statement is incorrect for the graph of the function $y=-3 \cos \left[\frac{\pi}{3}(x-4)\right]+7$ ?

1) The period is 6 .
2) The range is $[4,10]$.
3) The amplitude is 3 .
4) The midline is $y=-4$.

21 Algebraically determine the values of $x$ that satisfy the system of equations below.

$$
\begin{aligned}
& y=-2 x+1 \\
& y=-2 x^{2}+3 x+1
\end{aligned}
$$

22 The results of a poll of 200 students are shown in the table below:

|  | Preferred Music Style |  |  |
| :---: | :---: | :---: | :---: |
|  | Techno | Rap | Country |
| Female | 54 | 25 | 27 |
| Male | 36 | 40 | 18 |

For this group of students, do these data suggest that gender and preferred music styles are independent of each other? Justify your answer.

23 For the function $f(x)=(x-3)^{3}+1$, find $f^{-1}(x)$.
24 Given: $h(x)=\frac{2}{9} x^{3}+\frac{8}{9} x^{2}-\frac{16}{13} x+2$
$k(x)=-|0.7 x|+5$
State the solutions to the equation $h(x)=k(x)$, rounded to the nearest hundredth.
25 Algebraically prove that the difference of the squares of any two consecutive integers is an odd integer.
26 Rewrite the expression $\left(4 x^{2}+5 x\right)^{2}-5\left(4 x^{2}+5 x\right)-6$ as a product of four linear factors.

27 After sitting out of the refrigerator for a while, a turkey at room temperature $\left(68^{\circ} \mathrm{F}\right)$ is placed into an oven at 8 a.m., when the oven temperature is $325^{\circ} \mathrm{F}$. Newton's Law of Heating explains that the temperature of the turkey will increase proportionally to the difference between the temperature of the turkey and the temperature of the oven, as given by the formula below:

$$
T=T_{a}+\left(T_{0}-T_{a}\right) e^{-k t}
$$

$T_{a}=$ the temperature surrounding the object
$T_{0}=$ the initial temperature of the object
$t=$ the time in hours
$T=$ the temperature of the object after $t$ hours
$k=$ decay constant
The turkey reaches the temperature of approximately $100^{\circ} \mathrm{F}$ after 2 hours. Find the value of $k$, to the nearest thousandth, and write an equation to determine the temperature of the turkey after $t$ hours. Determine the Fahrenheit temperature of the turkey, to the nearest degree, at 3 p.m.

28 Seventy-two students are randomly divided into two equally-sized study groups. Each member of the first group (group 1) is to meet with a tutor after school twice each week for one hour. The second group (group 2), is given an online subscription to a tutorial account that they can access for a maximum of two hours each week. Students in both groups are given the same tests during the year. A summary of the two groups' final grades is shown below:

|  | Group 1 | Group 2 |
| :---: | :---: | :---: |
| $\overline{\mathrm{x}}$ | 80.16 | 83.8 |
| $S_{\mathrm{x}}$ | 6.9 | 5.2 |

Calculate the mean difference in the final grades (group 1-group 2) and explain its meaning in the context of the problem. A simulation was conducted in which the students’ final grades were rerandomized 500 times. The results are shown below.


Use the simulation to determine if there is a significant difference in the final grades. Explain your answer.

29 Given $z(x)=6 x^{3}+b x^{2}-52 x+15, z(2)=35$, and $z(-5)=0$, algebraically determine all the zeros of $z(x)$.
30 Two versions of a standardized test are given, an April version and a May version. The statistics for the April version show a mean score of 480 and a standard deviation of 24 . The statistics for the May version show a mean score of 510 and a standard deviation of 20. Assume the scores are normally distributed. Joanne took the April version and scored in the interval 510-540. What is the probability, to the nearest ten thousandth, that a test paper selected at random from the April version scored in the same interval? Maria took the May version. In what interval must Maria score to claim she scored as well as Joanne?

31 Titanium-44 is a radioactive isotope such that every 63 years, its mass decreases by half. For a sample of titanium- 44 with an initial mass of 100 grams, write a function that will give the mass of the sample remaining after any amount of time. Define all variables. Scientists sometimes use the average yearly decrease in mass for estimation purposes. Use the average yearly decrease in mass of the sample between year 0 and year 10 to predict the amount of the sample remaining after 40 years. Round your answer to the nearest tenth. Is the actual mass of the sample or the estimated mass greater after 40 years? Justify your answer.

## 0616AII Common Core State Standards

1 When $b>0$ and $d$ is a positive integer, the expression (3b) ${ }^{\frac{2}{d}}$ is equivalent to

1) $\frac{1}{(\sqrt[d]{3 b})^{2}}$
2) $(\sqrt{3 b})^{d}$
3) $\frac{1}{\sqrt{3 b^{d}}}$
4) $(\sqrt[d]{3 b})^{2}$

2 Julie averaged 85 on the first three tests of the semester in her mathematics class. If she scores 93 on each of the remaining tests, her average will be 90 . Which equation could be used to determine how many tests, $T$, are left in the semester?

1) $\frac{255+93 T}{3 T}=90$
2) $\frac{255+90 T}{3 T}=93$
3) $\frac{255+93 T}{T+3}=90$
4) $\frac{255+90 T}{T+3}=93$

3 Given $i$ is the imaginary unit, $(2-y i)^{2}$ in simplest form is

1) $y^{2}-4 y i+4$
2) $-y^{2}-4 y i+4$
3) $-y^{2}+4$
4) $y^{2}+4$

4 Which graph has the following characteristics?

- three real zeros
- as $x \rightarrow-\infty, f(x) \rightarrow-\infty$
- as $x \rightarrow \infty, f(x) \rightarrow \infty$

1) 


3)

4)

2)


5 The solution set for the equation $\sqrt{56-x}=x$ is

1) $\{-8,7\}$
2) $\{-7,8\}$
3) $\{7\}$
4) $\}$

6 The zeros for $f(x)=x^{4}-4 x^{3}-9 x^{2}+36 x$ are

1) $\{0, \pm 3,4\}$
2) $\{0,3,4\}$
3) $\{0, \pm 3,-4\}$
4) $\{0,3,-4\}$

7 Anne has a coin. She does not know if it is a fair coin. She flipped the coin 100 times and obtained 73 heads and 27 tails. She ran a computer simulation of 200 samples of 100 fair coin flips. The output of the proportion of heads is shown below.


Given the results of her coin flips and of her computer simulation, which statement is most accurate?

1) 73 of the computer's next 100 coin flips
2) Her coin is not fair. will be heads.
3) 50 of her next 100 coin flips will be
4) Her coin is fair. heads.

8 If $g(c)=1-c^{2}$ and $m(c)=c+1$, then which statement is not true?

1) $g(c) \cdot m(c)=1+c-c^{2}-c^{3}$
2) $g(c)+m(c)=2+c-c^{2}$
3) $m(c)-g(c)=c+c^{2}$
4) $\frac{m(c)}{g(c)}=\frac{-1}{1-c}$

9 The heights of women in the United States are normally distributed with a mean of 64 inches and a standard deviation of 2.75 inches. The percent of women whose heights are between 64 and 69.5 inches, to the nearest whole percent, is

1) 6
2) 48
3) 68
4) 95

10 The formula below can be used to model which scenario?

$$
\begin{aligned}
& a_{1}=3000 \\
& a_{n}=0.80 a_{n-1}
\end{aligned}
$$

1) The first row of a stadium has 3000 seats, and each row thereafter has 80 more seats than the row in front of it.
2) The last row of a stadium has 3000 seats, and each row before it has 80 fewer seats than the row behind it.
3) A bank account starts with a deposit of $\$ 3000$, and each year it grows by $80 \%$.
4) The initial value of a specialty toy is $\$ 3000$, and its value each of the following years is $20 \%$ less.

11 Sean's team has a baseball game tomorrow. He pitches $50 \%$ of the games. There is a $40 \%$ chance of rain during the game tomorrow. If the probability that it rains given that Sean pitches is $40 \%$, it can be concluded that these two events are

1) independent
2) mutually exclusive
3) dependent
4) complements

12 A solution of the equation $2 x^{2}+3 x+2=0$ is

1) $-\frac{3}{4}+\frac{1}{4} i \sqrt{7}$
2) $-\frac{3}{4}+\frac{1}{4} i$
3) $-\frac{3}{4}+\frac{1}{4} \sqrt{7}$
4) $\frac{1}{2}$

13 The Ferris wheel at the landmark Navy Pier in Chicago takes 7 minutes to make one full rotation. The height, $H$, in feet, above the ground of one of the six-person cars can be modeled by $H(t)=70 \sin \left(\frac{2 \pi}{7}(t-1.75)\right)+80$, where $t$ is time, in minutes. Using $H(t)$ for one full rotation, this car's minimum height, in feet, is

1) 150
2) 70
3) 10
4) 0

14 The expression $\frac{4 x^{3}+5 x+10}{2 x+3}$ is equivalent to

1) $2 x^{2}+3 x-7+\frac{31}{2 x+3}$
2) $2 x^{2}-3 x+7-\frac{11}{2 x+3}$
3) $2 x^{2}+2.5 x+5+\frac{15}{2 x+3}$
4) $2 x^{2}-2.5 x-5-\frac{20}{2 x+3}$

15 Which function represents exponential decay?

1) $y=2^{0.3 t}$
2) $y=1.2^{3 t}$
3) $y=\left(\frac{1}{2}\right)^{-t}$
4) $y=5^{-t}$

16 Given $f^{-1}(x)=-\frac{3}{4} x+2$, which equation represents $f(x)$ ?

1) $f(x)=\frac{4}{3} x-\frac{8}{3}$
2) $f(x)=-\frac{4}{3} x+\frac{8}{3}$
3) $f(x)=\frac{3}{4} x-2$
4) $f(x)=-\frac{3}{4} x+2$

17 A circle centered at the origin has a radius of 10 units. The terminal side of an angle, $\theta$, intercepts the circle in Quadrant II at point $C$. The $y$-coordinate of point $C$ is 8 . What is the value of $\cos \theta$ ?

1) $-\frac{3}{5}$
2) $-\frac{3}{4}$
3) $\frac{3}{5}$
4) $\frac{4}{5}$

18 Which statement about the graph of $c(x)=\log _{6} x$ is false?

1) The asymptote has equation $y=0$.
2) The domain is the set of positive reals.
3) The graph has no $y$-intercept.
4) The range is the set of all real numbers.

19 The equation $4 x^{2}-24 x+4 y^{2}+72 y=76$ is equivalent to

1) $4(x-3)^{2}+4(y+9)^{2}=76$
2) $4(x-3)^{2}+4(y+9)^{2}=121$
3) $4(x-3)^{2}+4(y+9)^{2}=166$
4) $4(x-3)^{2}+4(y+9)^{2}=436$

20 There was a study done on oxygen consumption of snails as a function of pH , and the result was a degree 4 polynomial function whose graph is shown below.


Which statement about this function is incorrect?

1) The degree of the polynomial is even. 3) At two pH values, there is a relative maximum value.
2) There is a positive leading coefficient.
3) There are two intervals where the function is decreasing.

21 Last year, the total revenue for Home Style, a national restaurant chain, increased $5.25 \%$ over the previous year. If this trend were to continue, which expression could the company's chief financial officer use to approximate their monthly percent increase in revenue? [Let $m$ represent months.]

1) $(1.0525)^{m}$
2) $(1.0525)^{\frac{12}{m}}$
3) $(1.00427)^{m}$
4) $(1.00427)^{\frac{m}{12}}$

22 Which value, to the nearest tenth, is not a solution of $p(x)=q(x)$ if $p(x)=x^{3}+3 x^{2}-3 x-1$ and $q(x)=3 x+8$ ?

1) -3.9
2) -1.1
3) 2.1
4) 4.7

23 The population of Jamesburg for the years 2010-2013, respectively, was reported as follows: $250,000 \quad 250,937 \quad 251,878 \quad 252,822$
How can this sequence be recursively modeled?

1) $j_{n}=250,000(1.00375)^{n-1}$
2) $j_{1}=250,000$
$j_{n}=1.00375 j_{n-1}$
3) $j_{n}=250,000+937^{(n-1)}$
4) $j_{1}=250,000$
$j_{n}=j_{n-1}+937$

24 The voltage used by most households can be modeled by a sine function. The maximum voltage is 120 volts, and there are 60 cycles every second. Which equation best represents the value of the voltage as it flows through the electric wires, where $t$ is time in seconds?

1) $V=120 \sin (t)$
2) $V=120 \sin (60 t)$
3) $V=120 \sin (60 \pi t)$
4) $V=120 \sin (120 \pi t)$

25 Solve for $x$ : $\frac{1}{x}-\frac{1}{3}=-\frac{1}{3 x}$
26 Describe how a controlled experiment can be created to examine the effect of ingredient $X$ in a toothpaste.
27 Determine if $x-5$ is a factor of $2 x^{3}-4 x^{2}-7 x-10$. Explain your answer.

28 On the axes below, graph one cycle of a cosine function with amplitude 3 , period $\frac{\pi}{2}$, midline $y=-1$, and passing through the point $(0,2)$.


29 A suburban high school has a population of 1376 students. The number of students who participate in sports is 649. The number of students who participate in music is 433 . If the probability that a student participates in either sports or music is $\frac{974}{1376}$, what is the probability that a student participates in both sports and music?

30 The directrix of the parabola $12(y+3)=(x-4)^{2}$ has the equation $y=-6$. Find the coordinates of the focus of the parabola.

31 Algebraically prove that $\frac{x^{3}+9}{x^{3}+8}=1+\frac{1}{x^{3}+8}$, where $x \neq-2$.

32 A house purchased 5 years ago for $\$ 100,000$ was just sold for $\$ 135,000$. Assuming exponential growth, approximate the annual growth rate, to the nearest percent.

33 Solve the system of equations shown below algebraically.

$$
\begin{aligned}
& (x-3)^{2}+(y+2)^{2}=16 \\
& 2 x+2 y=10
\end{aligned}
$$

34 Alexa earns $\$ 33,000$ in her first year of teaching and earns a $4 \%$ increase in each successive year. Write a geometric series formula, $S_{n}$, for Alexa's total earnings over $n$ years. Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the nearest cent.

35 Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have $50 \%$ for the DJ and $50 \%$ for the band. A simulation was run 200 times, each of sample size 55 , based on the premise that $60 \%$ of the students would prefer a DJ. The approximate normal simulation results are shown below.


Using the results of the simulation, determine a plausible interval containing the middle $95 \%$ of the data. Round all values to the nearest hundredth. Members of the prom committee are concerned that a vote of all students attending the prom may produce a $50 \%$ - $50 \%$ split. Explain what statistical evidence supports this concern.

36
Which function shown below has a greater average rate of change on the interval $[-2,4]$ ? Justify your answer.

| $\mathbf{x}$ | $\mathbf{f}(\mathbf{x})$ |
| :---: | :---: |
| -4 | 0.3125 |
| -3 | 0.625 |
| -2 | 1.25 |
| -1 | 2.5 |
| 0 | 5 |
| 1 | 10 |
| 2 | 20 |
| 3 | 40 |
| 4 | 80 |
| 5 | 160 |
| 6 | 320 |

$$
g(x)=4 x^{3}-5 x^{2}+3
$$

37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t)=N_{0}(e)^{-r t}$, where $N(t)$ is the amount left in the body, $N_{0}$ is the initial dosage, $r$ is the decay rate, and $t$ is time in hours. Patient $A, A(t)$, is given 800 milligrams of a drug with a decay rate of 0.347 . Patient $B, B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231 . Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient. Graph each function on the set of axes below.


To the nearest hour, $t$, when does the amount of the given drug remaining in patient $B$ begin to exceed the amount of the given drug remaining in patient $A$ ? The doctor will allow patient $A$ to take another 800 milligram dose of the drug once only $15 \%$ of the original dose is left in the body. Determine, to the nearest tenth of an hour, how long patient $A$ will have to wait to take another 800 milligram dose of the drug.

## 0816AII Common Core State Standards

1 Which equation has $1-i$ as a solution?

1) $x^{2}+2 x-2=0$
2) $x^{2}+2 x+2=0$
3) $x^{2}-2 x-2=0$
4) $x^{2}-2 x+2=0$

2 Which statement(s) about statistical studies is true?
I. A survey of all English classes in a high school would be a good sample to determine the number of hours students throughout the school spend studying.
II. A survey of all ninth graders in a high school would be a good sample to determine the number of student parking spaces needed at that high school.
III. A survey of all students in one lunch period in a high school would be a good sample to determine the number of hours adults spend on social media websites.
IV. A survey of all Calculus students in a high school would be a good sample to determine the number of students throughout the school who don't like math.

1) I, only
2) I and III
3) II, only
4) III and IV

3 To the nearest tenth, the value of $x$ that satisfies $2^{x}=-2 x+11$ is

1) 2.5
2) 2.6
3) 5.8
4) 5.9

4 The lifespan of a 60 -watt lightbulb produced by a company is normally distributed with a mean of 1450 hours and a standard deviation of 8.5 hours. If a 60 -watt lightbulb produced by this company is selected at random, what is the probability that its lifespan will be between 1440 and 1465 hours?

1) 0.3803
2) 0.4612
3) 0.8415
4) 0.9612

5 Which factorization is incorrect?

1) $4 k^{2}-49=(2 k+7)(2 k-7)$
2) $a^{3}-8 b^{3}=(a-2 b)\left(a^{2}+2 a b+4 b^{2}\right)$
3) $m^{3}+3 m^{2}-4 m+12=(m-2)^{2}(m+3)$
4) $t^{3}+5 t^{2}+6 t+t^{2}+5 t+6=(t+1)(t+2)(t+\varepsilon$

6 Sally's high school is planning their spring musical. The revenue, $R$, generated can be determined by the function $R(t)=-33 t^{2}+360 t$, where $t$ represents the price of a ticket. The production cost, $C$, of the musical is represented by the function $C(t)=700+5 t$. What is the highest ticket price, to the nearest dollar, they can charge in order to not lose money on the event?

1) $t=3$
2) $t=5$
3) $t=8$
4) $t=11$

7 The set of data in the table below shows the results of a survey on the number of messages that people of different ages text on their cell phones each month.

| Age Group | Text Messages per Month |  |  |
| :---: | :---: | :---: | :---: |
|  | $0-10$ | $11-50$ | Over 50 |
| $15-18$ | 4 | 37 | 68 |
| $19-22$ | 6 | 25 | 87 |
| $23-60$ | 25 | 47 | 157 |

If a person from this survey is selected at random, what is the probability that the person texts over 50 messages per month given that the person is between the ages of 23 and 60 ?

1) $\frac{157}{229}$
2) $\frac{157}{312}$
3) $\frac{157}{384}$
4) $\frac{157}{456}$

8 A recursive formula for the sequence $18,9,4.5, \ldots$ is

1) $g_{1}=18$
2) $g_{1}=18$

$$
g_{n}=\frac{1}{2} g_{n-1}
$$

$$
g_{n}=2 g_{n-1}
$$

2) $g_{n}=18\left(\frac{1}{2}\right)^{n-1}$
3) $g_{n}=18(2)^{n-1}$

9 Kristin wants to increase her running endurance. According to experts, a gradual mileage increase of $10 \%$ per week can reduce the risk of injury. If Kristin runs 8 miles in week one, which expression can help her find the total number of miles she will have run over the course of her 6 -week training program?

1) $\sum_{n=1}^{6} 8(1.10)^{n-1}$
2) $\sum_{n=1}^{6} 8(1.10)^{n}$
3) $\frac{8-8(1.10)^{6}}{0.90}$
4) $\frac{8-8(0.10)^{n}}{1.10}$

10 A sine function increasing through the origin can be used to model light waves. Violet light has a wavelength of 400 nanometers. Over which interval is the height of the wave decreasing, only?

1) $(0,200)$
2) $(100,300)$
3) $(200,400)$
4) $(300,400)$

11 The expression $\frac{x^{3}+2 x^{2}+x+6}{x+2}$ is equivalent to

1) $x^{2}+3$
2) $x^{2}+1+\frac{4}{x+2}$
3) $2 x^{2}+x+6$
4) $2 x^{2}+1+\frac{4}{x+2}$

12 A candidate for political office commissioned a poll. His staff received responses from 900 likely voters and $55 \%$ of them said they would vote for the candidate. The staff then conducted a simulation of 1000 more polls of 900 voters, assuming that $55 \%$ of voters would vote for their candidate. The output of the simulation is shown in the diagram below.


Given this output, and assuming a $95 \%$ confidence level, the margin of error for the poll is closest to

1) 0.01
2) 0.03
3) 0.06
4) 0.12

13 An equation to represent the value of a car after $t$ months of ownership is $v=32,000(0.81)^{\frac{t}{12}}$. Which statement is not correct?

1) The car lost approximately $19 \%$ of its value each month.
2) The value of the car when it was purchased was $\$ 32,000$.
3) The car maintained approximately $98 \%$ of its value each month.
4) The value of the car 1 year after it was purchased was $\$ 25,920$.

14 Which equation represents an odd function?

1) $y=\sin x$
2) $y=\cos x$
3) $y=(x+1)^{3}$
4) $y=e^{5 x}$

15 The completely factored form of $2 d^{4}+6 d^{3}-18 d^{2}-54 d$ is

1) $2 d\left(d^{2}-9\right)(d+3)$
2) $2 d\left(d^{2}+9\right)(d+3)$
3) $2 d(d+3)^{2}(d-3)$
4) $2 d(d-3)^{2}(d+3)$

Algebra II Regents Exam 0816
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16 Which diagram shows an angle rotation of 1 radian on the unit circle?
1)

3)

4)



17 The focal length, $F$, of a camera's lens is related to the distance of the object from the lens, $J$, and the distance to the image area in the camera, $W$, by the formula below.

$$
\frac{1}{J}+\frac{1}{W}=\frac{1}{F}
$$

When this equation is solved for $J$ in terms of $F$ and $W, J$ equals

1) $F-W$
2) $\frac{F W}{F-W}$
3) $\frac{F W}{W-F}$
4) $\frac{1}{F}-\frac{1}{W}$

18 The sequence $a_{1}=6, a_{n}=3 a_{n-1}$ can also be written as

1) $a_{n}=6 \cdot 3^{n}$
2) $a_{n}=6 \cdot 3^{n+1}$
3) $a_{n}=2 \cdot 3^{n}$
4) $a_{n}=2 \cdot 3^{n+1}$

19 Which equation represents the set of points equidistant from line $\ell$ and point $R$ shown on the graph below?


1) $y=-\frac{1}{8}(x+2)^{2}+1$
2) $y=-\frac{1}{8}(x+2)^{2}-1$
3) $y=-\frac{1}{8}(x-2)^{2}+1$
4) $y=-\frac{1}{8}(x-2)^{2}-1$

20 Mr. Farison gave his class the three mathematical rules shown below to either prove or disprove. Which rules can be proved for all real numbers?

I $\quad(m+p)^{2}=m^{2}+2 m p+p^{2}$
II $\quad(x+y)^{3}=x^{3}+3 x y+y^{3}$
III $\quad\left(a^{2}+b^{2}\right)^{2}=\left(a^{2}-b^{2}\right)^{2}+(2 a b)^{2}$

1) I, only
2) II and III
3) I and II
4) I and III

21 The graph of $p(x)$ is shown below.


What is the remainder when $p(x)$ is divided by $x+4$ ?

1) $x-4$
2) -4
3) 0
4) 4

22 A payday loan company makes loans between $\$ 100$ and $\$ 1000$ available to customers. Every 14 days, customers are charged $30 \%$ interest with compounding. In 2013, Remi took out a $\$ 300$ payday loan. Which expression can be used to calculate the amount she would owe, in dollars, after one year if she did not make payments?

1) $300(.30)^{\frac{14}{365}}$
2) $300(1.30)^{\frac{14}{365}}$
3) $300(.30)^{\frac{365}{14}}$
4) $300(1.30)^{\frac{365}{14}}$

23 Which value is not contained in the solution of the system shown below?

$$
\begin{aligned}
& a+5 b-c=-20 \\
& 4 a-5 b+4 c=19 \\
& -a-5 b-5 c=2
\end{aligned}
$$

1) -2
2) 2
3) 3
4) -3

24 In 2010, the population of New York State was approximately $19,378,000$ with an annual growth rate of $1.5 \%$. Assuming the growth rate is maintained for a large number of years, which equation can be used to predict the population of New York State $t$ years after 2010?

1) $P_{t}=19,378,000(1.5)^{t}$
2) $P_{t}=19,378,000(1.015)^{t-1}$
3) $P_{0}=19,378,000$
$P_{t}=19,378,000+1.015 P_{t-1}$
4) $P_{0}=19,378,000$
$P_{t}=1.015 P_{t-1}$

25 The volume of air in a person's lungs, as the person breathes in and out, can be modeled by a sine graph. A scientist is studying the differences in this volume for people at rest compared to people told to take a deep breath. When examining the graphs, should the scientist focus on the amplitude, period, or midline? Explain your choice.

26 Explain how $\left(3^{\frac{1}{5}}\right)^{2}$ can be written as the equivalent radical expression $\sqrt[5]{9}$.

27 Simplify $x i(i-7 i)^{2}$, where $i$ is the imaginary unit.

28 Using the identity $\sin ^{2} \theta+\cos ^{2} \theta=1$, find the value of $\tan \theta$, to the nearest hundredth, if $\cos \theta$ is -0.7 and $\theta$ is in Quadrant II.

29 Elizabeth waited for 6 minutes at the drive thru at her favorite fast-food restaurant the last time she visited. She was upset about having to wait that long and notified the manager. The manager assured her that her experience was very unusual and that it would not happen again. A study of customers commissioned by this restaurant found an approximately normal distribution of results. The mean wait time was 226 seconds and the standard deviation was 38 seconds. Given these data, and using a $95 \%$ level of confidence, was Elizabeth's wait time unusual? Justify your answer.

30 The $x$-value of which function's $x$-intercept is larger, $f$ or $h$ ? Justify your answer.

$$
f(x)=\log (x-4)
$$

| $\mathbf{x}$ | $\mathbf{h}(\mathbf{x})$ |
| :---: | :---: |
| -1 | 6 |
| 0 | 4 |
| 1 | 2 |
| 2 | 0 |
| 3 | -2 |

31 The distance needed to stop a car after applying the brakes varies directly with the square of the car's speed. The table below shows stopping distances for various speeds.

| Speed (mph) | 10 | 20 | 30 | 40 | 50 | 60 | 70 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Distance (ft) | 6.25 | 25 | 56.25 | 100 | 156.25 | 225 | 306.25 |

Determine the average rate of change in braking distance, in $\mathrm{ft} / \mathrm{mph}$, between one car traveling at 50 mph and one traveling at 70 mph . Explain what this rate of change means as it relates to braking distance.

32 Given events $A$ and $B$, such that $P(A)=0.6, P(B)=0.5$, and $P(A \cup B)=0.8$, determine whether $A$ and $B$ are independent or dependent.

33 Find algebraically the zeros for $p(x)=x^{3}+x^{2}-4 x-4$. On the set of axes below, graph $y=p(x)$.


34 One of the medical uses of Iodine-131 (I-131), a radioactive isotope of iodine, is to enhance x-ray images. The half-life of I-131 is approximately 8.02 days. A patient is injected with 20 milligrams of I-131. Determine, to the nearest day, the amount of time needed before the amount of I-131 in the patient's body is approximately 7 milligrams.

35 Solve the equation $\sqrt{2 x-7}+x=5$ algebraically, and justify the solution set.

Ayva designed an experiment to determine the effect of a new energy drink on a group of 20 volunteer students. Ten students were randomly selected to form group 1 while the remaining 10 made up group 2 . Each student in group 1 drank one energy drink, and each student in group 2 drank one cola drink. Ten minutes later, their times were recorded for reading the same paragraph of a novel. The results of the experiment are shown below.

| Group 1 <br> (seconds) | Group 2 <br> (seconds) |
| :---: | :---: |
| 17.4 | 23.3 |
| 18.1 | 18.8 |
| 18.2 | 22.1 |
| 19.6 | 12.7 |
| 18.6 | 16.9 |
| 16.2 | 24.4 |
| 16.1 | 21.2 |
| 15.3 | 21.2 |
| 17.8 | 16.3 |
| 19.7 | 14.5 |
| Mean $=17.7$ | Mean $=19.1$ |

Ayva thinks drinking energy drinks makes students read faster. Using information from the experimental design or the results, explain why Ayva's hypothesis may be incorrect. Using the given results, Ayva randomly mixes the 20 reading times, splits them into two groups of 10 , and simulates the difference of the means 232 times.


Ayva has decided that the difference in mean reading times is not an unusual occurrence. Support her decision using the results of the simulation. Explain your reasoning.

37 Seth's parents gave him $\$ 5000$ to invest for his 16th birthday. He is considering two investment options. Option $A$ will pay him $4.5 \%$ interest compounded annually. Option $B$ will pay him $4.6 \%$ compounded quarterly. Write a function of option $A$ and option $B$ that calculates the value of each account after $n$ years. Seth plans to use the money after he graduates from college in 6 years. Determine how much more money option $B$ will earn than option A to the nearest cent. Algebraically determine, to the nearest tenth of a year, how long it would take for option $B$ to double Seth's initial investment.

## 0117AII Common Core State Standards

1 Relative to the graph of $y=3 \sin x$, what is the shift of the graph of $y=3 \sin \left(x+\frac{\pi}{3}\right)$ ?

1) $\frac{\pi}{3}$ right
2) $\frac{\pi}{3}$ left
3) $\frac{\pi}{3}$ up
4) $\frac{\pi}{3}$ down

2 A rabbit population doubles every 4 weeks. There are currently five rabbits in a restricted area. If $t$ represents the time, in weeks, and $P(t)$ is the population of rabbits with respect to time, about how many rabbits will there be in 98 days?

1) 56
2) 152
3) 3688
4) 81,920

3 Factored completely, $m^{5}+m^{3}-6 m$ is equivalent to

1) $(m+3)(m-2)$
2) $\left(m^{2}+3 m\right)\left(m^{2}-2\right)$
3) $m\left(m^{4}+m^{2}-6\right)$
4) $m\left(m^{2}+3\right)\left(m^{2}-2\right)$

4 If $\sin ^{2}\left(32^{\circ}\right)+\cos ^{2}(M)=1$, then $M$ equals

1) $32^{\circ}$
2) $58^{\circ}$
3) $68^{\circ}$
4) $72^{\circ}$

5 What is the solution to the system of equations $y=3 x-2$ and $y=g(x)$ where $g(x)$ is defined by the function below?


1) $\{(0,-2)\}$
2) $\{(0,-2),(1,6)\}$
3) $\{(1,6)\}$
4) $\{(1,1),(6,16)\}$

6 Which statement about statistical analysis is false?

1) Experiments can suggest patterns and relationships in data.
2) Experiments can determine cause and effect relationships.
3) Observational studies can determine cause and effect relationships.
4) Observational studies can suggest patterns and relationships in data.

7 The expression $\left(\frac{m^{2}}{m^{\frac{1}{3}}}\right)^{-\frac{1}{2}}$ is equivalent to

1) $-\sqrt[6]{m^{5}}$
2) $\frac{1}{\sqrt[6]{m^{5}}}$
3) $-m \sqrt[5]{m}$
4) $\frac{1}{m \sqrt[5]{m}}$

8 What is the inverse of the function $y=\log _{3} x$ ?

1) $y=x^{3}$
2) $y=\log _{x} 3$
3) $y=3^{x}$
4) $x=3^{y}$

9 Gabriel performed an experiment to see if planting 13 tomato plants in black plastic mulch leads to larger tomatoes than if 13 plants are planted without mulch. He observed that the average weight of the tomatoes from tomato plants grown in black plastic mulch was 5 ounces greater than those from the plants planted without mulch. To determine if the observed difference is statistically significant, he rerandomized the tomato groups 100 times to study these random differences in the mean weights. The output of his simulation is summarized in the dotplot below.


## Differences in Mean Weight (oz.)

Given these results, what is an appropriate inference that can be drawn?

1) There was no effect observed between the two groups.
2) There was an effect observed that could be due to the random assignment of plants to the groups.
3) There is strong evidence to support the hypothesis that tomatoes from plants planted in black plastic mulch are larger than those planted without mulch.
4) There is strong evidence to support the hypothesis that tomatoes from plants planted without mulch are larger than those planted in black plastic mulch.

10 If $p(x)=a b^{x}$ and $r(x)=c d^{x}$, then $p(x) \bullet r(x)$ equals

1) $a c(b+d)^{x}$
2) $a c(b+d)^{2 x}$
3) $a c(b d)^{x}$
4) $a c(b d)^{x^{2}}$

11 The solution to the equation $18 x^{2}-24 x+87=0$ is

1) $-\frac{2}{3} \pm 6 i \sqrt{158}$
2) $-\frac{2}{3} \pm \frac{1}{6} i \sqrt{158}$
3) $\frac{2}{3} \pm 6 i \sqrt{158}$
4) $\frac{2}{3} \pm \frac{1}{6} i \sqrt{158}$

12 When $g(x)=\frac{2}{x+2}$ and $h(x)=\log (x+1)+3$ are graphed on the same set of axes, which coordinates best approximate their point of intersection?

1) $(-0.9,1.8)$
2) $(-0.9,1.9)$
3) $(1.4,3.3)$
4) $(1.4,3.4)$

13 The price of a postage stamp in the years since the end of World War I is shown in the scatterplot below.
Price of a Postage Stamp Since End of World War I


- Price of a postage stamp

The equation that best models the price, in cents, of a postage stamp based on these data is

1) $y=0.59 x-14.82$
2) $y=1.04(1.43)^{x}$
3) $y=1.43(1.04)^{x}$
4) $y=24 \sin (14 x)+25$

14 The eighth and tenth terms of a sequence are 64 and 100. If the sequence is either arithmetic or geometric, the ninth term can not be

1) -82
2) -80
3) 80
4) 82

15 The loudness of sound is measured in units called decibels (dB). These units are measured by first assigning an intensity $I_{0}$ to a very soft sound that is called the threshold sound. The sound to be measured is assigned an intensity, $I$, and the decibel rating, $d$, of this sound is found using $d=10 \log \frac{I}{I_{0}}$. The threshold sound audible to the average person is $1.0 \times 10^{-12} \mathrm{~W} / \mathrm{m}^{2}$ (watts per square meter). Consider the following sound level classifications:

| Moderate | $45-69 \mathrm{~dB}$ |
| :--- | :--- |
| Loud | $70-89 \mathrm{~dB}$ |
| Very loud | $90-109 \mathrm{~dB}$ |
| Deafening | $>110 \mathrm{~dB}$ |

How would a sound with intensity $6.3 \times 10^{-3} \mathrm{~W} / \mathrm{m}^{2}$ be classified?

1) moderate
2) very loud
3) loud
4) deafening

16 Pedro and Bobby each own an ant farm. Pedro starts with 100 ants and says his farm is growing exponentially at a rate of $15 \%$ per month. Bobby starts with 350 ants and says his farm is steadily decreasing by 5 ants per month. Assuming both boys are accurate in describing the population of their ant farms, after how many months will they both have approximately the same number of ants?

1) 7
2) 8
3) 13
4) 36

17 What is the solution, if any, of the equation $\frac{2}{x+3}-\frac{3}{4-x}=\frac{2 x-2}{x^{2}-x-12}$ ?

1) -1
2) all real numbers
3) -5
4) no real solution

18 In 2013, approximately 1.6 million students took the Critical Reading portion of the SAT exam. The mean score, the modal score, and the standard deviation were calculated to be 496, 430, and 115, respectively. Which interval reflects $95 \%$ of the Critical Reading scores?

1) $430 \pm 115$
2) $430 \pm 230$
3) $496 \pm 115$
4) $496 \pm 230$

19 Which statement regarding the graphs of the functions below is untrue?

$$
\begin{aligned}
& f(x)=3 \sin 2 x, \text { from }-\pi<x<\pi \\
& g(x)=(x-0.5)(x+4)(x-2) \\
& h(x)=\log _{2} x \\
& j(x)=-|4 x-2|+3
\end{aligned}
$$

1) $\quad f(x)$ and $j(x)$ have a maximum $y$-value of 3) $g(x)$ and $j(x)$ have the same end 3. behavior as $x \rightarrow-\infty$.
2) $\quad f(x), h(x)$, and $j(x)$ have one $y$-intercept.
3) $\quad g(x), h(x)$, and $j(x)$ have rational zeros.

20 When $g(x)$ is divided by $x+4$, the remainder is 0 . Given $g(x)=x^{4}+3 x^{3}-6 x^{2}-6 x+8$, which conclusion about $g(x)$ is true?

1) $g(4)=0$
2) $x-4$ is a factor of $g(x)$.
3) $g(-4)=0$
4) No conclusion can be made regarding $g(x)$.

21 Joelle has a credit card that has a $19.2 \%$ annual interest rate compounded monthly. She owes a total balance of $B$ dollars after $m$ months. Assuming she makes no payments on her account, the table below illustrates the balance she owes after $m$ months.

| $\mathbf{m}$ | $\mathbf{B}$ |
| :---: | :---: |
| 0 | 100.00 |
| 10 | 1172.00 |
| 19 | 1352.00 |
| 36 | 1770.80 |
| 60 | 2591.90 |
| 69 | 2990.00 |
| 72 | 3135.80 |
| 73 | 3186.00 |

Over which interval of time is her average rate of change for the balance on her credit card account the greatest?

1) month 10 to month 60
2) month 19 to month 69
3) month 36 to month 72
4) month 60 to month 73

22 Which graph represents a cosine function with no horizontal shift, an amplitude of 2 , and a period of $\frac{2 \pi}{3}$ ?
1)


2)
3)

4)


23 According to a pricing website, Indroid phones lose $58 \%$ of their cash value over 1.5 years. Which expression can be used to estimate the value of a $\$ 300$ Indroid phone in 1.5 years?

1) $300 e^{-0.87}$
2) $300 e^{-0.63}$
3) $300 e^{-0.58}$
4) $300 e^{-0.42}$

24 A cardboard box manufacturing company is building boxes with length represented by $x+1$, width by $5-x$, and height by $x-1$. The volume of the box is modeled by the function below.


Over which interval is the volume of the box changing at the fastest average rate?

1) $[1,2]$
2) $[1,3.5]$
3) $[1,5]$
4) $[0,3.5]$

25 Express $(1-i)^{3}$ in $a+b i$ form.
26 An orange-juice processing plant receives a truckload of oranges. The quality control team randomly chooses three pails of oranges, each containing 50 oranges, from the truckload. Identify the sample and the population in the given scenario. State one conclusion that the quality control team could make about the population if $5 \%$ of the sample was found to be unsatisfactory.

27 Using the unit circle below, explain why $\csc \theta=\frac{1}{y}$.


28 The function $M(t)$ represents the mass of radium over time, $t$, in years.

$$
M(t)=100 e^{\frac{\left(\ln \frac{1}{2}\right) t}{1590}}
$$

Determine if the function $M(t)$ represents growth or decay. Explain your reasoning.

29 On the grid below, sketch a cubic polynomial whose zeros are 1,3 , and -2 .


30 Given the equal terms $\sqrt[3]{x^{5}}$ and $y^{\frac{5}{6}}$, determine and state $y$, in terms of $x$.
31 The results of a survey of the student body at Central High School about television viewing preferences are shown below.

|  | Comedy Series | Drama Series | Reality Series | Total |
| :--- | :---: | :---: | :---: | :---: |
| Males | 95 | 65 | 70 | 230 |
| Females | 80 | 70 | 110 | 260 |
| Total | 175 | 135 | 180 | 490 |

Are the events "student is a male" and "student prefers reality series" independent of each other? Justify your answer.

32 Given $f(x)=3 x^{2}+7 x-20$ and $g(x)=x-2$, state the quotient and remainder of $\frac{f(x)}{g(x)}$, in the form $q(x)+\frac{r(x)}{g(x)}$.
33 Algebraically determine the values of $h$ and $k$ to correctly complete the identity stated below.

$$
2 x^{3}-10 x^{2}+11 x-7=(x-4)\left(2 x^{2}+h x+3\right)+k
$$

34 Elaina has decided to run the Buffalo half-marathon in May. She researched training plans on the Internet and is looking at two possible plans: Jillian's 12-week plan and Josh's 14 -week plan. The number of miles run per week for each plan is plotted below.


Which one of the plans follows an arithmetic pattern? Explain how you arrived at your answer. Write a recursive definition to represent the number of miles run each week for the duration of the plan you chose. Jillian's plan has an alternative if Elaina wanted to train instead for a full 26 -mile marathon. Week one would start at 13 miles and follow the same pattern for the half-marathon, but it would continue for 14 weeks. Write an explicit formula, in simplest form, to represent the number of miles run each week for the full-marathon training plan.

35 The guidance department has reported that of the senior class, $2.3 \%$ are members of key club, $K, 8.6 \%$ are enrolled in AP Physics, $P$, and $1.9 \%$ are in both. Determine the probability of $P$ given $K$, to the nearest tenth of a percent. The principal would like a basic interpretation of these results. Write a statement relating your calculated probabilities to student enrollment in the given situation.

36 Using the formula below, determine the monthly payment on a 5 -year car loan with a monthly percentage rate of $0.625 \%$ for a car with an original cost of $\$ 21,000$ and a $\$ 1000$ down payment, to the nearest cent.
$P_{n}=P M T\left(\frac{1-(1+i)^{-n}}{i}\right)$
$P_{n}=$ present amount borrowed
$n=$ number of monthly pay periods
$P M T=$ monthly payment
$i=$ interest rate per month
The affordable monthly payment is $\$ 300$ for the same time period. Determine an appropriate down payment, to the nearest dollar.

37 The speed of a tidal wave, $s$, in hundreds of miles per hour, can be modeled by the equation $s=\sqrt{t}-2 t+6$, where $t$ represents the time from its origin in hours. Algebraically determine the time when $s=0$. How much faster was the tidal wave traveling after 1 hour than 3 hours, to the nearest mile per hour? Justify your answer.

## 0617aii

1 The graph of the function $p(x)$ is sketched below.


Which equation could represent $p(x)$ ?

1) $p(x)=\left(x^{2}-9\right)(x-2)$
2) $p(x)=x^{3}-2 x^{2}+9 x+18$
3) $p(x)=\left(x^{2}+9\right)(x-2)$
4) $p(x)=x^{3}+2 x^{2}-9 x-18$

2 What is the solution to $8\left(2^{x+3}\right)=48$ ?

1) $x=\frac{\ln 6}{\ln 2}-3$
2) $x=0$
3) $x=\frac{\ln 48}{\ln 16}-3$
4) $x=\ln 4-3$

3 Cheap and Fast gas station is conducting a consumer satisfaction survey. Which method of collecting data would most likely lead to a biased sample?

1) interviewing every 5th customer to come into the station
2) interviewing customers who call an 800 number posted on the customers' receipts
3) interviewing customers chosen at random by a computer at the checkout
4) interviewing every customer who comes into the station on a day of the week chosen at random out of a hat

4 The expression $6 x i^{3}(-4 x i+5)$ is equivalent to

1) $2 x-5 i$
2) $-24 x^{2}-30 x i$
3) $-24 x^{2}+30 x-i$
4) $26 x-24 x^{2} i-5 i$

5 If $f(x)=3|x|-1$ and $g(x)=0.03 x^{3}-x+1$, an approximate solution for the equation $f(x)=g(x)$ is

1) 1.96
2) 11.29
3) $(-0.99,1.96)$
4) $(11.29,32.87)$

6 Given the parent function $p(x)=\cos x$, which phrase best describes the transformation used to obtain the graph of $g(x)=\cos (x+a)-b$, if $a$ and $b$ are positive constants?

1) right $a$ units, up $b$ units
2) left $a$ units, up $b$ units
3) right $a$ units, down $b$ units
4) left $a$ units, down $b$ units

7 The solution to the equation $4 x^{2}+98=0$ is

1) $\pm 7$
2) $\pm 7 i$
3) $\pm \frac{7 \sqrt{2}}{2}$
4) $\pm \frac{7 i \sqrt{2}}{2}$

8 Which equation is represented by the graph shown below?


1) $y=\frac{1}{2} \cos 2 x$
2) $y=\cos x$
3) $y=\frac{1}{2} \cos x$
4) $y=2 \cos \frac{1}{2} x$

9 A manufacturing company has developed a cost model, $C(x)=0.15 x^{3}+0.01 x^{2}+2 x+120$, where $x$ is the number of items sold, in thousands. The sales price can be modeled by $S(x)=30-0.01 x$. Therefore, revenue is modeled by $R(x)=x \bullet S(x)$. The company's profit, $P(x)=R(x)-C(x)$, could be modeled by

1) $0.15 x^{3}+0.02 x^{2}-28 x+120$
2) $-0.15 x^{3}-0.02 x^{2}+28 x-120$
3) $-0.15 x^{3}+0.01 x^{2}-2.01 x-120$
4) $-0.15 x^{3}+32 x+120$

10 A game spinner is divided into 6 equally sized regions, as shown in the diagram below.


For Miles to win, the spinner must land on the number 6. After spinning the spinner 10 times, and losing all 10 times, Miles complained that the spinner is unfair. At home, his dad ran 100 simulations of spinning the spinner 10 times, assuming the probability of winning each spin is $\frac{1}{6}$. The output of the simulation is shown in the diagram below.


Which explanation is appropriate for Miles and his dad to make?

1) The spinner was likely unfair, since the 3) The spinner was likely not unfair, since number 6 failed to occur in about $20 \%$ of the simulations. the number 6 failed to occur in about $20 \%$ of the simulations.
2) The spinner was likely unfair, since the spinner should have landed on the number 6 by the sixth spin.
3) The spinner was likely not unfair, since in the output the player wins once or twice in the majority of the simulations.

11 Which binomial is a factor of $x^{4}-4 x^{2}-4 x+8$ ?

1) $x-2$
2) $x+2$
3) $x-4$
4) $x+4$

12 Given that $\sin ^{2} \theta+\cos ^{2} \theta=1$ and $\sin \theta=-\frac{\sqrt{2}}{5}$, what is a possible value of $\cos \theta$ ?

1) $\frac{5+\sqrt{2}}{5}$
2) $\frac{\sqrt{23}}{5}$
3) $\frac{3 \sqrt{3}}{5}$
4) $\frac{\sqrt{35}}{5}$

13 A student studying public policy created a model for the population of Detroit, where the population decreased $25 \%$ over a decade. He used the model $P=714(0.75)^{d}$, where $P$ is the population, in thousands, $d$ decades after 2010. Another student, Suzanne, wants to use a model that would predict the population after $y$ years. Suzanne's model is best represented by

1) $P=714(0.6500)^{y}$
2) $P=714(0.8500)^{y}$
3) $P=714(0.9716)^{y}$
4) $P=714(0.9750)^{y}$

14 The probability that Gary and Jane have a child with blue eyes is 0.25 , and the probability that they have a child with blond hair is 0.5 . The probability that they have a child with both blue eyes and blond hair is 0.125 . Given this information, the events blue eyes and blond hair are

$$
\begin{array}{ll}
\text { I: } & \text { dependent } \\
\text { II: } & \text { independent } \\
\text { III: } & \text { mutually exclusive }
\end{array}
$$

1) I, only
2) I and III
3) II, only
4) II and III

15 Based on climate data that have been collected in Bar Harbor, Maine, the average monthly temperature, in degrees F , can be modeled by the equation $B(x)=23.914 \sin (0.508 x-2.116)+55.300$. The same governmental agency collected average monthly temperature data for Phoenix, Arizona, and found the temperatures could be modeled by the equation $P(x)=20.238 \sin (0.525 x-2.148)+86.729$. Which statement can not be concluded based on the average monthly temperature models $x$ months after starting data collection?

1) The average monthly temperature variation is more in Bar Harbor than in Phoenix.
2) The midline average monthly temperature for Bar Harbor is lower than the midline temperature for Phoenix.
3) The maximum average monthly temperature for Bar Harbor is $79^{\circ} \mathrm{F}$, to the nearest degree.
4) The minimum average monthly temperature for Phoenix is $20^{\circ} \mathrm{F}$, to the nearest degree.

16 For $x \neq 0$, which expressions are equivalent to one divided by the sixth root of $x$ ?

$$
\text { I. } \frac{\sqrt[6]{x}}{\sqrt[3]{x}} \text { II. } \frac{x^{\frac{1}{6}}}{x^{\frac{1}{3}}} \text { III. } x^{\frac{-1}{6}}
$$

1) I and II, only
2) II and III, only
3) I and III, only
4) I, II, and III

17 A parabola has its focus at $(1,2)$ and its directrix is $y=-2$. The equation of this parabola could be

1) $y=8(x+1)^{2}$
2) $y=\frac{1}{8}(x+1)^{2}$
3) $y=8(x-1)^{2}$
4) $y=\frac{1}{8}(x-1)^{2}$

18 The function $p(t)=110 e^{0.03922 t}$ models the population of a city, in millions, $t$ years after 2010. As of today, consider the following two statements:
I. The current population is 110 million.
II. The population increases continuously by approximately 3.9\% per year.

This model supports

1) I, only
2) both I and II
3) II, only
4) neither I nor II

19 To solve $\frac{2 x}{x-2}-\frac{11}{x}=\frac{8}{x^{2}-2 x}$, Ren multiplied both sides by the least common denominator. Which statement is true?

1) 2 is an extraneous solution.
2) 0 and 2 are extraneous solutions.
3) $\frac{7}{2}$ is an extraneous so1ution.
4) This equation does not contain any extraneous solutions.

20 Given $f(9)=-2$, which function can be used to generate the sequence $-8,-7.25,-6.5,-5.75, \ldots$ ?

1) $f(n)=-8+0.75 n$
2) $f(n)=-8-0.75(n-1)$
3) $f(n)=-8.75+0.75 n$
4) $f(n)=-0.75+8(n-1)$

21 The function $f(x)=2^{-0.25 x} \bullet \sin \left(\frac{\pi}{2} x\right)$ represents a damped sound wave function. What is the average rate of change for this function on the interval $[-7,7]$, to the nearest hundredth?

1) -3.66
2) -0.30
3) -0.26
4) 3.36

22 Mallory wants to buy a new window air conditioning unit. The cost for the unit is $\$ 329.99$. If she plans to run the unit three months out of the year for an annual operating cost of $\$ 108.78$, which function models the cost per year over the lifetime of the unit, $C(n)$, in terms of the number of years, $n$, that she owns the air conditioner.

1) $C(n)=329.99+108.78 n$
2) $C(n)=329.99+326.34 n$
3) $C(n)=\frac{329.99+108.78 n}{n}$
4) $C(n)=\frac{329.99+326.34 n}{n}$

23 The expression $\frac{-3 x^{2}-5 x+2}{x^{3}+2 x^{2}}$ can be rewritten as

1) $\frac{-3 x-3}{x^{2}+2 x}$
2) $\frac{-3 x-1}{x^{2}}$
3) $-3 x^{-1}+1$
4) $-3 x^{-1}+x^{-2}$

24 Jasmine decides to put $\$ 100$ in a savings account each month. The account pays $3 \%$ annual interest, compounded monthly. How much money, $S$, will Jasmine have after one year?

1) $S=100(1.03)^{12}$
2) $S=\frac{100-100(1.0025)^{12}}{1-1.0025}$
3) $S=100(1.0025)^{12}$
4) $S=\frac{100-100(1.03)^{12}}{1-1.03}$

25 Given $r(x)=x^{3}-4 x^{2}+4 x-6$, find the value of $r(2)$. What does your answer tell you about $x-2$ as a factor of $r(x)$ ? Explain.

26 The weight of a bag of pears at the local market averages 8 pounds with a standard deviation of 0.5 pound. The weights of all the bags of pears at the market closely follow a normal distribution. Determine what percentage of bags, to the nearest integer, weighed less than 8.25 pounds.

27 Over the set of integers, factor the expression $4 x^{3}-x^{2}+16 x-4$ completely.

Algebra II Regents Exam 0617
www.jmap.org
28 The graph below represents the height above the ground, $h$, in inches, of a point on a triathlete's bike wheel during a training ride in terms of time, $t$, in seconds.



Identify the period of the graph and describe what the period represents in this context.

29 Graph $y=400(.85)^{2 x}-6$ on the set of axes below.


30 Solve algebraically for all values of $x: \sqrt{x-4}+x=6$

31 Write $\sqrt[3]{x} \bullet \sqrt{x}$ as a single term with a rational exponent.

32 Data collected about jogging from students with two older siblings are shown in the table below.

|  | Neither Sibling <br> Jogs | One Sibling <br> Jogs | Both Siblings <br> Jogs |
| :---: | :---: | :---: | :---: |
| Student Does <br> Not Jog | 1168 | 1823 | 1380 |
| Student Jogs | 188 | 416 | 400 |

Using these data, determine whether a student with two older siblings is more likely to jog if one sibling jogs or if both siblings jog. Justify your answer.

33 Solve the following system of equations algebraically for all values of $x, y$, and $z$ :

$$
\begin{gathered}
x+y+z=1 \\
2 x+4 y+6 z=2 \\
-x+3 y-5 z=11
\end{gathered}
$$

34 Jim is looking to buy a vacation home for $\$ 172,600$ near his favorite southern beach. The formula to compute a mortgage payment, $M$, is $M=P \bullet \frac{r(1+r)^{N}}{(1+r)^{N}-1}$ where $P$ is the principal amount of the loan, $r$ is the monthly interest rate, and $N$ is the number of monthly payments. Jim's bank offers a monthly interest rate of $0.305 \%$ for a 15 -year mortgage. With no down payment, determine Jim's mortgage payment, rounded to the nearest dollar. Algebraically determine and state the down payment, rounded to the nearest dollar, that Jim needs to make in order for his mortgage payment to be $\$ 1100$.

Algebra II Regents Exam 0617
www.jmap.org
35 Graph $y=\log _{2}(x+3)-5$ on the set of axes below. Use an appropriate scale to include both intercepts.


Describe the behavior of the given function as $x$ approaches -3 and as $x$ approaches positive infinity.

36 Charlie's Automotive Dealership is considering implementing a new check-in procedure for customers who are bringing their vehicles for routine maintenance. The dealership will launch the procedure if $50 \%$ or more of the customers give the new procedure a favorable rating when compared to the current procedure. The dealership devises a simulation based on the minimal requirement that $50 \%$ of the customers prefer the new procedure. Each dot on the graph below represents the proportion of the customers who preferred the new check-in procedure, each of sample size 40 , simulated 100 times.


Assume the set of data is approximately normal and the dealership wants to be $95 \%$ confident of its results. Determine an interval containing the plausible sample values for which the dealership will launch the new procedure. Round your answer to the nearest hundredth. Forty customers are selected randomly to undergo the new check-in procedure and the proportion of customers who prefer the new procedure is $32.5 \%$. The dealership decides not to implement the new check-in procedure based on the results of the study. Use statistical evidence to explain this decision.

37 A radioactive substance has a mass of 140 g at $3 \mathrm{p} . \mathrm{m}$. and 100 g at $8 \mathrm{p} . \mathrm{m}$. Write an equation in the form $A=A_{0}\left(\frac{1}{2}\right)^{\frac{t}{h}}$ that models this situation, where $h$ is the constant representing the number of hours in the half-life, $A_{0}$ is the initial mass, and $A$ is the mass $t$ hours after 3 p.m. Using this equation, solve for $h$, to the nearest ten thousandth. Determine when the mass of the radioactive substance will be 40 g . Round your answer to the nearest tenth of an hour.

## 0817AII Common Core State Standards

1 The function $f(x)=\frac{x-3}{x^{2}+2 x-8}$ is undefined when $x$ equals

1) 2 or -4
2) 3, only
3) 4 or -2
4) 2 , only

2 Which expression is equivalent to $(3 k-2 i)^{2}$, where $i$ is the imaginary unit?

1) $9 k^{2}-4$
2) $9 k^{2}+4$
3) $9 k^{2}-12 k i-4$
4) $9 k^{2}-12 k i+4$

3 The roots of the equation $x^{2}+2 x+5=0$ are

1) -3 and 1
2) -1 , only
3) $-1+2 i$ and $-1-2 i$
4) $-1+4 i$ and $-1-4 i$

4 The solution set for the equation $\sqrt{x+14}-\sqrt{2 x+5}=1$ is

1) $\{-6\}$
2) $\{2\}$
3) $\{18\}$
4) $\{2,22\}$

5 As $x$ increases from 0 to $\frac{\pi}{2}$, the graph of the equation $y=2 \tan x$ will

1) increase from 0 to 2
2) increase without limit
3) decrease from 0 to -2
4) decrease without limit

6 Which equation represents a parabola with the focus at $(0,-1)$ and the directrix of $y=1$ ?

1) $x^{2}=-8 y$
2) $x^{2}=-4 y$
3) $x^{2}=8 y$
4) $x^{2}=4 y$

7 Which diagram represents an angle, $\alpha$, measuring $\frac{13 \pi}{20}$ radians drawn in standard position, and its reference angle, $\theta$ ?
1)

2)

3)

4)


8 What are the zeros of $P(m)=\left(m^{2}-4\right)\left(m^{2}+1\right)$ ?

1) 2 and -2 , only
2) $2,-2$, and -4
3) $-4, i$, and $-i$
4) $2,-2, i$, and $-i$

9 The value of a new car depreciates over time. Greg purchased a new car in June 2011. The value, $V$, of his car after $t$ years can be modeled by the equation $\log _{0.8}\left(\frac{V}{17000}\right)=t$. What is the average decreasing rate of change per year of the value of the car from June 2012 to June 2014, to the nearest ten dollars per year?

1) 1960
2) 2180
3) 2450
4) 2770

10 lridium-192 is an isotope of iridium and has a half-life of 73.83 days. If a laboratory experiment begins with 100 grams of Iridium-192, the number of grams, $A$, of Iridium-192 present after $t$ days would be $A=100\left(\frac{1}{2}\right)^{\frac{t}{73.83}}$. Which equation approximates the amount of Iridium-192 present after $t$ days?

1) $A=100\left(\frac{73.83}{2}\right)^{t}$
2) $A=100\left(\frac{1}{147.66}\right)^{t}$
3) $A=100(0.990656)^{t}$
4) $A=100(0.116381)^{t}$

11 The distribution of the diameters of ball bearings made under a given manufacturing process is normally distributed with a mean of 4 cm and a standard deviation of 0.2 cm . What proportion of the ball bearings will have a diameter less than 3.7 cm ?

1) 0.0668
2) 0.4332
3) 0.8664
4) 0.9500

12 A polynomial equation of degree three, $p(x)$, is used to model the volume of a rectangular box. The graph of $p(x)$ has $x$ intercepts at $-2,10$, and 14 . Which statements regarding $p(x)$ could be true?
A. The equation of $p(x)=(x-2)(x+10)(x+14)$.
B. The equation of $p(x)=-(x+2)(x-10)(x-14)$.
C. The maximum volume occurs when $x=10$.
D. The maximum volume of the box is approximately 56 .

1) $A$ and $C$
2) $A$ and $D$
3) $B$ and $C$
4) $B$ and $D$

13 Which expression is equivalent to $\frac{4 x^{3}+9 x-5}{2 x-1}$, where $x \neq \frac{1}{2}$ ?

1) $2 x^{2}+x+5$
2) $2 x^{2}+\frac{11}{2}+\frac{1}{2(2 x-1)}$
3) $2 x^{2}-x+5$
4) $2 x^{2}-x+4+\frac{1}{2 x-1}$

14 The inverse of the function $f(x)=\frac{x+1}{x-2}$ is

1) $f^{-1}(x)=\frac{x+1}{x+2}$
2) $f^{-1}(x)=\frac{2 x+1}{x-1}$
3) $f^{-1}(x)=\frac{x+1}{x-2}$
4) $f^{-1}(x)=\frac{x-1}{x+1}$

15 Which expression has been rewritten correctly to form a true statement?

1) $(x+2)^{2}+2(x+2)-8=(x+6) x$
2) $x^{3}+3 x^{2}-4 x y^{2}-12 y^{2}=(x-2 y)(x+3)^{2}$
3) $\left.x^{4}+4 x^{2}+9 x^{2} y^{2}-36 y^{2}=(x+3 y)^{2}(x-2)^{2} 4\right)$
) $\left(x^{2}-4\right)^{2}-5\left(x^{2}-4\right)-6=\left(x^{2}-7\right)\left(x^{2}-6\right)$

16 A study conducted in 2004 in New York City found that 212 out of 1334 participants had hypertension. Kim ran a simulation of 100 studies based on these data. The output of the simulation is shown in the diagram below.


At a 95\% confidence level, the proportion of New York City residents with hypertension and the margin of error are closest to

1) proportion $\approx .16$; margin of error $\approx .01$
2) proportion $\approx .01$; margin of error $\approx .16$
3) proportion $\approx .16$; margin of error $\approx .02$
4) proportion $\approx .02$; margin of error $\approx .16$

17 Which scenario is best described as an observational study?

1) For a class project, students in Health class ask every tenth student entering the school if they eat breakfast in the morning.
2) A social researcher wants to learn whether or not there is a link between attendance and grades. She gathers data from 15 school districts.
3) A researcher wants to learn whether or not there is a link between children's daily amount of physical activity and their overall energy level. During lunch at the local high school, she distributed a short questionnaire to students in the cafeteria.
4) Sixty seniors taking a course in Advanced Algebra Concepts are randomly divided into two classes. One class uses a graphing calculator all the time, and the other class never uses graphing calculators. A guidance counselor wants to determine whether there is a link between graphing calculator use and students' final exam grades.

18 Which sinusoid has the greatest amplitude?
1)

2) $y=3 \sin (\theta-3)+5$
3)

4) $y=-5 \sin (\theta-1)-3$

19 Consider the system shown below.

$$
\begin{gathered}
2 x-y=4 \\
(x+3)^{2}+y^{2}=8
\end{gathered}
$$

The two solutions of the system can be described as

1) both imaginary
2) both rational
3) both irrational
4) one rational and one irrational

20 Which binomial is not a factor of the expression $x^{3}-11 x^{2}+16 x+84$ ?

1) $x+2$
2) $x+4$
3) $x-6$
4) $x-7$

21 A ball is dropped from a height of 32 feet. It bounces and rebounds $80 \%$ of the height from which it was falling. What is the total downward distance, in feet, the ball traveled up to the 12th bounce?

1) 29
2) 58
3) 120
4) 149

22 A public opinion poll was conducted on behalf of Mayor Ortega's reelection campaign shortly before the election. 264 out of 550 likely voters said they would vote for Mayor Ortega; the rest said they would vote for his opponent. Which statement is least appropriate to make, according to the results of the poll?

1) There is a $48 \%$ chance that Mayor Ortega will win the election.
2) It is most likely that between $44 \%$ and $52 \%$ of voters will vote for Mayor Ortega.
3) The point estimate ( $\hat{\mathrm{p}}$ ) of voters who will vote for Mayor Ortega is $48 \%$.
4) Due to the margin of error, an inference cannot be made regarding whether Mayor Ortega or his opponent is most likely to win the election.

23 What does $\left(\frac{-54 x^{9}}{y^{4}}\right)^{\frac{2}{3}}$ equal?

1) $\frac{9 i x^{6} \sqrt[3]{4}}{y \sqrt[3]{y^{2}}}$
2) $\frac{9 i x^{63} \sqrt{4}}{y^{2} \sqrt[3]{y^{2}}}$
3) $\frac{9 x^{63} \sqrt{4}}{y \sqrt[3]{y}}$
4) $\frac{9 x^{63} \sqrt{4}}{y^{2} \sqrt[3]{y^{2}}}$

24 The Rickerts decided to set up an account for their daughter to pay for her college education. The day their daughter was born, they deposited $\$ 1000$ in an account that pays $1.8 \%$ compounded annually. Beginning with her first birthday, they deposit an additional $\$ 750$ into the account on each of her birthdays. Which expression correctly represents the amount of money in the account $n$ years after their daughter was born?

1) $a_{n}=1000(1.018)^{n}+750$
2) $a_{0}=1000$
$a_{n}=a_{n-1}(1.018)+750$
3) $a_{n}=1000(1.018)^{n}+750 n$
4) $a_{0}=1000$
$a_{n}=a_{n-1}(1.018)+750 n$

25 Explain how (-8) ${ }^{\frac{4}{3}}$ can be evaluated using properties of rational exponents to result in an integer answer.

26 A study was designed to test the effectiveness of a new drug. Half of the volunteers received the drug. The other half received a sugar pill. The probability of a volunteer receiving the drug and getting well was $40 \%$. What is the probability of a volunteer getting well, given that the volunteer received the drug?

27 Verify the following Pythagorean identity for all values of $x$ and $y$ :

$$
\left(x^{2}+y^{2}\right)^{2}=\left(x^{2}-y^{2}\right)^{2}+(2 x y)^{2}
$$

28 Mrs. Jones had hundreds of jelly beans in a bag that contained equal numbers of six different flavors. Her student randomly selected four jelly beans and they were all black licorice. Her student complained and said "What are the odds I got all of that kind?" Mrs. Jones replied, "simulate rolling a die 250 times and tell me if four black licorice jelly beans is unusual." Explain how this simulation could be used to solve the problem.

29 While experimenting with her calculator, Candy creates the sequence $4,9,19,39,79, \ldots$. Write a recursive formula for Candy's sequence. Determine the eighth term in Candy's sequence.

30 In New York State, the minimum wage has grown exponentially. In 1966, the minimum wage was $\$ 1.25$ an hour and in 2015, it was $\$ 8.75$. Algebraically determine the rate of growth to the nearest percent.

31 Algebraically determine whether the function $j(x)=x^{4}-3 x^{2}-4$ is odd, even, or neither.

32 On the axes below, sketch a possible function $p(x)=(x-a)(x-b)(x+c)$, where $a, b$, and $c$ are positive, $a>b$, and $p(x)$ has a positive $y$-intercept of $d$. Label all intercepts.


33 Solve for all values of $p: \frac{3 p}{p-5}-\frac{2}{p+3}=\frac{p}{p+3}$

34 Simon lost his library card and has an overdue library book. When the book was 5 days late, he owed $\$ 2.25$ to replace his library card and pay the fine for the overdue book. When the book was 21 days late, he owed $\$ 6.25$ to replace his library card and pay the fine for the overdue book. Suppose the total amount Simon owes when the book is $n$ days late can be determined by an arithmetic sequence. Determine a formula for $a_{n}$, the $n$th term of this sequence. Use the formula to determine the amount of money, in dollars, Simon needs to pay when the book is 60 days late.

35 a) On the axes below, sketch at least one cycle of a sine curve with an amplitude of 2 , a midline at $y=-\frac{3}{2}$, and a period of $2 \pi$.

b) Explain any differences between a sketch of $y=2 \sin \left(x-\frac{\pi}{3}\right)-\frac{3}{2}$ and the sketch from part a.

36 Using a microscope, a researcher observed and recorded the number of bacteria spores on a large sample of uniformly sized pieces of meat kept at room temperature. A summary of the data she recorded is shown in the table below.

| Hours (x) | Average Number <br> of Spores (y) |
| :---: | :---: |
| 0 | 4 |
| 0.5 | 10 |
| 1 | 15 |
| 2 | 60 |
| 3 | 260 |
| 4 | 1130 |
| 6 | 16,380 |

Using these data, write an exponential regression equation, rounding all values to the nearest thousandth. The researcher knows that people are likely to suffer from food-borne illness if the number of spores exceeds 100 . Using the exponential regression equation, determine the maximum amount of time, to the nearest quarter hour, that the meat can be kept at room temperature safely.

37 The value of a certain small passenger car based on its use in years is modeled by $V(t)=28482.698(0.684)^{t}$, where $V(t)$ is the value in dollars and $t$ is the time in years. Zach had to take out a loan to purchase the small passenger car. The function $Z(t)=22151.327(0.778)^{t}$, where $Z(t)$ is measured in dollars, and $t$ is the time in years, models the unpaid amount of Zach's loan over time. Graph $V(t)$ and $Z(t)$ over the interval $0 \leq t \leq 5$, on the set of axes below.


State when $V(t)=Z(t)$, to the nearest hundredth, and interpret its meaning in the context of the problem. Zach takes out an insurance policy that requires him to pay a $\$ 3000$ deductible in case of a collision. Zach will cancel the collision policy when the value of his car equals his deductible. To the nearest year, how long will it take Zach to cancel this policy? Justify your answer.

## 0118AII Common Core State Standards

1 The operator of the local mall wants to find out how many of the mall's employees make purchases in the food court when they are working. She hopes to use these data to increase the rent and attract new food vendors. In total, there are 1023 employees who work at the mall. The best method to obtain a random sample of the employees would be to survey

1) all 170 employees at each of the larger stores
2) $50 \%$ of the 90 employees of the food court
3) every employee
4) every 30th employee entering each mall entrance for one week

2 What is the solution set for $x$ in the equation below?

$$
\text { 1) } \quad\{1\}
$$

$$
\begin{aligned}
& \sqrt{x+1}-1=x \\
& \text { 3) }\{-1,0\} \\
& \text { 4) }\{0,1\}
\end{aligned}
$$

2) $\{0\}$

3 For the system shown below, what is the value of $z$ ?

$$
\begin{aligned}
& y=-2 x+14 \\
& \\
& 3 x-4 z=2 \\
& \\
& 3 x-y=16 \\
& \text { 3) } 6 \\
& \text { 4) } 4
\end{aligned}
$$

1) 5
2) 2

4 The hours of daylight, $y$, in Utica in days, $x$, from January 1, 2013 can be modeled by the equation $y=3.06 \sin (0.017 x-1.40)+12.23$. How many hours of daylight, to the nearest tenth, does this model predict for February 14, 2013?

1) 9.4
2) 10.4
3) 12.1
4) 12.2

5 A certain pain reliever is taken in 220 mg dosages and has a half-life of 12 hours. The function $A=220\left(\frac{1}{2}\right)^{\frac{t}{12}}$ can be used to model this situation, where $A$ is the amount of pain reliever in milligrams remaining in the body after $t$ hours. According to this function, which statement is true?

1) Every hour, the amount of pain reliever remaining is cut in half.
2) In 24 hours, there is no pain reliever remaining in the body.
3) In 12 hours, there is no pain reliever remaining in the body.
4) In 12 hours, 110 mg of pain reliever is remaining.

6 The expression $(x+a)(x+b)$ can not be written as

1) $a(x+b)+x(x+b)$
2) $x^{2}+a b x+a b$
3) $x^{2}+(a+b) x+a b$
4) $x(x+a)+b(x+a)$

7 There are 440 students at Thomas Paine High School enrolled in U.S. History. On the April report card, the students' grades are approximately normally distributed with a mean of 79 and a standard deviation of 7. Students who earn a grade less than or equal to 64.9 must attend summer school. The number of students who must attend summer school for U.S. History is closest to

1) 3
2) 5
3) 10
4) 22

8 For a given time, $x$, in seconds, an electric current, $y$, can be represented by $y=2.5\left(1-2.7^{-10 x}\right)$. Which equation is not equivalent?

1) $y=2.5-2.5\left(2.7^{-.10 x}\right)$
2) $y=2.5-2.5\left(\left(2.7^{2}\right)^{-.05 x}\right)$
3) $y=2.5-2.5\left(\frac{1}{2.7^{.10 x}}\right)$
4) $y=2.5-2.5\left(2.7^{-2}\right)\left(2.7^{.05 x}\right)$

9 What is the quotient when $10 x^{3}-3 x^{2}-7 x+3$ is divided by $2 x-1$ ?

1) $5 x^{2}+x+3$
2) $5 x^{2}-x+3$
3) $5 x^{2}-x-3$
4) $5 x^{2}+x-3$

10 Judith puts $\$ 5000$ into an investment account with interest compounded continuously. Which approximate annual rate is needed for the account to grow to $\$ 9110$ after 30 years?

1) $2 \%$
2) $2.2 \%$
3) $0.02 \%$
4) $0.022 \%$

11 If $n=\sqrt{a^{5}}$ and $m=a$, where $a>0$, an expression for $\frac{n}{m}$ could be

1) $a^{\frac{5}{2}}$
2) $a^{4}$
3) $\sqrt[3]{a^{2}}$
4) $\sqrt{a^{3}}$

12 The solutions to $x+3-\frac{4}{x-1}=5$ are

1) $\frac{3}{2} \pm \frac{\sqrt{17}}{2}$
2) $\frac{3}{2} \pm \frac{\sqrt{17}}{2} i$
3) $\frac{3}{2} \pm \frac{\sqrt{33}}{2}$
4) $\frac{3}{2} \pm \frac{\sqrt{33}}{2} i$

13 If $a e^{b t}=c$, where $a, b$, and $c$ are positive, then $t$ equals

1) $\ln \left(\frac{c}{a b}\right)$
2) $\ln \left(\frac{c b}{a}\right)$
3) $\frac{\ln \left(\frac{c}{a}\right)}{b}$
4) $\frac{\ln \left(\frac{c}{a}\right)}{\ln b}$

14 For which values of $x$, rounded to the nearest hundredth, will $\left|x^{2}-9\right|-3=\log _{3} x$ ?

1) 2.29 and 3.63
2) 2.37 and 3.54
3) 2.84 and 3.17
4) 2.92 and 3.06

15 The terminal side of $\theta$, an angle in standard position, intersects the unit circle at $P\left(-\frac{1}{3},-\frac{\sqrt{8}}{3}\right)$. What is the value of $\sec \theta$ ?

1) -3
2) $-\frac{3 \sqrt{8}}{8}$
3) $-\frac{1}{3}$
4) $-\frac{\sqrt{8}}{3}$

16 What is the equation of the directrix for the parabola $-8(y-3)=(x+4)^{2}$ ?

1) $y=5$
2) $y=1$
3) $y=-2$
4) $y=-6$

17 The function below models the average price of gas in a small town since January 1st.
$G(t)=-0.0049 t^{4}+0.0923 t^{3}-0.56 t^{2}+1.166 t+3.23$, where $0 \leq t \leq 10$.
If $G(t)$ is the average price of gas in dollars and $t$ represents the number of months since January 1st, the absolute maximum $G(t)$ reaches over the given domain is about

1) $\$ 1.60$
2) $\$ 3.92$
3) $\$ 4.01$
4) $\$ 7.73$

18 Written in simplest form, $\frac{c^{2}-d^{2}}{d^{2}+c d-2 c^{2}}$ where $c \neq d$, is equivalent to

1) $\frac{c+d}{d+2 c}$
2) $\frac{c-d}{d+2 c}$
3) $\frac{-c-d}{d+2 c}$
4) $\frac{-c+d}{d+2 c}$

19 If $p(x)=2 x^{3}-3 x+5$, what is the remainder of $p(x) \div(x-5)$ ?

1) -230
2) 0
3) 40
4) 240

20 The results of simulating tossing a coin 10 times, recording the number of heads, and repeating this 50 times are shown in the graph below.


Based on the results of the simulation, which statement is false?

1) Five heads occurred most often, which is 3) Obtaining three heads or fewer occurred consistent with the theoretical probability $28 \%$ of the time. of obtaining a heads.
2) Eight heads is unusual, as it falls outside the middle $95 \%$ of the data.
3) Seven heads is not unusual, as it falls within the middle $95 \%$ of the data.

21 What is the inverse of $f(x)=-6(x-2)$ ?

1) $f^{-1}(x)=-2-\frac{x}{6}$
2) $f^{-1}(x)=2-\frac{x}{6}$
3) $f^{-1}(x)=\frac{1}{-6(x-2)}$
4) $f^{-1}(x)=6(x+2)$

22 Brian deposited 1 cent into an empty non-interest bearing bank account on the first day of the month. He then additionally deposited 3 cents on the second day, 9 cents on the third day, and 27 cents on the fourth day. What would be the total amount of money in the account at the end of the 20th day if the pattern continued?

1) $\$ 11,622,614.67$
2) $\$ 17,433,922.00$
3) $\$ 116,226,146.80$
4) $\$ 1,743,392,200.00$

23 If the function $g(x)=a b^{x}$ represents exponential growth, which statement about $g(x)$ is false?

1) $a>0$ and $b>1$
2) The $y$-intercept is $(0, a)$.
3) The asymptote is $y=0$.
4) The $x$-intercept is $(b, 0)$.

24 At her job, Pat earns $\$ 25,000$ the first year and receives a raise of $\$ 1000$ each year. The explicit formula for the $n$th term of this sequence is $a_{n}=25,000+(n-1) 1000$. Which rule best represents the equivalent recursive formula?

1) $a_{n}=24,000+1000 n$
2) $a_{n}=25,000+1000 n$
3) $a_{1}=25,000, a_{n}=a_{n-1}+1000$
4) $a_{1}=25,000, a_{n}=a_{n+1}+1000$

25 Elizabeth tried to find the product of $(2+4 i)$ and $(3-i)$, and her work is shown below.

$$
\begin{aligned}
& (2+4 i)(3-i) \\
= & 6-2 i+12 i-4 i^{2} \\
= & 6+10 i-4 i^{2} \\
= & 6+10 i-4(1) \\
= & 6+10 i-4 \\
= & 2+10 i
\end{aligned}
$$

Identify the error in the process shown and determine the correct product of $(2+4 i)$ and $(3-i)$.

26 A runner is using a nine-week training app to prepare for a "fun run." The table below represents the amount of the program completed, $A$, and the distance covered in a session, $D$, in miles.

| A | $\frac{4}{9}$ | $\frac{5}{9}$ | $\frac{6}{9}$ | $\frac{8}{9}$ | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| D | 2 | 2 | 2.25 | 3 | 3.25 |

Based on these data, write an exponential regression equation, rounded to the nearest thousandth, to model the distance the runner is able to complete in a session as she continues through the nine-week program.

27 A formula for work problems involving two people is shown below.

$$
\frac{1}{t_{1}}+\frac{1}{t_{2}}=\frac{1}{t_{b}}
$$

$t_{1}=$ the time taken by the first person to complete the job
$t_{2}=$ the time taken by the second person to complete the job
$t_{b}=$ the time it takes for them working together to complete the job
Fred and Barney are carpenters who build the same model desk. It takes Fred eight hours to build the desk while it only takes Barney six hours. Write an equation that can be used to find the time it would take both carpenters working together to build a desk. Determine, to the nearest tenth of an hour, how long it would take Fred and Barney working together to build a desk.

28 Completely factor the following expression: $x^{2}+3 x y+3 x^{3}+y$

29 Researchers in a local area found that the population of rabbits with an initial population of 20 grew continuously at the rate of 5\% per month. The fox population had an initial value of 30 and grew continuously at the rate of $3 \%$ per month. Find, to the nearest tenth of a month, how long it takes for these populations to be equal.

30 Consider the function $h(x)=2 \sin (3 x)+1$ and the function $q$ represented in the table below.

| $\boldsymbol{x}$ | $\boldsymbol{q ( x )}$ |
| :---: | :---: |
| -2 | -8 |
| -1 | 0 |
| 0 | 0 |
| 1 | -2 |
| 2 | 0 |

Determine which function has the smaller minimum value for the domain [-2,2]. Justify your answer.

Algebra II CCSS Regents Exam 0118
www.jmap.org
31 The zeros of a quartic polynomial function $h$ are $-1, \pm 2$, and 3. Sketch a graph of $y=h(x)$ on the grid below.


32 Explain why $81^{\frac{3}{4}}$ equals 27.

33 Given: $f(x)=2 x^{2}+x-3$ and $g(x)=x-1$
Express $f(x) \bullet g(x)-[f(x)+g(x)]$ as a polynomial in standard form.

34
A student is chosen at random from the student body at a given high school. The probability that the student selects Math as the favorite subject is $\frac{1}{4}$. The probability that the student chosen is a junior is $\frac{116}{459}$. If the probability that the student selected is a junior or that the student chooses Math as the favorite subject is $\frac{47}{108}$, what is the exact probability that the student selected is a junior whose favorite subject is Math? Are the events "the student is a junior" and "the student's favorite subject is Math" independent of each other? Explain your answer.

35 In a random sample of 250 men in the United States, age 21 or older, 139 are married. The graph below simulated samples of 250 men, 200 times, assuming that 139 of the men are married.

a) Based on the simulation, create an interval in which the middle $95 \%$ of the number of married men may fall. Round your answer to the nearest integer.
b) A study claims " 50 percent of men 21 and older in the United States are married." Do your results from part a contradict this claim? Explain.

36 The graph of $y=f(x)$ is shown below. The function has a leading coefficient of 1.


Write an equation for $f(x)$. The function $g$ is formed by translating function $f$ left 2 units. Write an equation for $g(x)$.

37 The resting blood pressure of an adult patient can be modeled by the function $P$ below, where $P(t)$ is the pressure in millimeters of mercury after time $t$ in seconds.

$$
P(t)=24 \cos (3 \pi t)+120
$$

On the set of axes below, graph $y=P(t)$ over the domain $0 \leq t \leq 2$.


Determine the period of $P$. Explain what this value represents in the given context. Normal resting blood pressure for an adult is 120 over 80 . This means that the blood pressure oscillates between a maximum of 120 and a minimum of 80. Adults with high blood pressure (above 140 over 90 ) and adults with low blood pressure (below 90 over 60) may be at risk for health disorders. Classify the given patient's blood pressure as low, normal, or high and explain your reasoning.

## 2015 Algebra II Common Core State Standards Sample Items

## Answer Section

1 ANS: 1
The zeros of the polynomial are at $-b$, and $c$. The sketch of a polynomial of degree 3 with a negative leading coefficient should have end behavior showing as $x$ goes to negative infinity, $f(x)$ goes to positive infinity. The multiplicities of the roots are correctly represented in the graph.

PTS: 2 REF: spr1501aii NAT: F.IF.C. 7 TOP: Graphing Polynomial Functions
KEY: AII
2 ANS: 4

A parabola with a focus of $(0,4)$ and a directrix of $y=2$ is sketched as follows:
 is determined that the vertex of the parabola is $(0,3)$. It is also evident that the distance, $p$, between the vertex and the focus is 1 . It is possible to use the formula $(x-h)^{2}=4 p(y-k)$ to derive the equation of the parabola as follows: $(x-0)^{2}=4(1)(y-3)$

$$
\begin{aligned}
x^{2} & =4 y-12 \\
x^{2}+12 & =4 y \\
\frac{x^{2}}{4}+3 & =y
\end{aligned}
$$

or A point $(x, y)$ on the parabola must be the same distance from the focus as it is from the directrix. For any such point $(x, y)$, the distance to the focus is $\sqrt{(x-0)^{2}+(y-4)^{2}}$ and the distance to the directrix is $y-2$. Setting this equal leads to: $x^{2}+y^{2}-8 y+16=y^{2}-4 y+4$

$$
\begin{aligned}
& x^{2}+16=4 y+4 \\
& \frac{x^{2}}{4}+3=y
\end{aligned}
$$

PTS: 2 REF: spr1502aii NAT: G.GPE.A. 2 TOP: Graphing Quadratic Functions
3 ANS: 1
A reference triangle can be sketched using the coordinates $(-4,3)$ in the second quadrant to find the value of $\sin \theta$.


PTS: 2 REF: spr1503aii NAT: F.TF.A. 2 TOP: Determining Trigonometric Functions
KEY: extension to reals

4 ANS: 2
$B(t)=750\left(1.16^{\frac{1}{12}}\right)^{12 t} \approx 750(1.012)^{12 t} \quad B(t)=750\left(1+\frac{0.16}{12}\right)^{12 t}$ is wrong, because the growth is an annual rate that is not compounded monthly.

PTS: 2 REF: spr1504aii NAT: A.SSE.B. 3 TOP: Modeling Exponential Functions
KEY: AII
5 ANS:
$\frac{x^{\frac{8}{3}}}{x^{\frac{4}{3}}}=x^{y}$
$x^{\frac{4}{3}}=x^{y}$
$\frac{4}{3}=y$

PTS: 2
REF: spr1505aii NAT: N.RN.A. 2 TOP: Radicals and Rational Exponents
KEY: numbers
6 ANS:
$(4-3 i)(5+2 y i-5+2 y i)$
$(4-3 i)(4 y i)$
$16 y i-12 y i^{2}$
$12 y+16 y i$
PTS: 2 REF: spr1506aii NAT: N.CN.A. 2 TOP: Operations with Complex Numbers
7 ANS:
$f(4)=2(4)^{3}-5(4)^{2}-11(4)-4=128-80-44-4=0$ Any method that demonstrates 4 is a zero of $f(x)$ confirms
that $x-4$ is a factor, as suggested by the Remainder Theorem.


PTS: 2 REF: spr1507aii NAT: A.APR.B. 2 TOP: Remainder Theorem

8 ANS:


$$
\begin{aligned}
\sqrt{x-5} & =-x+7 \quad \sqrt{x-5}=-9+7=-2 \text { is extraneous. } \\
x-5 & =x^{2}-14 x+49 \\
0 & =x^{2}-15 x+54 \\
0 & =(x-6)(x-9) \\
x & =6,9
\end{aligned}
$$

PTS: 2
REF: spr1508aii NAT: A.REI.A. 2 TOP: Solving Radicals KEY: extraneous solutions

9


$$
720=\frac{120000\left(\frac{.048}{12}\right)\left(1+\frac{.048}{12}\right)^{n}}{\left(1+\frac{.048}{12}\right)^{n}-1} \frac{275.2}{12} \approx 23 \text { years }
$$

$$
\begin{aligned}
720(1.004)^{n}-720 & =480(1.004)^{n} \\
240(1.004)^{n} & =720 \\
1.004^{n} & =3 \\
n \log 1.004 & =\log 3 \\
n & \approx 275.2 \text { months }
\end{aligned}
$$

PTS: 4
REF: spr1509aii NAT: A.CED.A. 1 TOP: Exponential Growth

10 ANS:


$$
\begin{aligned}
& 6 x-3 y+2 z=-10 \quad x+3 y+5 z=45 \quad 4 x+10 z=62 \quad 4 x+4(7)=20 \\
& -2 x+3 y+8 z=72 \quad 6 x-3 y+2 z=-10 \quad 4 x+4 z=20 \quad 4 x=-8 \\
& 4 x+10 z=62 \quad 7 x+7 z=35 \quad 6 z=42 \quad x=-2
\end{aligned}
$$

$$
\begin{aligned}
6(-2)-3 y+2(7) & =-10 \\
-3 y & =-12 \\
y & =4
\end{aligned}
$$

PTS: 4 REF: spr1510aii NAT: A.REI.C. 6 TOP: Solving Linear Systems
KEY: three variables
11 ANS:
$a_{n}=x^{n-1}(x+1) x^{n-1}=0 x+1=0$

$$
x=0 \quad x=-1
$$

PTS: 4 REF: spr1511aii NAT: F.BF.A. 2 TOP: Sequences
12 ANS:
Yes. The margin of error from this simulation indicates that $95 \%$ of the observations fall within $\pm 0.12$ of the simulated proportion, 0.25 . The margin of error can be estimated by multiplying the standard deviation, shown to be 0.06 in the dotplot, by 2 , or applying the estimated standard error formula, $\left(\sqrt{\frac{p(1-p)}{n}}\right)$ or $\left(\sqrt{\frac{(0.25)(0.75)}{50}}\right)$ and multiplying by 2 . The interval $0.25 \pm 0.12$ includes plausible values for the true proportion of people who prefer Stephen's new product. The company has evidence that the population proportion could be at least $25 \%$. As seen in the dotplot, it can be expected to obtain a sample proportion of 0.18 ( 9 out of 50 ) or less several times, even when the population proportion is 0.25 , due to sampling variability. Given this information, the results of the survey do not provide enough evidence to suggest that the true proportion is not at least 0.25 , so the development of the product should continue at this time.

PTS: 4
REF: spr1512aii NAT: S.IC.B. 4
TOP: Analysis of Data

13 ANS:

This scenario can be modeled with a Venn Diagram:
 Since $P(S \cup I)_{c}=0.2, P(S \cup I)=0.8$. Then, $P(S \cap I)=P(S)+P(I)-P(S \cup I)$ If $S$ and $I$ are independent, then the

$$
\begin{aligned}
& =0.5+0.7-0.8 \\
& =0.4
\end{aligned}
$$

Product Rule must be satisfied. However, $(0.5)(0.7) \neq 0.4$. Therefore, salary and insurance have not been treated independently.

PTS: 4 REF: spr1513aii NAT: S.CP.A. 2 TOP: Theoretical Probability
14 ANS:


The amplitude, 12, can be interpreted from the situation, since the water level has a minimum of -12 and a maximum of 12 . The value of $A$ is -12 since at $8: 30$ it is low tide. The period of the function is 13 hours, and is expressed in the function through the parameter $B$. By experimentation with technology or using the relation $P=\frac{2 \pi}{B}$ (where $P$ is the period), it is determined that $B=\frac{2 \pi}{13}$.
$f(t)=-12 \cos \left(\frac{2 \pi}{13} t\right)$


In order to answer the question about when to fish, the student must interpret the function and determine which choice, 7:30 pm or 10:30 pm, is on an increasing interval. Since the function is increasing from $t=13$ to $t=19.5$ (which corresponds to 9:30 pm to 4:00 am), 10:30 is the appropriate choice.

PTS: 6 REF: spr1514aii NAT: F.IF.C. 7 TOP: Graphing Trigonometric Functions
KEY: graph

15 ANS: 4

$$
\underbrace{}_{(-3,5,-0.857)}
$$

PTS: 2
REF: fall1501aii NAT: A.REI.A. 2 TOP: Solving Rationals
KEY: rational solutions
16 ANS: 3
$f(x)=-f(x)$, so $f(x)$ is odd. $g(-x) \neq g(x)$, so $g(x)$ is not even. $g(-x) \neq-g(x)$, so $g(x)$ is not odd. $h(-x)=h(x)$, so $h(x)$ is even.

PTS: 2 REF: fall1502aii NAT: F.BF.B. 3 TOP: Even and Odd Functions
17 ANS: 1

$$
\begin{gathered}
2 x + 3 \longdiv { 6 x ^ { 3 } + 1 7 x ^ { 2 } + 1 0 x + 2 } \\
\frac{6 x^{3}+9 x^{2}}{8 x^{2}+10 x} \\
\frac{8 x^{2}+12 x}{-2 x+2} \\
\frac{-2 x-3}{5}
\end{gathered}
$$

PTS: 2
REF: fall1503aii NAT: A.APR.D. 6 TOP: Rational Expressions

18 ANS: 3


PTS: 2 REF: fall1504aii NAT: A.REI.B. 4 TOP: Solving Quadratics
KEY: complex solutions | completing the square
19 ANS: 4
$k^{4}-4 k^{2}+8 k^{3}-32 k+12 k^{2}-48$
$k^{2}\left(k^{2}-4\right)+8 k\left(k^{2}-4\right)+12\left(k^{2}-4\right)$
$\left(k^{2}-4\right)\left(k^{2}+8 k+12\right)$
$(k+2)(k-2)(k+6)(k+2)$
PTS: 2
REF: fall1505aii NAT: A.SSE.A. 2 TOP: Factoring Polynomials
KEY: factoring by grouping
20 ANS: 4


As the range is [4,10], the midline is $y=\frac{4+10}{2}=7$.

PTS: 2
REF: fall1506aii NAT: F.IF.C. 7 TOP: Graphing Trigonometric Functions
KEY: mixed

21 ANS:


$$
\begin{aligned}
-2 x+1 & =-2 x^{2}+3 x+1 \\
2 x^{2}-5 x & =0 \\
x(2 x-5) & =0 \\
x & =0, \frac{5}{2}
\end{aligned}
$$

PTS: 2
REF: fall1507aii NAT: A.REI.C. 7 TOP: Quadratic-Linear Systems
KEY: AII
22 ANS:
Based on these data, the two events do not appear to be independent. $P(F)=\frac{106}{200}=0.53$, while $P(F \mid T)=\frac{54}{90}=0.6, P(F \mid R)=\frac{25}{65}=0.39$, and $P(F \mid C)=\frac{27}{45}=0.6$. The probability of being female are not the same as the conditional probabilities. This suggests that the events are not independent.

PTS: 2 REF: fall1508aii NAT: S.CP.A. 4 TOP: Conditional Probability
23 ANS:
$x=(y-3)^{3}+1$
$x-1=(y-3)^{3}$
$\sqrt[3]{x-1}=y-3$
$\sqrt[3]{x-1}+3=y$
$f^{-1}(x)=\sqrt[3]{x-1}+3$
PTS: 2 REF: fall1509aii NAT: F.BF.B. 4 TOP: Inverse of Functions
KEY: equations

24 ANS:
(-1.13/4.21)
PTS: 2
REF: fall1510aii NAT: A.REI.D. 11 TOP: Other Systems
KEY: AII
25 ANS:
Let $x$ equal the first integer and $x+1$ equal the next. $(x+1)^{2}-x^{2}=x^{2}+2 x+1-x^{2}=2 x+1.2 x+1$ is an odd integer.

PTS: 2 REF: fall1511aii NAT: A.APR.C. 4 TOP: Polynomial Identities
26 ANS:
The expression is of the form $y^{2}-5 y-6$ or $(y-6)(y+1)$. Let $y=4 x^{2}+5 x$ :
$\left(4 x^{2}+5 x-6\right)\left(4 x^{2}+5 x+1\right)$
$(4 x-3)(x+2)(4 x+1)(x+1)$
PTS: 2
REF: fall1512aii NAT: A.SSE.A. 2 TOP: Factoring Polynomials
KEY: $\mathrm{a}>1$
27


$$
\begin{array}{rl}
100 & =325+(68-325) e^{-2 k} \\
-225 & T=325-257 e^{-0.066 t} \\
& =-257 e^{-2 k} \\
& T=325-257 e^{-0.066(7)} \approx 163 \\
k & =\frac{\ln \left(\frac{-225}{-257}\right)}{-2} \\
k & \approx 0.066
\end{array}
$$

PTS: 4
REF: fall1513aii NAT: F.LE.A. 4 TOP: Exponential Growth

28
ANS:
The mean difference between the students' final grades in group 1 and group 2 is -3.64 . This value indicates that students who met with a tutor had a mean final grade of 3.64 points less than students who used an on-line subscription. One can infer whether this difference is due to the differences in intervention or due to which students were assigned to each group by using a simulation to rerandomize the students’ final grades many (500) times. If the observed difference - 3.64 is the result of the assignment of students to groups alone, then a difference of -3.64 or less should be observed fairly regularly in the simulation output. However, a difference of -3 or less occurs in only about $2 \%$ of the rerandomizations. Therefore, it is quite unlikely that the assignment to groups alone accounts for the difference; rather, it is likely that the difference between the interventions themselves accounts for the difference between the two groups' mean final grades.

PTS: 4 REF: fall1514aii NAT: S.IC.B. 5 TOP: Analysis of Data
29 ANS:

$$
\begin{aligned}
& 0=6(-5)^{3}+b(-5)^{2}-52(-5)+15 \quad z(x)=6 x^{3}+19 x^{2}-52 x+15 \\
& 0=-750+25 b+260+15
\end{aligned}
$$

$$
475=25 b
$$

$$
19=b
$$

| -5 | 6 | 19 | -52 | 15 |
| :---: | :---: | :---: | :---: | :---: |
|  |  | -30 | 55 | 15 |
|  | 6 | -11 | 3 | 0 |
| $6 x^{2}-11 x+3=0$ |  |  |  |  |

$(2 x-3)(3 x-1)=0$

$$
x=\frac{3}{2}, \frac{1}{3},-5
$$

PTS: 4 REF: fall1515aii NAT: A.APR.B. 2 TOP: Remainder Theorem
30 ANS:
$\operatorname{normcdf}(510,540,480,24)=0.0994 \quad z=\frac{510-480}{24}=1.251 .25=\frac{x-510}{20} 2.5=\frac{x-510}{20} 535-560$

$$
z=\frac{540-480}{24}=2.5 \quad x=535 \quad x=560
$$

PTS: 4 REF: fall1516aii NAT: S.ID.A. 4 TOP: Normal Distributions
KEY: probability
31 ANS:
$A(t)=100(0.5)^{\frac{t}{63}}$, where $t$ is time in years, and $A(t)$ is the amount of titanium-44 left after $t$ years.
$\frac{A(10)-A(0)}{10-0}=\frac{89.58132-100}{10}=-1.041868$ The estimated mass at $t=40$ is $100-40(-1.041868) \approx 58.3$. The
actual mass is $A(40)=100(0.5)^{\frac{40}{63}} \approx 64.3976$. The estimated mass is less than the actual mass.
PTS: 6 REF: fall1517aii NAT: F.LE.A. 2 TOP: Modeling Exponential Functions
KEY: AII

## 0616AII Common Core State Standards

## Answer Section

| 1 | ANS: 4 | PTS: 2 | REF: 061601aii | NAT: N.RN.A. 2 |
| :--- | :--- | :---: | :--- | :--- |
|  | TOP: Radicals and Rational Exponents | KEY: variables |  |  |
| 2 | ANS: 3 | PTS: 2 | REF: 061602aii | NAT: A.CED.A. 1 |
|  | TOP: Modeling Rationals |  |  |  |

3 ANS: 2
$(2-y i)(2-y i)=4-4 y i+y^{2} i^{2}=-y^{2}-4 y i+4$
PTS: 2 REF: 061603aii NAT: N.CN.A. 2 TOP: Operations with Complex Numbers
4 ANS: 3
The graph shows three real zeros, and has end behavior matching the given end behavior.
PTS: 2 REF: 061604aii NAT: F.IF.C. 7 TOP: Graphing Polynomial Functions
KEY: AII
5 ANS: 3


$$
\sqrt{56-x}=x \quad-8 \text { is extraneous. }
$$

$$
\begin{aligned}
56-x & =x^{2} \\
0 & =x^{2}+x-56 \\
0 & =(x+8)(x-7) \\
x & =7
\end{aligned}
$$

PTS: 2
REF: 061605aii NAT: A.REI.A. 2
TOP: Solving Radicals
KEY: extraneous solutions

6 ANS: 1


$$
x^{4}-4 x^{3}-9 x^{2}+36 x=0
$$

$$
\begin{aligned}
x^{3}(x-4)-9 x(x-4) & =0 \\
\left(x^{3}-9 x\right)(x-4) & =0 \\
x\left(x^{2}-9\right)(x-4) & =0 \\
x(x+3)(x-3)(x-4) & =0 \\
x & =0, \pm 3,4
\end{aligned}
$$

PTS: 2
REF: 061606aii
NAT: A.APR.B. 3 TOP: Zeros of Polynomials
KEY: AII
7 ANS: 3
PTS: 2
REF: 061607aii NAT: S.IC.A. 2
TOP: Analysis of Data
8 ANS: 4
$\frac{m(c)}{g(c)}=\frac{c+1}{1-c^{2}}=\frac{c+1}{(1+c)(1-c)}=\frac{1}{1-c}$
PTS: 2 REF: 061608aii NAT: F.BF.A. 1 TOP: Operations with Functions
9 ANS: 2

$\bar{x}+2 \sigma$ represents approximately $48 \%$ of the data.
PTS: 2
REF: 061609aii NAT: S.ID.A. 4 TOP: Normal Distributions
KEY: percent
10 ANS: 4
The scenario represents a decreasing geometric sequence with a common ratio of 0.80 .
PTS: 2 REF: 061610aii NAT: F.BF.A. 2 TOP: Sequences
11 ANS: 1
The probability of rain equals the probability of rain, given that Sean pitches.
PTS: 2
REF: 061611aii
NAT: S.CP.A. 3 TOP: Conditional Probability

12 ANS: 1


$$
x=\frac{-3 \pm \sqrt{3^{2}-4(2)(2)}}{2(2)}=\frac{-3 \pm \sqrt{-7}}{4}=-\frac{3}{4} \pm \frac{i \sqrt{7}}{4}
$$

PTS: 2 REF: 061612aii NAT: A.REI.B. 4 TOP: Solving Quadratics
KEY: complex solutions | quadratic formula
13 ANS: 3

$H(t)$ is at a minimum at $70(-1)+80=10$
PTS: 2
REF: 061613aii NAT: F.IF.B. 4 TOP: Graphing Trigonometric Functions
KEY: maximum/minimum
14 ANS: 2
$2 x + 3 \longdiv { 4 x ^ { 3 } + 0 x ^ { 2 } + 5 x + 1 0 }$

$$
\begin{aligned}
& \frac{4 x^{3}+6 x^{2}}{-6 x^{2}+5 x} \\
& \frac{-6 x^{2}-9 x}{14 x+10} \\
& \frac{14 x+21}{-11}
\end{aligned}
$$

PTS: 2 REF: 061614aii NAT: A.APR.D. 6 TOP: Rational Expressions
15 ANS: 4


PTS: 2
REF: 061615aii
NAT: F.IF.C. 8
TOP: Modeling Exponential Functions

16
ANS: 2

$$
\begin{aligned}
x & =-\frac{3}{4} y+2 \\
-4 x & =3 y-8 \\
-4 x+8 & =3 y \\
-\frac{4}{3} x+\frac{8}{3} & =y
\end{aligned}
$$

PTS: 2
REF: 061616aii
NAT: F.BF.B. 4
TOP: Inverse of Functions
KEY: equations
17 ANS: 1

$$
\cos \theta=-\frac{6}{10}=-\frac{3}{5}
$$



PTS: 2 REF: 061617aii NAT: F.TF.A. 2 TOP: Determining Trigonometric Functions
KEY: extension to reals
18 ANS: 1


PTS: 2 REF: 061618aii NAT: F.IF.C. 7 TOP: Graphing Logarithmic Functions
19 ANS: 4
$4\left(x^{2}-6 x+9\right)+4\left(y^{2}+18 y+81\right)=76+36+324$

$$
4(x-3)^{2}+4(y+9)^{2}=436
$$

PTS: 2
REF: 061619aii
NAT: G.GPE.A. 1 TOP: Equations of Circles
KEY: completing the square
20 ANS: 2
PTS: 2
REF: 061620aii NAT: F.IF.B. 4
TOP: Graphing Polynomial Functions

21 ANS: 3
$1.0525^{\frac{1}{12}} \approx 1.00427$
PTS: 2 REF: 061621aii NAT: F.BF.A. 1 TOP: Modeling Exponential Functions
KEY: AII
22 ANS: 4


PTS: 2
REF: 061622aii
KEY: AII
23 ANS: 3
TOP: Sequences
PTS: 2
NAT: A.REI.D. 11 TOP: Other Systems

ANS: 4
period $=\frac{2 \pi}{B}$
$\frac{1}{60}=\frac{2 \pi}{B}$
$B=120 \pi$
PTS: 2 REF: 061624aii NAT: F.TF.B. 5 TOP: Modeling Trigonometric Functions
25 ANS:


$$
\begin{aligned}
\frac{3-x}{3 x} & =-\frac{1}{3 x} \\
3-x & =-1 \\
x & =4
\end{aligned}
$$

PTS: 2
REF: 061625aii
NAT: A.REI.A. 2 TOP: Solving Rationals KEY: rational solutions

ANS:
Randomly assign participants to two groups. One group uses the toothpaste with ingredient $X$ and the other group uses the toothpaste without ingredient $X$.

PTS: 2 REF: 061626aii NAT: S.IC.B. 3 TOP: Analysis of Data
KEY: type
27 ANS:

$x - 5 \longdiv { 2 x ^ { 3 } - 4 x ^ { 2 } - 7 x - 1 0 }$ Since there is a remainder, $x-5$ is not a factor.
$\underline{2 x^{3}-10 x^{2}}$
$6 x^{2}-7 x$
$6 x^{2}-30 x$
$23 x-10$
$\underline{23 x-115}$
105
PTS: 2 REF: 061627aii NAT: A.APR.B. 2 TOP: Remainder Theorem
28 ANS:


PTS: 2 REF: 061628aii NAT: F.IF.C. 7 TOP: Graphing Trigonometric Functions KEY: graph
29
ANS:
$P(S \cap M)=P(S)+P(M)-P(S \cup M)=\frac{649}{1376}+\frac{433}{1376}-\frac{974}{1376}=\frac{108}{1376}$
PTS: 2
REF: 061629aii NAT: S.CP.B. 7 TOP: Theoretical Probability

30 ANS:


The vertex of the parabola is $(4,-3)$. The $x$-coordinate of the focus and the vertex is the same. Since the distance from the vertex to the directrix is 3 , the distance from the vertex to the focus is 3 , so the $y$-coordinate of the focus is 0 . The coordinates of the focus are $(4,0)$.

PTS: 2
REF: 061630aii
NAT: G.GPE.A. 2 TOP: Graphing Quadratic Functions
31 ANS:
$\frac{x^{3}+9}{x^{3}+8}=\frac{x^{3}+8}{x^{3}+8}+\frac{1}{x^{3}+8}$
$\frac{x^{3}+9}{x^{3}+8}=\frac{x^{3}+9}{x^{3}+8}$
PTS: 2 REF: 061631aii NAT: A.APR.C. 4 TOP: Polynomial Identities
32 ANS:

$$
\begin{aligned}
A & =P e^{r t} \\
135000 & =100000 e^{5 r} \\
1.35 & =e^{5 r} \\
\ln 1.35 & =\ln e^{5 r} \\
\ln 1.35 & =5 r \\
.06 & \approx r \text { or } 6 \%
\end{aligned}
$$

PTS: 2 REF: 061632aii NAT: F.LE.A. 4 TOP: Exponential Growth

33 ANS:


$$
\begin{aligned}
y & =-x+5 \quad y=-7+5=-2 \\
(x-3)^{2}+(-x+5+2)^{2} & =16 \quad y=-3+5=2 \\
x^{2}-6 x+9+x^{2}-14 x+49 & =16 \\
2 x^{2}-20 x+42 & =0 \\
x^{2}-10 x+21 & =0 \\
(x-7)(x-3) & =0 \\
x & =7,3
\end{aligned}
$$

PTS: 4 REF: 061633aii NAT: A.REI.C. 7 TOP: Quadratic-Linear Systems
KEY: AII
34 ANS:
$S_{n}=\frac{33000-33000(1.04)^{n}}{1-1.04} S_{15}=\frac{33000-33000(1.04)^{15}}{1-1.04} \approx 660778.39$
PTS: 4 REF: 061634aii NAT: A.SSE.B. 4 TOP: Series
35 ANS:
$0.602 \pm 2 \cdot 0.066=0.47-0.73$. Since 0.50 falls within the $95 \%$ interval, this supports the concern there may be an even split.

PTS: 4 REF: 061635aii NAT: S.IC.B. 5 TOP: Analysis of Data
36 ANS:
$\frac{f(4)-f(-2)}{4--2}=\frac{80-1.25}{6}=13.125 g(x)$ has a greater rate of change
$\frac{g(4)-g(-2)}{4--2}=\frac{179--49}{6}=38$
PTS: 4
REF: 061636aii NAT: F.IF.B. 6 TOP: Rate of Change
KEY: AII

37 ANS:

$A(t)=800 e^{-0.347 t}$
$B(t)=400 e^{-0.231 t}$
$800 e^{-0.347 t}=400 e^{-0.231 t}$
$\ln 2 e^{-0.347 t}=\ln e^{-0.231 t} \quad \ln 0.15=\ln e^{-0.347 t}$

$$
\ln 2+\ln e^{-0.347 t}=\ln e^{-0.231 t} \quad \ln 0.15=-0.347 t \cdot \ln e
$$

$$
\ln 2-0.347 t=-0.231 t
$$

$$
5.5 \approx t
$$

$$
\begin{aligned}
\ln 2 & =0.116 t \\
6 & \approx t
\end{aligned}
$$



PTS: 6 REF: 061637aii NAT: A.REI.D. 11 TOP: Other Systems KEY: AII

## 0816AII Common Core State Standards

## Answer Section

1 ANS: 4
If $1-i$ is one solution, the other is $1+i . \quad(x-(1-i))(x-(1+i))=0$

$$
\begin{array}{r}
x^{2}-x-i x-x+i x+\left(1-i^{2}\right)=0 \\
x^{2}-2 x+2=0
\end{array}
$$

PTS: 2 REF: 081601aii NAT: A.REI.B. 4 TOP: Complex Conjugate Root Theorem
2 ANS: 1
II. Ninth graders drive to school less often; III.Students know little about adults; IV. Calculus students love math!

PTS: 2 REF: 081602aii NAT: S.IC.B. 3 TOP: Analysis of Data
KEY: bias
3 ANS: 2


PTS: 2
REF: 081603aii NAT: A.REI.D. 11 TOP: Other Systems
KEY: AII
4 ANS: 3


PTS: 2 REF: 081604aii NAT: S.ID.A. 4 TOP: Normal Distributions
KEY: probability
5 ANS: 3
$(m-2)^{2}(m+3)=\left(m^{2}-4 m+4\right)(m+3)=m^{3}+3 m^{2}-4 m^{2}-12 m+4 m+12=m^{3}-m^{2}-8 m+12$
PTS: 2
REF: 081605aii NAT: A.SSE.A. 2 TOP: Factoring Polynomials
KEY: factoring by grouping

6 ANS: 3

$$
\begin{aligned}
-33 t^{2}+360 t & =700+5 t \\
-33 t^{2}+355 t-700 & =0 \\
t & =\frac{-355 \pm \sqrt{355^{2}-4(-33)(-700)}}{2(-33)} \approx 3,8
\end{aligned}
$$

PTS: 2 REF: 081606aii NAT: A.REI.D. 11 TOP: Quadratic-Linear Systems
KEY: AII
7 ANS: 1
$\frac{157}{25+47+157}$
PTS: 2 REF: 081607aii NAT: S.CP.A. 4 TOP: Conditional Probability
8 ANS: 1
(2) is not recursive

PTS: 2 REF: 081608aii NAT: F.BF.A. 2 TOP: Sequences
9 ANS: 1 PTS: 2 REF: 081609aii NAT: F.BF.B. 6
TOP: Sigma Notation KEY: represen
10 ANS: 2 PTS: 2 REF: 081610aii NAT: F.IF.B. 4
TOP: Graphing Trigonometric Functions KEY: increasing/decreasing
11 ANS: 2

$$
\begin{array}{r}
\frac{x^{2}+0 x+1}{x + 2 \longdiv { x ^ { 3 } + 2 x ^ { 2 } + x + 6 }} \\
\frac{x^{3}+2 x^{2}}{0 x^{2}+x} \\
\frac{0 x^{2}+0 x}{x+6} \\
\frac{x+2}{4}
\end{array}
$$

PTS: 2 REF: 081611aii NAT: A.APR.D. 6 TOP: Rational Expressions
12 ANS: 2
$M E=\left(z \sqrt{\frac{p(1-p)}{n}}\right)=\left(1.96 \sqrt{\frac{(0.55)(0.45)}{900}}\right) \approx 0.03$
PTS: 2 REF: 081612aii NAT: S.IC.B. 4 TOP: Analysis of Data

13 ANS: 1
The car lost approximately $19 \%$ of its value each year.
PTS: 2 REF: 081613aii NAT: F.LE.B. 5 TOP: Modeling Exponential Functions
14 ANS: 1
The graph of $y=\sin x$ is unchanged when rotated $180^{\circ}$ about the origin.
PTS: 2 REF: 081614aii NAT: F.BF.B. 3 TOP: Even and Odd Functions
15 ANS: 3

$$
\begin{gathered}
2 d\left(d^{3}+3 d^{2}-9 d-27\right) \\
2 d\left(d^{2}(d+3)-9(d+3)\right) \\
2 d\left(d^{2}-9\right)(d+3) \\
2 d(d+3)(d-3)(d+3) \\
2 d(d+3)^{2}(d-3)
\end{gathered}
$$

PTS: 2 REF: 081615aii NAT: A.SSE.A. 2 TOP: Factoring Polynomials
KEY: factoring by grouping
16 ANS: 1 PTS: 2 REF: 081616aii NAT: F.TF.A. 1
TOP: Unit Circle
17 ANS: 3
$\frac{1}{J}=\frac{1}{F}-\frac{1}{W}$
$\frac{1}{J}=\frac{W-F}{F W}$
$J=\frac{F W}{W-F}$
PTS: 2 REF: 081617aii NAT: A.REI.A. 2 TOP: Solving Rationals
KEY: rational solutions
18 ANS: 3 PTS: 2 REF: 081618aii NAT: F.LE.A. 2
TOP: Sequences
19 ANS: 4
The vertex is $(2,-1)$ and $p=2 . y=-\frac{1}{4(2)}(x-2)^{2}-1$
PTS: 2 REF: 081619aii NAT: G.GPE.A. 2 TOP: Graphing Quadratic Functions
20 ANS: 4
$(x+y)^{3}=x^{3}+3 x^{2} y+3 x y^{2}+y^{3} \neq x^{3}+3 x y+y^{3}$
PTS: 2 REF: 081620aii NAT: A.APR.C. 4 TOP: Polynomial Identities
21 ANS: 3
Since $x+4$ is a factor of $p(x)$, there is no remainder.
PTS: 2
REF: 081621aii NAT: A.APR.B. 2 TOP: Remainder Theorem

22 ANS: 4 PTS: 2 REF: 081622aii NAT: F.BF.A. 1
TOP: Modeling Exponential Functions KEY: AII
23 ANS: 2
Combining (1) and (3): $-6 c=-18$ Combining (1) and (2): $5 a+3 c=-1$ Using (3): $-(-2)-5 b-5(3)=2$

$$
\begin{aligned}
& c=3 \\
& 5 a+3(3)=-1 \\
& 5 a=-10 \\
& 2-5 b-15=2 \\
& b=-3 \\
& a=-2
\end{aligned}
$$

PTS: 2 REF: 081623aii NAT: A.REI.C. 6 TOP: Solving Linear Systems
KEY: three variables
24 ANS: 4 PTS: 2 REF: 081624aii NAT: F.BF.A. 2
TOP: Sequences
25 ANS:
Amplitude, because the height of the graph shows the volume of the air.
PTS: 2 REF: 081625aii NAT: F.IF.C. 7 TOP: Graphing Trigonometric Functions
KEY: mixed
26 ANS:
Applying the commutative property, $\left(3^{\frac{1}{5}}\right)^{2}$ can be rewritten as $\left(3^{2}\right)^{\frac{1}{5}}$ or $9^{\frac{1}{5}}$. A fractional exponent can be rewritten as a radical with the denominator as the index, or $9^{\frac{1}{5}}=\sqrt[5]{9}$.

PTS: 2 REF: 081626aii NAT: N.RN.A. 1 TOP: Radicals and Rational Exponents
27 ANS:
$x i(-6 i)^{2}=x i\left(36 i^{2}\right)=36 x i^{3}=-36 x i$
PTS: 2 REF: 081627aii NAT: N.CN.A. 2 TOP: Operations with Complex Numbers
28 ANS:
$\sin ^{2} \theta+(-0.7)^{2}=1 \quad$ Since $\theta$ is in Quadrant II, $\sin \theta=\sqrt{.51}$ and $\tan \theta=\frac{\sin \theta}{\cos \theta}=\frac{\sqrt{.51}}{-0.7} \approx-1.02$ $\sin ^{2} \theta=.51$ $\sin \theta= \pm \sqrt{.51}$

PTS: 2 REF: 081628aii NAT: F.TF.C. 8 TOP: Determining Trigonometric Functions
29 ANS:
Using a $95 \%$ level of confidence, $x \pm 2$ standard deviations sets the usual wait time as $150-302$ seconds. 360 seconds is unusual.

PTS: 2 REF: 081629aii NAT: S.IC.B. 6 TOP: Analysis of Data

30 ANS:
$0=\log _{10}(x-4)$ The $x$-intercept of $h$ is $(2,0) . f$ has the larger value.

$$
\begin{aligned}
10^{0} & =x-4 \\
1 & =x-4 \\
x & =5
\end{aligned}
$$

PTS: 2 REF: 081630aii NAT: F.IF.C. 9 TOP: Comparing Functions
KEY: AII
31 ANS:
$\frac{156.25-56.25}{70-50}=\frac{150}{20}=7.5$ Between $50-70 \mathrm{mph}$, each additional mph in speed requires 7.5 more feet to stop.
PTS: 2 REF: 081631aii NAT: F.IF.B. 6 TOP: Rate of Change
KEY: AII
32 ANS:
$P(A \cup B)=P(A)+P(B)-P(A \cap B) A$ and $B$ are independent since $P(A \cap B)=P(A) \cdot P(B)$

$$
\begin{array}{rlrl}
0.8 & =0.6+0.5-P(A \cap B) & 0.3 & =0.6 \cdot 0.5 \\
P(A \cap B) & =0.3 & 0.3 & =0.3
\end{array}
$$

PTS: 2 REF: 081632aii NAT: S.CP.A. 2 TOP: Theoretical Probability
33 ANS:
$0=x^{2}(x+1)-4(x+1)$

$0=\left(x^{2}-4\right)(x+1)$
$0=(x+2)(x-2)(x+1)$
$x=-2,-1,2$
PTS: 4 REF: 081633aii NAT: F.IF.C. 7 TOP: Graphing Polynomial Functions

34 ANS:

$$
\begin{aligned}
7 & =20(0.5)^{\frac{t}{8.02}} \\
\log 0.35 & =\log 0.5^{\frac{t}{8.02}} \\
\log 0.35 & =\frac{t \log 0.5}{8.02} \\
\frac{8.02 \log 0.35}{\log 0.5} & =t \\
t & \approx 12
\end{aligned}
$$

PTS: 4 REF: 081634aii NAT: F.LE.A. 4 TOP: Exponential Decay
35 ANS:

$$
\begin{array}{rlrl}
(\sqrt{2 x-7})^{2} & =(5-x)^{2} & \sqrt{2(4)-7}+4 & =5 \\
2 x-7 & =25-10 x+x^{2} & \sqrt{2(8)-7}+8 & =5 \\
0 & =x^{2}-12 x+32 \\
0 & =(x-8)(x-4) \\
x & =4,8
\end{array}
$$

PTS: 4 REF: 081635aii NAT: A.REI.A. 2 TOP: Solving Radicals
KEY: extraneous solutions
36
ANS:
Some of the students who did not drink energy drinks read faster than those who did drink energy drinks.
$17.7-19.1=-1.4$ Differences of -1.4 and less occur $\frac{25}{232}$ or about $10 \%$ of the time, so the difference is not unusual.

PTS: 4
REF: 081636aii NAT: S.IC.B. 5 TOP: Analysis of Data
$A=5000(1.045)^{n} \quad 5000\left(1+\frac{.046}{4}\right)^{4(6)}-5000(1.045)^{6} \approx 6578.87-6511.30 \approx 67.5710000=5000\left(1+\frac{.046}{4}\right)^{4 n}$
$B=5000\left(1+\frac{.046}{4}\right)^{4 n}$

$$
\begin{aligned}
2 & =1.0115^{4 n} \\
\log 2 & =4 n \cdot \log 1.0115 \\
n & =\frac{\log 2}{4 \log 1.0115} \\
n & \approx 15.2
\end{aligned}
$$

PTS: 6
REF: 081637aii NAT: A.CED.A. 1 TOP: Exponential Growth

## 0117AII Common Core State Standards

## Answer Section

1 ANS: 2
PTS: 2
REF: 011701aii
NAT: F.IF.B. 4
TOP: Graphing Trigonometric Functions
2 ANS: 1
$P(28)=5(2)^{\frac{98}{28}} \approx 56$
PTS: 2 REF: 011702aii NAT: F.LE.A. 2 TOP: Modeling Exponential Functions
KEY: AII
3 ANS: 4
$m^{5}+m^{3}-6 m=m\left(m^{4}+m^{2}-6\right)=m\left(m^{2}+3\right)\left(m^{2}-2\right)$

PTS: 2 REF: 011703aii NAT: A.SSE.A. 2 TOP: Factoring Polynomials
KEY: higher power AII
4 ANS: 1 PTS: 2 REF: 011704aii NAT: F.TF.C. 8
TOP: Simplifying Trigonometric Expressions
5 ANS: 4
$y=g(x)=(x-2)^{2} \quad(x-2)^{2}=3 x-2 \quad y=3(6)-2=16$
$x^{2}-4 x+4=3 x-2 \quad y=3(1)-2=1$
$x^{2}-7 x+6=0$
$(x-6)(x-1)=0$
$x=6,1$

PTS: 2 REF: 011705aii NAT: A.REI.C. 7 TOP: Quadratic-Linear Systems
KEY: AII
6 ANS: 3
PTS: 2
REF: 011706aii NAT: S.IC.B. 3
TOP: Analysis of Data
KEY: type
7 ANS: 2
$\left(m^{\frac{5}{3}}\right)^{-\frac{1}{2}}=m^{-\frac{5}{6}}=\frac{1}{\sqrt[6]{m^{5}}}$

PTS: 2 REF: 011707aii NAT: N.RN.A. 2 TOP: Radicals and Rational Exponents
KEY: variables
8 ANS: 3
PTS: 2
TOP: Inverse of Functions
9 ANS: $2 \quad$ PTS: 2
08ail
NAT: F.BF.B. 4
KEY: equations
REF: 011709aii
NAT: S.IC.B. 5
TOP: Analysis of Data
10 ANS: 3 PTS: 2
REF: 011710aii
NAT: F.BF.A. 1
TOP: Operations with Functions

11 ANS: 4
$x=\frac{8 \pm \sqrt{(-8)^{2}-4(6)(29)}}{2(6)}=\frac{8 \pm \sqrt{-632}}{12}=\frac{8 \pm i \sqrt{4} \sqrt{158}}{12}=\frac{2}{3} \pm \frac{1}{6} i \sqrt{158}$
PTS: 2 REF: 011711aii NAT: A.REI.B. 4 TOP: Solving Quadratics
KEY: complex solutions | quadratic formula
12 ANS: 2


PTS: 2
REF: 011712aii NAT: A.REI.D. 11 TOP: Other Systems
KEY: AII
13 ANS: 3
The pattern suggests an exponential pattern, not linear or sinusoidal. A 4\% growth rate is accurate, while a $43 \%$ growth rate is not.

PTS: 2 REF: 011713aii NAT: S.ID.B. 6 TOP: Regression
KEY: choose model
14 ANS: 1
$d=18 ; r= \pm \frac{5}{4}$
PTS: 2 REF: 011714aii NAT: F.IF.A. 3 TOP: Sequences
KEY: term
15 ANS: 3
$d=10 \log \frac{6.3 \times 10^{-3}}{1.0 \times 10^{-12}} \approx 98$
PTS: 2 REF: 011715aii NAT: F.IF.B. 4 TOP: Evaluating Logarithmic Expressions
16 ANS: 2


PTS: 2 REF: 011716aii NAT: A.REI.D. 11 TOP: Other Systems
KEY: AII

17 ANS: 1

$$
\begin{aligned}
\frac{2(x-4)}{(x+3)(x-4)}+\frac{3(x+3)}{(x-4)(x+3)} & =\frac{2 x-2}{x^{2}-x-12} \\
2 x-8+3 x+9 & =2 x-2 \\
3 x & =-3 \\
x & =-1
\end{aligned}
$$

PTS: 2 REF: 011717aii NAT: A.REI.A. 2 TOP: Solving Rationals
KEY: rational solutions
18 ANS: 4
$496 \pm 2(115)$
PTS: 2 REF: 011718aii NAT: S.ID.A. 4 TOP: Normal Distributions
KEY: interval
19 ANS: 2
$h(x)$ does not have a $y$-intercept.
PTS: 2 REF: 011719aii NAT: F.IF.C. 9 TOP: Comparing Functions
20 ANS: 2 PTS: 2 REF: 011720aii NAT: A.APR.B. 2
TOP: Remainder Theorem
21 ANS: 4
(1) $\frac{B(60)-B(10)}{60-10} \approx 28 \%$ (2) $\frac{B(69)-B(19)}{69-19} \approx 33 \%$ (3) $\frac{B(72)-B(36)}{72-36} \approx 38 \%$ (4) $\frac{B(73)-B(60)}{73-60} \approx 46 \%$

PTS: 2 REF: 011721aii NAT: F.IF.B. 6 TOP: Rate of Change
KEY: AII
22 ANS: 3
(3) repeats 3 times over $2 \pi$.

PTS: 2 REF: 011722aii NAT: F.IF.C. 7 TOP: Graphing Trigonometric Functions
KEY: recognize
23 ANS: 1

$$
\begin{aligned}
\frac{A}{P} & =e^{r t} \\
0.42 & =e^{r t} \\
\ln 0.42 & =\ln e^{r t} \\
-0.87 & \approx r t
\end{aligned}
$$

PTS: 2 REF: 011723aii NAT: F.BF.A. 1 TOP: Modeling Exponential Functions
KEY: AII

24 ANS: 1
(1) $\frac{9-0}{2-1}=9$ (2) $\frac{17-0}{3.5-1}=6.8$ (3) $\frac{0-0}{5-1}=0$ (4) $\frac{17--5}{3.5-1} \approx 6.3$

PTS: 2 REF: 011724aii NAT: F.IF.B. 6 TOP: Rate of Change
KEY: AII
25 ANS:
$(1-i)(1-i)(1-i)=\left(1-2 i+i^{2}\right)(1-i)=-2 i(1-i)=-2 i+2 i^{2}=-2-2 i$
PTS: 2 REF: 011725aii NAT: N.CN.A. 2 TOP: Operations with Complex Numbers
26 ANS:
sample: pails of oranges; population: truckload of oranges. It is likely that about $5 \%$ of all the oranges are unsatisfactory.

PTS: 2 REF: 011726aii NAT: S.IC.A. 2 TOP: Analysis of Data
27 ANS:
$\csc \theta=\frac{1}{\sin \theta}$, and $\sin \theta$ on a unit circle represents the $y$ value of a point on the unit circle. Since $y=\sin \theta$, $\csc \theta=\frac{1}{y}$.

PTS: 2 REF: 011727aii NAT: F.TF.A. 2 TOP: Reciprocal Trigonometric Relationships
28 ANS:
$\frac{\left(\ln \frac{1}{2}\right)}{1590}$ is negative, so $M(t)$ represents decay.
PTS: 2 REF: 011728aii NAT: F.IF.C. 8 TOP: Modeling Exponential Functions
29 ANS:


PTS: 2
REF: 011729aii NAT: F.IF.C. 7
TOP: Graphing Polynomial Functions

30 ANS:

$$
\begin{aligned}
\left(x^{\frac{5}{3}}\right)^{\frac{6}{5}} & =\left(y^{\frac{5}{6}}\right)^{\frac{6}{5}} \\
x^{2} & =y
\end{aligned}
$$

PTS: 2
REF: 011730aii NAT: N.RN.A. 2 TOP: Radicals and Rational Exponents
KEY: variables
31 ANS:
No, because $P(M / R) \neq P(M)$

$$
\begin{aligned}
& \frac{70}{180} \neq \frac{230}{490} \\
& 0.38 \neq 0.47
\end{aligned}
$$

PTS: 2 REF: 011731aii NAT: S.CP.A. 4 TOP: Conditional Probability
32 ANS:

$$
\begin{gathered}
x - 2 \longdiv { 3 x + 1 3 } \begin{array} { c } 
{ \frac { 3 x ^ { 2 } + 7 x - 2 0 } { 3 x ^ { 2 } - 6 x } } \\
{ 1 3 x - 2 0 } \\
{ \underline { 1 3 x - 2 6 } }
\end{array}
\end{gathered}
$$

6

PTS: 2 REF: 011732aii NAT: A.APR.D. 6 TOP: Rational Expressions
33 ANS:
$\begin{array}{rlrl}2 x^{3}-10 x^{2}+11 x-7 & =2 x^{3}+h x^{2}+3 x-8 x^{2}-4 h x-12+k & h & =-2 \\ -2 x^{2}+8 x+5 & =h x^{2}-4 h x+k & k & =5\end{array}$
PTS: 4 REF: 011733aii NAT: A.APR.C. 4 TOP: Polynomial Identities
34 ANS:
Jillian's plan, because distance increases by one mile each week. $a_{1}=10 \quad a_{n}=n+12$

$$
a_{n}=a_{n-1}+1
$$

PTS: 4 REF: 011734aii NAT: F.LE.A. 2 TOP: Sequences
35 ANS:
$P(P / K)=\frac{P\left(P^{\wedge} K\right)}{P(K)}=\frac{1.9}{2.3} \approx 82.6 \%$ A key club member has an $82.6 \%$ probability of being enrolled in AP Physics.
PTS: 4 REF: 011735aii NAT: S.CP.B. 6 TOP: Conditional Probability

36 ANS:
$20000=P M T\left(\frac{1-(1+.00625)^{-60}}{0.00625}\right) 21000-x=300\left(\frac{1-(1+.00625)^{-60}}{0.00625}\right)$
$P M T \approx 400.76$
$x \approx 6028$
PTS: 4 REF: 011736aii NAT: A.SSE.B. 4 TOP: Series
37 ANS:

$$
\begin{aligned}
& 0=\sqrt{t}-2 t+6 \quad 2\left(\frac{9}{4}\right)-6<0, \text { so } \frac{9}{4} \text { is extraneous. } \\
& 2 t-6=\sqrt{t} \\
& 4 t^{2}-24 t+36=t \\
& 4 t^{2}-25 t+36=0 \\
&(4 t-9)(t-4)=0 \\
& t=\frac{9}{4}, 4 \\
&(\sqrt{1}-2(1)+6)-(\sqrt{3}-2(3)+6)=5-\sqrt{3} \approx 3.268327 \mathrm{mph}
\end{aligned}
$$

PTS: 6
REF: 011737aii NAT: A.REI.A. 2 TOP: Solving Radicals KEY: context

## 0617aii

Answer Section
1 ANS: $1 \quad$ PTS: 2
TOP: Zeros of Polynomials
REF: 061701aii NAT: A.APR.B. 3

2 ANS: 1

$$
\begin{aligned}
8\left(2^{x+3}\right) & =48 \\
2^{x+3} & =6 \\
(x+3) \ln 2 & =\ln 6 \\
x+3 & =\frac{\ln 6}{\ln 2} \\
x & =\frac{\ln 6}{\ln 2}-3
\end{aligned}
$$

PTS: 2 REF: 061702aii NAT: F.LE.A. 4 TOP: Exponential Equations
KEY: without common base
3 ANS: 3
Self selection causes bias.
PTS: 2
REF: 061703aii
NAT: S.IC.B. 3 TOP: Analysis of Data
KEY: bias
4 ANS: 2
$6 x i^{3}(-4 x i+5)=-24 x^{2} i^{4}+30 x i^{3}=-24 x^{2}(1)+30 x(-1)=-24 x^{2}-30 x i$
PTS: 2 REF: 061704aii NAT: N.CN.A. 2 TOP: Operations with Complex Numbers
5 ANS: 2


PTS: 2
REF: 061705aii
NAT: A.REI.D. 11 TOP: Other Systems
KEY: AII
6 ANS: 4
PTS: 2
REF: 061706aii NAT: F.IF.B. 4
TOP: Graphing Trigonometric Functions

7 ANS: 4
$4 x^{2}=-98$
$x^{2}=-\frac{98}{4}$
$x^{2}=-\frac{49}{2}$
$x= \pm \sqrt{-\frac{49}{2}}= \pm \frac{7 i}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}= \pm \frac{7 i \sqrt{2}}{2}$
PTS: 2 REF: 061707aii NAT: A.REI.B. 4 TOP: Solving Quadratics
KEY: complex solutions | taking square roots
8 ANS: 1 PTS: 2 REF: 061708aii NAT: F.IF.C. 7
TOP: Graphing Trigonometric Functions KEY: identify
9 ANS: 2
$x(30-0.01 x)-\left(0.15 x^{3}+0.01 x^{2}+2 x+120\right)=30 x-0.01 x^{2}-0.15 x^{3}-0.01 x^{2}-2 x-120$ $=-0.15 x^{3}-0.02 x^{2}+28 x-120$

PTS: 2 REF: 061709aii NAT: F.BF.A. 1 TOP: Operations with Functions
10 ANS: 3 PTS: 2 REF: 061710aii NAT: S.IC.A. 2
TOP: Analysis of Data
11 ANS: 1

2 | 1 | 0 | -4 | -4 | 8 |
| ---: | ---: | ---: | ---: | ---: |
|  | 2 | 4 | 0 | -8 |
| 1 | 2 | 0 | -4 | 0 |

Since there is no remainder when the quartic is divided by $x-2$, this binomial is a factor.
PTS: 2
REF: 061711aii NAT: A.APR.B. 2 TOP: Remainder Theorem
12 ANS: 2
$\cos \theta= \pm \sqrt{1-\left(\frac{-\sqrt{2}}{5}\right)^{2}}= \pm \sqrt{\frac{25}{25}-\frac{2}{25}}= \pm \frac{\sqrt{23}}{5}$
PTS: 2 REF: 061712aii NAT: F.TF.C. 8 TOP: Determining Trigonometric Functions
13 ANS: 3
$0.75^{\frac{1}{10}} \approx .9716$
PTS: 2
REF: 061713aii NAT: A.SSE.B. 3 TOP: Modeling Exponential Functions
KEY: AII

14 ANS: 2
The events are independent because $P(A$ and $B)=P(A) \cdot P(B)$.
$0.125=0.5 \cdot 0.25$
If $P(A$ or $B)=P(A)+P(B)-P(A$ and $B)=0.25+0.5-.125=0.625$, then the events are not mutually exclusive because $P(A$ or $B)=P(A)+P(B)$

$$
0.625 \neq 0.5+0.25
$$

PTS: 2
REF: 061714aii
NAT: S.CP.B. 7 TOP: Theoretical Probability
15 ANS: 4

|  | Bar Harbor | Phoenix |
| :--- | :--- | :--- |
| Minimum | 31.386 | 66.491 |
| Midline | 55.3 | 86.729 |
| Maximum | 79.214 | 106.967 |
| Range | 47.828 | 40.476 |

PTS: 2
REF: 061715aii NAT: F.IF.B. 4
TOP: Graphing Trigonometric Functions
KEY: maximum/minimum
16 ANS: $4 \quad$ PTS: 2
REF: 061716aii
NAT: N.RN.A. 2
TOP: Radicals and Rational Exponents
KEY: variables
17 ANS: 4
The vertex is $(1,0)$ and $p=2 . y=\frac{1}{4(2)}(x-1)^{2}+0$

PTS: 2 REF: 061717aii NAT: G.GPE.A. 2 TOP: Graphing Quadratic Functions
18 ANS: 2
The 2010 population is 110 million.
PTS: 2 REF: 061718aii NAT: F.LE.B. 5 TOP: Modeling Exponential Functions
19 ANS: 1
$\frac{2 x}{x-2}\left(\frac{x}{x}\right)-\frac{11}{x}\left(\frac{x-2}{x-2}\right)=\frac{8}{x^{2}-2 x}$

$$
\begin{aligned}
2 x^{2}-11 x+22 & =8 \\
2 x^{2}-11 x+14 & =0 \\
(2 x-7)(x-2) & =0 \\
x & =\frac{7}{2}, 2
\end{aligned}
$$

PTS: 2
20 ANS: 3
TOP: Sequences

REF: 061719aii
PTS: 2
KEY: AII

NAT: A.REI.A. 2 TOP: Solving Rationals
REF: 061720aii NAT: F.LE.A. 2

21 ANS: 3
$\frac{f(7)-f(-7)}{7--7}=\frac{=2^{-0.25(7)} \cdot \sin \left(\frac{\pi}{2}(7)\right)-2^{-0.25(-7)} \cdot \sin \left(\frac{\pi}{2}(-7)\right)}{14} \approx-0.26$
PTS: 2 REF: 061721aii NAT: F.IF.B. 6 TOP: Rate of Change
KEY: AII
22 PTS: 2 REF: 061722aii NAT: A.CED.A. 1
TOP: Modeling Rationals
23 ANS: 4
$\frac{-3 x^{2}-5 x+2}{x^{3}+2 x^{2}}=\frac{(-3 x+1)(x+2)}{x^{2}(x+2)}=\frac{-3 x}{x^{2}}+\frac{1}{x^{2}}=-3 x^{-1}+x^{-2}$
PTS: 2 REF: 061723aii NAT: A.APR.D. 6 TOP: Expressions with Negative Exponents
KEY: variables
24 ANS: 2 PTS: 2 REF: 061724aii NAT: A.SSE.B. 4
TOP: Series
25 ANS:
$r(2)=-6$. Since there is a remainder when the cubic is divided by $x-2$, this binomial is not a factor.

| 1 | -4 | 4 | $4$ | 6 |
| :---: | :---: | :---: | :---: | :---: |
|  | 2 | -4 |  |  |
|  | -2 |  |  |  |

PTS: 2 REF: 061725aii NAT: A.APR.B. 2 TOP: Remainder Theorem
26 ANS:


PTS: 2 REF: 061726aii NAT: S.ID.A. 4 TOP: Normal Distributions
KEY: percent
27 ANS:
$x^{2}(4 x-1)+4(4 x-1)=\left(x^{2}+4\right)(4 x-1)$
PTS: 2
REF: 061727aii
NAT: A.SSE.A. 2 TOP: Factoring Polynomials
KEY: factoring by grouping
28
ANS:
period is $\frac{2}{3}$. The wheel rotates once every $\frac{2}{3}$ second.
PTS: 2 REF: 061728aii NAT: F.IF.C. 7 TOP: Graphing Trigonometric Functions
KEY: period

29 ANS:


PTS: 2
REF: 061729aii
NAT: F.IF.C. 7 TOP: Graphing Exponential Functions
KEY: AII
30 ANS:

$$
\begin{aligned}
\sqrt{x-4} & =-x+6 \quad \sqrt{x-4}=-8+6=-2 \text { is extraneous. } \\
x-4 & =x^{2}-12 x+36 \\
0 & =x^{2}-13 x+40 \\
0 & =(x-8)(x-5) \\
x & =5,8
\end{aligned}
$$

PTS: 2 REF: 061730aii NAT: A.REI.A. 2 TOP: Solving Radicals
KEY: extraneous solutions
31 ANS:
$\sqrt[3]{x} \cdot \sqrt{x}=x^{\frac{1}{3}} \bullet x^{\frac{1}{2}}=x^{\frac{3}{6}} \bullet x^{\frac{3}{6}}=x^{\frac{5}{6}}$
PTS: 2
REF: 061731aii NAT: N.RN.A. 2 TOP: Operations with Radicals
KEY: with variables, index > 2
32
ANS:
A student is more likely to jog if both siblings jog. 1 jogs: $\frac{416}{2239} \approx 0.19$. both jog: $\frac{400}{1780} \approx 0.22$
PTS: 2 REF: 061732aii NAT: S.CP.A. 4 TOP: Conditional Probability
33 ANS:
$x+y+z=1 \quad 2 x+2 y+2 z=2-2 z-z=3 \quad y-(-1)=3 x+2-1=1$
$\underline{\underline{-x+3 y-5 z=11}} \underline{\underline{2 x+4 y+6 z=2}} \quad-3 z=3 \quad y=2 \quad x=0$
$4 y-4 z=12 \quad 2 y+4 z=0 \quad z=-1$
$y-z=3 \quad y+2 z=0$
$y=-2 z$
PTS: 4 REF: 061733aii NAT: A.REI.C. 6 TOP: Solving Linear Systems
KEY: three variables

34 ANS:

$$
\begin{aligned}
M=172600 \bullet \frac{0.00305(1+0.00305)^{12 \cdot 15}}{(1+0.00305)^{12 \cdot 15}-1} \approx 1247 \quad 1100 & =(172600-x) \bullet \frac{0.00305(1+0.00305)^{12 \cdot 15}}{(1+0.00305)^{12 \cdot 15}-1} \\
1100 & \approx(172600-x) \bullet(0.007228) \\
152193 & \approx 172600-x \\
20407 & \approx x
\end{aligned}
$$

PTS: 4
REF: 061734ai
35 ANS:


$$
\text { As } x \rightarrow-3, y \rightarrow-\infty . \text { As } x \rightarrow \infty, y \rightarrow \infty .
$$

PTS: 4
REF: 061735aii NAT: F.IF.C. 7 TOP: Graphing Logarithmic Functions
36 ANS:
$0.506 \pm 2 \cdot 0.078=0.35-0.66$. The $32.5 \%$ value falls below the $95 \%$ confidence level.
PTS: 4 REF: 061736aii NAT: S.IC.B. 5 TOP: Analysis of Data
37 ANS:
$100=140\left(\frac{1}{2}\right)^{\frac{5}{h}} \log \frac{100}{140}=\log \left(\frac{1}{2}\right)^{\frac{5}{h}} \quad 40=140\left(\frac{1}{2}\right)^{\frac{t}{10.3002}}$

$$
\begin{aligned}
\log \frac{5}{7}=\frac{5}{h} \log \frac{1}{2} & \log \frac{2}{7}
\end{aligned}=\log \left(\frac{1}{2}\right)^{\frac{t}{10.3002}} \quad \begin{aligned}
& \log \frac{5}{7} \log \frac{1}{2} \\
& h=10.3002 \\
& \log \frac{2}{7}=\frac{t \log \left(\frac{1}{2}\right)}{10.3002} \\
& t=\frac{10.3002 \log \frac{2}{7}}{\log \frac{1}{2}} \approx 18.6
\end{aligned}
$$

PTS: 6
REF: 061737aii NAT: F.LE.A. 4 TOP: Exponential Decay

## 0817AII Common Core State Standards

## Answer Section

1 ANS: 1

$$
\begin{aligned}
x^{2}+2 x-8 & =0 \\
(x+4)(x-2) & =0 \\
x & =-4,2
\end{aligned}
$$

PTS: 2 REF: 081701aii NAT: A.APR.D. 6 TOP: Undefined Rationals
2 ANS: 3
$(3 k-2 i)^{2}=9 k^{2}-12 k i+4 i^{2}=9 k^{2}-12 k i-4$
PTS: 2 REF: 081702aii NAT: N.CN.A. 2 TOP: Operations with Complex Numbers
3 ANS: 3
$x^{2}+2 x+1=-5+1$

$$
\begin{aligned}
(x+1)^{2} & =-4 \\
x+1 & = \pm 2 i \\
x & =-1 \pm 2 i
\end{aligned}
$$

PTS: 2 REF: 081703aii NAT: A.REI.B. 4 TOP: Solving Quadratics
KEY: complex solutions | completing the square
4 ANS: 2

$$
\begin{array}{rlr}
\sqrt{x+14} & =\sqrt{2 x+5}+1 & \sqrt{22+14}-\sqrt{2(22)+5}=1 \\
x+14 & =2 x+5+2 \sqrt{2 x+5}+1 & 6-7 \neq 1 \\
-x+8 & =2 \sqrt{2 x+5} &
\end{array}
$$

$x^{2}-16 x+64=8 x+20$
$x^{2}-24 x+44=0$
$(x-22)(x-2)=0$

$$
x=2,22
$$

PTS: 2
KEY: advanced
5 ANS: 3
PTS: 2
TOP: Graphing Trigonometric Functions

NAT: A.REI.A. 2 TOP: Solving Radicals
REF: 081705aii NAT: F.IF.B. 4
KEY: increasing/decreasing

6 ANS: 2
The vertex of the parabola is $(0,0)$. The distance, $p$, between the vertex and the focus or the vertex and the directrix is 1. $y=\frac{-1}{4 p}(x-h)^{2}+k$

$$
\begin{aligned}
& y=\frac{-1}{4(1)}(x-0)^{2}+0 \\
& y=-\frac{1}{4} x^{2}
\end{aligned}
$$

PTS: 2 REF: 081706aii NAT: G.GPE.A. 2 TOP: Graphing Quadratic Functions
7 ANS: 4 PTS: 2
REF: 081707aii NAT: F.TF.A. 2
TOP: Reference Angles
8 ANS: 4 PTS: 2
REF: 081708aii NAT: A.APR.B. 3
TOP: Zeros of Polynomials
KEY: AII
9 ANS: 3
$\log _{0.8}\left(\frac{V}{17000}\right)=t \quad \frac{17,000(0.8)^{3}-17,000(0.8)^{1}}{3-1} \approx-2450$

$$
\begin{aligned}
0.8^{t} & =\frac{V}{17000} \\
V & =17000(0.8)^{t}
\end{aligned}
$$

PTS: 2 REF: 081709aii NAT: F.IF.B. 6 TOP: Rate of Change
KEY: AII
10 ANS: 3
$\left(\frac{1}{2}\right)^{\frac{1}{73.83}} \approx 0.990656$
PTS: 2 REF: 081710aii NAT: A.SSE.B. 3 TOP: Modeling Exponential Functions
KEY: AII
11 ANS: 1


PTS: 2
REF: 081711aii
NAT: S.ID.A. 4 TOP: Normal Distributions
KEY: percent

12 ANS: 4
The maximum volume of $p(x)=-(x+2)(x-10)(x-14)$ is about 56 , at $x=12.1$
PTS: 2 REF: 081712aii NAT: F.IF.B. 4 TOP: Graphing Polynomial Functions
13 ANS: 1
$2 x - 1 \longdiv { 4 x ^ { 3 } + 0 x ^ { 2 } + 9 x - 5 }$

$$
\begin{aligned}
& \frac{4 x^{3}-2 x^{2}}{2 x^{2}+9 x} \\
& \frac{2 x^{2}-x}{10 x-5} \\
& \underline{10 x-5}
\end{aligned}
$$

PTS: 2 REF: 081713aii NAT: A.APR.D. 6 TOP: Rational Expressions
14 ANS: 2

$$
\begin{aligned}
x & =\frac{y+1}{y-2} \\
x y-2 x & =y+1 \\
x y-y & =2 x+1 \\
y(x-1) & =2 x+1 \\
y & =\frac{2 x+1}{x-1}
\end{aligned}
$$

PTS: 2 REF: 081714aii NAT: F.BF.B. 4 TOP: Inverse of Functions
KEY: equations
15 ANS: 1

1) let $y=x+2$, then $y^{2}+2 y-8$

$$
\begin{aligned}
& (y+4)(y-2) \\
& (x+2+4)(x+2-2) \\
& (x+6) x
\end{aligned}
$$

PTS: 2 REF: 081715aii NAT: A.SSE.A. 2 TOP: Factoring Polynomials
KEY: multivariable
16 ANS: 2
$M E=\left(z \sqrt{\frac{p(1-p)}{n}}\right)=\left(1.96 \sqrt{\frac{(0.16)(0.84)}{1334}}\right) \approx 0.02$
PTS: 2
REF: 081716ai NAT: S.IC.B. 4

TOP: Analysis of Data

17 ANS: $2 \quad$ PTS: 2
TOP: Analysis of Data
18 ANS: 4 PTS: 2
TOP: Graphing Trigonometric Functions
REF: 081718aii
KEY: amplitude
19 ANS: 1

$$
\begin{array}{rlc}
(x+3)^{2}+(2 x-4)^{2} & =8 & b^{2}-4 a c \\
x^{2}+6 x+9+4 x^{2}-16 x+16 & =8 & 100-4(5)(17)<0 \\
5 x^{2}-10 x+17 & =0
\end{array}
$$

PTS: 2
REF: 081719aii NAT: A.REI.C. 7 TOP: Quadratic-Linear Systems
KEY: AII
20 ANS: 2

$-4 |$| 1 | -11 | 16 | 84 |
| ---: | ---: | ---: | ---: |
|  | -4 | 60 | -304 |
| 1 | -15 | 76 |  |

Since there is a remainder when the cubic is divided by $x+4$, this binomial is not a factor.
PTS: 2 REF: 081720aii NAT: A.APR.B. 2 TOP: Remainder Theorem
21 ANS: 4
$d=32(.8)^{b-1} S_{n}=\frac{32-32(.8)^{12}}{1-.8} \approx 149$
$\begin{array}{llll}\text { PTS: } 2 & \text { REF: 081721aii } & \text { NAT: A.SSE.B. } 4 & \text { TOP: Series }\end{array}$
22 ANS: 1 PTS: 2 REF: 081722aii NAT: S.IC.B. 6
TOP: Analysis of Data
23 ANS: 4
$\left(\frac{-54 x^{9}}{y^{4}}\right)^{\frac{2}{3}}=\frac{(2 \cdot-27)^{\frac{2}{3}} x^{\frac{18}{3}}}{y^{\frac{8}{3}}}=\frac{2^{\frac{2}{3}} \cdot 9 x^{6}}{y^{2} \cdot y^{\frac{2}{3}}}=\frac{9 x^{63} \sqrt{4}}{y^{2} \sqrt[3]{y^{2}}}$
PTS: 2 REF: 081723aii NAT: N.RN.A. 2 TOP: Radicals and Rational Exponents
KEY: variables
24 ANS: 3 PTS: 2 REF: 081724aii NAT: F.BF.A. 2
TOP: Sequences
25 ANS:
Rewrite $\frac{4}{3}$ as $\frac{1}{3} \cdot \frac{4}{1}$, using the power of a power rule.

PTS: 2 REF: 081725aii NAT: N.RN.A. 1 TOP: Radicals and Rational Exponents
26 ANS:
$P(W / D)=\frac{P\left(W^{\wedge} D\right)}{P(D)}=\frac{.4}{.5} \approx .8$

PTS: 2 REF: 081726aii NAT: S.CP.B. 6 TOP: Conditional Probability

27 ANS:

$$
\begin{aligned}
\left(x^{2}+y^{2}\right)^{2} & =\left(x^{2}-y^{2}\right)^{2}+(2 x y)^{2} \\
x^{4}+2 x^{2} y^{2}+y^{4} & =x^{4}-2 x^{2} y^{2}+y^{4}+4 x^{2} y^{2} \\
x^{4}+2 x^{2} y^{2}+y^{4} & =x^{4}+2 x^{2} y^{2}+y^{4}
\end{aligned}
$$

PTS: 2
REF: 081727aii
NAT: A.APR.C. 4 TOP: Polynomial Identities
28 ANS:
Since there are six flavors, each flavor can be assigned a number, 1-6. Use the simulation to see the number of times the same number is rolled 4 times in a row.

PTS: 2 REF: 081728aii NAT: S.IC.A. 2 TOP: Analysis of Data
29 ANS:
$a_{1}=4 \quad a_{8}=639$
$a_{n}=2 a_{n-1}+1$
PTS: 2 REF: 081729aii NAT: F.LE.A. 2 TOP: Sequences
30 ANS:

$$
\begin{aligned}
8.75 & =1.25 x^{49} \\
7 & =x^{49} \\
x & =\sqrt[49]{7} \approx 1.04
\end{aligned}
$$

PTS: 2 REF: 081730aii NAT: F.LE.A. 4 TOP: Exponential Growth
31 ANS:
$j(-x)=(-x)^{4}-3(-x)^{2}-4=x^{2}-3 x^{2}-4$ Since $j(x)=j(-x)$, the function is even.
PTS: 2 REF: 081731aii NAT: F.BF.B. 3 TOP: Even and Odd Functions
32 ANS:


PTS: 2
REF: 081732aii NAT: F.IF.C. 7 TOP: Graphing Polynomial Functions
KEY: AII

33
ANS:

$$
\begin{aligned}
\frac{3 p}{p-5} & =\frac{p+2}{p+3} \\
3 p^{2}+9 p & =p^{2}-3 p-10 \\
2 p^{2}+12 p+10 & =0 \\
p^{2}+6 p+5 & =0 \\
(p+5)(p+1) & =0 \\
p & =-5,-1
\end{aligned}
$$

PTS: 4 REF: 081733aii NAT: A.REI.A. 2 TOP: Solving Rationals
KEY: rational solutions
34 ANS:
$\frac{6.25-2.25}{21-5}=\frac{4}{16}=\$ .25$ fine per day. $2.25-5(.25)=\$ 1$ replacement fee. $a_{n}=1.25+(n-1)(.25) . a_{60}=\$ 16$
PTS: 4 REF: 081734aii NAT: F.LE.A. 2 TOP: Sequences
35


Part a sketch is shifted $\frac{\pi}{3}$ units right.

PTS: 4 REF: 081735aii NAT: F.IF.C. 7 TOP: Graphing Trigonometric Functions KEY: graph

36 ANS:
$y=4.168(3.981)^{x} . \quad 100=4.168(3.981)^{x}$

$$
\begin{gathered}
\log \frac{100}{4.168}=\log (3.981)^{x} \\
\log \frac{100}{4.168}=x \log (3.981) \\
\frac{\log \frac{100}{4.168}}{\log (3.981)}=x \\
x \approx 2.25
\end{gathered}
$$

PTS: 4 REF: 081736aii NAT: S.ID.B. 6 TOP: Regression
KEY: exponential AII
37 ANS:



At 1.95 years, the value of the car equals the loan
balance. Zach can cancel the policy after 6 years.
PTS: 4
REF: 081737aii NAT: A.REI.D. 11 TOP: Other Systems
KEY: AII

## 0118AII Common Core State Standards

Answer Section

1 ANS: 4
PTS: 2
REF: 011801aii
NAT: S.IC.B. 3
TOP: Analysis of Data
KEY: bias
2 ANS: 3
$\sqrt{x+1}=x+1$
$x+1=x^{2}+2 x+1$
$0=x^{2}+x$
$0=x(x+1)$
$x=-1,0$

PTS: 2 REF: 011802aii NAT: A.REI.A. 2 TOP: Solving Radicals
KEY: extraneous solutions
3 ANS: 4
$3 x-(-2 x+14)=163(6)-4 z=2$

$$
\begin{array}{rlrl}
5 x & =30 & -4 z & =-16 \\
x & =6 & z & =4
\end{array}
$$

PTS: 2 REF: 011803aii NAT: A.REI.C. 6 TOP: Solving Linear Systems
KEY: three variables
4 ANS: 2 PTS: 2
REF: 011804aii NAT: F.TF.A. 2
TOP: Determining Trigonometric Functions KEY: radians
5 ANS: 4 PTS: 2
REF: 011805aii NAT: F.LE.B. 5
TOP: Modeling Exponential Functions
6 ANS: 2 PTS: 2 REF: 011806aii NAT: A.APR.C. 4
TOP: Polynomial Identities
7 ANS: 3
$440 \times 2.3 \% \approx 10$

PTS: 2
REF: 011807aii
NAT: S.ID.A. 4 TOP: Normal Distributions
KEY: predict
8 ANS: 4
PTS: 2
REF: 011808aii NAT: A.SSE.B. 3
TOP: Modeling Exponential Functions KEY: AII

9 ANS: 4

$$
\begin{gathered}
2 x - 1 \longdiv { 5 x ^ { 2 } + x - 3 } \\
\frac{10 x^{3}-5 x^{2}}{2 x^{2}-7 x+3} \\
\underline{2 x^{2}-x} \\
-6 x+3 \\
\underline{-6 x+3}
\end{gathered}
$$

PTS: 2 REF: 011809aii NAT: A.APR.D. 6 TOP: Rational Expressions
10 ANS: 1

$$
9110=5000 e^{30 r}
$$

$\ln \frac{911}{500}=\ln e^{30 r}$
$\frac{\ln \frac{911}{500}}{30}=r$

$$
r \approx .02
$$

PTS: 2 REF: 011810aii NAT: F.LE.A. 4 TOP: Exponential Growth
11 ANS: 4
$\frac{n}{m}=\frac{\sqrt{a^{5}}}{a}=\frac{a^{\frac{5}{2}}}{a^{\frac{2}{2}}}=a^{\frac{3}{2}}=\sqrt{a^{3}}$
PTS: 2 REF: 011811aii NAT: N.RN.A. 2 TOP: Radicals and Rational Exponents
KEY: variables
12 ANS: 1

$$
\begin{aligned}
x-\frac{4}{x-1} & =2 \quad x=\frac{3 \pm \sqrt{(-3)^{2}-4(1)(-2)}}{2(1)}=\frac{3 \pm \sqrt{17}}{2} \\
x(x-1)-4 & =2(x-1) \\
x^{2}-x-4 & =2 x-2 \\
x^{2}-3 x-2 & =0
\end{aligned}
$$

PTS: 2 REF: 011812aii NAT: A.REI.A. 2 TOP: Solving Rationals KEY: rational solutions

13 ANS: 3
$e^{b t}=\frac{c}{a}$
$\ln e^{b t}=\ln \frac{c}{a}$
$b t \ln e=\ln \frac{c}{a}$
$t=\frac{\ln \frac{c}{a}}{b}$
PTS: 2 REF: 011813aii NAT: F.LE.A. 4 TOP: Exponential Growth
14 ANS: 1
PTS: 2
TOP: Other Systems
REF: 011814aii NAT: A.REI.D. 11
KEY: AII
15 ANS: $1 \quad$ PTS: 2
REF: 011815aii NAT: F.TF.A. 2
TOP: Unit Circle
16 ANS: 1
In vertex form, the parabola is $y=-\frac{1}{4(2)}(x+4)^{2}+3$. The vertex is $(-4,3)$ and $p=2.3+2=5$
PTS: 2 REF: 011816aii NAT: G.GPE.A. 2 TOP: Graphing Quadratic Functions
17 ANS: 3


PTS: 2 REF: 011817aii NAT: F.IF.B. 4 TOP: Graphing Polynomial Functions
18 ANS: 3
$\frac{c^{2}-d^{2}}{d^{2}+c d-2 c^{2}}=\frac{(c+d)(c-d)}{(d+2 c)(d-c)}=\frac{-(c+d)}{d+2 c}=\frac{-c-d}{d+2 c}$

PTS: 2 REF: 011818aii NAT: A.APR.D. 6 TOP: Rational Expressions
KEY: a > 0
19 ANS: 4
$p(5)=2(5)^{3}-3(5)+5=240$
PTS: 2 REF: 011819aii NAT: A.APR.B. 2 TOP: Remainder Theorem
20 ANS: 2 PTS: 2 REF: 011820aii NAT: S.IC.A. 2
TOP: Analysis of Data

21 ANS: 2

$$
\begin{aligned}
x & =-6(y-2) \\
-\frac{x}{6} & =y-2 \\
-\frac{x}{6}+2 & =y
\end{aligned}
$$

PTS: 2 REF: 011821aii NAT: F.BF.B. 4 TOP: Inverse of Functions
KEY: equations
22 ANS: 2
$S_{20}=\frac{.01-.01(3)^{20}}{1-3}=17,433,922$
PTS: 2 REF: 011822aii NAT: A.SSE.B. 4 TOP: Series
23 ANS: 4
There is no $x$-intercept.
PTS: 2 REF: 011823aii NAT: F.IF.C. 7 TOP: Graphing Exponential Functions
KEY: AII
24 ANS: 3
PTS: 2
REF: 011824aii NAT: F.BF.A. 2
TOP: Sequences
25 ANS:
$i^{2}=-1$, and not $1 ; 10+10 i$
PTS: 2 REF: 011825aii NAT: N.CN.A. 2 TOP: Operations with Complex Numbers
26 ANS:
$D=1.223(2.652)^{A}$
PTS: 2 REF: 011826aii NAT: S.ID.B. 6 TOP: Regression
KEY: exponential AII
27
$\frac{1}{8}+\frac{1}{6}=\frac{1}{t_{b}} ; \frac{24 t_{b}}{8}+\frac{24 t_{b}}{6}=\frac{24 t_{b}}{t_{b}}$

$$
\begin{aligned}
3 t_{b}+4 t_{b} & =24 \\
t_{b} & =\frac{24}{7} \approx 3.4
\end{aligned}
$$

PTS: 2 REF: 011827aii NAT: A.CED.A. 1 TOP: Modeling Rationals
$3 x^{3}+x^{2}+3 x y+y=x^{2}(3 x+1)+y(3 x+1)=\left(x^{2}+y\right)(3 x+1)$
PTS: 2 REF: 011828aii NAT: A.SSE.A. 2 TOP: Factoring Polynomials
KEY: factoring by grouping

29 ANS:
$20 e^{.05 t}=30 e^{.03 t}$
$\frac{\frac{2}{3} e^{.05 t}}{e^{.05 t}}=\frac{e^{.03 t}}{e^{.05 t}}$
$\ln \frac{2}{3}=\ln e^{-.02 t}$
$\ln \frac{2}{3}=-.02 t \ln e$
$\frac{\ln \frac{2}{3}}{-.02}=t$
$20.3 \approx t$
PTS: 2 REF: 011829aii NAT: A.REI.D. 11 TOP: Other Systems
KEY: AII
30 ANS:
$q$ has the smaller minimum value for the domain $[-2,2]$. h's minimum is $-1(2(-1)+1)$ and $q$ 's minimum is -8 .
PTS: 2
REF: 011830aii NAT: F.IF.C. 9 TOP: Comparing Functions
KEY: AII
31 ANS:


PTS: 2
REF: 011831aii
NAT: F.IF.C. 7
TOP: Graphing Polynomial Functions
32 ANS:
The denominator of the rational exponent represents the index of a root, and the 4 th root of 81 is 3 and $3^{3}$ is 27 .
PTS: 2
REF: 011832ai
NAT: N.RN.A. 1 TOP: Radicals and Rational Exponents

33 ANS:

$$
\begin{gathered}
\left(2 x^{2}+x-3\right) \cdot(x-1)-\left[\left(2 x^{2}+x-3\right)+(x-1)\right] \\
\left(2 x^{3}-2 x^{2}+x^{2}-x-3 x+3\right)-\left(2 x^{2}+2 x-4\right) \\
2 x^{3}-3 x^{2}-6 x+7
\end{gathered}
$$

PTS: 4 REF: 011833aii NAT: F.BF.A. 1 TOP: Operations with Functions
34 ANS:
$\frac{47}{108}=\frac{1}{4}+\frac{116}{459}-P(M$ and $J) ;$ No, because $\frac{31}{459} \neq \frac{1}{4} \cdot \frac{116}{459}$
$P(M$ and $J)=\frac{31}{459}$
PTS: 4 REF: 011834aii NAT: S.CP.A. 3 TOP: Conditional Probability
35 ANS:
$138.905 \pm 2 \cdot 7.95=123-155$. No, since 125 ( $50 \%$ of 250 ) falls within the $95 \%$ interval.
PTS: 4 REF: 011835aii NAT: S.IC.A. 2 TOP: Analysis of Data
36 ANS:
$f(x)=x^{2}(x+4)(x-3) ; g(x)=(x+2)^{2}(x+6)(x-1)$
PTS: 4 REF: 011836aii NAT: A.APR.B. 3 TOP: Zeros of Polynomials
37 ANS:


The period of $P$ is $\frac{2}{3}$, which means the patient's blood pressure reaches a high every $\frac{2}{3}$
second and a low every $\frac{2}{3}$ second. The patient's blood pressure is high because 144 over 96 is greater than 120 over 80 .

PTS: 6 REF: 011837aii NAT: F.IF.C. 7 TOP: Graphing Trigonometric Functions KEY: graph

