# JMAP REGENTS BY STATE STANDARD: TOPIC

NY Algebra II Regents Exam Questions from Spring 2015 to January 2023 Sorted by State Standard: Topic

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#### Algebra II Regents Exam Questions by State Standard: Topic

### **RATE**

#### F.IF.B.6: RATE OF CHANGE

1 Joelle has a credit card that has a 19.2% annual interest rate compounded monthly. She owes a total balance of *B* dollars after *m* months. Assuming she makes no payments on her account, the table below illustrates the balance she owes after *m* months.

| m  | В       |
|----|---------|
| 0  | 100.00  |
| 10 | 1172.00 |
| 19 | 1352.00 |
| 36 | 1770.80 |
| 60 | 2591.90 |
| 69 | 2990.00 |
| 72 | 3135.80 |
| 73 | 3186.00 |

Over which interval of time is her average rate of change for the balance on her credit card account the greatest?

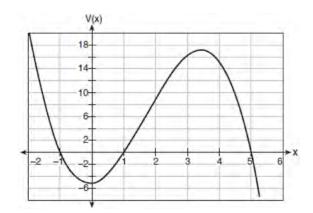
1 month 10 to month 60

3 month 36 to month 72

2 month 19 to month 69

4 month 60 to month 73

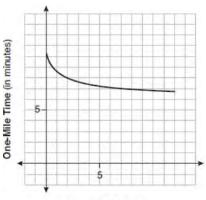
2 A cardboard box manufacturing company is building boxes with length represented by x + 1, width by 5 - x, and height by x - 1. The volume of the box is modeled by the function below.



Over which interval is the volume of the box changing at the fastest average rate?

- 1 [1,2]
- 2 [1,3.5]
- 3 [1,5]
- 4 [0,3.5]

3 Irma initially ran one mile in over ten minutes. She then began a training program to reduce her one-mile time. She recorded her one-mile time once a week for twelve consecutive weeks, as modeled in the graph below.



Number of Weeks

Which statement regarding Irma's one-mile training program is correct?

- 1 Her one-mile speed increased as the number of weeks increased.
- 2 Her one-mile speed decreased as the number of weeks increased.
- 3 If the trend continues, she will run under a six-minute mile by week thirteen.
- 4 She reduced her one-mile time the most between weeks ten and twelve.
- 4 The function  $N(t) = 100e^{-0.023t}$  models the number of grams in a sample of cesium-137 that remain after t years. On which interval is the sample's average rate of decay the fastest?
  - 1 [1,10]
  - 2 [10,20]
  - 3 [15,25]
  - 4 [1,30]

- 5 The function  $N(x) = 90(0.86)^x + 69$  can be used to predict the temperature of a cup of hot chocolate in degrees Fahrenheit after x minutes. What is the approximate average rate of change of the temperature of the hot chocolate, in degrees per minute, over the interval [0,6]?
  - 1 8.93
  - 2 -0.11
  - 3 0.11
  - 4 8.93
- 6 The value of a new car depreciates over time. Greg purchased a new car in June 2011. The value, *V*, of his car after *t* years can be modeled by the equation

$$\log_{0.8} \left( \frac{V}{17000} \right) = t$$
. What is the average decreasing

rate of change per year of the value of the car from June 2012 to June 2014, to the *nearest ten dollars* per year?

- 1 1960
- 2 2180
- 3 2450
- 4 2770

7 The equation 
$$t = \frac{1}{0.0105} \ln \left( \frac{A}{5000} \right)$$
 relates time,  $t$ ,

in years, to the amount of money, *A*, earned by a \$5000 investment. Which statement accurately describes the relationship between the average rates of change of *t* on the intervals [6000, 8000] and [9000, 12,000]?

- 1 A comparison cannot be made because the intervals are different sizes.
- 2 The average rate of change is equal for both intervals.
- 3 The average rate of change is larger for the interval [6000, 8000].
- 4 The average rate of change is larger for the interval [9000, 12,000].

8 The function 
$$f(x) = 2^{-0.25x} \bullet \sin\left(\frac{\pi}{2}x\right)$$
 represents a

damped sound wave function. What is the average rate of change for this function on the interval [-7,7], to the *nearest hundredth*?

- 1 -3.66
- 2 -0.30
- 3 -0.26
- 4 3.36
- 9 The distance needed to stop a car after applying the brakes varies directly with the square of the car's speed. The table below shows stopping distances for various speeds.

| Speed (mph)          | 10   | 20 | 30    | 40  | 50     | 60  | 70     |
|----------------------|------|----|-------|-----|--------|-----|--------|
| <b>Distance</b> (ft) | 6.25 | 25 | 56.25 | 100 | 156.25 | 225 | 306.25 |

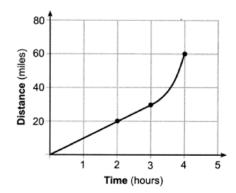
Determine the average rate of change in braking distance, in ft/mph, between one car traveling at 50 mph and one traveling at 70 mph. Explain what this rate of change means as it relates to braking distance.

10 The table below shows the number of hours of daylight on the first day of each month in Rochester, NY.

| Month | Hours of Daylight |
|-------|-------------------|
| Jan.  | 9.4               |
| Feb.  | 10.6              |
| March | 11.9              |
| April | 13.9              |
| May   | 14.7              |
| June  | 15.4              |
| July  | 15.1              |
| Aug.  | 13.9              |
| Sept. | 12.5              |
| Oct.  | 11.1              |
| Nov.  | 9.7               |
| Dec.  | 9.0               |

Given the data, what is the average rate of change in hours of daylight per month from January 1st to April 1st? Interpret what this means in the context of the problem.

Determine the average rate of change, in mph, from 2 to 4 hours on the graph shown below.



12 The world population was 2560 million people in 1950 and 3040 million in 1960 and can be modeled by the function  $p(t) = 2560e^{0.017185t}$ , where t is time in years after 1950 and p(t) is the population in millions. Determine the average rate of change of p(t) in millions of people per year, from  $4 \le t \le 8$ . Round your answer to the *nearest hundredth*.

13 The average monthly high temperature in Buffalo, in degrees Fahrenheit, can be modeled by the function

 $B(t) = 25.29 \sin(0.4895t - 1.9752) + 55.2877$ , where t is the month number (January = 1). State, to the *nearest tenth*, the average monthly rate of temperature change between August and November. Explain its meaning in the given context.

14 The monthly high temperature (°F) in Buffalo, New York can be modeled by

 $B(m) = 24.9 \sin(0.5m - 2.05) + 55.25$ , where m is the number of the month and January = 1. Find the average rate of change in the monthly high temperature between June and October, to the *nearest hundredth*. Explain what this value represents in the given context.

## **QUADRATICS**

#### **A.REI.B.4: SOLVING QUADRATICS**

- 15 What is the solution when the equation  $wx^2 + w = 0$  is solved for x, where w is a positive integer?
  - 1 1
  - 2 0
  - 3 6
  - $4 \pm i$
- 16 The solution to the equation  $4x^2 + 98 = 0$  is
  - 1 ±7
  - $2 \pm 7i$
  - $3 \pm \frac{7\sqrt{2}}{2}$
  - $4 \quad \pm \frac{7i\sqrt{2}}{2}$
- 17 The roots of the equation  $x^2 + 2x + 5 = 0$  are
  - 1 -3 and 1
  - 2 -1, only
  - 3 -1 + 2i and -1 2i
  - 4 -1 + 4i and -1 4i
- 18 The solutions to the equation  $-\frac{1}{2}x^2 = -6x + 20$  are
  - 1  $-6 \pm 2i$
  - $2 -6 \pm 2\sqrt{19}$
  - $3 \quad 6 \pm 2i$
  - 4  $6 \pm 2\sqrt{19}$

- 19 A solution of the equation  $2x^2 + 3x + 2 = 0$  is
  - $1 \quad -\frac{3}{4} + \frac{1}{4}i\sqrt{7}$
  - $2 \qquad -\frac{3}{4} + \frac{1}{4}i$
  - $3 \quad -\frac{3}{4} + \frac{1}{4}\sqrt{7}$
  - $4 \frac{1}{2}$
- 20 The roots of the equation  $3x^2 + 2x = -7$  are
  - 1  $-2, -\frac{1}{3}$
  - $2 -\frac{7}{3}, 1$
  - $3 \quad -\frac{1}{3} \pm \frac{2i\sqrt{5}}{3}$
  - $4 \quad -\frac{1}{3} \pm \frac{\sqrt{11}}{3}$
- 21 The solutions to the equation  $5x^2 2x + 13 = 9$  are
  - $1 \qquad \frac{1}{5} \pm \frac{\sqrt{21}}{5}$
  - $2 \quad \frac{1}{5} \pm \frac{\sqrt{19}}{5} i$
  - $3 \quad \frac{1}{5} \pm \frac{\sqrt{66}}{5} i$
  - $4 \frac{1}{5} \pm \frac{\sqrt{66}}{5}$

22 The solution to the equation  $18x^2 - 24x + 87 = 0$  is

$$1 \quad -\frac{2}{3} \pm 6i\sqrt{158}$$

$$2 \quad -\frac{2}{3} \pm \frac{1}{6} i \sqrt{158}$$

$$3 \quad \frac{2}{3} \pm 6i\sqrt{158}$$

$$4 \quad \frac{2}{3} \pm \frac{1}{6} i \sqrt{158}$$

23 If a solution of  $2(2x-1) = 5x^2$  is expressed in simplest a + bi form, the value of b is

$$1 \quad \frac{\sqrt{6}}{5}$$

$$2 \quad \frac{\sqrt{6}}{5}$$

$$3 \quad \frac{1}{5}i$$

$$4 \frac{1}{5}$$

24 Solve the equation  $2x^2 + 5x + 8 = 0$ . Express the answer in a + bi form.

25 a) Algebraically determine the roots, in simplest a + bi form, to the equation below.

$$x^2 - 2x + 7 = 4x - 10$$

b) Consider the system of equations below.

$$y = x^2 - 2x + 7$$
$$y = 4x - 10$$

The graph of this system confirms the solution from part a is imaginary. Explain why.

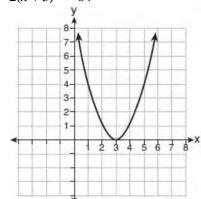
#### A.REI.B.4: USING THE DISCRIMINANT

26 Which representation of a quadratic has imaginary roots?

| X    | У  |
|------|----|
| -2.5 | 2  |
| -2.0 | 0  |
| -1.5 | -1 |
| -1.0 | -1 |
| -0.5 | 0  |
| 0.0  | 2  |

 $2 \qquad 2(x+3)^2 = 64$ 

1



 $3 4 2x^2 + 32 = 0$ 

27 Does the equation  $x^2 - 4x + 13 = 0$  have imaginary solutions? Justify your answer.

## A.REI.B.4: COMPLEX CONJUGATE ROOT THEOREM

28 Which equation has roots of 3 + i and 3 - i?

$$1 \qquad x^2 - 6x + 10 = 0$$

$$2 \qquad x^2 + 6x - 10 = 0$$

$$3 \quad x^2 - 10x + 6 = 0$$

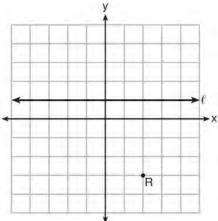
$$4 \qquad x^2 + 10x - 6 = 0$$

- 29 Which equation has 1 i as a solution?
  - $1 \quad x^2 + 2x 2 = 0$
  - $2 \quad x^2 + 2x + 2 = 0$
  - $3 \quad x^2 2x 2 = 0$
  - $4 \quad x^2 2x + 2 = 0$

# G.GPE.A.2: GRAPHING QUADRATIC FUNCTIONS

- 30 A parabola has a directrix of y = 3 and a vertex at (2,1). Which ordered pair is the focus of the parabola?
  - 1 (2,-1)
  - 2 (2,0)
  - 3 (2,2)
  - 4 (2,5)
- 31 The parabola described by the equation  $y = \frac{1}{12}(x-2)^2 + 2$  has the directrix at y = -1. The focus of the parabola is
  - 1 (2,-1)
  - 2 (2,2)
  - 3 (2,3)
  - 4 (2,5)
- 32 What is the equation of the directrix for the parabola  $-8(y-3) = (x+4)^2$ ?
  - 1 y = 5
  - 2 y = 1
  - y = -2
  - y = -6

Which equation represents the set of points equidistant from line  $\ell$  and point R shown on the graph below?



- $1 \qquad y = -\frac{1}{8}(x+2)^2 + 1$
- $2 \qquad y = -\frac{1}{8}(x+2)^2 1$
- $3 \qquad y = -\frac{1}{8} (x 2)^2 + 1$
- 4  $y = -\frac{1}{8}(x-2)^2 1$
- Which equation represents a parabola with the focus at (0,-1) and the directrix of y = 1?

$$1 x^2 = -8y$$

$$2 x^2 = -4y$$

$$3 \quad x^2 = 8y$$

$$4 x^2 = 4y$$

Which equation represents a parabola with a focus of (0,4) and a directrix of y = 2?

$$1 \qquad y = x^2 + 3$$

$$2 \qquad y = -x^2 + 1$$

$$3 \qquad y = \frac{x^2}{2} + 3$$

$$4 \qquad y = \frac{x^2}{4} + 3$$

36 A parabola has its focus at (1,2) and its directrix is y = -2. The equation of this parabola could be

$$1 \qquad y = 8(x+1)^2$$

$$2 \qquad y = \frac{1}{8} (x+1)^2$$

$$3 y = 8(x-1)^2$$

$$4 \qquad y = \frac{1}{8} (x - 1)^2$$

Which equation represents the equation of the parabola with focus (-3,3) and directrix y = 7?

$$1 \qquad y = \frac{1}{8} (x+3)^2 - 5$$

$$2 \qquad y = \frac{1}{8} \left( x - 3 \right)^2 + 5$$

$$3 \qquad y = -\frac{1}{8} (x+3)^2 + 5$$

$$4 \qquad y = -\frac{1}{8}(x-3)^2 + 5$$

38 A parabola that has a vertex at (2,1) and a focus of (2,-3) has an equation of

$$1 \qquad y = \frac{1}{16} \left( x - 2 \right)^2 + 1$$

$$2 \qquad y = -\frac{1}{16} (x+2)^2 - 1$$

$$3 \qquad y = -\frac{1}{16} (x - 2)^2 + 1$$

4 
$$y = -\frac{1}{16}(x-2)^2 - 3$$

Which equation represents a parabola with a focus of (-2,5) and a directrix of y = 9?

$$1 \quad (y-7)^2 = 8(x+2)$$

$$2 \quad (y-7)^2 = -8(x+2)$$

$$3 \quad (x+2)^2 = 8(y-7)$$

$$4 \qquad (x+2)^2 = -8(y-7)$$

Which equation represents a parabola with a focus of (4,-3) and directrix of y = 1?

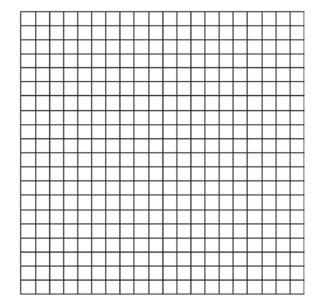
1 
$$(x-1)^2 = 4(y+3)$$

$$2 (x-1)^2 = -8(y-3)$$

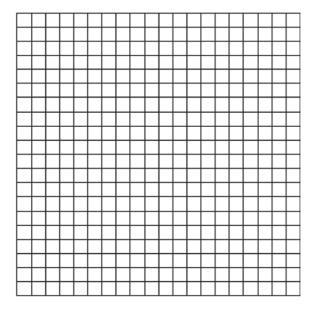
$$3 \quad (x+4)^2 = 4(y-3)$$

4 
$$(x-4)^2 = -8(y+1)$$

- 41 The directrix of the parabola  $12(y+3) = (x-4)^2$  has the equation y = -6. Find the coordinates of the focus of the parabola.
- 42 The parabola  $y = -\frac{1}{20}(x-3)^2 + 6$  has its focus at (3,1). Determine and state the equation of the directrix. (The use of the grid below is optional.)



Determine an equation for the parabola with focus (4,-1) and directrix y=-5. (Use of the grid below is optional.)



#### **POWERS**

A.SSE.B.3: MODELING EXPONENTIAL FUNCTIONS

44 A study of the annual population of the red-winged blackbird in Ft. Mill, South Carolina, shows the population, B(t), can be represented by the function  $B(t) = 750(1.16)^t$ , where the t represents the number of years since the study began. In terms of the monthly rate of growth, the population of red-winged blackbirds can be best approximated by the function

1 
$$B(t) = 750(1.012)^t$$

$$2 B(t) = 750(1.012)^{12t}$$

3 
$$B(t) = 750(1.16)^{12t}$$

$$4 \qquad B(t) = 750(1.16)^{\frac{t}{12}}$$

45 A student studying public policy created a model for the population of Detroit, where the population decreased 25% over a decade. He used the model  $P = 714(0.75)^d$ , where P is the population, in thousands, d decades after 2010. Another student, Suzanne, wants to use a model that would predict the population after y years. Suzanne's model is best represented by

1 
$$P = 714(0.6500)^y$$

$$P = 714(0.8500)^{y}$$

$$P = 714(0.9716)^{y}$$

4 
$$P = 714(0.9750)^y$$

46 Iridium-192 is an isotope of iridium and has a half-life of 73.83 days. If a laboratory experiment begins with 100 grams of Iridium-192, the number of grams, *A*, of Iridium-192 present after *t* days

would be 
$$A = 100 \left(\frac{1}{2}\right)^{\frac{t}{73.83}}$$
. Which equation

approximates the amount of Iridium-192 present after *t* days?

$$1 \qquad A = 100 \left( \frac{73.83}{2} \right)^{t}$$

$$2 \qquad A = 100 \left( \frac{1}{147.66} \right)^t$$

$$3 \quad A = 100(0.990656)^t$$

4 
$$A = 100(0.116381)^t$$

## Algebra II Regents Exam Questions by State Standard: Topic www.jmap.org

- 47 On average, college seniors graduating in 2012 could compute their growing student loan debt using the function  $D(t) = 29,400(1.068)^t$ , where t is time in years. Which expression is equivalent to  $29,400(1.068)^t$  and could be used by students to identify an approximate daily interest rate on their loans?
  - $1 \quad 29,400 \left( 1.068^{\frac{1}{365}} \right)^{t}$
  - $2 \quad 29,400 \left(\frac{1.068}{365}\right)^{365t}$
  - $3 \qquad 29,400 \left(1 + \frac{0.068}{365}\right)^t$
  - $4 \qquad 29,400 \left(1.068^{\frac{1}{365}}\right)^{365t}$
- 48 Stephanie found that the number of white-winged cross bills in an area can be represented by the formula  $C = 550(1.08)^t$ , where t represents the number of years since 2010. Which equation correctly represents the number of white-winged cross bills in terms of the monthly rate of population growth?
  - 1  $C = 550(1.00643)^t$
  - $2 C = 550(1.00643)^{12t}$
  - $3 \qquad C = 550(1.00643)^{\frac{t}{12}}$
  - 4  $C = 550(1.00643)^{t+12}$

- 49 Julia deposits \$2000 into a savings account that earns 4% interest per year. The exponential function that models this savings account is  $y = 2000(1.04)^t$ , where t is the time in years. Which equation correctly represents the amount of money in her savings account in terms of the monthly growth rate?
  - 1  $y = 166.67(1.04)^{0.12t}$
  - $2 \qquad y = 2000(1.01)^t$
  - $3 \quad v = 2000(1.0032737)^{12t}$
  - 4  $y = 166.67(1.0032737)^t$
- 50 Kelly-Ann has \$20,000 to invest. She puts half of the money into an account that grows at an annual rate of 0.9% compounded monthly. At the same time, she puts the other half of the money into an account that grows continuously at an annual rate of 0.8%. Which function represents the value of Kelly-Ann's investments after *t* years?
  - $1 f(t) = 10,000(1.9)^t + 10,000e^{0.8t}$
  - $2 f(t) = 10,000(1.009)^{t} + 10,000e^{0.008t}$
  - 3  $f(t) = 10,000(1.075)^{12t} + 10,000e^{0.8t}$
  - $4 f(t) = 10,000(1.00075)^{12t} + 10,000e^{0.008t}$
- 51 A study of black bears in the Adirondacks reveals that their population can be represented by the function  $P(t) = 3500(1.025)^t$ , where t is the number of years since the study began. Which function is correctly rewritten to reveal the monthly growth rate of the black bear population?
  - 1  $P(t) = 3500(1.00206)^{12t}$
  - $P(t) = 3500(1.00206)^{\frac{t}{12}}$
  - $3 \quad P(t) = 3500(1.34489)^{12t}$
  - 4  $P(t) = 3500(1.34489)^{\frac{t}{12}}$

- 52 The growth of a \$500 investment can be modeled by the function  $P(t) = 500(1.03)^t$ , where t represents time in years. In terms of the monthly rate of growth, the value of the investment can be best approximated by
  - 1  $P(t) = 500(1.00247)^{12t}$
  - $P(t) = 500(1.00247)^{t}$
  - 3  $P(t) = 500(1.03)^{12t}$
  - $4 P(t) = 500(1.03)^{\frac{t}{12}}$
- 53 The amount of a substance, A(t), that remains after t days can be given by the equation
  - $A(t) = A_0(0.5)^{\frac{t}{0.0803}}$ , where  $A_0$  represents the initial amount of the substance. An equivalent form of this equation is
  - 1  $A(t) = A_0(0.000178)^t$
  - 2  $A(t) = A_0(0.945861)^t$
  - 3  $A(t) = A_0(0.04015)^t$
  - 4  $A(t) = A_0(1.08361)^t$
- Mia has a student loan that is in deferment, meaning that she does not need to make payments right now. The balance of her loan account during her deferment can be represented by the function  $f(x) = 35,000(1.0325)^x$ , where x is the number of years since the deferment began. If the bank decides to calculate her balance showing a monthly growth rate, an approximately equivalent function would be
  - 1  $f(x) = 35,000(1.0027)^{12x}$
  - 2  $f(x) = 35,000(1.0027)^{\frac{x}{12}}$
  - $f(x) = 35,000(1.0325)^{12x}$
  - 4  $f(x) = 35,000(1.0325)^{\frac{x}{12}}$

For a given time, x, in seconds, an electric current, y, can be represented by  $y = 2.5(1 - 2.7^{-1.0x})$ .

Which equation is *not* equivalent?

1 
$$y = 2.5 - 2.5(2.7^{-1.10x})$$

2 
$$y = 2.5 - 2.5 \left( \left( 2.7^2 \right)^{-.05x} \right)$$

$$3 \qquad y = 2.5 - 2.5 \left( \frac{1}{2.7^{.10x}} \right)$$

4 
$$y = 2.5 - 2.5(2.7^{-2})(2.7^{.05x})$$

## F.BF.A.1: MODELING EXPONENTIAL FUNCTIONS

- 56 Last year, the total revenue for Home Style, a national restaurant chain, increased 5.25% over the previous year. If this trend were to continue, which expression could the company's chief financial officer use to approximate their monthly percent increase in revenue? [Let *m* represent months.]
  - $1 \quad (1.0525)^m$
  - $(1.0525)^{\frac{12}{m}}$
  - $(1.00427)^m$
  - 4  $(1.00427)^{\frac{m}{12}}$

- 57 A payday loan company makes loans between \$100 and \$1000 available to customers. Every 14 days, customers are charged 30% interest with compounding. In 2013, Remi took out a \$300 payday loan. Which expression can be used to calculate the amount she would owe, in dollars, after one year if she did not make payments?
  - 1  $300(.30)^{\frac{14}{365}}$
  - $2 \quad 300(1.30)^{\frac{14}{365}}$
  - $3 \quad 300(.30)^{\frac{365}{14}}$
  - $4 \quad 300(1.30)^{\frac{365}{14}}$
- 58 Camryn puts \$400 into a savings account that earns 6% annually. The amount in her account can be modeled by  $C(t) = 400(1.06)^t$  where t is the time in years. Which expression best approximates the amount of money in her account using a weekly growth rate?
  - 1 400(1.001153846)<sup>t</sup>
  - 2 400(1.001121184)<sup>t</sup>
  - 3  $400(1.001153846)^{52t}$
  - 4  $400(1.001121184)^{52t}$
- 59 Susan won \$2,000 and invested it into an account with an annual interest rate of 3.2%. If her investment were compounded monthly, which expression best represents the value of her investment after *t* years?
  - 1  $2000(1.003)^{12t}$
  - 2  $2000(1.032)^{\frac{t}{12}}$
  - 3  $2064^{\frac{t}{12}}$
  - $4 \quad \frac{2000(1.032)^t}{12}$

- 60 According to a pricing website, Indroid phones lose 58% of their cash value over 1.5 years. Which expression can be used to estimate the value of a \$300 Indroid phone in 1.5 years?
  - 1  $300e^{-0.87}$
  - $2 \quad 300e^{-0.63}$
  - $3 \quad 300e^{-0.58}$
  - 4  $300e^{-0.42}$
- 61 Biologists are studying a new bacterium. They create a culture with 100 of the bacteria and anticipate that the number of bacteria will double every 30 hours. Write an equation for the number of bacteria, *B*, in terms of the number of hours, *t*, since the experiment began.

## F.LE.A.2: MODELING EXPONENTIAL EQUATIONS

- A rabbit population doubles every 4 weeks. There are currently five rabbits in a restricted area. If t represents the time, in weeks, and P(t) is the population of rabbits with respect to time, about how many rabbits will there be in 98 days?
  - 1 56
  - 2 152
  - 3 3688
  - 4 81,920
- 63 If \$5000 is put into a savings account that pays 3.5% interest compounded monthly, how much money, to the *nearest ten cents*, would be in that account after 6 years, assuming no money was added or withdrawn?
  - 1 \$5177.80
  - 2 \$5941.30
  - 3 \$6146.30
  - 4 \$6166.50

64 Sodium iodide-131, used to treat certain medical conditions, has a half-life of 1.8 hours. The data table below shows the amount of sodium iodide-131, rounded to the nearest thousandth, as the dose fades over time.

| Number<br>of<br>Half Lives        | 1       | 2      | 3      | 4      | 5     |
|-----------------------------------|---------|--------|--------|--------|-------|
| Amount of<br>Sodium<br>Iodide-131 | 139.000 | 69.500 | 34.750 | 17.375 | 8.688 |

What approximate amount of sodium iodide-131 will remain in the body after 18 hours?

1 0.001

3 0.271

2 0.136

4 0.543

- 65 A population of 950 bacteria grows continuously at a rate of 4.75% per day. Write an exponential function, N(t), that represents the bacterial population after t days and explain the reason for your choice of base. Determine the bacterial population after 36 hours, to the *nearest bacterium*.
- 66 Titanium-44 is a radioactive isotope such that every 63 years, its mass decreases by half. For a sample of titanium-44 with an initial mass of 100 grams, write a function that will give the mass of the sample remaining after any amount of time. Define all variables. Scientists sometimes use the average yearly decrease in mass for estimation purposes. Use the average yearly decrease in mass of the sample between year 0 and year 10 to predict the amount of the sample remaining after 40 years. Round your answer to the *nearest tenth*. Is the actual mass of the sample or the estimated mass greater after 40 years? Justify your answer.

## F.LE.B.5: MODELING EXPONENTIAL EQUATIONS

67 The value of an automobile *t* years after it was purchased is given by the function

 $V = 38,000(0.84)^{t}$ . Which statement is true?

- 1 The value of the car increases 84% each year.
- 2 The value of the car decreases 84% each year.
- 3 The value of the car increases 16% each year.
- 4 The value of the car decreases 16% each year.
- An equation to represent the value of a car after t months of ownership is  $v = 32,000(0.81)^{\frac{t}{12}}$ . Which statement is *not* correct?
  - 1 The car lost approximately 19% of its value each month.
  - 2 The car maintained approximately 98% of its value each month.
  - 3 The value of the car when it was purchased was \$32,000.
  - 4 The value of the car 1 year after it was purchased was \$25,920.

## Algebra II Regents Exam Questions by State Standard: Topic www.jmap.org

69 A certain pain reliever is taken in 220 mg dosages and has a half-life of 12 hours. The function

$$A = 220 \left(\frac{1}{2}\right)^{\frac{t}{12}}$$
 can be used to model this situation,

where A is the amount of pain reliever in milligrams remaining in the body after t hours.

According to this function, which statement is true?

- 1 Every hour, the amount of pain reliever remaining is cut in half.
- 2 In 12 hours, there is no pain reliever remaining in the body.
- 3 In 24 hours, there is no pain reliever remaining in the body.
- 4 In 12 hours, 110 mg of pain reliever is remaining.
- 70 The amount of a substance, A(t), in grams, remaining after t days is modeled by

$$A(t) = 50(0.5)^{\frac{t}{3}}$$
. Which statement is false?

- 1 In 20 days, there is no substance remaining.
- 2 After two half-lives, there is 25% of the substance remaining.
- 3 The amount of the substance remaining can also be modeled by

$$A(t) = 50(2)^{\frac{-t}{3}}.$$

4 After one week, there is less than 10g of the substance remaining.

71 A savings account, *S*, has an initial value of \$50. The account grows at a 2% interest rate compounded *n* times per year, *t*, according to the function below.

$$S(t) = 50\left(1 + \frac{.02}{n}\right)^{nt}$$

Which statement about the account is correct?

- 1 As the value of *n* increases, the amount of interest per year decreases.
- As the value of *n* increases, the value of the account approaches the function  $S(t) = 50e^{0.02t}$ .
- 3 As the value of *n* decreases to one, the amount of interest per year increases.
- 4 As the value of *n* decreases to one, the value of the account approaches the function  $S(t) = 50(1 0.02)^{t}$ .
- 72 The function  $p(t) = 110e^{0.03922t}$  models the population of a city, in millions, t years after 2010. As of today, consider the following two statements:
  - I. The current population is 110 million.
  - II. The population increases continuously by approximately 3.9% per year.

This model supports

- 1 I, only
- 2 II, only
- 3 both I and II
- 4 neither I nor II
- 73 If  $f(t) = 50(.5)^{\frac{t}{5715}}$  represents a mass, in grams, of carbon-14 remaining after t years, which statement(s) must be true?
  - I. The mass of the carbon-14 is decreasing by half each year.

II. The mass of the original sample is 50 g.

- 1 I, only
- 2 II, only
- 3 I and II
- 4 neither I nor II

## F.IF.B.4: EVALUATING EXPONENTIAL EXPRESSIONS

74 Monthly mortgage payments can be found using the formula below, where M is the monthly payment, P is the amount borrowed, r is the annual interest rate, and n is the total number of monthly payments.

$$M = \frac{P\left(\frac{r}{12}\right)\left(1 + \frac{r}{12}\right)^n}{\left(1 + \frac{r}{12}\right)^n - 1}$$

If Adam takes out a 15-year mortgage, borrowing \$240,000 at an annual interest rate of 4.5%, his monthly payment will be

- 1 \$1379.09
- 2 \$1604.80
- 3 \$1835.98
- 4 \$9011.94
- 75 Using the formula below, determine the monthly payment on a 5-year car loan with a monthly percentage rate of 0.625% for a car with an original cost of \$21,000 and a \$1000 down payment, to the *nearest cent*.

$$P_n = PMT \left( \frac{1 - (1+i)^{-n}}{i} \right)$$

 $P_n$  = present amount borrowed

n = number of monthly pay periods PMT = monthly payment

i = interest rate per month

The affordable monthly payment is \$300 for the same time period. Determine an appropriate down payment, to the *nearest dollar*.

76 Jim is looking to buy a vacation home for \$172,600 near his favorite southern beach. The formula to compute a mortgage payment, M, is

$$M = P \bullet \frac{r(1+r)^N}{(1+r)^N - 1}$$
 where *P* is the principal

amount of the loan, r is the monthly interest rate, and N is the number of monthly payments. Jim's bank offers a monthly interest rate of 0.305% for a 15-year mortgage. With no down payment, determine Jim's mortgage payment, rounded to the *nearest dollar*. Algebraically determine and state the down payment, rounded to the *nearest dollar*, that Jim needs to make in order for his mortgage payment to be \$1100.

77 The Wells family is looking to purchase a home in a suburb of Rochester with a 30-year mortgage that has an annual interest rate of 3.6%. The house the family wants to purchase is \$152,500 and they will make a \$15,250 down payment and borrow the remainder. Use the formula below to determine their monthly payment, to the *nearest dollar*.

$$M = \frac{P\left(\frac{r}{12}\right)\left(1 + \frac{r}{12}\right)^n}{\left(1 + \frac{r}{12}\right)^n - 1}$$

M =monthly payment

P = amount borrowed

r = annual interest rate

n = total number of monthly payments

#### F.IF.B.4: EVALUATING LOGARITHMIC EXPRESSIONS

78 The loudness of sound is measured in units called decibels (dB). These units are measured by first assigning an intensity  $I_0$  to a very soft sound that is called the threshold sound. The sound to be measured is assigned an

intensity, I, and the decibel rating, d, of this sound is found using  $d = 10 \log \frac{I}{I_0}$ . The threshold sound audible to

the average person is  $1.0 \times 10^{-12}~\text{W/m}^2$  (watts per square meter). Consider the following sound level classifications:

| Moderate  | 45-69 dB  |
|-----------|-----------|
| Loud      | 70-89 dB  |
| Very loud | 90-109 dB |
| Deafening | >110 dB   |

How would a sound with intensity  $6.3 \times 10^{-3} \text{ W/m}^2$  be classified?

1 moderate

3 very loud

2 loud

4 deafening

# F.IF.C.7: GRAPHING EXPONENTIAL FUNCTIONS

79 Which function represents exponential decay?

1 
$$y = 2^{0.3t}$$

$$y = 1.2^{3t}$$

$$y = \left(\frac{1}{2}\right)^{-1}$$

$$4 y = 5^{-t}$$

80 Which function represents exponential decay?

$$1 \qquad p(x) = \left(\frac{1}{4}\right)^{-x}$$

$$2 q(x) = 1.8^{-x}$$

$$r(x) = 2.3^{2x}$$

$$4 \qquad s(x) = 4^{\frac{x}{2}}$$

81 If the function  $g(x) = ab^x$  represents exponential growth, which statement about g(x) is *false*?

1 
$$a > 0$$
 and  $b > 1$ 

2 The y-intercept is 
$$(0,a)$$
.

3 The asymptote is 
$$y = 0$$
.

4 The *x*-intercept is 
$$(b,0)$$
.

82 Which statement is true about the graph of

$$f(x) = \left(\frac{1}{8}\right)^x ?$$

4 The graph has an asymptote, 
$$x = 0$$
.

83 Given  $f(x) = 3^{x-1} + 2$ , as  $x \to -\infty$ 

$$1 f(x) \to -1$$

$$2 \quad f(x) \to 0$$

$$3 \quad f(x) \to 2$$

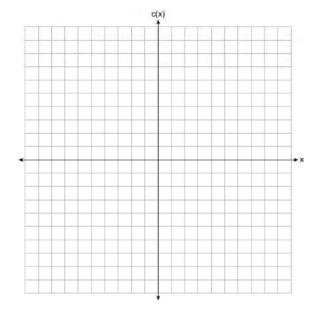
$$4 f(x) \to -\infty$$

84 The function M(t) represents the mass of radium over time, t, in years.

$$M(t) = 100e^{\frac{\left(\ln\frac{1}{2}\right)t}{1590}}$$

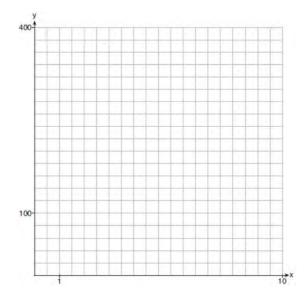
Determine if the function M(t) represents growth or decay. Explain your reasoning.

- 85 Describe the transformation applied to the graph of  $p(x) = 2^x$  that forms the new function  $q(x) = 2^{x-3} + 4$ .
- 86 Graph  $c(x) = -9(3)^{x-4} + 2$  on the axes below.



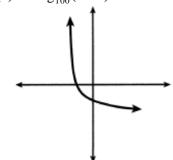
Describe the end behavior of c(x) as x approaches positive infinity. Describe the end behavior of c(x) as x approaches negative infinity.

87 Graph  $y = 400(.85)^{2x} - 6$  on the set of axes below.

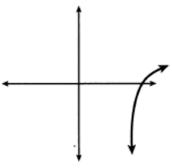


# F.IF.C.7: GRAPHING LOGARITHMIC FUNCTIONS

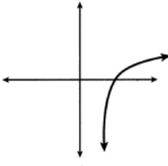
Which sketch could represent the function  $m(x) = -\log_{100}(x-2)$ ?



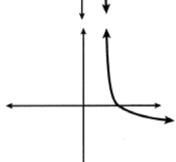
1



2

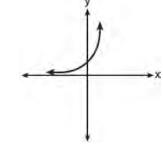


3

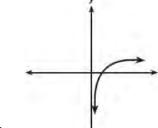


4

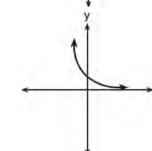
89 Which sketch best represents the graph of  $x = 3^y$ ?



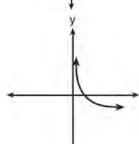
1



2



3

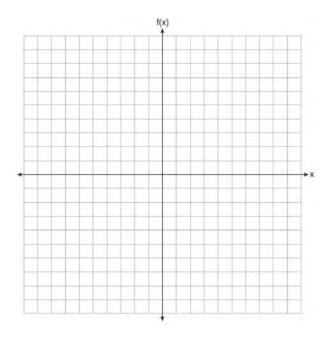


4

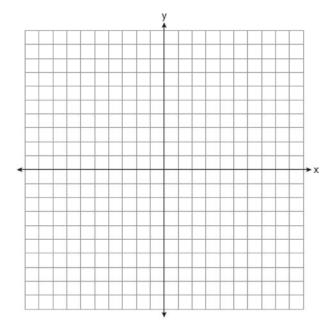
- 90 The graph of  $y = \log_2 x$  is translated to the right 1 unit and down 1 unit. The coordinates of the *x*-intercept of the translated graph are
  - 1 (0,0)
  - 2 (1,0)
  - 3(2,0)
  - 4 (3,0)

- 91 If  $f(x) = \log_3 x$  and g(x) is the image of f(x) after a translation five units to the left, which equation represents g(x)?
  - $1 \qquad g(x) = \log_3(x+5)$
  - $2 \qquad g(x) = \log_3 x + 5$
  - $3 \qquad g(x) = \log_3(x 5)$
  - $4 \qquad g(x) = \log_3 x 5$
- 92 Which statement about the graph of  $c(x) = \log_6 x$  is *false*?
  - 1 The asymptote has equation y = 0.
  - 2 The graph has no y-intercept.
  - 3 The domain is the set of positive reals.
  - 4 The range is the set of all real numbers.
- 93 Which statement below about the graph of  $f(x) = -\log(x+4) + 2$  is true?
  - 1 f(x) has a y-intercept at (0,2).
  - 2 -f(x) has a y-intercept at (0,2).
  - 3 As  $x \to \infty$ ,  $f(x) \to \infty$ .
  - $4 \quad x \to -4, f(x) \to \infty.$

94 Graph  $f(x) = \log_2(x+6)$  on the set of axes below.

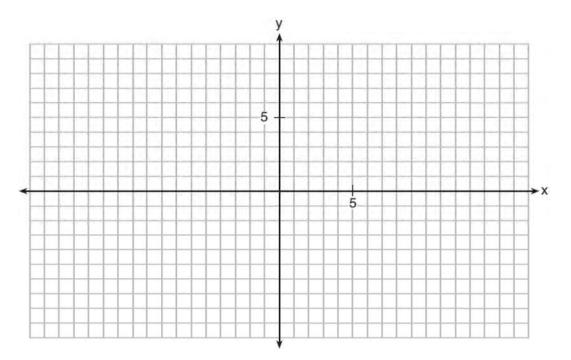


95 Graph the following function on the axes below.  $f(x) = \log_3(2-x)$ 

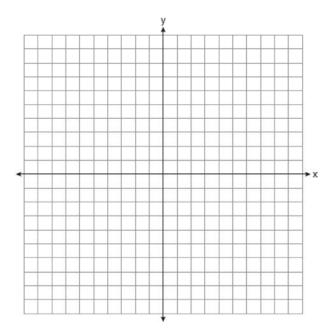


State the domain of f. State the equation of the asymptote.

96 On the grid below, graph the function  $y = \log_2(x-3) + 1$ 



97 Graph  $y = \log_2(x+3) - 5$  on the set of axes below. Use an appropriate scale to include *both* intercepts.



Describe the behavior of the given function as x approaches -3 and as x approaches positive infinity.

#### A.CED.A.1: EXPONENTIAL EQUATIONS

98 Given a > 0, solve the equation  $a^{x+1} = \sqrt[3]{a^2}$  for x algebraically.

#### A.CED.A.1: EXPONENTIAL GROWTH

99 Monthly mortgage payments can be found using the formula below:

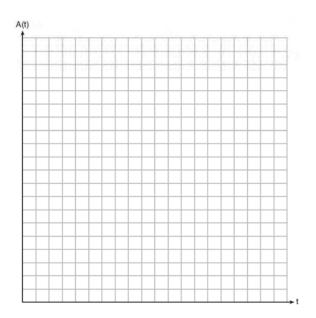
$$M = \frac{P\left(\frac{r}{12}\right)\left(1 + \frac{r}{12}\right)^n}{\left(1 + \frac{r}{12}\right)^n - 1}$$

M = monthly payment P = amount borrowed r = annual interest rate n = number of monthly payments

The Banks family would like to borrow \$120,000 to purchase a home. They qualified for an annual interest rate of 4.8%. Algebraically determine the *fewest* number of whole years the Banks family would need to include in the mortgage agreement in order to have a monthly payment of no more than \$720.

100 Seth's parents gave him \$5000 to invest for his 16th birthday. He is considering two investment options. Option *A* will pay him 4.5% interest compounded annually. Option *B* will pay him 4.6% compounded quarterly. Write a function of option *A* and option *B* that calculates the value of each account after *n* years. Seth plans to use the money after he graduates from college in 6 years. Determine how much more money option *B* will earn than option *A* to the *nearest cent*. Algebraically determine, to the *nearest tenth of a year*, how long it would take for option *B* to double Seth's initial investment.

- 101 Carla wants to start a college fund for her daughter Lila. She puts \$63,000 into an account that grows at a rate of 2.55% per year, compounded monthly. Write a function, C(t), that represents the amount of money in the account t years after the account is opened, given that no more money is deposited into or withdrawn from the account. Calculate algebraically the number of years it will take for the account to reach \$100,000, to the *nearest hundredth of a year*.
- 102 Tony is evaluating his retirement savings. He currently has \$318,000 in his account, which earns an interest rate of 7% compounded annually. He wants to determine how much he will have in the account in the future, even if he makes no additional contributions to the account. Write a function, A(t), to represent the amount of money that will be in his account in t years. Graph A(t) where  $0 \le t \le 20$  on the set of axes below.



Tony's goal is to save \$1,000,000. Determine algebraically, to the *nearest year*, how many years it will take for him to achieve his goal. Explain how your graph of A(t) confirms your answer.

#### A.CED.A.1: EXPONENTIAL DECAY

103 Objects cool at different rates based on the formula below.

$$T = (T_0 - T_R)e^{-rt} + T_R$$

 $T_0$ : initial temperature

 $T_R$ : room temperature

r: rate of cooling of the object

t: time in minutes that the object

cools to a temperature, T

Mark makes T-shirts using a hot press to transfer designs to the shirts. He removes a shirt from a press that heats the shirt to 400°F. The rate of cooling for the shirt is 0.0735 and the room temperature is 75°F. Using this information, write an equation for the temperature of the shirt, T, after t minutes. Use the equation to find the temperature of the shirt, to the nearest degree, after five minutes. At the same time, Mark's friend Jeanine removes a hoodie from a press that heats the hoodie to 450°F. After eight minutes, the hoodie measured 270°F. The room temperature is still 75°F. Determine the rate of cooling of the hoodie, to the nearest ten thousandth. The T-shirt and hoodie were removed at the same time. Determine when the temperature will be the same, to the *nearest* minute.

#### F.LE.A.4: EXPONENTIAL EQUATIONS

- 104 If  $ae^{bt} = c$ , where a, b, and c are positive, then t equals
  - 1  $\ln\left(\frac{c}{ab}\right)$
  - $2 \ln \left(\frac{cb}{a}\right)$
  - $3 \quad \frac{\ln\left(\frac{c}{a}\right)}{b}$
  - $\frac{\ln\left(\frac{c}{a}\right)}{\ln b}$
- 105 What is the solution to  $8(2^{x+3}) = 48$ ?
  - $1 \qquad x = \frac{\ln 6}{\ln 2} 3$
  - $2 \quad x = 0$
  - $3 \qquad x = \frac{\ln 48}{\ln 16} 3$
  - $4 \qquad x = \ln 4 3$
- 106 The solution to the equation  $5e^{x+2} = 7$  is
  - $1 \qquad -2 + \ln\left(\frac{7}{5}\right)$
  - $2 \qquad \left(\frac{\ln 7}{\ln 5}\right) 2$
  - $3 \frac{-3}{5}$
  - $4 -2 + \ln(2)$
- 107 The solution of  $87e^{0.3x} = 5918$ , to the *nearest thousandth*, is
  - 1 0.583
  - 2 1.945
  - 3 4.220
  - 4 14.066

- 108 To the *nearest tenth*, the solution to the equation  $4300e^{0.07x} 123 = 5000$  is
  - 1 1.1
  - 2 2.5
  - 3 6.3
  - 4 68.5
- 109 Which expression is *not* a solution to the equation

$$2^t = \sqrt{10}?$$

- $1 \quad \frac{1}{2}\log_2 10$
- $2 \quad \log_2 \sqrt{10}$
- $3 \quad \log_4 10$
- $4 \log_{10} 4$

#### F.LE.A.4: EXPONENTIAL GROWTH

- 110 A local university has a current enrollment of 12,000 students. The enrollment is increasing continuously at a rate of 2.5% each year. Which logarithm is equal to the number of years it will take for the population to increase to 15,000 students?
  - 1  $\frac{\ln 1.25}{0.25}$
  - $2 \frac{\ln 3000}{0.025}$
  - $3 \frac{\ln 1.25}{2.5}$
  - $4 \frac{\ln 1.25}{0.025}$
- 111 Judith puts \$5000 into an investment account with interest compounded continuously. Which approximate annual rate is needed for the account to grow to \$9110 after 30 years?
  - 1 2%
  - 2 2.2%
  - 3 0.02%
  - 4 0.022%

- 112 A house purchased 5 years ago for \$100,000 was just sold for \$135,000. Assuming exponential growth, approximate the annual growth rate, to the *nearest percent*.
- 113 In New York State, the minimum wage has grown exponentially. In 1966, the minimum wage was \$1.25 an hour and in 2015, it was \$8.75. Algebraically determine the rate of growth to the *nearest percent*.
- 114 Determine, to the *nearest tenth of a year*, how long it would take an investment to double at a  $3\frac{3}{4}$  % interest rate, compounded continuously.
- 115 When observed by researchers under a microscope, a smartphone screen contained approximately 11,000 bacteria per square inch. Bacteria, under normal conditions, double in population every 20 minutes.
  - a) Assuming an initial value of 11,000 bacteria, write a function, p(t), that can be used to model the population of bacteria, p, on a smartphone screen, where t represents the time in minutes after it is first observed under a microscope.
  - b) Using p(t) from part a, determine algebraically, to the *nearest hundredth of a minute*, the amount of time it would take for a smartphone screen that was not touched or cleaned to have a population of 1,000,000 bacteria per square inch.

116 After sitting out of the refrigerator for a while, a turkey at room temperature (68°F) is placed into an oven at 8 a.m., when the oven temperature is 325°F. Newton's Law of Heating explains that the temperature of the turkey will increase proportionally to the difference between the temperature of the turkey and the temperature of the oven, as given by the formula below:

$$T = T_a + \left(T_0 - T_a\right)e^{-kt}$$

 $T_a$  = the temperature surrounding the object

 $T_0$  = the initial temperature of the object

t =the time in hours

T = the temperature of the object after t hours

k = decay constant

The turkey reaches the temperature of approximately  $100^{\circ}$  F after 2 hours. Find the value of k, to the *nearest thousandth*, and write an equation to determine the temperature of the turkey after t hours. Determine the Fahrenheit temperature of the turkey, to the *nearest degree*, at 3 p.m.

#### F.LE.A.4: EXPONENTIAL DECAY

The half-life of iodine-131 is 8 days. The percent of the isotope left in the body d days after being

introduced is  $I = 100 \left(\frac{1}{2}\right)^{\frac{d}{8}}$ . When this equation is

written in terms of the number e, the base of the natural logarithm, it is equivalent to  $I = 100e^{kd}$ . What is the approximate value of the constant, k?

- 1 -0.087
- 2 0.087
- 3 -11.542
- 4 11.542

118 The Fahrenheit temperature, F(t), of a heated object at time t, in minutes, can be modeled by the function below.  $F_s$  is the surrounding temperature,  $F_0$  is the initial temperature of the object, and k is a constant.

$$F(t) = F_{s} + (F_{0} - F_{s})e^{-kt}$$

Coffee at a temperature of 195°F is poured into a container. The room temperature is kept at a constant 68°F and k = 0.05. Coffee is safe to drink when its temperature is, at most, 120°F. To the *nearest minute*, how long will it take until the coffee is safe to drink?

- 1 7
- 2 10
- 3 11
- 4 18
- One of the medical uses of Iodine–131 (I–131), a radioactive isotope of iodine, is to enhance x-ray images. The half-life of I–131 is approximately 8.02 days. A patient is injected with 20 milligrams of I–131. Determine, to the *nearest day*, the amount of time needed before the amount of I–131 in the patient's body is approximately 7 milligrams.
- 120 The half-life of a radioactive substance is 15 years. Write an equation that can be used to determine the amount, s(t), of 200 grams of this substance that remains after t years. Determine algebraically, to the *nearest year*, how long it will take for  $\frac{1}{10}$  of this substance to remain.

121 A radioactive substance has a mass of 140 g at 3 p.m. and 100 g at 8 p.m. Write an equation in the

form 
$$A = A_0 \left(\frac{1}{2}\right)^{\frac{t}{h}}$$
 that models this situation,

where h is the constant representing the number of hours in the half-life,  $A_0$  is the initial mass, and A is the mass t hours after 3 p.m. Using this equation, solve for h, to the *nearest ten thousandth*. Determine when the mass of the radioactive substance will be 40 g. Round your answer to the *nearest tenth of an hour*.

#### A.SSE.A.2: FACTORING POLYNOMIALS

- When the expression  $(x+2)^2 + 4(x+2) + 3$  is rewritten as the product of two binomials, the result is
  - 1 (x+3)(x+1)
  - 2(x+5)(x+3)
  - 3 (x+2)(x+2)
  - 4 (x+6)(x+1)
- 123 Which expression is equivalent to

$$(x+2)^2 - 5(x+2) + 6$$
?

- 1 x(x-1)
- 2(x-3)(x-2)
- 3 (x-4)(x+3)
- 4 (x-6)(x+1)
- 124 The expression  $(x + a)^2 + 5(x + a) + 4$  is equivalent
  - 1 (a+1)(a+4)
  - 2(x+1)(x+4)
  - 3 (x+a+1)(x+a+4)
  - $4 \quad x^2 + a^2 + 5x + 5a + 4$

- 125 When factored completely,  $m^5 + m^3 6m$  is equivalent to
  - 1 (m+3)(m-2)
  - $2 (m^2 + 3m)(m^2 2)$
  - $3 m(m^4 + m^2 6)$
  - 4  $m(m^2+3)(m^2-2)$
- 126 If  $(a^3 + 27) = (a + 3)(a^2 + ma + 9)$ , then *m* equals
  - 1 -9
  - 2 -3
  - 3 3
  - 4 6
- 127 The completely factored form of

$$2d^4 + 6d^3 - 18d^2 - 54d$$
 is

- 1  $2d(d^2-9)(d+3)$
- $2 2d(d^2+9)(d+3)$
- $3 \quad 2d(d+3)^2(d-3)$
- 4  $2d(d-3)^2(d+3)$
- 128 What is the completely factored form of

$$k^4 - 4k^2 + 8k^3 - 32k + 12k^2 - 48$$
?

- 1 (k-2)(k-2)(k+3)(k+4)
- 2 (k-2)(k-2)(k+6)(k+2)
- 3 (k+2)(k-2)(k+3)(k+4)
- 4 (k+2)(k-2)(k+6)(k+2)
- 129 The completely factored form of

$$n^4 - 9n^2 + 4n^3 - 36n - 12n^2 + 108$$
 is

- 1  $(n^2-9)(n+6)(n-2)$
- 2 (n+3)(n-3)(n+6)(n-2)
- 3 (n-3)(n-3)(n+6)(n-2)
- 4 (n+3)(n-3)(n-6)(n+2)

130 Which expression is equivalent to

$$x^{6}y^{4}(x^{4}-16)-9(x^{4}-16)$$
?

- 1  $x^{10}y^4 16x^6y^4 9x^4 144$
- $(x^6y^4-9)(x+2)^3(x-2)$
- 3  $(x^3y^2+3)(x^3y^2-3)(x+2)^2(x-2)^2$
- 4  $(x^3y^2+3)(x^3y^2-3)(x^2+4)(x^2-4)$
- 131 Which factorization is *incorrect*?
  - $1 \quad 4k^2 49 = (2k+7)(2k-7)$
  - 2  $a^3 8b^3 = (a 2b)(a^2 + 2ab + 4b^2)$
  - $3 m^3 + 3m^2 4m + 12 = (m-2)^2(m+3)$
  - 4  $t^3 + 5t^2 + 6t + t^2 + 5t + 6 = (t+1)(t+2)(t+3)$
- 132 Which expression has been rewritten correctly to form a true statement?
  - 1  $(x+2)^2 + 2(x+2) 8 = (x+6)x$
  - 2  $x^4 + 4x^2 + 9x^2y^2 36y^2 = (x+3y)^2(x-2)^2$
  - 3 x<sup>3</sup> + 3x<sup>2</sup> 4xy<sup>2</sup> 12y<sup>2</sup> = (x 2y)(x + 3)<sup>2</sup>
  - 4  $(x^2-4)^2-5(x^2-4)-6=(x^2-7)(x^2-6)$
- 133 Over the set of integers, factor the expression  $x^4 4x^2 12$ .
- 134 Over the set of integers, completely factor  $x^4 5x^2 + 4$
- 135 Factor the expression  $x^3 2x^2 9x + 18$  completely.
- 136 Factor completely over the set of integers:  $-2x^4 + x^3 + 18x^2 9x$

- 137 Over the set of integers, factor the expression  $4x^3 x^2 + 16x 4$  completely.
- 138 Completely factor the following expression:  $x^2 + 3xy + 3x^3 + y$
- 139 Rewrite the expression  $(4x^2 + 5x)^2 5(4x^2 + 5x) 6$  as a product of four linear factors.

# A.APR.B.3: SOLVING POLYNOMIAL EQUATIONS

- 140 What are the zeros of  $P(m) = (m^2 4)(m^2 + 1)$ ?
  - 1 2 and -2, only
  - $2 \quad 2, -2, \text{ and } -4$
  - 3 -4, i, and -i
  - 4 2, -2, i, and -i
- 141 The zeros for  $f(x) = x^4 4x^3 9x^2 + 36x$  are
  - 1  $\{0,\pm 3,4\}$
  - 2 {0,3,4}
  - $3 \{0,\pm 3,-4\}$
  - 4 {0,3,-4}
- 142 Given  $c(m) = m^3 2m^2 + 4m 8$ , the solution of c(m) = 0 is
  - $1 \pm 2$
  - 2 2, only
  - 3 2i, 2
  - 4  $\pm 2i, 2$

143 When factoring to reveal the roots of the equation  $x^3 + 2x^2 - 9x - 18 = 0$ , which equations can be used?

I. 
$$x^2(x+2) - 9(x+2) = 0$$

II. 
$$x(x^2-9)+2(x^2-9)=0$$

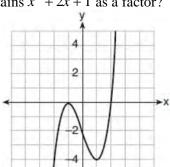
III. 
$$(x-2)(x^2-9)=0$$

- 1 I and II, only
- 2 I and III, only
- 3 II and III, only
- 4 I, II, and III
- 144 Which statement regarding polynomials and their zeros is true?
  - 1  $f(x) = (x^2 1)(x + a)$  has zeros of 1 and -a, only.
  - 2  $f(x) = x^3 ax^2 + 16x 16a$  has zeros of 4 and a, only.
  - 3  $f(x) = (x^2 + 25)(x + a)$  has zeros of  $\pm 5$  and -a.
  - 4  $f(x) = x^3 ax^2 9x + 9a$  has zeros of  $\pm 3$  and a.
- 145 Algebraically determine the zeros of the function below.

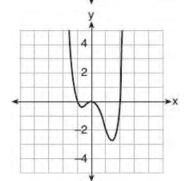
$$r(x) = 3x^3 + 12x^2 - 3x - 12$$

## A.APR.B.3: GRAPHING POLYNOMIAL EQUATIONS

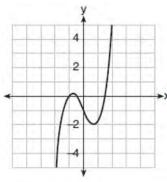
146 Which graph represents a polynomial function that contains  $x^2 + 2x + 1$  as a factor?



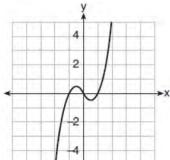
1



2

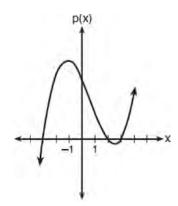


3



4

147 The graph of the function p(x) is sketched below.



Which equation could represent p(x)?

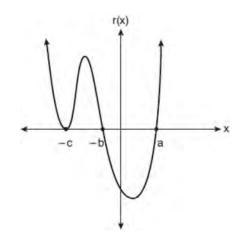
1 
$$p(x) = (x^2 - 9)(x - 2)$$

$$2 p(x) = x^3 - 2x^2 + 9x + 18$$

3 
$$p(x) = (x^2 + 9)(x - 2)$$

$$4 \qquad p(x) = x^3 + 2x^2 - 9x - 18$$

148 A sketch of r(x) is shown below.



An equation for r(x) could be

$$1 r(x) = (x-a)(x+b)(x+c)$$

2 
$$r(x) = (x+a)(x-b)(x-c)^2$$

3 
$$r(x) = (x+a)(x-b)(x-c)$$

4 
$$r(x) = (x-a)(x+b)(x+c)^2$$

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149 Evan graphed a cubic function,

 $f(x) = ax^3 + bx^2 + cx + d$ , and determined the roots of f(x) to be  $\pm 1$  and 2. What is the value of b, if a = 1?

150 Consider a cubic polynomial with the characteristics below.

• as 
$$x \to \infty$$
,  $f(x) \to -\infty$ 

Given a > 0 and b > 0, which equation represents a cubic polynomial with these characteristics?

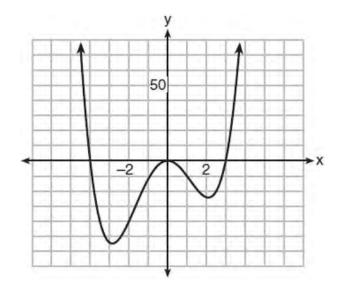
1 
$$f(x) = (x-a)(x^2+b)$$

2 
$$f(x) = (a-x)(x^2+b)$$

3 
$$f(x) = (a - x^2)(x^2 + b)$$

4 
$$f(x) = (x-a)(b-x^2)$$

151 The graph of y = f(x) is shown below. The function has a leading coefficient of 1.



Write an equation for f(x). The function g is formed by translating function f left 2 units. Write an equation for g(x).

#### F.IF.B.4: GRAPHING POLYNOMIAL FUNCTIONS

152 Given  $f(x) = x^4 - x^3 - 6x^2$ , for what values of x will f(x) > 0?

1 
$$x < -2$$
, only

2 
$$x < -2 \text{ or } x > 3$$

3 
$$x < -2 \text{ or } 0 \le x \le 3$$

$$4 x > 3$$
, only

153 Which description could represent the graph of

$$f(x) = 4x^2(x+a) - x - a$$
, if a is an integer?  
1 As  $x \to -\infty$ ,  $f(x) \to \infty$ , as  $x \to \infty$ ,  $f(x) \to \infty$ ,

2 As 
$$x \to -\infty$$
,  $f(x) \to -\infty$ , as  $x \to \infty$ ,  $f(x) \to \infty$ , and the graph has 3 *x*-intercepts.

3 As 
$$x \to -\infty$$
,  $f(x) \to \infty$ , as  $x \to \infty$ ,  $f(x) \to -\infty$ , and the graph has 4 *x*-intercepts.

4 As 
$$x \to -\infty$$
,  $f(x) \to -\infty$ , as  $x \to \infty$ ,  $f(x) \to \infty$ , and the graph has 4 *x*-intercepts.

154 The function below models the average price of gas in a small town since January 1st.

$$G(t) = -0.0049t^4 + 0.0923t^3 - 0.56t^2 + 1.166t + 3.23$$
, where  $0 \le t \le 10$ .

If G(t) is the average price of gas in dollars and t represents the number of months since January 1st, the absolute maximum G(t) reaches over the given domain is about

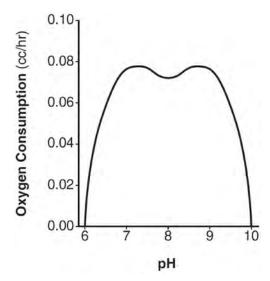
155 An estimate of the number of milligrams of a medication in the bloodstream *t* hours after 400 mg has been taken can be modeled by the function below.

$$I(t) = 0.5t^4 + 3.45t^3 - 96.65t^2 + 347.7t$$
,  
where  $0 \le t \le 6$ 

Over what time interval does the amount of medication in the bloodstream strictly increase?

- 1 0 to 2 hours
- 2 0 to 3 hours
- 3 2 to 6 hours
- 4 3 to 6 hours
- 156 A polynomial equation of degree three, p(x), is used to model the volume of a rectangular box. The graph of p(x) has x intercepts at -2, 10, and 14. Which statements regarding p(x) could be true?
  - A. The equation of p(x) = (x-2)(x+10)(x+14).
  - B. The equation of p(x) = -(x+2)(x-10)(x-14).
  - C. The maximum volume occurs when x = 10.
  - D. The maximum volume of the box is approximately 56.
  - 1 *A* and *C*
  - A and D
  - 3 B and C
  - 4 B and D

157 There was a study done on oxygen consumption of snails as a function of pH, and the result was a degree 4 polynomial function whose graph is shown below.



Which statement about this function is *incorrect*?

- 1 The degree of the polynomial is even.
- 2 There is a positive leading coefficient.
- 3 At two pH values, there is a relative maximum value.
- 4 There are two intervals where the function is decreasing.

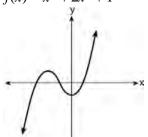
158 Consider the end behavior description below.

• as 
$$x \to -\infty$$
,  $f(x) \to \infty$ 

• as 
$$x \to \infty$$
,  $f(x) \to -\infty$ 

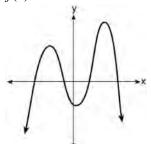
Which function satisfies the given conditions?

$$1 \qquad f(x) = x^4 + 2x^2 + 1$$



2

$$3 \qquad f(x) = -x^3 + 2x - 6$$



4

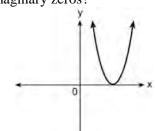
159 Factor completely over the set of integers:  $16x^4 - 81$ . Sara graphed the polynomial

 $y = 16x^4 - 81$  and stated "All the roots of

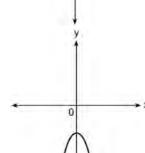
 $y = 16x^4 - 81$  and stated "All the roots of  $y = 16x^4 - 81$  are real." Is Sara correct? Explain your reasoning.

## F.IF.C.7: GRAPHING POLYNOMIAL FUNCTIONS

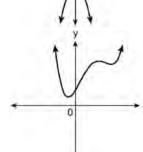
160 Which graph shows a quadratic function with two imaginary zeros?



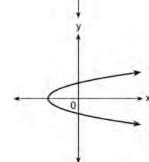
1



2



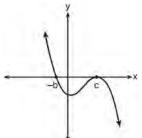
3



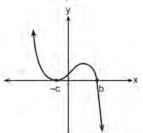
4

161 If a, b, and c are all positive real numbers, which graph could represent the sketch of the graph of

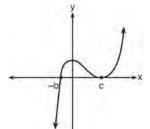
 $p(x) = -a(x+b)(x^2 - 2cx + c^2)?$ 



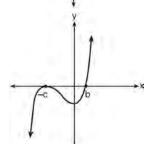
1



2

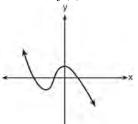


3

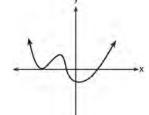


4

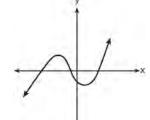
- 162 Which graph has the following characteristics?
  - three real zeros
  - as  $x \to -\infty$ ,  $f(x) \to -\infty$
  - as  $x \to \infty$ ,  $f(x) \to \infty$



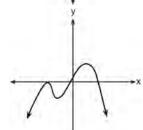
1



2

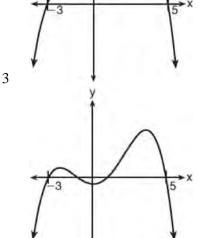


3

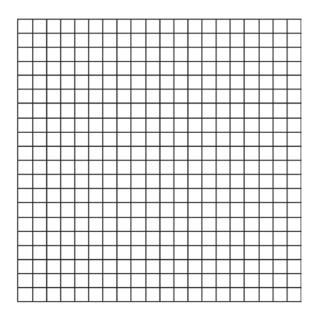


4

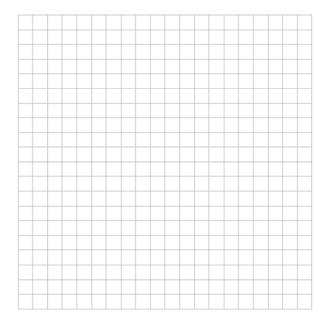
163 A 4th degree polynomial has zeros –5, 3, *i*, and –*i*. Which graph could represent the function defined by this polynomial?



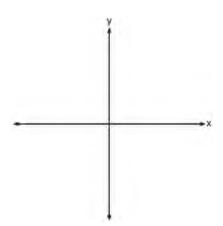
164 The zeros of a quartic polynomial function h are  $-1, \pm 2$ , and 3. Sketch a graph of y = h(x) on the grid below.



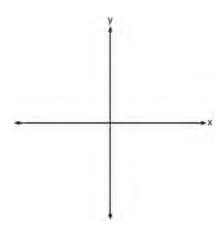
165 On the grid below, sketch a cubic polynomial whose zeros are 1, 3, and -2.



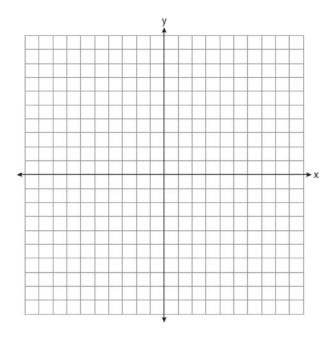
166 The zeros of a quartic polynomial function are 2, −2, 4, and −4. Use the zeros to construct a possible sketch of the function, on the set of axes below.



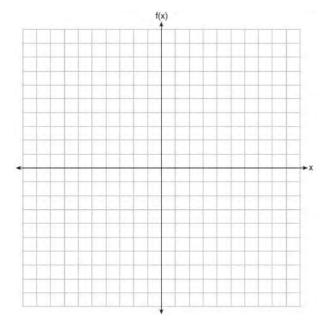
On the axes below, sketch a possible function p(x) = (x - a)(x - b)(x + c), where a, b, and c are positive, a > b, and p(x) has a positive y-intercept of d. Label all intercepts.



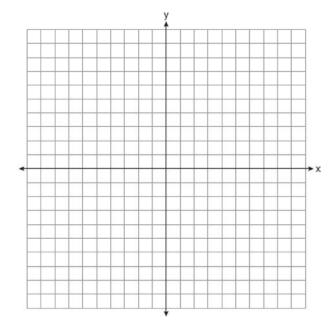
Find algebraically the zeros for  $p(x) = x^3 + x^2 - 4x - 4$ . On the set of axes below, graph y = p(x).



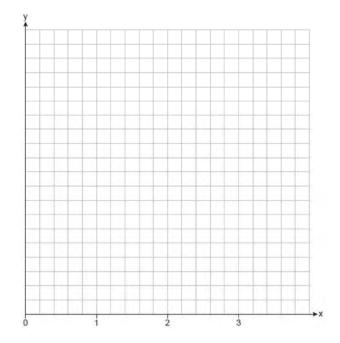
169 On the grid below, graph the function  $f(x) = x^3 - 6x^2 + 9x + 6$  on the domain  $-1 \le x \le 4$ .



170 Graph  $y = x^3 - 4x^2 + 2x + 7$  on the set of axes below.



171 The function v(x) = x(3-x)(x+4) models the volume, in cubic inches, of a rectangular solid for  $0 \le x \le 3$ . Graph y = v(x) over the domain  $0 \le x \le 3$ .

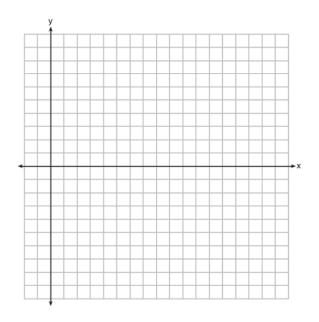


To the *nearest tenth of a cubic inch*, what is the maximum volume of the rectangular solid?

172 A major car company analyzes its revenue, R(x), and costs C(x), in millions of dollars over a fifteen-year period. The company represents its revenue and costs as a function of time, in years, x, using the given functions.

over the domain  $2 \le x \le 16$ .

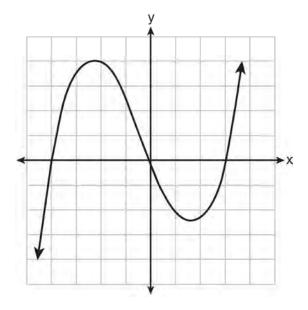
$$R(x) = 550x^3 - 12,000x^2 + 83,000x + 7000$$
  
 $C(x) = 880x^3 - 21,000x^2 + 150,000x - 160,000$   
The company's profits can be represented as the difference between its revenue and costs. Write the profit function,  $P(x)$ , as a polynomial in standard form. Graph  $y = P(x)$  on the set of axes below



Over the given domain, state when the company was the least profitable and the most profitable, to the *nearest year*. Explain how you determined your answer.

## A.APR.B.2: REMAINDER AND FACTOR THEOREMS

173 The graph of p(x) is shown below.



What is the remainder when p(x) is divided by

$$x + 4$$
?

$$1 \quad x-4$$

$$\begin{array}{ccc} 2 & -4 \\ 3 & 0 \end{array}$$

174 Which binomial is *not* a factor of the expression

$$x^3 - 11x^2 + 16x + 84$$
?

1 
$$x + 2$$

$$2 x + 4$$

$$3 \quad x-6$$

$$4 x - 7$$

175 Given  $P(x) = x^3 - 3x^2 - 2x + 4$ , which statement is true?

1 
$$(x-1)$$
 is a factor because  $P(-1) = 2$ .

2 
$$(x+1)$$
 is a factor because  $P(-1) = 2$ .

3 
$$(x+1)$$
 is a factor because  $P(1) = 0$ .

4 
$$(x-1)$$
 is a factor because  $P(1) = 0$ .

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- 176 Which binomial is a factor of  $x^4 4x^2 4x + 8$ ?
  - $1 \quad x-2$
  - 2 x+2
  - 3 x 4
  - 4 x + 4
- 177 If  $p(x) = 2x^3 3x + 5$ , what is the remainder of  $p(x) \div (x 5)$ ?
  - 1 -230
  - 2 0
  - 3 40
  - 4 240
- 178 If x 1 is a factor of  $x^3 kx^2 + 2x$ , what is the value of k?
  - 1 0
  - 2 2
  - 3 3
  - 4 –3
- 179 When g(x) is divided by x + 4, the remainder is 0. Given  $g(x) = x^4 + 3x^3 - 6x^2 - 6x + 8$ , which conclusion about g(x) is true?
  - 1 g(4) = 0
  - 2 g(-4) = 0
  - 3 x-4 is a factor of g(x).
  - 4 No conclusion can be made regarding g(x).
- 180 For the polynomial p(x), if p(3) = 0, it can be concluded that
  - 1 x + 3 is a factor of p(x)
  - 2 x-3 is a factor of p(x)
  - 3 when p(x) is divided by 3, the remainder is zero
  - 4 when p(x) is divided by -3, the remainder is zero

- 181 Consider the function  $f(x) = 2x^3 + x^2 18x 9$ . Which statement is true?
  - 1 2x-1 is a factor of f(x).
  - 2 x-3 is a factor of f(x).
  - $3 \quad f(3) \neq f\left(-\frac{1}{2}\right)$
  - $4 \qquad f\left(\frac{1}{2}\right) = 0$
- 182 If  $f(x) = 2x^4 x^3 16x + 8$ , then  $f\left(\frac{1}{2}\right)$ 
  - 1 equals 0 and 2x + 1 is a factor of f(x)
  - 2 equals 0 and 2x 1 is a factor of f(x)
  - 3 does not equal 0 and 2x + 1 is not a factor of f(x)
  - 4 does not equal 0 and 2x 1 is a factor of f(x)
- 183 Use an appropriate procedure to show that x 4 is a factor of the function  $f(x) = 2x^3 5x^2 11x 4$ . Explain your answer.
- 184 Determine if x 5 is a factor of  $2x^3 4x^2 7x 10$ . Explain your answer.
- 185 Given  $r(x) = x^3 4x^2 + 4x 6$ , find the value of r(2). What does your answer tell you about x 2 as a factor of r(x)? Explain.
- Determine for which polynomial(s) (x + 2) is a factor. Explain your answer.

$$P(x) = x^4 - 3x^3 - 16x - 12$$

$$Q(x) = x^3 - 3x^2 - 16x - 12$$

- 187 Show why x 3 is a factor of  $m(x) = x^3 x^2 5x 3$ . Justify your answer.
- 188 Evaluate j(-1) given  $j(x) = 2x^4 x^3 35x^2 + 16x + 48$ . Explain what your answer tells you about x + 1 as a factor. Algebraically find the remaining zeros of j(x).
- 189 Given  $z(x) = 6x^3 + bx^2 52x + 15$ , z(2) = 35, and z(-5) = 0, algebraically determine all the zeros of z(x).

#### A.APR.C.4: POLYNOMIAL IDENTITIES

190 Emmeline is working on one side of a polynomial identity proof used to form Pythagorean triples. Her work is shown below:

$$(5x)^2 + (5x^2 - 5)^2$$

Step 1:  $25x^2 + (5x^2 - 5)^2$ 

Step 2:  $25x^2 + 25x^2 + 25$ 

Step 3:  $50x^2 + 25$ 

Step 4:  $75x^2$ 

What statement is true regarding Emmeline's work?

- 1 Emmeline's work is entirely correct.
- 2 There is a mistake in step 2, only.
- 3 There are mistakes in step 2 and step 4.
- 4 There is a mistake in step 4, only.

- 191 The expression (x + a)(x + b) can *not* be written as
  - $1 \quad a(x+b) + x(x+b)$
  - $2 \quad x^2 + abx + ab$
  - $3 \quad x^2 + (a+b)x + ab$
  - 4 x(x+a)+b(x+a)
- 192 Which expression can be rewritten as (x+7)(x-1)?
  - 1  $(x+3)^2-16$
  - 2  $(x+3)^2 10(x+3) 2(x+3) + 20$
  - $3 \quad \frac{(x-1)(x^2-6x-7)}{(x+1)}$
  - $4 \frac{(x+7)(x^2+4x+3)}{(x+3)}$
- 193 Which equation represents a polynomial identity?
  - 1  $x^3 + y^3 = (x+y)^3$
  - 2  $x^3 + y^3 = (x + y)(x^2 xy + y^2)$
  - $3 x^3 + y^3 = (x + y)(x^2 xy y^2)$
  - 4  $x^3 + y^3 = (x y)(x^2 + xy + y^2)$

Given the polynomial identity  $x^6 + y^6 = (x^2 + y^2)(x^4 - x^2y^2 + y^4)$ , which equation must also be true for all values of x and y?

1 
$$x^6 + y^6 = x^2(x^4 - x^2y^2 + y^4) + y^2(x^4 - x^2y^2 + y^4)$$

2 
$$x^6 + y^6 = (x^2 + y^2)(x^2 - y^2)(x^2 - y^2)$$

3 
$$(x^3 + y^3)^2 = (x^2 + y^2)(x^4 - x^2y^2 + y^4)$$

4 
$$(x^6 + y^6) - (x^2 + y^2) = x^4 - x^2y^2 + y^4$$

195 Which statement(s) are true for all real numbers?

I 
$$(x-y)^2 = x^2 + y^2$$
  
II  $(x+y)^3 = x^3 + 3xy + y^3$ 

- 1 I, only
- 2 II, only
- 3 I and II
- 4 neither I nor II

196 Mr. Farison gave his class the three mathematical rules shown below to either prove or disprove.
Which rules can be proved for all real numbers?

I 
$$(m+p)^2 = m^2 + 2mp + p^2$$
  
II  $(x+y)^3 = x^3 + 3xy + y^3$   
III  $(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$ 

- 1 I, only
- 2 I and II
- 3 II and III
- 4 I and III

197 Given the following polynomials

$$x = (a+b+c)^{2}$$

$$y = a^{2} + b^{2} + c^{2}$$

$$z = ab + bc + ac$$

Which identity is true?

- $1 \quad x = y z$
- $2 \quad x = y + z$
- $3 \quad x = y 2z$
- $4 \quad x = y + 2z$

198 Erin and Christa were working on cubing binomials for math homework. Erin believed they could save time with a shortcut. She wrote down the rule below for Christa to follow.

$$(a+b)^3 = a^3 + b^3$$

Does Erin's shortcut always work? Justify your result algebraically.

199 Algebraically prove that 
$$\frac{x^3 + 9}{x^3 + 8} = 1 + \frac{1}{x^3 + 8}$$
, where  $x \neq -2$ .

200 Verify the following Pythagorean identity for all values of *x* and *y*:

$$(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$$

201 Algebraically determine the values of *h* and *k* to correctly complete the identity stated below.

$$2x^3 - 10x^2 + 11x - 7 = (x - 4)(2x^2 + hx + 3) + k$$

202 Algebraically prove that the difference of the squares of any two consecutive integers is an odd integer.

### **RADICALS**

#### N.RN.A.2: OPERATIONS WITH RADICALS

203 The expression  $\left(a\sqrt[3]{2b^2}\right)\left(\sqrt[3]{4a^2b}\right)$  is equivalent

- to
- 1  $2ab\sqrt[3]{a^2}$
- 2 2ah
- 3  $2ab\sqrt[3]{2a^2}$
- 4  $2a^2b\sqrt[3]{2b}$

Given y > 0, the expression  $\sqrt{3x^2y} \bullet \sqrt[3]{27x^3y^2}$  is equivalent to

- $1 \quad 81x^5y^3$
- $2 \quad 3^{1.5}x^2y$
- $3 \quad 3^{\frac{5}{2}} x^2 y^{\frac{5}{3}}$
- $4 \quad 3^{\frac{3}{2}} x^2 y^{\frac{7}{6}}$

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205 For positive values of x, which expression is

equivalent to  $\sqrt{16x^2} \bullet x^{\frac{2}{3}} + \sqrt[3]{8x^5}$ 

- 1  $6\sqrt[5]{x^3}$

- 206 For x > 0, which expression is equivalent to

 $\frac{\sqrt[3]{x^2} \bullet \sqrt{x^5}}{\sqrt[6]{x}}?$ 

- 1 x
- 2
- $3 \quad x^3$
- 207 Write  $\sqrt[3]{x} \cdot \sqrt{x}$  as a single term with a rational exponent.

### A.REI.A.2: SOLVING RADICALS

- 208 What is the solution set of  $x = \sqrt{3x + 40}$ ?
  - $1 \{-5,8\}$
  - 2 {8}
  - $3 \{-4,10\}$
  - 4 { }
- 209 The solution set for the equation  $\sqrt{56-x} = x$  is
  - $\{-8,7\}$ 1
  - $2 \{-7,8\}$
  - 3 {7}
  - 4 { }

- 210 The solution set for the equation  $b = \sqrt{2b^2 64}$  is
  - 1  $\{-8\}$
  - 2 {8}
  - 3  $\{\pm 8\}$
  - { }
- 211 What is the solution set for x in the equation below?

$$\sqrt{x+1} - 1 = x$$

- 1 {1}
- 2 {0}
- 3  $\{-1,0\}$
- $\{0,1\}$
- 212 The value(s) of x that satisfy

 $\sqrt{x^2 - 4x - 5} = 2x - 10$  are

- **{5**}
- 2 {7}
- 3 {5,7}
- 4 {3,5,7}
- 213 The solution set for the equation

$$\sqrt{x+14} - \sqrt{2x+5} = 1 \text{ is}$$

- 1 {-6}
- 2 {2}
- 3 {18}
- {2,22}
- 214 Solve the given equation algebraically for all values of x.  $3\sqrt{x} - 2x = -5$
- 215 Solve algebraically for all values of x:

$$\sqrt{x-5} + x = 7$$

216 Solve algebraically for all values of *x*: 
$$\sqrt{x-4} + x = 6$$

217 Solve the equation 
$$\sqrt{2x-7} + x = 5$$
 algebraically, and justify the solution set.

218 Solve algebraically for all values of *x*: 
$$\sqrt{4x+1} = 11-x$$

219 Solve the equation 
$$\sqrt{49 - 10x} + 5 = 2x$$
 algebraically.

220 Solve algebraically for all values of *x*: 
$$\sqrt{6-2x} + x = 2(x+15) - 9$$

221 The speed of a tidal wave, 
$$s$$
, in hundreds of miles per hour, can be modeled by the equation  $s = \sqrt{t - 2t + 6}$ , where  $t$  represents the time from its origin in hours. Algebraically determine the time when  $s = 0$ . How much faster was the tidal wave traveling after 1 hour than 3 hours, to the nearest mile per hour? Justify your answer.

A Foucault pendulum can be used to demonstrate that the Earth rotates. The time, 
$$t$$
, in seconds, that it takes for one swing or period of the pendulum can be modeled by the equation  $t = 2\pi \sqrt{\frac{L}{g}}$  where

L is the length of the pendulum in meters and g is a constant of 9.81 m/s<sup>2</sup>. The first Foucault pendulum was constructed in 1851 and has a pendulum length of 67 m. Determine, to the *nearest tenth of a second*, the time it takes this pendulum to complete one swing. Another Foucault pendulum at the United Nations building takes 9.6 seconds to complete one swing. Determine, to the *nearest tenth of a meter*, the length of this pendulum.

The Beaufort Wind Scale was devised by British Rear Admiral Sir Francis Beaufort, in 1805 based upon observations of the effects of the wind. Beaufort numbers, B, are determined by the equation  $B = 1.69\sqrt{s + 4.45} - 3.49$ , where s is the speed of the wind in mph, and B is rounded to the nearest integer from 0 to 12.

| Beaufort Wind Scale |                 |  |
|---------------------|-----------------|--|
| Beaufort Number     | Force of Wind   |  |
| 0                   | Calm            |  |
| 1                   | Light air       |  |
| 2                   | Light breeze    |  |
| 3                   | Gentle breeze   |  |
| 4                   | Moderate breeze |  |
| 5                   | Fresh breeze    |  |
| 6                   | Steady breeze   |  |
| 7                   | Moderate gale   |  |
| 8                   | Fresh gale      |  |
| 9                   | Strong gale     |  |
| 10                  | Whole gale      |  |
| 11                  | Storm           |  |
| 12                  | Hurricane       |  |

Using the table above, classify the force of wind at a speed of 30 mph. Justify your answer. In 1946, the scale was extended to accommodate strong hurricanes. A strong hurricane received a *B* value of exactly 15. Algebraically determine the value of *s*, to the *nearest mph*. Any *B* values that round to 10 receive a Beaufort number of 10. Using technology, find an approximate range of wind speeds, to the *nearest mph*, associated with a Beaufort number of 10.

# N.RN.A.1: RADICALS AND RATIONAL EXPONENTS

224 Explain what a rational exponent, such as  $\frac{5}{2}$  means.

Use this explanation to evaluate  $9^{\frac{3}{2}}$ .

225 Explain why  $81^{\frac{3}{4}}$  equals 27.

226 Explain how (-8) <sup>4</sup>/<sub>3</sub> can be evaluated using properties of rational exponents to result in an integer answer.

227 Explain how 
$$\left(3^{\frac{1}{5}}\right)^2$$
 can be written as the equivalent radical expression  $\sqrt[5]{9}$ .

### N.RN.A.2: RADICALS AND RATIONAL **EXPONENTS**

- 228 For all positive values of x, which expression is equivalent to  $x^{\frac{3}{4}}$ ?

  - $2 \sqrt[3]{x^4}$
  - $3 \left(x^3\right)^4$
  - $4 \quad 3(x^4)$
- 229 The expression  $\sqrt[4]{81x^8y^6}$  is equivalent to
  - $\begin{array}{ccc}
    1 & 3x^2y^{\frac{3}{2}} \\
    2 & 3x^4y^2
    \end{array}$
- 230 For  $x \ge 0$ , which equation is *false*?
  - $1 \qquad (x^{\frac{3}{2}})^2 = \sqrt[4]{x^3}$
  - $2 \qquad (x^3)^{\frac{1}{4}} = \sqrt[4]{x^3}$
  - 3  $\left(x^{\frac{3}{2}}\right)^{\frac{1}{2}} = \sqrt[4]{x^3}$ 4  $\left(x^{\frac{2}{3}}\right)^2 = \sqrt[3]{x^4}$

231 When b > 0 and d is a positive integer, the expression  $(3b)^{\frac{2}{d}}$  is equivalent to

- $2 \left(\sqrt{3b}\right)^d$
- $3 \quad \frac{1}{\sqrt{3h^d}}$
- $4 \left(\sqrt[d]{3b}\right)^2$
- 232 If  $n = \sqrt{a^5}$  and m = a, where a > 0, an expression for  $\frac{n}{m}$  could be
- 233 Given x > 0, the expression  $\frac{x^{\frac{1}{5}}}{x^{\frac{1}{2}}}$  can be rewritten as

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234 The expression 
$$\left(\frac{m^2}{\frac{1}{m^3}}\right)^{-\frac{1}{2}}$$
 is equivalent to

1 
$$-\sqrt[6]{m^5}$$

$$2 \quad \frac{1}{\sqrt[6]{m^5}}$$

$$3 -m\sqrt[5]{m}$$

$$4 \quad \frac{1}{m \sqrt[5]{m}}$$

235 What does 
$$\left(\frac{-54x^9}{y^4}\right)^{\frac{2}{3}}$$
 equal?

1 
$$\frac{9ix^6\sqrt[3]{4}}{y\sqrt[3]{y^2}}$$

$$2 \frac{9ix^{6}\sqrt[3]{4}}{y^{2}\sqrt[3]{y^{2}}}$$

$$3 \quad \frac{9x^{6}\sqrt[3]{4}}{y\sqrt[3]{y}}$$

$$4 \frac{9x^{6}\sqrt[3]{4}}{y^{2}\sqrt[3]{y^{2}}}$$

For  $x \neq 0$ , which expressions are equivalent to one divided by the sixth root of x?

I. 
$$\frac{\sqrt[6]{x}}{\sqrt[3]{x}}$$
 II.  $\frac{x^{\frac{1}{6}}}{\sqrt[1]{3}}$  III.  $x^{\frac{-1}{6}}$ 

- 1 I and II, only
- 2 I and III, only
- 3 II and III, only
- 4 I, II, and III

- Given the equal terms  $\sqrt[3]{x^5}$  and  $y^{\frac{5}{6}}$ , determine and state y, in terms of x.
- 238 Kenzie believes that for  $x \ge 0$ , the expression  $\left(\sqrt[7]{x^2}\right)\left(\sqrt[5]{x^3}\right)$  is equivalent to  $\sqrt[35]{x^6}$ . Is she correct? Justify your response algebraically.

239 For 
$$n$$
 and  $p > 0$ , is the expression
$$\left(p^2 n^{\frac{1}{2}}\right)^8 \sqrt{p^5 n^4} \text{ equivalent to } p^{18} n^6 \sqrt{p}?$$
Justify your answer.

240 For 
$$x \neq 0$$
 and  $y \neq 0$ ,  $\sqrt[3]{81x^{15}y^9} = 3^a x^5 y^3$ . Determine the value of *a*.

241 Use the properties of rational exponents to determine the value of 
$$y$$
 for the equation: 
$$\frac{\sqrt[3]{x^8}}{\left(x^4\right)^{\frac{1}{3}}} = x^y, \ x > 1$$

242 Justify why 
$$\frac{\sqrt[3]{x^2y^5}}{\sqrt[4]{x^3y^4}}$$
 is equivalent to  $x^{\frac{-1}{12}}y^{\frac{2}{3}}$  using

properties of rational exponents, where  $x \neq 0$  and  $y \neq 0$ .

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243 Express the fraction 
$$\frac{2x^{\frac{3}{2}}}{\left(16x^4\right)^{\frac{1}{4}}}$$
 in simplest radical form.

form.

### N.CN.A.2: OPERATIONS WITH COMPLEX NUMBERS

- 244 If A = -3 + 5i, B = 4 2i, and C = 1 + 6i, where i is the imaginary unit, then A - BC equals
  - 1 5-17i
  - 2 + 27i
  - 3 -19 17i
  - 4 -19 + 27i
- 245 The expression  $6xi^3(-4xi+5)$  is equivalent to
  - 1 2x-5i
  - $2 -24x^2 30xi$
  - $3 -24x^2 + 30x i$
  - 4  $26x 24x^2i 5i$
- 246 Given i is the imaginary unit,  $(2-yi)^2$  in simplest form is
  - $1 \quad y^2 4yi + 4$
  - $2 -y^2 4yi + 4$
  - $3 -y^2 + 4$
  - $4 v^2 + 4$
- 247 Given that i is the imaginary unit, the expression  $(x-2i)^2$  is equivalent to
  - 1  $x^2 + 4$
  - $2 x^2 4$
  - $3 \quad x^2 2xi 4$
  - $4 \quad x^2 4xi 4$

- 248 Which expression is equivalent to  $(3k-2i)^2$ , where *i* is the imaginary unit?
  - $1 \quad 9k^2 4$
  - $2 9k^2 + 4$
  - $9k^2 12ki 4$
  - $4 \quad 9k^2 12ki + 4$
- 249 The expression  $6 (3x 2i)^2$  is equivalent to
  - $1 -9x^2 + 12xi + 10$
  - $2 9x^2 12xi + 2$
  - $3 -9x^2 + 10$
  - $4 -9x^2 + 12xi 4i + 6$
- 250 Expressed in simplest a + bi form,

$$(7-3i) + (x-2i)^2 - (4i+2x^2)$$
 is

- 1  $(3-x^2)-(4x+7)i$
- 2  $(3+3x^2)-(4x+7)i$
- $3 (3-x^2)-7i$
- 4  $(3+3x^2)-7i$
- 251 Where i is the imaginary unit, the expression  $(x+3i)^2 - (2x-3i)^2$  is equivalent to

$$1 -3x^2$$

$$2 -3x^2 - 18$$

$$3 -3x^2 + 18xi$$

$$4 -3x^2 - 6xi - 18$$

252 Which expression is equivalent to

 $(2x-i)^2 - (2x-i)(2x+3i)$  where i is the imaginary unit and x is a real number?

- -4 8xi
- 2 -4-4xi
- 4 8x 4i

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- 253 Which expression is equivalent to  $(x+yi)(x^2-xyi-y^2)$ , where i is the imaginary unit?
  - 1  $x^3 + y^3 i$
  - $2 \quad x^3 xy^2 (xy^2 + y^3)i$
  - $3 \quad x^3 2xy^2 y^3i$
  - $4 \quad x^3 y^3 i$
- 254 If  $(6-ki)^2 = 27-36i$ , the value of k is
  - 1 -36
  - 2 -3
  - 3 3
- 255 Simplify  $xi(i-7i)^2$ , where i is the imaginary unit.
- 256 Express  $(1-i)^3$  in a+bi form.
- 257 Write (5+2yi)(4-3i)-(5-2yi)(4-3i) in a+biform, where y is a real number.
- 258 Write  $-\frac{1}{2}i^{3}(\sqrt{-9}-4)-3i^{2}$  in simplest a+bi

259 Elizabeth tried to find the product of (2+4i) and (3-i), and her work is shown below.

$$(2+4i)(3-i)$$

$$= 6-2i+12i-4i^{2}$$

$$= 6+10i-4i^{2}$$

$$= 6+10i-4(1)$$

$$= 6+10i-4$$

$$= 2+10i$$

Identify the error in the process shown and determine the correct product of (2+4i) and (3-i).

## **RATIONALS**

#### A.APR.D.6: UNDEFINED RATIONALS

- 260 The function  $f(x) = \frac{x-3}{x^2 + 2x 8}$  is undefined when
  - x equals
  - $1 \quad 2 \text{ or } -4$
  - $2 4 {or} 2$
  - 3 3, only
  - 2, only

### A.APR.D.6: EXPRESSIONS WITH NEGATIVE **EXPONENTS**

261 The expression  $\frac{-3x^2 - 5x + 2}{x^3 + 2x^2}$  can be rewritten as

$$1 \qquad \frac{-3x-3}{x^2+2x}$$

$$2 \quad \frac{-3x-1}{x^2}$$

$$3 \quad -3x^{-1} + 1 \\
4 \quad -3x^{-1} + x^{-2}$$

$$4 \quad -3x^{-1} + x^{-2}$$

262 Given that 
$$\left(\frac{\frac{17}{y^{\frac{8}{8}}}}{\frac{5}{4}}\right)^{-4} = y^n$$
, where  $y > 0$ , determine the value of  $n$ .

### A.APR.D.6: RATIONAL EXPRESSIONS

263 Which expression(s) are equivalent to  $\frac{x^2 - 4x}{2x}$ , where  $x \neq 0$ ?

I. 
$$\frac{x}{2} - 2$$
 II.  $\frac{x-4}{2}$  III.  $\frac{x-1}{2} - \frac{3}{2}$ 

- 1 II, only
- 2 I and II
- 3 II and III
- 4 I, II, and III
- 264 For all values of x for which the expression is defined,  $\frac{x^2 + 3x}{x^2 + 5x + 6}$  is equivalent to

$$1 \quad 1 - \frac{x}{x+2}$$

$$2 \frac{x}{x+2}$$

$$3 \qquad \frac{3x}{5x+6}$$

$$4 1 + \frac{1}{2x+6}$$

265 Written in simplest form, the fraction  $\frac{x^3 - 9x}{9 - x^2}$ , where  $x \neq \pm 3$ , is equivalent to

$$3 \quad \frac{-x(x+3)}{(3+x)}$$

$$4 \quad \frac{x(x-3)}{(3-x)}$$

266 Written in simplest form,  $\frac{c^2 - d^2}{d^2 + cd - 2c^2}$  where  $c \neq d$ , is equivalent to

$$1 \qquad \frac{c+d}{d+2c}$$

$$2 \qquad \frac{c-d}{d+2c}$$

$$3 \qquad \frac{-c-d}{d+2c}$$

$$4 \qquad \frac{-c+d}{d+2c}$$

267 For all values of x for which the expression is defined,  $\frac{x^3 + 2x^2 - 9x - 18}{x^3 - x^2 - 6x}$ , in simplest form, is equivalent to

$$2 -\frac{17}{2}$$

$$3 \frac{x+3}{x}$$

$$4 \frac{x^2-9}{x(x-3)}$$

268 Given  $x \ne -2$ , the expression  $\frac{2x^2 + 5x + 8}{x + 2}$  is equivalent to

$$1 \qquad 2x^2 + \frac{9}{x+2}$$

$$2 \qquad 2x + \frac{7}{x+2}$$

$$3 \qquad 2x+1+\frac{6}{x+2}$$

4 
$$2x+9-\frac{10}{x+2}$$

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- 269 The expression  $\frac{x^3 + 2x^2 + x + 6}{x + 2}$  is equivalent to

  - $2 \quad x^2 + 1 + \frac{4}{x+2}$
  - $3 \quad 2x^2 + x + 6$
  - 4  $2x^2 + 1 + \frac{4}{x+2}$
- 270 Given  $x \ne -3$ , the expression  $\frac{2x^3 + 7x^2 3x 25}{x + 3}$  is equivalent to
  - 1  $2x^2 + x 6 \frac{7}{x+3}$
  - 2  $2x^2 + 13x 36 + \frac{83}{x+3}$
  - $3 \quad 2x^2 + x 13$
  - 4  $x^2 + 4x 15 + \frac{20}{x+3}$
- 271 Which expression is equivalent to

$$\frac{2x^4 + 8x^3 - 25x^2 - 6x + 14}{x + 6}$$
?

- 1  $2x^3 + 4x^2 + x 12 + \frac{86}{x+6}$
- $2 \quad 2x^3 4x^2 x + 14$
- 3  $2x^3 4x^2 x + \frac{14}{x+6}$
- $4 \quad 2x^3 4x^2 x$
- 272 What is the quotient when  $10x^3 3x^2 7x + 3$  is divided by 2x - 1?
  - 1  $5x^2 + x + 3$
  - $2 5x^2 x + 3$
  - $3 \quad 5x^2 x 3$
  - $4 5x^2 + x 3$

- 273 The expression  $\frac{6x^3 + 17x^2 + 10x + 2}{2x + 3}$  equals
  - 1  $3x^2 + 4x 1 + \frac{5}{2x-3}$
  - 2  $6x^2 + 8x 2 + \frac{5}{2x + 3}$
  - 3  $6x^2 x + 13 \frac{37}{2x + 3}$
  - 4  $3x^2 + 13x + \frac{49}{2} + \frac{151}{2x+3}$
  - 274 The expression  $\frac{x^2 + 12}{x^2 + 3}$  can be rewritten as
    - $1 \frac{10}{x^2 + 3}$
    - $\begin{array}{ccc}
      2 & 1 + \frac{9}{x^2 + 3} \\
      3 & x + 9 \\
      4 & 4
      \end{array}$
  - 275 Which expression is equivalent to  $\frac{x^3-2}{x-2}$ ?

    - 2  $x^2 + 2x + 4 + \frac{6}{x-2}$

    - 4  $x^2 2x + 4 \frac{10}{x^2}$
  - 276 The expression  $\frac{9x^2-2}{3x+1}$  is equivalent to
    - 1  $3x-1-\frac{1}{3x+1}$
    - 2  $3x-1+\frac{1}{3x+1}$
    - $3 \quad 3x + 1 \frac{1}{3x + 1}$
    - 4  $3x+1+\frac{1}{3x+1}$

- 277 The expression  $\frac{4x^3 + 5x + 10}{2x + 3}$  is equivalent to
  - $1 \qquad 2x^2 + 3x 7 + \frac{31}{2x + 3}$
  - $2 \qquad 2x^2 3x + 7 \frac{11}{2x + 3}$
  - 3  $2x^2 + 2.5x + 5 + \frac{15}{2x+3}$
  - 4  $2x^2 2.5x 5 \frac{20}{2x + 3}$
- 278 Which expression is equivalent to  $\frac{4x^3 + 9x 5}{2x 1}$ ,
  - where  $x \neq \frac{1}{2}$ ?
  - $1 \qquad 2x^2 + x + 5$
  - $2 \qquad 2x^2 + \frac{11}{2} + \frac{1}{2(2x-1)}$
  - $3 \quad 2x^2 x + 5$
  - 4  $2x^2 x + 4 + \frac{1}{2x 1}$
- 279 The expression  $\frac{x^4 5x^2 + 4x + 14}{x + 2}$  is equivalent to
  - 1  $x^3 2x^2 x + 6 + \frac{2}{x+2}$
  - $2 \quad x^3 5x + 4 \frac{14}{x+2}$
  - 3  $x^3 + 2x^2 x + 2 + \frac{18}{x+2}$
  - 4  $x^3 + 2x^2 9x + 22 \frac{30}{x+2}$
- 280 Given  $f(x) = 3x^2 + 7x 20$  and g(x) = x 2, state the quotient and remainder of  $\frac{f(x)}{g(x)}$ , in the form  $q(x) + \frac{r(x)}{g(x)}$ .

- 281 Given  $f(x) = 3x^3 4x^2 + 2x 1$  and g(x) = x 4, state the quotient and remainder of  $\frac{f(x)}{g(x)}$ , in the form  $q(x) + \frac{r(x)}{g(x)}$ . Is x = 4 a root of f(x)? Explain your answer.
- 282 Determine the quotient and remainder when  $(6a^3 + 11a^2 4a 9)$  is divided by (3a 2). Express your answer in the form  $q(a) + \frac{r(a)}{d(a)}$ .
- 283 Given  $a(x) = x^4 + 2x^3 + 4x 10$  and b(x) = x + 2, determine  $\frac{a(x)}{b(x)}$  in the form  $q(x) + \frac{r(x)}{b(x)}$ . Is b(x) a factor of a(x)? Explain.
- When the function p(x) is divided by x 1 the quotient is  $x^2 + 7 + \frac{5}{x 1}$ . State p(x) in standard form.

## A.APR.D.7: ADDITION AND SUBTRACTION OF RATIONALS

- 285 The expression  $2 \frac{x-1}{x+2}$  is equivalent to
  - $1 \qquad 1 \frac{3}{x+2}$
  - $2 \qquad 1 + \frac{3}{x+2}$
  - $3 \qquad 1 \frac{1}{x+2}$
  - $4 \qquad 1 + \frac{1}{x+2}$

#### A.CED.A.1: MODELING RATIONALS

- 286 A number, minus twenty times its reciprocal, equals eight. The number is
  - 1 10 or -2
  - 2 10 or 2
  - 3 -10 or -2
  - 4 -10 or 2
- Julie averaged 85 on the first three tests of the semester in her mathematics class. If she scores 93 on each of the remaining tests, her average will be 90. Which equation could be used to determine how many tests, *T*, are left in the semester?
  - $1 \quad \frac{255 + 93T}{3T} = 90$
  - $2 \frac{255 + 90T}{3T} = 93$
  - $3 \quad \frac{255 + 93T}{T + 3} = 90$
  - $4 \qquad \frac{255 + 90T}{T + 3} = 93$
- 288 A manufacturing plant produces two different-sized containers of peanuts. One container weighs *x* ounces and the other weighs *y* pounds. If a gift set can hold one of each size container, which expression represents the number of gift sets needed to hold 124 ounces?
  - $1 \quad \frac{124}{16x + y}$
  - $2 \quad \frac{x+16y}{124}$
  - $3 \qquad \frac{124}{x + 16y}$
  - $4 \qquad \frac{16x + y}{124}$

- 289 A rush-hour commuter train has arrived on time 64 of its first 80 days. As arrivals continue, which equation can be used to find *x*, the number of consecutive days that the train must arrive on schedule to raise its on-time performance rate to 90%?
  - $1 \qquad \frac{64}{80+x} = \frac{90}{100}$
  - $2 \qquad \frac{64+x}{80+x} = \frac{90}{100}$
  - $3 \quad \frac{64+x}{80} = \frac{90}{100}$
  - $4 \qquad \frac{x}{80+x} = \frac{90}{100}$
- 290 Mallory wants to buy a new window air conditioning unit. The cost for the unit is \$329.99. If she plans to run the unit three months out of the year for an annual operating cost of \$108.78, which function models the cost per year over the lifetime of the unit, C(n), in terms of the number of years, n, that she owns the air conditioner.
  - 1 C(n) = 329.99 + 108.78n
  - C(n) = 329.99 + 326.34n
  - $3 \qquad C(n) = \frac{329.99 + 108.78n}{n}$
  - $4 \qquad C(n) = \frac{329.99 + 326.34n}{n}$

#### A.REI.A.2: SOLVING RATIONALS

291 The focal length, F, of a camera's lens is related to the distance of the object from the lens, J, and the distance to the image area in the camera, W, by the formula below.

$$\frac{1}{I} + \frac{1}{W} = \frac{1}{F}$$

When this equation is solved for J in terms of F and W, J equals

- $1 \quad F-W$
- $2 \frac{FW}{F-W}$
- $3 \frac{FW}{W-F}$
- $4 \qquad \frac{1}{F} \frac{1}{W}$
- 292 What is the solution set of the equation

$$\frac{2}{x} - \frac{3x}{x+3} = \frac{x}{x+3}$$
?

- 1 {3]
- $2 \quad \left\{ \frac{3}{2} \right\}$
- 3 {-2,3}
- $4 \quad \left\{-1, \frac{3}{2}\right\}$

293 What is the solution set of the equation

$$\frac{2}{3x+1} = \frac{1}{x} - \frac{6x}{3x+1}$$
?

- $1 \quad \left\{-\frac{1}{3}, \frac{1}{2}\right\}$
- $2 \quad \left\{-\frac{1}{3}\right\}$
- $3 \quad \left\{\frac{1}{2}\right\}$
- $4 \quad \left\{\frac{1}{3}, -2\right\}$
- 294 What is the solution set of the equation

$$\frac{x+2}{x} + \frac{x}{3} = \frac{2x^2 + 6}{3x}$$
?

- $1 \quad \{-3\}$
- $2 \{-3,0\}$
- 3 {3}
- 4 {0,3}
- 295 What is the solution set of the equation

$$\frac{10}{x^2 - 2x} + \frac{4}{x} = \frac{5}{x - 2}$$
?

- $1 \{0,2\}$
- 2 {0}
- 3 {2}
- 4 {}
- 296 What is the solution set of the equation

$$\frac{4}{k^2 - 8k + 12} = \frac{k}{k - 2} + \frac{1}{k - 6}?$$

- $1 \{-1,6\}$
- $2 \quad \{1,-6\}$
- 3 {-1}
- 4 {1}

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297 What is the solution, if any, of the equation

$$\frac{2}{x+3} - \frac{3}{4-x} = \frac{2x-2}{x^2 - x - 12}$$
?

- 1 -1
- 2 –5
- 3 all real numbers
- 4 no real solution
- 298 The solutions to  $x + 3 \frac{4}{x 1} = 5$  are

$$1 \quad \frac{3}{2} \pm \frac{\sqrt{17}}{2}$$

- $2 \quad \frac{3}{2} \pm \frac{\sqrt{17}}{2} i$
- $3 \quad \frac{3}{2} \pm \frac{\sqrt{33}}{2}$
- $4 \quad \frac{3}{2} \pm \frac{\sqrt{33}}{2} i$
- 299 What is the solution set of the equation

$$\frac{3x+25}{x+7} - 5 = \frac{3}{x}?$$

- $1 \quad \left\{\frac{3}{2}, 7\right\}$
- $2 \quad \left\{\frac{7}{2}, -3\right\}$
- $3 \left\{-\frac{3}{2},7\right\}$
- $4 \quad \left\{-\frac{7}{2}, -3\right\}$

300 To solve  $\frac{2x}{x-2} - \frac{11}{x} = \frac{8}{x^2 - 2x}$ , Ren multiplied

both sides by the least common denominator.

Which statement is true?

- 1 2 is an extraneous solution.
- 2  $\frac{7}{2}$  is an extraneous so 1 ution.
- 3 0 and 2 are extraneous solutions.
- 4 This equation does not contain any extraneous solutions.
- 301 Solve algebraically for n:  $\frac{2}{n^2} + \frac{3}{n} = \frac{4}{n^2}$
- 302 Solve for all values of p:  $\frac{3p}{p-5} \frac{2}{p+3} = \frac{p}{p+3}$
- 303 Solve for x:  $\frac{1}{x} \frac{1}{3} = -\frac{1}{3x}$
- 304 Algebraically solve for x:  $\frac{-3}{x+3} + \frac{1}{2} = \frac{x}{6} \frac{1}{2}$
- 305 Algebraically solve for x:  $\frac{7}{2x} \frac{2}{x+1} = \frac{1}{4}$

306 A formula for work problems involving two people is shown below.

$$\frac{1}{t_1} + \frac{1}{t_2} = \frac{1}{t_b}$$

 $t_1$  = the time taken by the first person to complete the job

 $t_2$  = the time taken by the second person to complete the job

 $t_b$  = the time it takes for them working together to complete the job

Fred and Barney are carpenters who build the same model desk. It takes Fred eight hours to build the desk while it only takes Barney six hours. Write an equation that can be used to find the time it would take both carpenters working together to build a desk. Determine, to the *nearest tenth of an hour*, how long it would take Fred and Barney working together to build a desk.

307 Sarah is fighting a sinus infection. Her doctor prescribed a nasal spray and an antibiotic to fight the infection. The active ingredients, in milligrams, remaining in the bloodstream from the nasal spray, n(t), and the antibiotic, a(t), are modeled in the functions below, where t is the time in hours since the medications were taken.

$$n(t) = \frac{t+1}{t+5} + \frac{18}{t^2 + 8t + 15}$$

$$a(t) = \frac{9}{t+3}$$

Determine which drug is made with a greater initial amount of active ingredient. Justify your answer. Sarah's doctor told her to take both drugs at the same time. Determine algebraically the number of hours after taking the medications when both medications will have the same amount of active ingredient remaining in her bloodstream.

### **SYSTEMS**

#### A.REI.C.6: SOLVING LINEAR SYSTEMS

308 Consider the system of equations below?

$$x + 2y - z = 1$$

$$-x - 3y + 2z = 0$$

$$2x - 4y + z = 10$$

What is the solution to the given system of equations?

- 1 (1,1,2)
- 2 (3,-1,0)
- 3 (5,-1,2)
- 4 (3,5,8)

309 For the system shown below, what is the value of z?

$$y = -2x + 14$$

$$3x - 4z = 2$$

$$3x - y = 16$$

- 1 5
- 2 2
- 3 6
- 4 4

310 Which value is *not* contained in the solution of the system shown below?

$$a + 5b - c = -20$$

$$4a - 5b + 4c = 19$$

$$-a - 5b - 5c = 2$$

- 1 -2
- 2 2
- 3 3
- 4 –

311 Consider the system of equations below:

$$x + y - z = 6$$

$$2x - 3y + 2z = -19$$

$$-x + 4y - z = 17$$

- Which number is *not* the value of any variable in the solution of the system?
- 1 –1
- 2 2
- 3 3
- 4 –4
- 312 Consider the system below.

$$x + y + z = 9$$

$$x - y - z = -1$$

$$x - y + z = 21$$

- Which value is *not* in the solution, (x,y,z), of the system?
- 1 -8
- 2 -6
- 3 11
- 4 4
- 313 Solve the following system of equations algebraically for all values of x, y, and z:

$$x + 3y + 5z = 45$$

$$6x - 3y + 2z = -10$$

$$-2x + 3y + 8z = 72$$

314 Solve the following system of equations algebraically for all values of x, y, and z:

$$x + y + z = 1$$

$$2x + 4y + 6z = 2$$

$$-x + 3y - 5z = 11$$

315 Solve the following system of equations algebraically for all values of x, y, and z:

$$2x + 3y - 4z = -1$$

$$x - 2y + 5z = 3$$

$$-4x + y + z = 16$$

316 Solve the following system of equations algebraically for all values of *a*, *b*, and *c*.

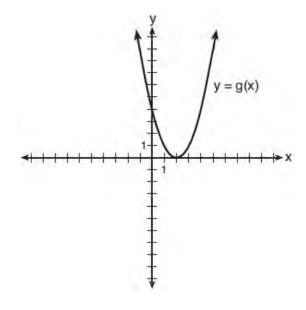
$$a + 4b + 6c = 23$$

$$a + 2b + c = 2$$

$$6b + 2c = a + 14$$

#### A.REI.C.7: QUADRATIC-LINEAR SYSTEMS

What is the solution to the system of equations y = 3x - 2 and y = g(x) where g(x) is defined by the function below?



- $1 \{(0,-2)\}$
- $2 \{(0,-2),(1,6)\}$
- 3 {(1,6)}
- 4 {(1,1),(6,16)}

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318 How many real solutions exist for the system of equations below?

$$y = \frac{1}{4}x - 8$$

$$y = \frac{1}{2}x^2 + 2x$$

- 1 1
- 2 2
- 3 3
- 4 0
- 319 The graphs of the equations  $y = x^2 + 4x 1$  and y + 3 = x are drawn on the same set of axes. One solution of this system is
  - $1 \quad (-5, -2)$
  - 2(-1,-4)
  - 3 (1,4)
  - 4(-2,-1)
- 320 What is the solution set of the following system of equations?

$$y = 3x + 6$$

$$y = (x+4)^2 - 10$$

- 1 {(-5,-9)}
- 2 {(5,21)}
- $3 \{(0,6),(-5,-9)\}$
- $4 \{(0,6),(5,21)\}$
- What are the solution(s) to the system of equations shown below?

$$x^2 + y^2 = 5$$

$$y = 2x$$

- 1 x = 1 and x = -1
- 2 x = 1
- 3 (1,2) and (-1,-2)
- 4 (1,2), only

322 Consider the system shown below.

$$2x - y = 4$$

$$(x+3)^2 + y^2 = 8$$

The two solutions of the system can be described as

- 1 both imaginary
- 2 both irrational
- 3 both rational
- 4 one rational and one irrational
- 323 Algebraically determine the values of *x* that satisfy the system of equations below.

$$y = -2x + 1$$

$$y = -2x^2 + 3x + 1$$

324 Algebraically determine the solution set for the system of equations below.

$$y = 2x^2 - 7x + 4$$

$$y = 11 - 2x$$

325 Solve the following system of equations algebraically.  $x^2 + y^2 = 400$ 

$$y = x - 28$$

326 Solve the system of equations algebraically.

$$x^2 + y^2 = 25$$

$$y + 5 = 2x$$

327 Algebraically solve the following system of equations.

$$(x-2)^2 + (y-3)^2 = 16$$

$$x + y - 1 = 0$$

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328 Solve the system of equations shown below algebraically.

$$(x-3)^2 + (y+2)^2 = 16$$
$$2x + 2y = 10$$

A.REI.D.11: QUADRATIC-LINEAR SYSTEMS

329 Sally's high school is planning their spring musical. The revenue, R, generated can be determined by the function  $R(t) = -33t^2 + 360t$ , where t represents the price of a ticket. The production cost, C, of the musical is represented by the function C(t) = 700 + 5t. What is the highest ticket price, to the *nearest dollar*, they can charge in order to *not* lose money on the event?

> 1 t = 3

2 *t* = 5

3 t = 8

t = 11

#### A.REI.D.11: OTHER SYSTEMS

330 Selected values for the functions f and g are shown in the tables below.

| X     | f(x)  | X     | g(x)  |
|-------|-------|-------|-------|
| -3.12 | -4.88 | -2.01 | -1.01 |
| 0     | -6    | 0     | 0.58  |
| 1.23  | -4.77 | 8.52  | 2.53  |
| 8.52  | 2.53  | 13.11 | 3.01  |
| 9.01  | 3.01  | 16.52 | 3.29  |

A solution to the equation f(x) = g(x) is

1 0 2

2.53

3 3.01

8.52

4

331 Which value, to the *nearest tenth*, is *not* a solution of p(x) = q(x) if  $p(x) = x^3 + 3x^2 - 3x - 1$  and q(x) = 3x + 8?

$$q(x) = 3x + 1$$

332 If f(x) = 3|x| - 1 and  $g(x) = 0.03x^3 - x + 1$ , an approximate solution for the equation f(x) = g(x) is

> 1 1.96

2 11.29

3 (-0.99, 1.96)

(11.29, 32.87)

333 Given:  $h(x) = \frac{2}{9}x^3 + \frac{8}{9}x^2 - \frac{16}{13}x + 2$ 

$$k(x) = -|0.7x| + 5$$

State the solutions to the equation h(x) = k(x), rounded to the *nearest hundredth*.

334 To the *nearest tenth*, the value of x that satisfies

$$2^x = -2x + 11$$
 is

- 1 2.5
- 2 2.6
- 3 5.8
- 4 5.9
- 335 For which approximate value(s) of *x* will log(x+5) = |x-1| 3?
  - 1 5, 1
  - 2 -2.41, 0.41
  - 3 -2.41, 5
  - 4 5, only
- 336 After examining the functions  $f(x) = \ln(x+2)$  and  $g(x) = e^{x-1}$  over the interval (-2,3], Lexi determined that the correct number of solutions to the equation f(x) = g(x) is
  - 1 1
  - 2 2
  - 3 3
  - 4 0
- 337 If  $p(x) = 2\ln(x) 1$  and  $m(x) = \ln(x+6)$ , then what is the solution for p(x) = m(x)?
  - 1 1.65
  - 2 3.14
  - 3 5.62
  - 4 no solution

- 338 For which values of x, rounded to the *nearest* hundredth, will  $|x^2 9| 3 = \log_3 x$ ?
  - 1 2.29 and 3.63
  - 2 2.37 and 3.54
  - 3 2.84 and 3.17
  - 4 2.92 and 3.06
- 339 How many solutions exist for

$$\frac{1}{1-x^2} = -|3x-2| + 5?$$

- 1 1
- 2 2
- 3 3
- 4 4
- 340 What is the total number of points of intersection of the graphs of the equations  $y = e^x$  and xy = 20?
  - 1
  - 2 2
  - 3 3
  - 4 0
- 341 When  $g(x) = \frac{2}{x+2}$  and  $h(x) = \log(x+1) + 3$  are

graphed on the same set of axes, which coordinates best approximate their point of intersection?

- $1 \quad (-0.9, 1.8)$
- $2 \quad (-0.9, 1.9)$
- 3 (1.4, 3.3)
- 4 (1.4, 3.4)
- 342 Which value, to the *nearest tenth*, is the *smallest* solution of f(x) = g(x) if  $f(x) = 3\sin\left(\frac{1}{2}x\right) 1$  and

$$g(x) = x^3 - 2x + 1$$
?

- 1 -3.6
- 2 -2.1
- 3 -1.8
- 4 1.4

343 The populations of two small towns at the beginning of 2018 and their annual population growth rate are shown in the table below.

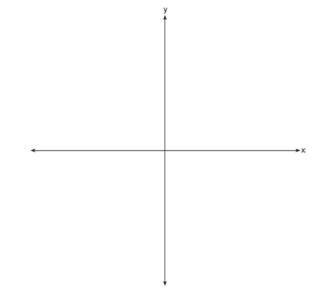
| Town         | Population | Annual Population<br>Growth Rate |
|--------------|------------|----------------------------------|
| Jonesville   | 1240       | 6% increase                      |
| Williamstown | 890        | 11% increase                     |

Assuming the trend continues, approximately how many years after the beginning of 2018 will it take for the populations to be equal?

1 7 2 20

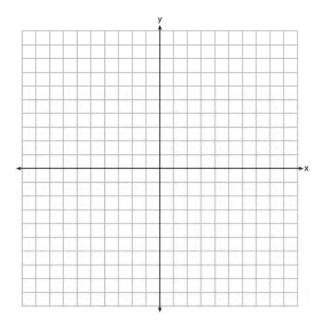
- 3 68 4 125
- Pedro and Bobby each own an ant farm. Pedro starts with 100 ants and says his farm is growing exponentially at a rate of 15% per month. Bobby starts with 350 ants and says his farm is steadily decreasing by 5 ants per month. Assuming both boys are accurate in describing the population of their ant farms, after how many months will they both have approximately the same number of ants?
  - 1 7
  - 2 8
  - 3 13
  - 4 36
- Researchers in a local area found that the population of rabbits with an initial population of 20 grew continuously at the rate of 5% per month. The fox population had an initial value of 30 and grew continuously at the rate of 3% per month. Find, to the *nearest tenth of a month*, how long it takes for these populations to be equal.

346 Sketch the graphs of  $r(x) = \frac{1}{x}$  and a(x) = |x| - 3 on the set of axes below. Determine, to the *nearest tenth*, the positive solution of r(x) = a(x).



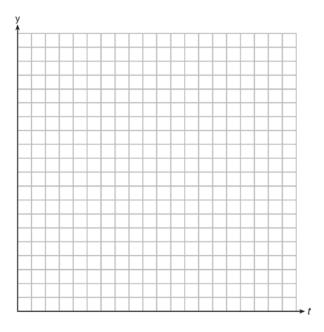
On the set of axes below, graph y = f(x) and y = g(x) for the given functions.

$$f(x) = x^3 - 3x^2$$
$$g(x) = 2x - 5$$



State the number of solutions to the equation f(x) = g(x).

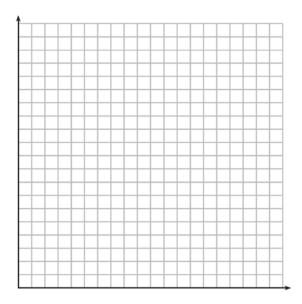
348 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function  $N(t) = N_0(e)^{-rt}$ , where N(t) is the amount left in the body,  $N_0$  is the initial dosage, r is the decay rate, and t is time in hours. Patient A, A(t), is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, B(t), is given 400 milligrams of another drug with a decay rate of 0.231. Write two functions, A(t) and B(t), to represent the breakdown of the respective drug given to each patient. Graph each function on the set of axes below.



To the *nearest hour*, *t*, when does the amount of the given drug remaining in patient *B* begin to exceed the amount of the given drug remaining in patient *A*? The doctor will allow patient *A* to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour*, how long patient *A* will have to wait to take another 800 milligram dose of the drug.

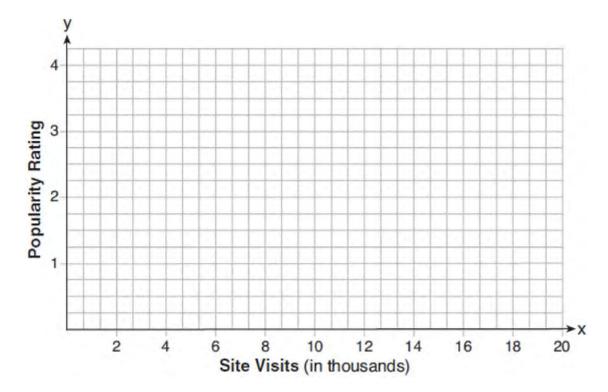
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349 The value of a certain small passenger car based on its use in years is modeled by  $V(t) = 28482.698(0.684)^t$ , where V(t) is the value in dollars and t is the time in years. Zach had to take out a loan to purchase the small passenger car. The function  $Z(t) = 22151.327(0.778)^t$ , where Z(t) is measured in dollars, and t is the time in years, models the unpaid amount of Zach's loan over time. Graph V(t) and Z(t) over the interval  $0 \le t \le 5$ , on the set of axes below.



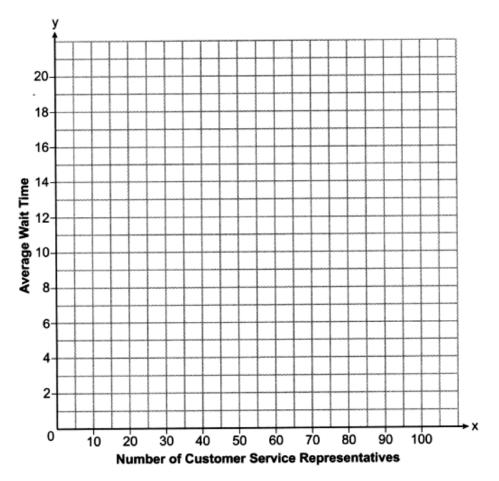
State when V(t) = Z(t), to the *nearest hundredth*, and interpret its meaning in the context of the problem. Zach takes out an insurance policy that requires him to pay a \$3000 deductible in case of a collision. Zach will cancel the collision policy when the value of his car equals his deductible. To the *nearest year*, how long will it take Zach to cancel this policy? Justify your answer.

Website popularity ratings are often determined using models that incorporate the number of visits per week a website receives. One model for ranking websites is  $P(x) = \log(x - 4)$ , where x is the number of visits per week in thousands and P(x) is the website's popularity rating. According to this model, if a website is visited 16,000 times in one week, what is its popularity rating, rounded to the *nearest tenth*? Graph y = P(x) on the axes below.



An alternative rating model is represented by  $R(x) = \frac{1}{2}x - 6$ , where x is the number of visits per week in thousands. Graph R(x) on the same set of axes. For what number of weekly visits will the two models provide the same rating?

A technology company is comparing two plans for speeding up its technical support time. Plan *A* can be modeled by the function  $A(x) = 15.7(0.98)^x$  and plan *B* can be modeled by the function  $B(x) = 11(0.99)^x$  where *x* is the number of customer service representatives employed by the company and A(x) and B(x) represent the average wait time, in minutes, of each customer. Graph A(x) and B(x) in the interval  $0 \le x \le 100$  on the set of axes below.



To the *nearest integer*, solve the equation A(x) = B(x). Determine, to the *nearest minute*, B(100) - A(100). Explain what this value represents in the given context.

## **FUNCTIONS**

### F.BF.A.1: OPERATIONS WITH FUNCTIONS

- 352 If  $p(x) = ab^x$  and  $r(x) = cd^x$ , then  $p(x) \bullet r(x)$  equals
  - 1  $ac(b+d)^x$
  - $2 \quad ac(b+d)^{2x}$
  - $3 \quad ac(bd)^x$
  - 4  $ac(bd)^{x^2}$
- 353 If  $f(x) = x^2 + 9$  and g(x) = x + 3, which operation would not result in a polynomial expression?
  - 1 f(x) + g(x)
  - $2 \qquad f(x) g(x)$
  - 3  $f(x) \bullet g(x)$
  - 4  $f(x) \div g(x)$
- 354 If  $g(c) = 1 c^2$  and m(c) = c + 1, then which statement is *not* true?
  - 1  $g(c) \cdot m(c) = 1 + c c^2 c^3$
  - 2  $g(c) + m(c) = 2 + c c^2$
  - 3  $m(c) g(c) = c + c^2$
  - $4 \qquad \frac{m(c)}{g(c)} = \frac{-1}{1-c}$
- 355 For all real values of x, if  $f(x) = (x-3)^2$  and  $g(x) = (x+3)^2$ , what is f(x) g(x)?
  - 1 18
  - 2 0
  - 3 -12x
  - 4  $2x^2 12x 18$

- 356 A manufacturing company has developed a cost model,  $C(x) = 0.15x^3 + 0.01x^2 + 2x + 120$ , where x is the number of items sold, in thousands. The sales price can be modeled by S(x) = 30 0.01x. Therefore, revenue is modeled by  $R(x) = x \cdot S(x)$ . The company's profit, P(x) = R(x) C(x), could be modeled by
  - 1  $0.15x^3 + 0.02x^2 28x + 120$
  - $2 -0.15x^3 0.02x^2 + 28x 120$
  - $3 -0.15x^3 + 0.01x^2 2.01x 120$
  - 4  $-0.15x^3 + 32x + 120$
- 357 The profit function, p(x), for a company is the cost function, c(x), subtracted from the revenue function, r(x). The profit function for the Acme Corporation is  $p(x) = -0.5x^2 + 250x 300$  and the revenue function is  $r(x) = -0.3x^2 + 150x$ . The cost function for the Acme Corporation is
  - $1 \quad c(x) = 0.2x^2 100x + 300$
  - $2 \qquad c(x) = 0.2x^2 + 100x + 300$
  - $3 \quad c(x) = -0.2x^2 + 100x 300$
  - $4 \qquad c(x) = -0.8x^2 + 400x 300$
- 358 Chet has \$1200 invested in a bank account modeled by the function  $P(n) = 1200(1.002)^n$ , where P(n) is the value of his account, in dollars, after n months. Chet's debt is modeled by the function Q(n) = 100n, where Q(n) is the value of debt, in dollars, after n months. After n months, which function represents Chet's net worth, R(n)?
  - $1 \qquad R(n) = 1200(1.002)^n + 100n$
  - $2 R(n) = 1200(1.002)^{12n} + 100n$
  - $R(n) = 1200(1.002)^n 100n$
  - $4 \qquad R(n) = 1200(1.002)^{12n} 100n$

- 359 Given:  $f(x) = 2x^2 + x 3$  and g(x) = x 1Express  $f(x) \cdot g(x) - [f(x) + g(x)]$  as a polynomial in standard form.
- 360 Write the expression  $A(x) \bullet B(x) 3C(x)$  as a polynomial in standard form.

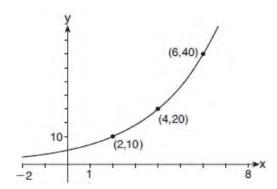
$$A(x) = x^3 + 2x - 1$$

$$B(x) = x^2 + 7$$

$$C(x) = x^4 - 5x$$

### F.LE.A.2: FAMILIES OF FUNCTIONS

361 The graph of y = f(x) is shown below.



Which expression defines f(x)?

- $1 \quad 2x$
- $2 5(2^x)$
- $3 \quad 5(2^{\frac{x}{2}})$
- 4  $5(2^{2x})$

362 Which table best represents an exponential relationship?

| x | У   |
|---|-----|
| 1 | 8   |
| 2 | 4   |
| 3 | 2   |
| 4 | 1   |
| 5 | 1 2 |
|   |     |

1

2

3

|    | - |
|----|---|
| x  | у |
| 8  | 0 |
| 4  | 1 |
| 0  | 2 |
| -4 | 3 |
| -8 | 4 |

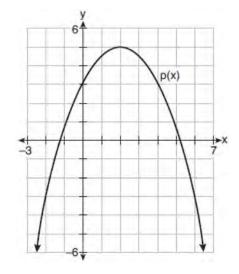
| X | у   |
|---|-----|
| 0 | 0   |
| 1 | 1   |
| 2 | 4   |
| 3 | 9   |
| 4 | 16  |
|   | 1 2 |

| X | У   |
|---|-----|
| 1 | 1   |
| 2 | 8   |
| 3 | 27  |
| 4 | 64  |
| 5 | 125 |

- 363 Perry invested in property that cost him \$1500. Five years later it was worth \$3000, and 10 years from his original purchase, it was worth \$6000. Assuming the growth rate remains the same, which type of function could he create to find the value of his investment 30 years from his original purchase?
  - 1 exponential function
  - 2 linear function
  - 3 quadratic function
  - 4 trigonometric function

### F.IF.C.9: COMPARING FUNCTIONS

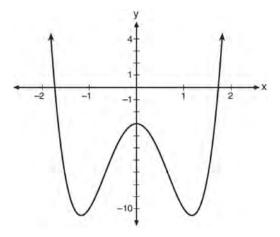
364 Consider  $f(x) = 4x^2 + 6x - 3$ , and p(x) defined by the graph below.



The difference between the values of the maximum of p and minimum of f is

- 1 0.25
- 2 1.25
- 3 3.25
- 4 10.25

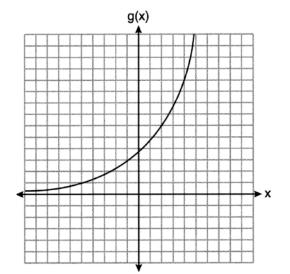
365 Consider the function  $p(x) = 3x^3 + x^2 - 5x$  and the graph of y = m(x) below.



Which statement is true?

- 1 p(x) has three real roots and m(x) has two real roots.
- 2 p(x) has one real root and m(x) has two real roots.
- 3 p(x) has two real roots and m(x) has three real roots.
- 4 p(x) has three real roots and m(x) has four real roots.

366 Consider the graph of g and the table representing t below.



Over the interval [2,4], which statement regarding the average rate of change for g and t is true?

- 1 g has a greater average rate of change.
- The average rate of change for g is twice the average rate of change for t.

t(x)

3 5

2

-5

-1

3

0

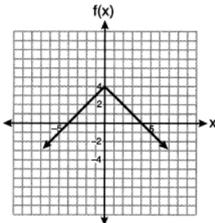
2

3

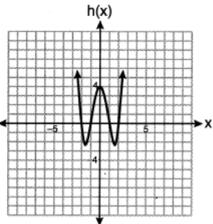
4

- 2 The average rates of change are equal.
- 4 The average rate of change for *g* is half the average rate of change for *t*.

367 Which function has a maximum *y*-value of 4 and a midline of y = 1?



 $\begin{array}{ccc}
1 & & \\
2 & g(x) = -3\cos(x) + 1
\end{array}$ 



3  $4 j(x) = 4\sin(x) + 1$ 

368 Which statement regarding the graphs of the functions below is *untrue*?

$$f(x) = 3\sin 2x$$
, from  $-\pi < x < \pi$ 

$$g(x) = (x - 0.5)(x + 4)(x - 2)$$

$$h(x) = \log_2 x$$

$$j(x) = -|4x - 2| + 3$$

- 1 f(x) and j(x) have a maximum y-value of 3.
- 2 f(x), h(x), and j(x) have one y-intercept.
- 3 g(x) and j(x) have the same end behavior as  $x \to -\infty$ .
- 4 g(x), h(x), and j(x) have rational zeros.

369 Which function shown below has a greater average rate of change on the interval [-2,4]? Justify your answer.

| X  | f(x)   |
|----|--------|
| -4 | 0.3125 |
| -3 | 0.625  |
| -2 | 1.25   |
| -1 | 2.5    |
| 0  | 5      |
| 1  | 10     |
| 2  | 20     |
| 3  | 40     |
| 4  | 80     |
| 5  | 160    |
| 6  | 320    |
|    |        |

$$g(x) = 4x^3 - 5x^2 + 3$$

370 The x-value of which function's x-intercept is larger, f or h? Justify your answer.

$$f(x) = \log(x - 4)$$

| X  | h(x) |
|----|------|
| -1 | 6    |
| 0  | 4    |
| 1  | 2    |
| 2  | 0    |
| 3  | -2   |

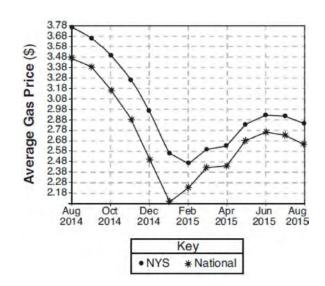
371 Consider the function  $h(x) = 2\sin(3x) + 1$  and the function q represented in the table below.

| x  | q(x) |
|----|------|
| -2 | -8   |
| -1 | 0    |
| 0  | 0    |
| 1  | -2   |
| 2  | 0    |

Determine which function has the *smaller* minimum value for the domain [-2,2]. Justify your answer.

### <u>F.BF.B.3: TRANSFORMATIONS WITH</u> <u>FUNCTIONS</u>

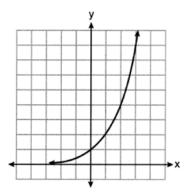
372 The graph below represents national and New York State average gas prices.



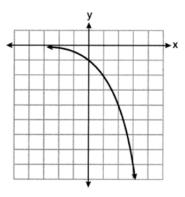
If New York State's gas prices are modeled by G(x) and C > 0, which expression best approximates the national average x months from August 2014?

- 1 G(x+C)
- 2 G(x) + C
- G(x-C)
- 4 G(x) C

373 Consider the function y = h(x), defined by the graph below.



Which equation could be used to represent the graph shown below?



- $1 \qquad y = h(x) 2$
- $2 \qquad y = h(x-2)$
- 3 y = -h(x)
- 4 y = h(-x)

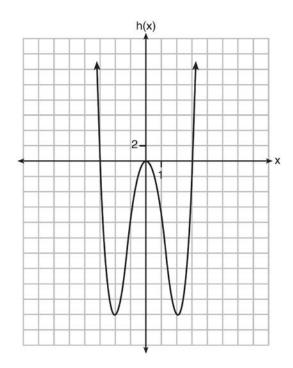
### F.BF.B.3: EVEN AND ODD FUNCTIONS

- 374 If f(x) is an even function, which function must also be even?
  - 1 f(x-2)
  - 2 f(x) + 3
  - $3 \qquad f(x+1)$
  - $4 \quad f(x+1) + 3$

375 Functions f, g, and h are given below.

$$f(x) = \sin(2x)$$

$$g(x) = f(x) + 1$$



Which statement is true about functions f, g, and h?

- 1 f(x) and g(x) are odd, h(x) is even.
- 2 f(x) and g(x) are even, h(x) is odd.
- 3 f(x) is odd, g(x) is neither, h(x) is even.
- 4 f(x) is even, g(x) is neither, h(x) is odd.
- 376 Which equation represents an odd function?
  - 1  $y = \sin x$
  - $y = \cos x$
  - $3 \qquad y = (x+1)^3$
  - $4 y = e^{5x}$

- 377 Which function is even?
  - 1  $f(x) = \sin x$
  - $2 \qquad f(x) = x^2 4$
  - 3 f(x) = |x 2| + 5
  - $4 \qquad f(x) = x^4 + 3x^3 + 4$
- 378 Algebraically determine whether the function  $j(x) = x^4 3x^2 4$  is odd, even, or neither.

#### F.BF.B.4: INVERSE OF FUNCTIONS

- 379 What is the inverse of f(x) = -6(x-2)?
  - $1 \qquad f^{-1}(x) = -2 \frac{x}{6}$
  - $2 \qquad f^{-1}(x) = 2 \frac{x}{6}$
  - 3  $f^{-l}(x) = \frac{1}{-6(x-2)}$
  - 4  $f^{-1}(x) = 6(x+2)$
- 380 Given  $f(x) = \frac{1}{2}x + 8$ , which equation represents the inverse, g(x)?
  - $1 \qquad g(x) = 2x 8$
  - $2 \qquad g(x) = 2x 16$
  - $3 \qquad g(x) = -\frac{1}{2}x + 8$
  - $4 \qquad g(x) = -\frac{1}{2}x 16$
- 381 What is the inverse of the function y = 4x + 5?
  - $1 \qquad x = \frac{1}{4}y \frac{5}{4}$
  - $2 \qquad y = \frac{1}{4}x \frac{5}{4}$
  - $3 \qquad y = 4x 5$
  - $4 \qquad y = \frac{1}{4x + 5}$

- 382 The inverse of  $f(x) = -6x + \frac{1}{2}$  is
  - $1 \qquad f^{-1}(x) = 6x \frac{1}{2}$
  - $2 \qquad f^{-1}(x) = \frac{1}{-6x + \frac{1}{2}}$
  - 3  $f^{-1}(x) = -\frac{1}{6}x + \frac{1}{12}$
  - $4 \qquad f^{-1}(x) = -\frac{1}{6}x + 2$
- 383 If  $f(x) = \frac{1}{2}x + 2$ , then the inverse function is
  - $1 \qquad f^{-1}(x) = -\frac{1}{2}x 2$
  - $2 \qquad f^{-1}(x) = \frac{1}{2}x 1$
  - 3  $f^{-1}(x) = 2x 4$
  - $4 f^{-1}(x) = 2x + 2$
- 384 Given  $f^{-1}(x) = -\frac{3}{4}x + 2$ , which equation represents f(x)?
  - $1 \qquad f(x) = \frac{4}{3}x \frac{8}{3}$
  - 2  $f(x) = -\frac{4}{3}x + \frac{8}{3}$
  - $3 \qquad f(x) = \frac{3}{4}x 2$
  - $4 \qquad f(x) = -\frac{3}{4}x + 2$

- 385 Given  $f(x) = -\frac{2}{5}x + 4$ , which statement is true of the inverse function  $f^{-1}(x)$ ?
  - 1  $f^{-1}(x)$  is a line with slope  $\frac{5}{2}$ .
  - 2  $f^{-1}(x)$  is a line with slope  $\frac{2}{5}$ .
  - 3  $f^{-1}(x)$  passes through the point (6,-5).
  - 4  $f^{-1}(x)$  has a y-intercept at (0,-4).
- 386 What is the inverse of  $f(x) = x^3 2$ ?
  - $1 f^{-1}(x) = \sqrt[3]{x} + 2$
  - $2 f^{-1}(x) = \pm \sqrt[3]{x} + 2$
  - $3 f^{-1}(x) = \sqrt[3]{x+2}$
  - 4  $f^{-1}(x) = \pm \sqrt[3]{x+2}$
- 387 If  $f(x) = a^x$  where a > 1, then the inverse of the function is
  - $1 \qquad f^{-1}(x) = \log_x a$
  - $2 \qquad f^{-1}(x) = a \log x$
  - $3 \qquad f^{-1}(x) = \log_a x$
  - $4 \qquad f^{-1}(x) = x \log a$
- 388 What is the inverse of the function  $y = \log_3 x$ ?
  - $1 \qquad y = x^3$
  - $2 y = \log_x 3$
  - $3 y = 3^x$
  - 4  $x = 3^y$

## Algebra II Regents Exam Questions by State Standard: Topic

389 The inverse of the function  $f(x) = \frac{x+1}{x-2}$  is

1 
$$f^{-1}(x) = \frac{x+1}{x+2}$$

$$2 f^{-1}(x) = \frac{2x+1}{x-1}$$

$$3 f^{-1}(x) = \frac{x+1}{x-2}$$

$$4 f^{-1}(x) = \frac{x-1}{x+1}$$

390 What is the inverse of  $f(x) = \frac{x}{x+2}$ , where  $x \neq -2$ ?

1 
$$f^{-1}(x) = \frac{2x}{x-1}$$

$$2 f^{-1}(x) = \frac{-2x}{x-1}$$

$$3 f^{-1}(x) = \frac{x}{x-2}$$

4 
$$f^{-1}(x) = \frac{-x}{x-2}$$

391 For the function  $f(x) = (x-3)^3 + 1$ , find  $f^{-1}(x)$ .

## SEQUENCES AND SERIES

### F.LE.A.2: SEQUENCES

392 Given f(9) = -2, which function can be used to generate the sequence  $-8, -7.25, -6.5, -5.75, \dots$ ?

$$1 \qquad f(n) = -8 + 0.75n$$

2 
$$f(n) = -8 - 0.75(n-1)$$

$$3 \qquad f(n) = -8.75 + 0.75n$$

4 
$$f(n) = -0.75 + 8(n-1)$$

393 A recursive formula for the sequence  $18,9,4.5,\ldots$ 

is

$$1 g_1 = 18$$

$$g_n = \frac{1}{2} g_{n-1}$$

$$2 \qquad g_n = 18 \left(\frac{1}{2}\right)^{n-1}$$

$$g_1 = 18$$

$$g_n = 2g_{n-1}$$

4 
$$g_n = 18(2)^{n-1}$$

394 The sequence  $a_1 = 6$ ,  $a_n = 3a_{n-1}$  can also be written as

$$1 a_n = 6 \cdot 3^n$$

$$2 \quad a_n = 6 \cdot 3^{n+1}$$

$$a_n = 2 \cdot 3^n$$

$$4 \qquad a_n = 2 \cdot 3^{n+1}$$

395 A recursive formula for the sequence 40,30,22.5,... is

$$1 \qquad g_n = 40 \left(\frac{3}{4}\right)^n$$

$$g_1 = 40$$

$$g_n = g_{n-1} - 10$$

$$3 \qquad g_n = 40 \left(\frac{3}{4}\right)^{n-1}$$

4 
$$g_1 = 40$$

$$g_n = \frac{3}{4} g_{n-1}$$

396 A recursive formula for the sequence 64,48,36,...

is

$$1 \quad a_n = 64(0.75)^{n-1}$$

 $a_1 = 64$ 

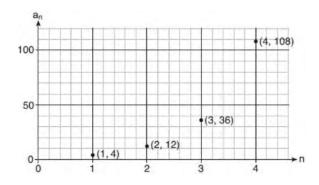
$$a_n = a_{n-1} - 16$$

3 
$$a_n = 64 + (n-1)(-16)$$

 $4 a_1 = 64$ 

$$a_n = 0.75a_{n-1}$$

397 Write a recursive formula,  $a_n$ , to describe the sequence graphed below.



- Write a recursive formula for the sequence 6,9,13.5,20.25,...
- 399 While experimenting with her calculator, Candy creates the sequence 4, 9, 19, 39, 79, .... Write a recursive formula for Candy's sequence.

  Determine the eighth term in Candy's sequence.
- 400 Write an explicit formula for  $a_n$ , the *n*th term of the recursively defined sequence below.

$$a_1 = x + 1$$

$$a_n = x(a_{n-1})$$

For what values of x would  $a_n = 0$  when n > 1?

401 The population, in millions of people, of the United States can be represented by the recursive formula below, where  $a_0$  represents the population in 1910 and n represents the number of years since 1910.

$$a_0 = 92.2$$

$$a_n = 1.015a_{n-1}$$

Identify the percentage of the annual rate of growth from the equation  $a_n = 1.015a_{n-1}$ . Write an exponential function, P, where P(t) represents the United States population in millions of people, and t is the number of years since 1910. According to this model, determine algebraically the number of years it takes for the population of the United States to be approximately 300 million people. Round your answer to the *nearest year*.

#### F.IF.A.3: SEQUENCES

- 402 A function is defined as  $a_n = a_{n-1} + \log_{n+1}(n-1)$ , where  $a_1 = 8$ . What is the value of  $a_3$ ?
  - 1 8
  - 2 8.5
  - 3 9.2
  - 4 10
- 403 When a ball bounces, the heights of consecutive bounces form a geometric sequence. The height of the first bounce is 121 centimeters and the height of the third bounce is 64 centimeters. To the *nearest centimeter*, what is the height of the fifth bounce?
  - 25
     34
  - 3 36
  - 4 42

#### Algebra II Regents Exam Questions by State Standard: Topic www.jmap.org

404 Consider the following patterns:

I. 16, -12, 9, -6.75, . . .

II. 1,4,9,16,...

III. 6, 18, 30, 42, . . .

 $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \dots$ IV.

Which pattern is geometric?

1 I

2 II

Ш 3

IV

405 The eighth and tenth terms of a sequence are 64 and 100. If the sequence is either arithmetic or geometric, the ninth term can not be

> 1 -82

2 -80

3 80

4 82

406 The recursive formula to describe a sequence is shown below.

$$a_1 = 3$$

$$a_n = 1 + 2a_{n-1}$$

State the first four terms of this sequence. Can this sequence be represented using an explicit geometric formula? Justify your answer.

#### F.BF.A.2: SEQUENCES

407 After Roger's surgery, his doctor administered pain medication in the following amounts in milligrams over four days.

| Day (n)    | <b>Day</b> (n) 1 2 |      | 3      | 4      |  |
|------------|--------------------|------|--------|--------|--|
| Dosage (m) | 2000               | 1680 | 1411.2 | 1185.4 |  |

How can this sequence best be modeled recursively?

$$1 m_1 = 2000$$

$$3 m_1 = 2000$$

$$m_n = m_{n-1} - 320$$

$$m_n = (0.84)m_{n-1}$$

$$2 m_n = 2000(0.84)^{n-1}$$

$$m_n = (0.84)m_{n-1}$$

$$4 m_n = 2000(0.84)^{n+1}$$

408 At her job, Pat earns \$25,000 the first year and receives a raise of \$1000 each year. The explicit formula for the *n*th term of this sequence is  $a_n = 25,000 + (n-1)1000$ . Which rule best represents the equivalent recursive formula?

$$a_n = 24,000 + 1000n$$

$$2 a_n = 25,000 + 1000n$$

3 
$$a_1 = 25,000, a_n = a_{n-1} + 1000$$

4 
$$a_1 = 25,000, a_n = a_{n+1} + 1000$$

- 409 The Rickerts decided to set up an account for their daughter to pay for her college education. The day their daughter was born, they deposited \$1000 in an account that pays 1.8% compounded annually. Beginning with her first birthday, they deposit an additional \$750 into the account on each of her birthdays. Which expression correctly represents the amount of money in the account *n* years after their daughter was born?
  - 1  $a_n = 1000(1.018)^n + 750$
  - $2 a_n = 1000(1.018)^n + 750n$
  - $a_0 = 1000$

$$a_n = a_{n-1}(1.018) + 750$$

 $a_0 = 1000$ 

$$a_n = a_{n-1}(1.018) + 750n$$

- 410 In 2010, the population of New York State was approximately 19,378,000 with an annual growth rate of 1.5%. Assuming the growth rate is maintained for a large number of years, which equation can be used to predict the population of New York State *t* years after 2010?
  - 1  $P_t = 19,378,000(1.5)^t$
  - $P_0 = 19,378,000$

$$P_t = 19,378,000 + 1.015P_{t-1}$$

- 3  $P_t = 19,378,000(1.015)^{t-1}$
- 4  $P_0 = 19,378,000$

$$P_t = 1.015 P_{t-1}$$

411 The population of Jamesburg for the years 2010-2013, respectively, was reported as follows: 250,000 250,937 251,878 252,822 How can this sequence be recursively modeled?

1 
$$j_n = 250,000(1.00375)^{n-1}$$

$$j_n = 250,000 + 937^{(n-1)}$$

$$j_1 = 250,000$$

$$j_n = 1.00375 j_{n-1}$$

4 
$$j_1 = 250,000$$

$$j_n = j_{n-1} + 937$$

412 The average depreciation rate of a new boat is approximately 8% per year. If a new boat is purchased at a price of \$75,000, which model is a recursive formula representing the value of the boat *n* years after it was purchased?

1 
$$a_n = 75,000(0.08)^n$$

$$a_0 = 75,000$$

$$a_n = (0.92)^n$$

$$a_n = 75,000(1.08)^n$$

4 
$$a_0 = 75,000$$

$$a_n = 0.92(a_{n-1})$$

413 Savannah just got contact lenses. Her doctor said she can wear them 2 hours the first day, and can then increase the length of time by 30 minutes each day. If this pattern continues, which formula would *not* be appropriate to determine the length of time, in either minutes or hours, she could wear her contact lenses on the *n*th day?

$$1 \quad a_1 = 120$$

$$a_n = a_{n-1} + 30$$

$$a_n = 90 + 30n$$

$$a_1 = 2$$

$$a_n = a_{n-1} + 0.5$$

4 
$$a_n = 2.5 + 0.5n$$

414 A tree farm initially has 150 trees. Each year, 20% of the trees are cut down and 80 seedlings are planted. Which recursive formula models the number of trees,  $a_n$ , after n years?

$$1 \quad a_1 = 150$$

$$a_n = a_{n-1}(0.2) + 80$$

$$a_1 = 150$$

$$a_n = a_{n-1}(0.8) + 80$$

$$a_n = 150(0.2)^n + 80$$

4 
$$a_n = 150(0.8)^n + 80$$

415 The formula below can be used to model which scenario?

$$a_1 = 3000$$

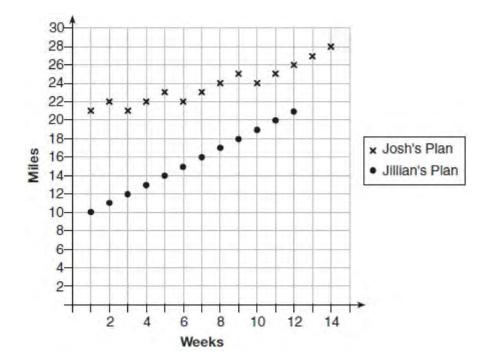
$$a_n = 0.80a_{n-1}$$

- 1 The first row of a stadium has 3000 seats, and each row thereafter has 80 more seats than the row in front of it.
- 2 The last row of a stadium has 3000 seats, and each row before it has 80 fewer seats than the row behind it.
- A bank account starts with a deposit of \$3000, and each year it grows by 80%.
- 4 The initial value of a specialty toy is \$3000, and its value each of the following years is 20% less.

- 416 Which situation could be modeled using a geometric sequence?
  - 1 A cell phone company charges \$30.00 per month for 2 gigabytes of data and \$12.50 for each additional gigabyte of data.
  - 2 The temperature in your car is 79°. You lower the temperature of your air conditioning by 2° every 3 minutes in order to find a comfortable temperature.
  - David's parents have set a limit of 50 minutes per week that he may play online games during the school year. However, they will increase his time by 5% per week for the next ten weeks.
  - 4 Sarah has \$100.00 in her piggy bank and saves an additional \$15.00 each week.
- 417 Simon lost his library card and has an overdue library book. When the book was 5 days late, he owed \$2.25 to replace his library card and pay the fine for the overdue book. When the book was 21 days late, he owed \$6.25 to replace his library card and pay the fine for the overdue book. Suppose the total amount Simon owes when the book is n days late can be determined by an arithmetic sequence. Determine a formula for  $a_n$ , the nth term of this sequence. Use the formula to determine the amount of money, in dollars, Simon needs to pay when the book is 60 days late.

## Algebra II Regents Exam Questions by State Standard: Topic www.jmap.org

Elaina has decided to run the Buffalo half-marathon in May. She researched training plans on the Internet and is looking at two possible plans: Jillian's 12-week plan and Josh's 14-week plan. The number of miles run per week for each plan is plotted below.



Which one of the plans follows an arithmetic pattern? Explain how you arrived at your answer. Write a recursive definition to represent the number of miles run each week for the duration of the plan you chose. Jillian's plan has an alternative if Elaina wanted to train instead for a full 26-mile marathon. Week one would start at 13 miles and follow the same pattern for the half-marathon, but it would continue for 14 weeks. Write an explicit formula, in *simplest form*, to represent the number of miles run each week for the full-marathon training plan.

#### F.BF.B.6: SIGMA NOTATION

419 A company fired several employees in order to save money. The amount of money the company saved per year over five years following the loss of employees is shown in the table below.

| Year | <b>Amount Saved</b> |  |  |
|------|---------------------|--|--|
|      | (in dollars)        |  |  |
| 1    | 59,000              |  |  |
| 2    | 64,900              |  |  |
| 3    | 71,390              |  |  |
| 4    | 78,529              |  |  |
| 5    | 86,381.9            |  |  |

Which expression determines the total amount of money saved by the company over 5 years?

$$1 \quad \frac{59,000 - 59,000(1.1)^5}{1 - 1.1}$$

$$3 \qquad \sum_{n=1}^{5} 59,000(1.1)^n$$

$$2 \quad \frac{59,000 - 59,000(0.1)^5}{1 - 0.1}$$

$$4 \qquad \sum_{n=1}^{5} 59,000(0.1)^{n-1}$$

420 Kristin wants to increase her running endurance.

According to experts, a gradual mileage increase of 10% per week can reduce the risk of injury. If Kristin runs 8 miles in week one, which expression can help her find the total number of miles she will have run over the course of her 6-week training program?

$$1 \qquad \sum_{n=1}^{6} 8(1.10)^{n-1}$$

$$2 \qquad \sum_{n=1}^{6} 8(1.10)^n$$

$$3 \quad \frac{8 - 8(1.10)^6}{0.90}$$

$$4 \frac{8-8(0.10)^n}{1.10}$$

#### A.SSE.B.4: SERIES

421 A ball is dropped from a height of 32 feet. It bounces and rebounds 80% of the height from which it was falling. What is the total downward distance, in feet, the ball traveled up to the 12th bounce?

422 The first term of a geometric sequence is 8 and the fourth term is 216. What is the sum of the first 12 terms of the corresponding series?

## Algebra II Regents Exam Questions by State Standard: Topic www.jmap.org

$$-2 + 6 - 18 + 54 - \dots$$
 is

$$2 -59$$

# 424 Brian deposited 1 cent into an empty non-interest bearing bank account on the first day of the month. He then additionally deposited 3 cents on the second day, 9 cents on the third day, and 27 cents on the fourth day. What would be the total amount of money in the account at the end of the 20th day if the pattern continued?

$$1 \quad S = 100(1.03)^{12}$$

$$2 S = \frac{100 - 100(1.0025)^{12}}{1 - 1.0025}$$

$$S = 100(1.0025)^{12}$$

$$4 \qquad S = \frac{100 - 100(1.03)^{12}}{1 - 1.03}$$

| 427 | Jake wants to buy a car and hopes to save at least \$5000 for a down payment. | The table below summarizes the |
|-----|---|--------------------------------|
|     | amount of money he plans to save each week.                                   |                                |

| Week                    | 1 | 2 | 3    | 4     | 5 |
|-------------------------|---|---|------|-------|---|
| Money Saved, in Dollars | 2 | 5 | 12.5 | 31.25 |   |

Based on this plan, which expression should he use to determine how much he has saved in *n* weeks?

$$1 \quad \frac{2 - 2(2.5^n)}{1 - 2.5}$$

$$3 \frac{1-2.5^n}{1-2.5}$$

$$2 \quad \frac{2-2(2.5^{n-1})}{1-2.5}$$

$$4 \quad \frac{1 - 2.5^{n-1}}{1 - 2.5}$$

428 Given the geometric series 
$$300+360+432+518.4+...$$
, write a geometric series formula,  $S_n$ , for the sum of the first  $n$  terms. Use the formula to find the sum of the first  $10$  terms, to the *nearest tenth*.

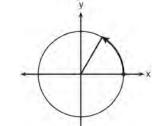
429 Alexa earns \$33,000 in her first year of teaching and earns a 4% increase in each successive year. Write a geometric series formula, 
$$S_n$$
, for Alexa's total earnings over  $n$  years. Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent*.

- 430 Rowan is training to run in a race. He runs 15 miles in the first week, and each week following, he runs 3% more than the week before. Using a geometric series formula, find the total number of miles Rowan runs over the first ten weeks of training, rounded to the *nearest thousandth*.
- 431 Sonja is cutting wire to construct a mobile. She cuts 100 inches for the first piece, 80 inches for the second piece, and 64 inches for the third piece. Assuming this pattern continues, write an explicit equation for  $a_n$ , the length in inches of the nth piece. Sonja only has 40 feet of wire to use for the project and wants to cut 20 pieces total for the mobile using her pattern. Will she have enough wire? Justify your answer.
- 432 The initial push of a child on a swing causes the swing to travel a total of 6 feet. Each successive swing travels 80% of the distance of the previous swing. Determine the total distance, to the *nearest hundredth of a foot*, a child travels in the first five swings.

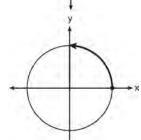
## TRIGONOMETRY

F.TF.A.1: UNIT CIRCLE

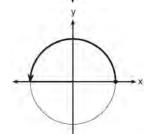
433 Which diagram shows an angle rotation of 1 radian on the unit circle?



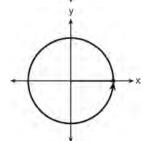
1



2

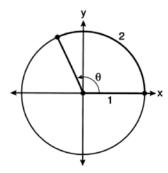


3



4

434 An angle,  $\theta$ , is rotated counterclockwise on the unit circle, with its terminal side in the second quadrant, as shown in the diagram below.



Which value represents the radian measure of angle  $\theta$ ?

- 1 1
- 2 2
- 3 65.4
- 4 114.6

#### F.TF.A.2: UNIT CIRCLE

435 The terminal side of  $\theta$ , an angle in standard position, intersects the unit circle at  $P\left(-\frac{1}{3}, -\frac{\sqrt{8}}{3}\right)$ 

What is the value of  $\sec \theta$ ?

$$2 \quad -\frac{3\sqrt{8}}{8}$$

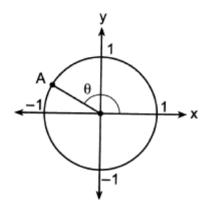
$$3 -\frac{1}{3}$$

$$4 \quad -\frac{\sqrt{8}}{3}$$

436 In the diagram of a unit circle below, point A,

$$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$
, represents the point where the

terminal side of  $\theta$  intersects the unit circle.

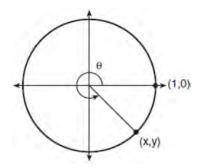


What is  $m\angle\theta$ ?

- 1 30°
- 2 120°
- 3 135°
- 4 150°
- 437 Point  $M\left(t, \frac{4}{7}\right)$  is located in the second quadrant on the unit circle. Determine the exact value of t.

## F.TF.A.2: RECIPROCAL TRIGONOMETRIC RELATIONSHIPS

438 Using the unit circle below, explain why  $\csc \theta = \frac{1}{v}$ .



# F.TF.A.2: FINDING THE TERMINAL SIDE OF AN ANGLE

439 Natalia's teacher has given her the following information about angle  $\theta$ .

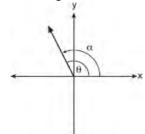
• 
$$\pi < \theta < 2\pi$$

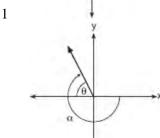
• 
$$\cos \theta = \frac{\sqrt{3}}{4}$$

Explain how Natalia can determine if the value of  $\tan \theta$  is positive or negative.

#### F.TF.A.2: REFERENCE ANGLES

440 Which diagram represents an angle,  $\alpha$ , measuring  $\frac{13\pi}{20}$  radians drawn in standard position, and its reference angle,  $\theta$ ?

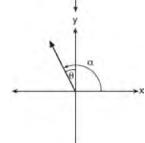


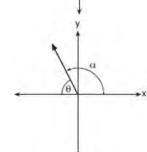


2

3

4





#### F.TF.A.2: DETERMINING TRIGONOMETRIC **FUNCTIONS**

- 441 If the terminal side of angle  $\theta$ , in standard position, passes through point (-4,3), what is the numerical value of  $\sin \theta$ ?

  - $\frac{3}{5}$   $\frac{4}{5}$   $-\frac{3}{5}$   $-\frac{4}{5}$
- 442 Given  $\cos \theta = \frac{7}{25}$ , where  $\theta$  is an angle in standard position terminating in quadrant IV, and  $\sin^2 \theta + \cos^2 \theta = 1$ , what is the value of  $\tan \theta$ ?
- 443 The hours of daylight, y, in Utica in days, x, from January 1, 2013 can be modeled by the equation  $y = 3.06 \sin(0.017x - 1.40) + 12.23$ . How many hours of daylight, to the nearest tenth, does this model predict for February 14, 2013?
  - 1 9.4
  - 2 10.4
  - 3 12.1
  - 12.2

- 444 The temperature, in degrees Fahrenheit, in Times Square during a day in August can be predicted by the function  $T(x) = 8\sin(0.3x - 3) + 74$ , where x is the number of hours after midnight. According to this model, the predicted temperature, to the nearest degree Fahrenheit, at 7 P.M. is
  - 1 68
  - 2 74
  - 3 77
  - 4 81
- 445 An angle,  $\theta$ , is in standard position and its terminal side passes through the point (2,-1). Find the *exact* value of  $\sin \theta$ .

#### F.TF.C.8: DETERMINING TRIGONOMETRIC **FUNCTIONS**

- 446 A circle centered at the origin has a radius of 10 units. The terminal side of an angle,  $\theta$ , intercepts the circle in Quadrant II at point C. The y-coordinate of point C is 8. What is the value of  $\cos \theta$ ?

#### Algebra II Regents Exam Questions by State Standard: Topic www.jmap.org

- 447 If  $\cos \theta = -\frac{3}{4}$  and  $\theta$  is in Quadrant III, then  $\sin \theta$  is equivalent to

  - $\begin{array}{ccc}
    2 & \frac{\sqrt{7}}{4} \\
    3 & -\frac{5}{4} \\
    4 & \frac{5}{4}
    \end{array}$
- 448 If  $\cos A = \frac{\sqrt{5}}{3}$  and  $\tan A < 0$ , what is the value of  $\sin A$ ?

  - $2 \quad -\frac{\sqrt{5}}{3}$
- 449 Given that  $\sin^2 \theta + \cos^2 \theta = 1$  and  $\sin \theta = -\frac{\sqrt{2}}{5}$ , what is a possible value of  $\cos \theta$ ?
  - $1 \quad \frac{5+\sqrt{2}}{5}$

  - $3 \quad \frac{3\sqrt{3}}{5}$

- 450 Given  $\tan \theta = \frac{7}{24}$ , and  $\theta$  terminates in Quadrant III, determine the value of  $\cos \theta$ .
- 451 Using the identity  $\sin^2 \theta + \cos^2 \theta = 1$ , find the value of  $\tan \theta$ , to the *nearest hundredth*, if  $\cos \theta$  is -0.7and  $\theta$  is in Quadrant II.
- 452 Given  $\cos A = \frac{3}{\sqrt{10}}$  and  $\cot A = -3$ , determine the value of  $\sin A$  in radical form.

#### F.TF.C.8: SIMPLIFYING TRIGONOMETRIC **EXPRESSIONS**

- 453 If  $\sin^2(32^\circ) + \cos^2(M) = 1$ , then *M* equals

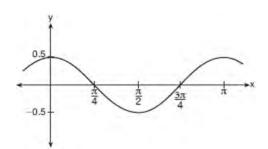
  - 2 58°
  - 3 68°
  - 72°

#### F.TF.B.5: MODELING TRIGONOMETRIC **FUNCTIONS**

- The voltage used by most households can be modeled by a sine function. The maximum voltage is 120 volts, and there are 60 cycles every second. Which equation best represents the value of the voltage as it flows through the electric wires, where t is time in seconds?
  - $V = 120\sin(t)$ 1
  - $V = 120 \sin(60t)$
  - $V = 120 \sin(60\pi t)$
  - $V = 120 \sin(120\pi t)$

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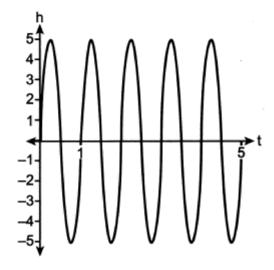
Which equation is represented by the graph shown below?



- $1 y = \frac{1}{2}\cos 2x$  $2 y = \cos x$

- 3  $y = \frac{1}{2}\cos x$ 4  $y = 2\cos\frac{1}{2}x$

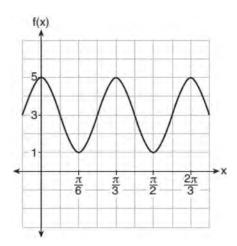
456 A cyclist pedals a bike at a rate of 60 revolutions per minute. The height, h, of a pedal at time t, in seconds, is plotted below.



The graph can be modeled by the function  $h(t) = 5\sin(kt)$ , where k is equal to

- 1 1
- 2  $2\pi$
- 3 60
- 4

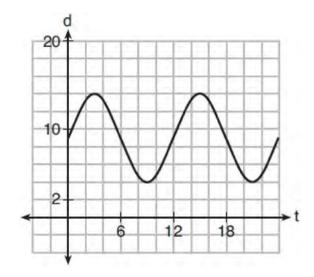
457 The function  $f(x) = a \cos bx + c$  is plotted on the graph shown below.



What are the values of a, b, and c?

- 1 a = 2, b = 6, c = 3
- 2 a = 2, b = 3, c = 1
- a = 4, b = 6, c = 5
- 4  $a = 4, b = \frac{\pi}{3}, c = 3$

458 The depth of the water at a marker 20 feet from the shore in a bay is depicted in the graph below.



If the depth, d, is measured in feet and time, t, is measured in hours since midnight, what is an equation for the depth of the water at the marker?

$$1 \qquad d = 5\cos\left(\frac{\pi}{6}t\right) + 9$$

$$2 \qquad d = 9\cos\left(\frac{\pi}{6}t\right) + 5$$

$$3 \qquad d = 9\sin\left(\frac{\pi}{6}t\right) + 5$$

$$4 \qquad d = 5\sin\left(\frac{\pi}{6}t\right) + 9$$

# F.IF.B.4: GRAPHING TRIGONOMETRIC FUNCTIONS

- 459 Given the parent function  $p(x) = \cos x$ , which phrase best describes the transformation used to obtain the graph of  $g(x) = \cos(x+a) b$ , if a and b are positive constants?
  - 1 right a units, up b units
  - 2 right a units, down b units
  - 3 left *a* units, up *b* units
  - 4 left a units, down b units

- 460 Relative to the graph of  $y = 3\sin x$ , what is the shift of the graph of  $y = 3\sin\left(x + \frac{\pi}{3}\right)$ ?
  - 1  $\frac{\pi}{3}$  right
  - $2 \frac{\pi}{3}$  left
  - $3 \quad \frac{\pi}{3} \text{ up}$
  - $4 \frac{\pi}{3}$  down
- Which function's graph has a period of 8 and reaches a maximum height of 1 if at least one full period is graphed?
  - $1 \qquad y = -4\cos\left(\frac{\pi}{4}x\right) 3$
  - $2 \qquad y = -4\cos\left(\frac{\pi}{4}x\right) + 5$
  - $3 \quad y = -4\cos(8x) 3$
  - $4 \qquad y = -4\cos(8x) + 5$
- 462 The Ferris wheel at the landmark Navy Pier in Chicago takes 7 minutes to make one full rotation. The height, *H*, in feet, above the ground of one of the six-person cars can be modeled by

$$H(t) = 70 \sin\left(\frac{2\pi}{7} (t - 1.75)\right) + 80$$
, where t is time,

in minutes. Using H(t) for one full rotation, this car's minimum height, in feet, is

- 1 150
- 2 70
- 3 10
- 4 0

- 463 A sine function increasing through the origin can be used to model light waves. Violet light has a wavelength of 400 nanometers. Over which interval is the height of the wave *decreasing*, only?
  - 1 (0,200)
  - 2 (100,300)
  - 3 (200,400)
  - 4 (300,400)
- 464 As  $\theta$  increases from  $-\frac{\pi}{2}$  to 0 radians, the value of  $\cos \theta$  will
  - 1 decrease from 1 to 0
  - 2 decrease from 0 to -1
  - 3 increase from -1 to 0
  - 4 increase from 0 to 1
- 465 Given  $p(\theta) = 3\sin\left(\frac{1}{2}\theta\right)$  on the interval

$$-\pi < \theta < \pi$$
, the function p

- 1 decreases, then increases
- 2 increases, then decreases
- 3 decreases throughout the interval
- 4 increases throughout the interval
- 466 As x increases from 0 to  $\frac{\pi}{2}$ , the graph of the equation  $y = 2 \tan x$  will
  - 1 increase from 0 to 2
  - 2 decrease from 0 to -2
  - 3 increase without limit
  - 4 decrease without limit

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467 The depth of the water, d(t), in feet, on a given day at Thunder Bay, t hours after midnight is modeled by  $d(t) = 5 \sin \left( \frac{\pi}{6} (t - 5) \right) + 7$ . Which statement

about the Thunder Bay tide is *false*?

- 1 A low tide occurred at 2 a.m.
- 2 The maximum depth of the water was 12 feet.
- The water depth at 9 a.m. was approximately 11 feet.
- 4 The difference in water depth between high tide and low tide is 14 feet.
- 468 Based on climate data that have been collected in Bar Harbor, Maine, the average monthly temperature, in degrees F, can be modeled by the equation

 $B(x) = 23.914\sin(0.508x - 2.116) + 55.300$ . The same governmental agency collected average monthly temperature data for Phoenix, Arizona, and found the temperatures could be modeled by the equation

 $P(x) = 20.238 \sin(0.525x - 2.148) + 86.729$ . Which statement can *not* be concluded based on the average monthly temperature models x months after starting data collection?

- 1 The average monthly temperature variation is more in Bar Harbor than in Phoenix.
- The midline average monthly temperature for Bar Harbor is lower than the midline temperature for Phoenix.
- 3 The maximum average monthly temperature for Bar Harbor is 79° F, to the nearest degree.
- 4 The minimum average monthly temperature for Phoenix is 20° F, to the nearest degree.
- 469 A person's lung capacity can be modeled by the function  $C(t) = 250 \sin\left(\frac{2\pi}{5}t\right) + 2450$ , where C(t)

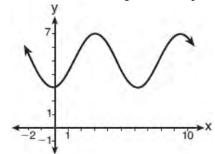
represents the volume in mL present in the lungs after *t* seconds. State the maximum value of this function over one full cycle, and explain what this value represents.

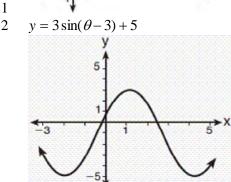
470 The height, h(t) in cm, of a piston, is given by the equation  $h(t) = 12\cos\left(\frac{\pi}{3}t\right) + 8$ , where t represents

the number of seconds since the measurements began. Determine the average rate of change, in cm/sec, of the piston's height on the interval  $1 \le t \le 2$ . At what value(s) of t, to the *nearest tenth of a second*, does h(t) = 0 in the interval  $1 \le t \le 5$ ? Justify your answer.

## F.IF.C.7: GRAPHING TRIGONOMETRIC FUNCTIONS

471 Which sinusoid has the greatest amplitude?





 $\begin{array}{ll}
3 \\
4 & y = -5\sin(\theta - 1) - 3
\end{array}$ 

## Algebra II Regents Exam Questions by State Standard: Topic www.jmap.org

472 The equation below can be used to model the height of a tide in feet, H(t), on a beach at t hours.

$$H(t) = 4.8 \sin\left(\frac{\pi}{6} (t+3)\right) + 5.1$$

Using this function, the amplitude of the tide is

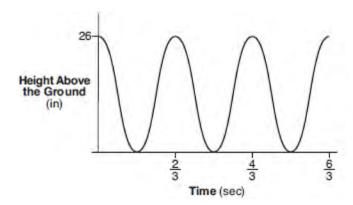
- $1 \frac{\pi}{6}$
- 2 4.8
- 3 3
- 4 5.1

- 473 On July 21, 2016, the water level in Puget Sound, WA reached a high of 10.1 ft at 6 a.m. and a low of -2 ft at 12:30 p.m. Across the country in Long Island, NY, Shinnecock Bay's water level reached a high of 2.5 ft at 10:42 p.m. and a low of -0.1 ft at 5:31 a.m. The water levels of both locations are affected by the tides and can be modeled by sinusoidal functions. Determine the difference in amplitudes, in feet, for these two locations.
- 474 The height above ground for a person riding a Ferris wheel after *t* seconds is modeled by

$$h(t) = 150 \sin\left(\frac{\pi}{45}t + 67.5\right) + 160$$
 feet. How many

seconds does it take to go from the bottom of the wheel to the top of the wheel?

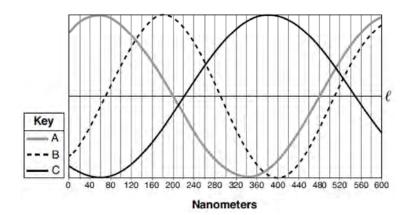
- 1 10
- 2 45
- 3 90
- 4 150
- 475 The graph below represents the height above the ground, *h*, in inches, of a point on a triathlete's bike wheel during a training ride in terms of time, *t*, in seconds.





Identify the period of the graph and describe what the period represents in this context.

476 Visible light can be represented by sinusoidal waves. Three visible light waves are shown in the graph below. The midline of each wave is labeled  $\ell$ .



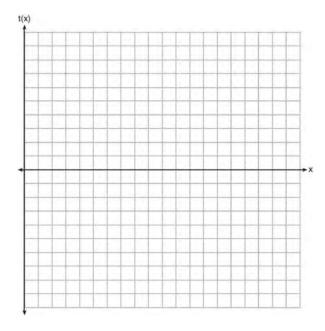
Based on the graph, which light wave has the longest period? Justify your answer.

- 477 Which statement is *incorrect* for the graph of the function  $y = -3\cos\left[\frac{\pi}{3}(x-4)\right] + 7$ ?
  - 1 The period is 6.
  - 2 The amplitude is 3.
  - 3 The range is [4,10].
  - 4 The midline is y = -4.
- 478 Tides are a periodic rise and fall of ocean water.
  On a typical day at a seaport, to predict the time of the next high tide, the most important value to have would be the
  - 1 time between consecutive low tides
  - 2 time when the tide height is 20 feet
  - 3 average depth of water over a 24-hour period
  - 4 difference between the water heights at low and high tide

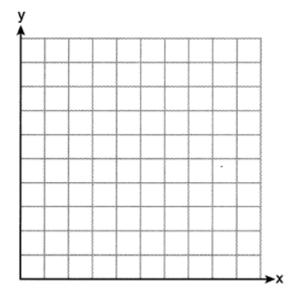
- 479 The average monthly temperature of a city can be modeled by a cosine graph. Melissa has been living in Phoenix, Arizona, where the average annual temperature is 75°F. She would like to move, and live in a location where the average annual temperature is 62°F. When examining the graphs of the average monthly temperatures for various locations, Melissa should focus on the
  - 1 amplitude
  - 2 horizontal shift
  - 3 period
  - 4 midline
- 480 The volume of air in a person's lungs, as the person breathes in and out, can be modeled by a sine graph. A scientist is studying the differences in this volume for people at rest compared to people told to take a deep breath. When examining the graphs, should the scientist focus on the amplitude, period, or midline? Explain your choice.

Algebra II Regents Exam Questions by State Standard: Topic  $\underline{www.jmap.org}$ 

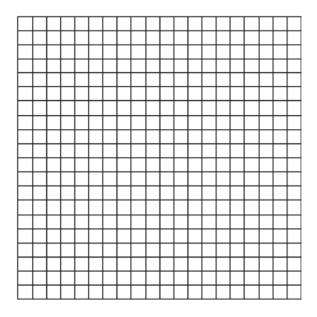
481 Graph  $t(x) = 3\sin(2x) + 2$  over the domain  $[0, 2\pi]$  on the set of axes below.



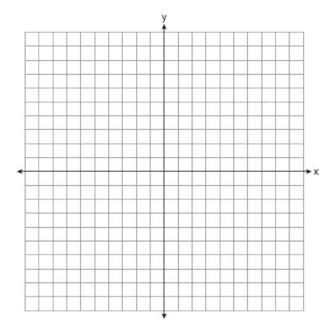
482 Graph  $y = 2\cos\left(\frac{1}{2}x\right) + 5$  on the interval  $[0, 2\pi]$ , using the axes below.



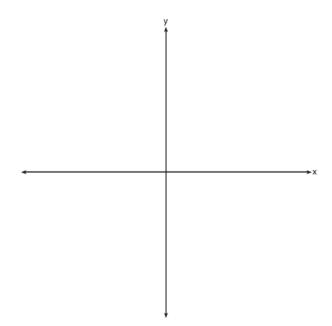
483 Write an equation for a sine function with an amplitude of 2 and a period of  $\frac{\pi}{2}$ . On the grid below, sketch the graph of the equation in the interval 0 to  $2\pi$ .



484 On the axes below, graph *one* cycle of a cosine function with amplitude 3, period  $\frac{\pi}{2}$ , midline y = -1, and passing through the point (0,2).



485 a) On the axes below, sketch *at least one* cycle of a sine curve with an amplitude of 2, a midline at  $y = -\frac{3}{2}$ , and a period of  $2\pi$ .

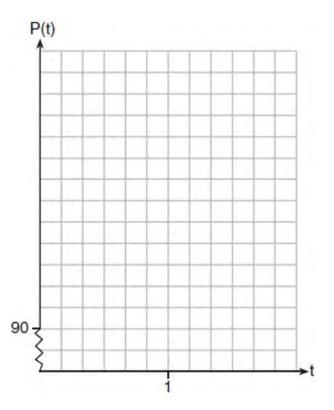


b) Explain any differences between a sketch of  $y = 2\sin\left(x - \frac{\pi}{3}\right) - \frac{3}{2}$  and the sketch from part a.

486 The resting blood pressure of an adult patient can be modeled by the function P below, where P(t) is the pressure in millimeters of mercury after time t in seconds.

$$P(t) = 24\cos(3\pi t) + 120$$

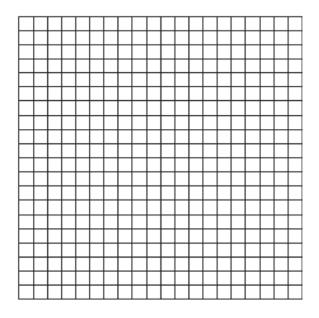
On the set of axes below, graph y = P(t) over the domain  $0 \le t \le 2$ .



Determine the period of *P*. Explain what this value represents in the given context. Normal resting blood pressure for an adult is 120 over 80. This means that the blood pressure oscillates between a maximum of 120 and a minimum of 80. Adults with high blood pressure (above 140 over 90) and adults with low blood pressure (below 90 over 60) may be at risk for health disorders. Classify the given patient's blood pressure as low, normal, or high and explain your reasoning.

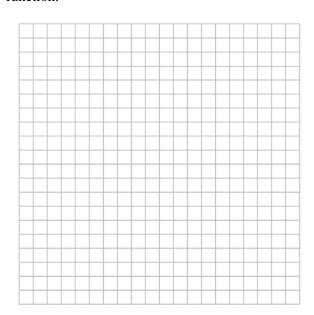
487 Griffin is riding his bike down the street in Churchville, N.Y. at a constant speed, when a nail gets caught in one of his tires. The height of the nail above the ground, in inches, can be represented by the trigonometric function

 $f(t) = -13\cos(0.8\pi t) + 13$ , where t represents the time (in seconds) since the nail first became caught in the tire. Determine the period of f(t). Interpret what the period represents in this context. On the grid below, graph at least one cycle of f(t) that includes the y-intercept of the function.



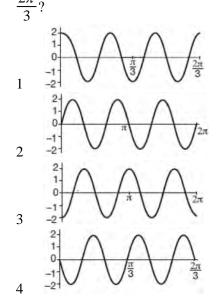
Does the height of the nail ever reach 30 inches above the ground? Justify your answer.

488 The ocean tides near Carter Beach follow a repeating pattern over time, with the amount of time between each low and high tide remaining relatively constant. On a certain day, low tide occurred at 8:30 a.m. and high tide occurred at 3:00 p.m. At high tide, the water level was 12 inches above the average local sea level; at low tide it was 12 inches below the average local sea level. Assume that high tide and low tide are the maximum and minimum water levels each day, respectively. Write a cosine function of the form  $f(t) = A\cos(Bt)$ , where A and B are real numbers, that models the water level, f(t), in inches above or below the average Carter Beach sea level, as a function of the time measured in t hours since 8:30 a.m. On the grid below, graph one cycle of this function.



People who fish in Carter Beach know that a certain species of fish is most plentiful when the water level is increasing. Explain whether you would recommend fishing for this species at 7:30 p.m. or 10:30 p.m. using evidence from the given context.

489 Which graph represents a cosine function with no horizontal shift, an amplitude of 2, and a period of  $2\pi$ 



## **CONICS**

G.GPE.A.1: EQUATIONS OF CIRCLES

490 The equation  $4x^2 - 24x + 4y^2 + 72y = 76$  is equivalent to

$$1 \quad 4(x-3)^2 + 4(y+9)^2 = 76$$

2 
$$4(x-3)^2 + 4(y+9)^2 = 121$$

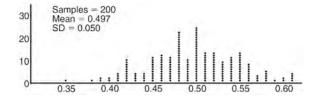
$$3 \quad 4(x-3)^2 + 4(y+9)^2 = 166$$

$$4 \quad 4(x-3)^2 + 4(y+9)^2 = 436$$

## **GRAPHS AND STATISTICS**

#### S.IC.A.2: ANALYSIS OF DATA

491 Anne has a coin. She does not know if it is a fair coin. She flipped the coin 100 times and obtained 73 heads and 27 tails. She ran a computer simulation of 200 samples of 100 fair coin flips. The output of the proportion of heads is shown below.



Given the results of her coin flips and of her computer simulation, which statement is most accurate?

- 1 73 of the computer's next 100 coin flips will be heads.
- 2 50 of her next 100 coin flips will be heads.
- 3 Her coin is not fair.
- 4 Her coin is fair.

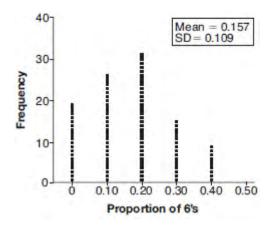
492 A game spinner is divided into 6 equally sized regions, as shown in the diagram below.



For Miles to win, the spinner must land on the number 6. After spinning the spinner 10 times, and losing all 10 times, Miles complained that the spinner is unfair. At home, his dad ran 100 simulations of spinning the spinner 10 times,

assuming the probability of winning each spin is  $\frac{1}{6}$ .

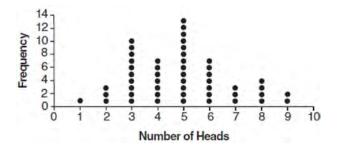
The output of the simulation is shown in the diagram below.



Which explanation is appropriate for Miles and his dad to make?

- 1 The spinner was likely unfair, since the number 6 failed to occur in about 20% of the simulations.
- 2 The spinner was likely unfair, since the spinner should have landed on the number 6 by the sixth spin.
- 3 The spinner was likely not unfair, since the number 6 failed to occur in about 20% of the simulations.
- 4 The spinner was likely not unfair, since in the output the player wins once or twice in the majority of the simulations.

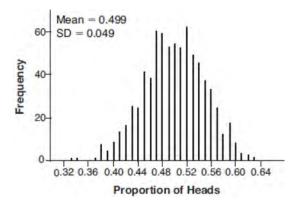
493 The results of simulating tossing a coin 10 times, recording the number of heads, and repeating this 50 times are shown in the graph below.



Based on the results of the simulation, which statement is *false*?

- 1 Five heads occurred most often, which is consistent with the theoretical probability of obtaining a heads.
- 2 Eight heads is unusual, as it falls outside the middle 95% of the data.
- 3 Obtaining three heads or fewer occurred 28% of the time.
- 4 Seven heads is not unusual, as it falls within the middle 95% of the data.
- 494 An orange-juice processing plant receives a truckload of oranges. The quality control team randomly chooses three pails of oranges, each containing 50 oranges, from the truckload. Identify the sample and the population in the given scenario. State *one* conclusion that the quality control team could make about the population if 5% of the sample was found to be unsatisfactory.
- 495 Mrs. Jones had hundreds of jelly beans in a bag that contained equal numbers of six different flavors. Her student randomly selected four jelly beans and they were all black licorice. Her student complained and said "What are the odds I got all of that kind?" Mrs. Jones replied, "simulate rolling a die 250 times and tell me if four black licorice jelly beans is unusual." Explain how this simulation could be used to solve the problem.

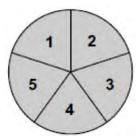
496 Robin flips a coin 100 times. It lands heads up 43 times, and she wonders if the coin is unfair. She runs a computer simulation of 750 samples of 100 fair coin flips. The output of the proportion of heads is shown below.



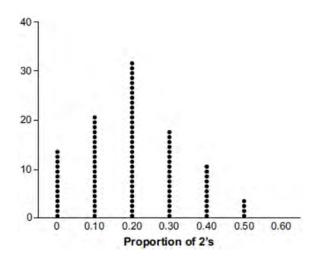
Do the results of the simulation provide strong evidence that Robin's coin is unfair? Explain your answer.

## Algebra II Regents Exam Questions by State Standard: Topic www.jmap.org

497 Joette is playing a carnival game. To win a prize, one has to correctly guess which of five equally sized regions a spinner will land on, as shown in the diagram below.

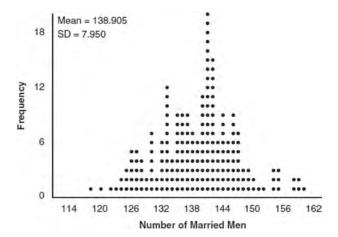


She complains that the game is unfair because her favorite number, 2, has only been spun once in ten times she played the game. State the proportion of 2's that were spun. State the theoretical probability of spinning a 2. The simulation output below shows the results of simulating ten spins of a fair spinner, repeated 100 times.



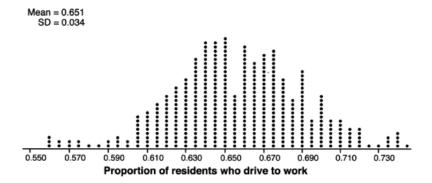
Does the output indicate that the carnival game was unfair? Explain your answer.

498 In a random sample of 250 men in the United States, age 21 or older, 139 are married. The graph below simulated samples of 250 men, 200 times, assuming that 139 of the men are married.



- a) Based on the simulation, create an interval in which the middle 95% of the number of married men may fall. Round your answer to the *nearest integer*.
- b) A study claims "50 percent of men 21 and older in the United States are married." Do your results from part a contradict this claim? Explain.

In order to decrease the percentage of its residents who drive to work, a large city launches a campaign to encourage people to use public transportation instead. Before starting the campaign, the city's Department of Transportation uses census data to estimate that 65% of its residents drive to work. The Department of Transportation conducts a simulation, shown below, run 400 times based on this estimate. Each dot represents the proportion of 200 randomly selected residents who drive to work.

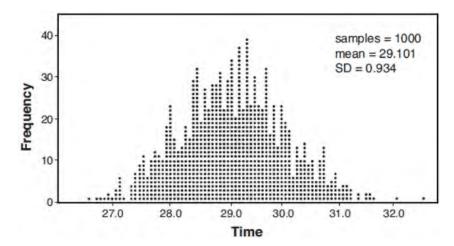


Use the simulation results to construct a plausible interval containing the middle 95% of the data. Round your answer to the *nearest hundredth*. One year after launching the campaign, the Department of Transportation conducts a survey of 200 randomly selected city residents and finds that 122 of them drive to work. Should the department conclude that the city's campaign was effective? Use statistical evidence from the simulation to explain your answer.

A radio station claims to its advertisers that the mean number of minutes commuters listen to the station is 30. The station conducted a survey of 500 of their listeners who commute. The sample statistics are shown below.

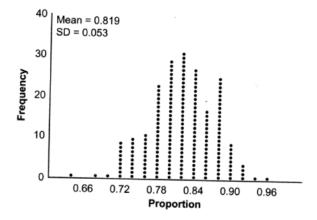
| X              | 29.11  |
|----------------|--------|
| S <sub>x</sub> | 20.718 |

A simulation was run 1000 times based upon the results of the survey. The results of the simulation appear below.



Based on the simulation results, is the claim that commuters listen to the station on average 30 minutes plausible? Explain your response including an interval containing the middle 95% of the data, rounded to the *nearest hundredth*.

501 State officials claim 82% of a community want to repeal the 30 mph speed limit on an expressway. A community organization devises a simulation based on the claim that 82% of the community supports the repeal. Each dot on the graph below represents the proportion of community members who support the repeal. The graph shows 200 simulated surveys, each of sample size 60.



Based on the simulation, determine an interval containing the middle 95% of plausible proportions. Round your answer to the *nearest thousandth*. The community organization conducted its own sample survey of 60 people and found 70% supported the repeal. Based on the results of the simulation, explain why the organization should question the State officials' claim.

#### S.IC.B.3: ANALYSIS OF DATA

502 Which statement about statistical analysis is *false*?

- 1 Experiments can suggest patterns and relationships in data.
- 2 Experiments can determine cause and effect relationships.
- 3 Observational studies can determine cause and effect relationships.
- 4 Observational studies can suggest patterns and relationships in data.

- 503 Which statement about data collection is most accurate?
  - 1 A survey about parenting styles given to every tenth student entering the library will provide unbiased results.
  - 2 An observational study allows a researcher to determine the cause of an outcome.
  - 3 Margin of error increases as sample size increases.
  - 4 A survey collected from a random sample of students in a school can be used to represent the opinions of the school population.
- 504 A researcher randomly divides 50 bean plants into two groups. He puts one group by a window to receive natural light and the second group under artificial light. He records the growth of the plants weekly. Which data collection method is described in this situation?
  - 1 observational study
  - 2 controlled experiment
  - 3 survey
  - 4 systematic sample
- 505 Which investigation technique is most often used to determine if a single variable has an impact on a given population?
  - 1 observational study
  - 2 random survey
  - 3 controlled experiment
  - 4 formal interview
- 506 A researcher wants to determine if room-darkening shades cause people to sleep longer. Which method of data collection is most appropriate?
  - 1 census
  - 2 survey
  - 3 observation study
  - 4 controlled experiment

- 507 A sociologist reviews randomly selected surveillance videos from a public park over a period of several years and records the amount of time people spent on a smartphone. The statistical procedure the sociologist used is called
  - 1 a census
  - 2 an experiment
  - 3 an observational study
  - 4 a sample survey
- A veterinary pharmaceutical company plans to test a new drug to treat a common intestinal infection among puppies. The puppies are randomly assigned to two equal groups. Half of the puppies will receive the drug, and the other half will receive a placebo. The veterinarians monitor the puppies. This is an example of which study method?
  - 1 census
  - 2 observational study
  - 3 survey
  - 4 controlled experiment
- 509 In watching auditions for lead singer in a band, Liem became curious as to whether there is an association between how animated the leadsinger is and the amount of applause from the audience. He decided to watch each singer and rate the singer on a scale of 1 to 5, where 1 is the least animated and 5 is the most animated. He did this for all 5 nights of auditions and found that the more animated singers did receive louder applause. The study Liem conducted would be best described as
  - 1 experimental
  - 2 observational
  - 3 a sample survey
  - 4 a random assignment

- 510 According to a study, 45% of Americans have type O blood. If a random number generator produces three-digit values from 000 to 999, which values would represent those having type O blood?
  - 1 between 000 and 045, inclusive
  - 2 between 000 and 444, inclusive
  - 3 between 000 and 449, inclusive
  - 4 between 000 and 450, inclusive
- of its customers at its location at the Staten Island ferry terminal. After the survey is completed, the statistical consultant states that 70% of customers who took the survey said the most important factor in choosing where to get their coffee is how fast they are served. Based on this result, Hot and Tasty Coffee can infer that
  - 1 most of its customers in New York State care most about being served quickly
  - 2 coffee drinkers care less about taste and more about being served quickly
  - 3 most of its customers at the Staten Island ferry terminal care most about being served quickly
  - 4 most of its customers at transportation terminals and stations care most about being served quickly

## Algebra II Regents Exam Questions by State Standard: Topic www.jmap.org

- 512 Which scenario is best described as an observational study?
  - 1 For a class project, students in Health class ask every tenth student entering the school if they eat breakfast in the morning.
  - 2 A social researcher wants to learn whether or not there is a link between attendance and grades. She gathers data from 15 school districts.
  - 3 A researcher wants to learn whether or not there is a link between children's daily amount of physical activity and their overall energy level. During lunch at the local high school, she distributed a short questionnaire to students in the cafeteria.
  - 4 Sixty seniors taking a course in Advanced Algebra Concepts are randomly divided into two classes. One class uses a graphing calculator all the time, and the other class never uses graphing calculators. A guidance counselor wants to determine whether there is a link between graphing calculator use and students' final exam grades.
- 513 Describe how a controlled experiment can be created to examine the effect of ingredient *X* in a toothpaste.
- 514 A random sample of 100 people that would best estimate the proportion of all registered voters in a district who support improvements to the high school football field should be drawn from registered voters in the district at a
  - 1 football game
  - 2 supermarket
  - 3 school fund-raiser
  - 4 high school band concert

- 515 Mrs. Favata's statistics class wants to conduct a survey to see how students feel about changing the school mascot's name. Which plan is the best process for gathering an appropriate sample?
  - 1 Survey students in a random sample of senior homerooms.
  - 2 Survey every tenth student entering art classes in the school.
  - 3 Survey every fourth student entering the cafeteria during each lunch period.
  - 4 Survey all members of the school's varsity sports teams.
- 516 Cheap and Fast gas station is conducting a consumer satisfaction survey. Which method of collecting data would most likely lead to a biased sample?
  - 1 interviewing every 5th customer to come into the station
  - 2 interviewing customers chosen at random by a computer at the checkout
  - 3 interviewing customers who call an 800 number posted on the customers' receipts
  - 4 interviewing every customer who comes into the station on a day of the week chosen at random out of a hat
- 517 The operator of the local mall wants to find out how many of the mall's employees make purchases in the food court when they are working. She hopes to use these data to increase the rent and attract new food vendors. In total, there are 1023 employees who work at the mall. The best method to obtain a random sample of the employees would be to survey
  - 1 all 170 employees at each of the larger stores
  - 2 50% of the 90 employees of the food court
  - 3 every employee
  - 4 every 30th employee entering each mall entrance for one week

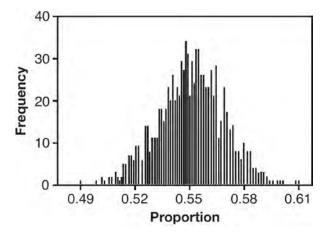
- 518 Which statement(s) about statistical studies is true?
  - I. A survey of all English classes in a high school would be a good sample to determine the number of hours students throughout the school spend studying.
  - II. A survey of all ninth graders in a high school would be a good sample to determine the number of student parking spaces needed at that high school.
  - III. A survey of all students in one lunch period in a high school would be a good sample to determine the number of hours adults spend on social media websites.
  - IV. A survey of all Calculus students in a high school would be a good sample to determine the number of students throughout the school who don't like math.
  - 1 I, only
  - 2 II, only
  - 3 I and III
  - 4 III and IV
- 519 Chuck's Trucking Company has decided to initiate an Employee of the Month program. To determine the recipient, they put the following sign on the back of each truck.



The driver who receives the highest number of positive comments will win the recognition. Explain *one* statistical bias in this data collection method.

#### S.IC.B.4: ANALYSIS OF DATA

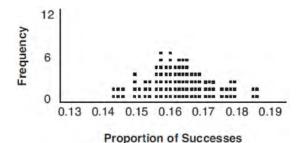
520 A candidate for political office commissioned a poll. His staff received responses from 900 likely voters and 55% of them said they would vote for the candidate. The staff then conducted a simulation of 1000 more polls of 900 voters, assuming that 55% of voters would vote for their candidate. The output of the simulation is shown in the diagram below.



Given this output, and assuming a 95% confidence level, the margin of error for the poll is closest to

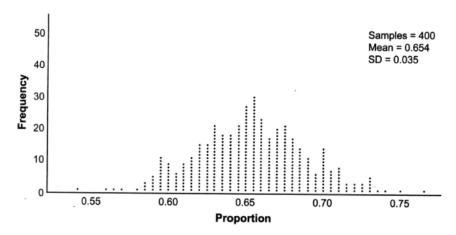
- 1 0.01
- 2 0.03
- 3 0.06
- 4 0.12

521 A study conducted in 2004 in New York City found that 212 out of 1334 participants had hypertension. Kim ran a simulation of 100 studies based on these data. The output of the simulation is shown in the diagram below.



At a 95% confidence level, the proportion of New York City residents with hypertension and the margin of error are closest to

- 1 proportion  $\approx$  .16; margin of error  $\approx$  .01
- 2 proportion  $\approx$  .16; margin of error  $\approx$  .02
- 3 proportion  $\approx .01$ ; margin of error  $\approx .16$
- 4 proportion  $\approx .02$ ; margin of error  $\approx .16$
- Betty conducted a survey of her class to see if they like pizza. She gathered 200 responses and 65% of the voters said they did like pizza. Betty then ran a simulation of 400 more surveys, each with 200 responses, assuming that 65% of the voters would like pizza. The output of the simulation is shown below.



Considering the middle 95% of the data, what is the margin of error for the simulation?

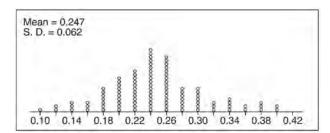
1 0.01

3 0.05

2 0.02

4 0.07

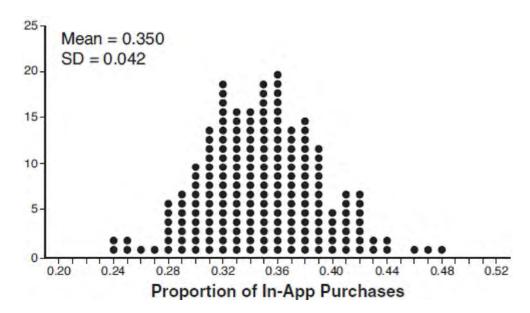
Stephen's Beverage Company is considering whether to produce a new brand of cola. The company will launch the product if at least 25% of cola drinkers will buy the product. Fifty cola drinkers are randomly selected to take a blind taste-test of products *A*, *B*, and the new product. Nine out of fifty participants preferred Stephen's new cola to products *A* and *B*. The company then devised a simulation based on the requirement that 25% of cola drinkers will buy the product. Each dot in the graph shown below represents the proportion of people who preferred Stephen's new product, each of sample size 50, simulated 100 times.



**Proportion Preferring Stephen's Product** 

Assume the set of data is approximately normal and the company wants to be 95% confident of its results. Does the sample proportion obtained from the blind taste-test, nine out of fifty, fall within the margin of error developed from the simulation? Justify your answer. The company decides to continue developing the product even though only nine out of fifty participants preferred its brand of cola in the taste-test. Describe how the simulation data could be used to support this decision.

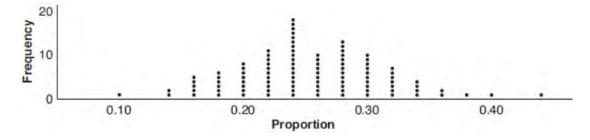
524 Some smart-phone applications contain "in-app" purchases, which allow users to purchase special content within the application. A random sample of 140 users found that 35 percent made in-app purchases. A simulation was conducted with 200 samples of 140 users assuming 35 percent of the samples make in-app purchases. The approximately normal results are shown below.



Considering the middle 95% of the data, determine the margin of error, to the *nearest hundredth*, for the simulated results. In the given context, explain what this value represents.

### S.IC.B.5: ANALYSIS OF DATA

A group of students was trying to determine the proportion of candies in a bag that are blue. The company claims that 24% of candies in bags are blue. A simulation was run 100 times with a sample size of 50, based on the premise that 24% of the candies are blue. The approximately normal results of the simulation are shown in the dot plot below.



The simulation results in a mean of 0.254 and a standard deviation of 0.060. Based on this simulation, what is a plausible interval containing the middle 95% of the data?

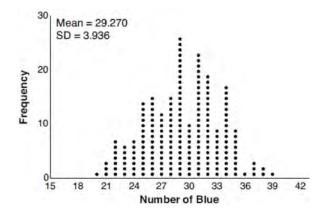
1 (0.194, 0.314)

 $3 \quad (-0.448, 0.568)$ 

2 (0.134, 0.374)

4 (0.254, 0.374)

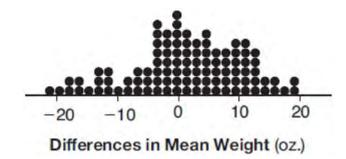
526 The J&B candy company claims that 45% of the candies it produces are blue, 30% are brown, and 25% are yellow. Each bag holds 65 candies. A simulation was run 200 times, each of sample size 65, based on the premise that 45% of the candies are blue. The results of the simulation are shown below.



Bonnie purchased a bag of J& B's candy and counted 24 blue candies. What inference can be made regarding a bag of J& B's with only 24 blue candies?

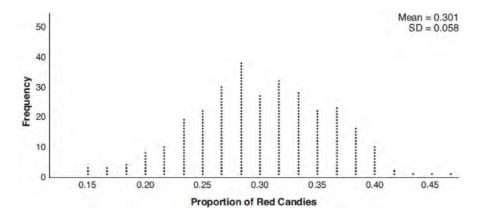
- 1 The company is not meeting their production standard.
- 2 Bonnie's bag was a rarity and the company should not be concerned.
- 3 The company should change their claim to 37% blue candies are produced.
- 4 Bonnie's bag is within the middle 95% of the simulated data supporting the company's claim.

527 Gabriel performed an experiment to see if planting 13 tomato plants in black plastic mulch leads to larger tomatoes than if 13 plants are planted without mulch. He observed that the average weight of the tomatoes from tomato plants grown in black plastic mulch was 5 ounces greater than those from the plants planted without mulch. To determine if the observed difference is statistically significant, he rerandomized the tomato groups 100 times to study these random differences in the mean weights. The output of his simulation is summarized in the dotplot below.



Given these results, what is an appropriate inference that can be drawn?

- 1 There was no effect observed between the two groups.
- 2 There was an effect observed that could be due to the random assignment of plants to the groups.
- There is strong evidence to support the hypothesis that tomatoes from plants planted in black plastic mulch are larger than those planted without mulch.
- 4 There is strong evidence to support the hypothesis that tomatoes from plants planted without mulch are larger than those planted in black plastic mulch.
- Mary bought a pack of candy. The manufacturer claims that 30% of the candies manufactured are red. In her pack, 14 of the 60 candies are red. She ran a simulation of 300 samples, assuming the manufacturer is correct. The results are shown below.



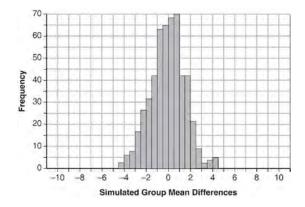
Based on the simulation, determine the middle 95% of plausible values that the proportion of red candies in a pack is within. Based on the simulation, is it unusual that Mary's pack had 14 red candies out of a total of 60? Explain.

# Algebra II Regents Exam Questions by State Standard: Topic www.jmap.org

529 Seventy-two students are randomly divided into two equally-sized study groups. Each member of the first group (group 1) is to meet with a tutor after school twice each week for one hour. The second group (group 2), is given an online subscription to a tutorial account that they can access for a maximum of two hours each week. Students in both groups are given the same tests during the year. A summary of the two groups' final grades is shown below:

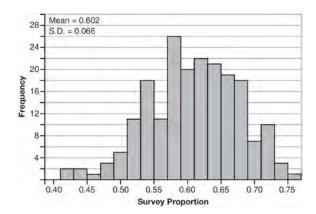
|             | Group 1 | Group 2 |
|-------------|---------|---------|
| X           | 80.16   | 83.8    |
| $S_{\rm x}$ | 6.9     | 5.2     |

Calculate the mean difference in the final grades (group 1 - group 2) and explain its meaning in the context of the problem. A simulation was conducted in which the students' final grades were rerandomized 500 times. The results are shown below.



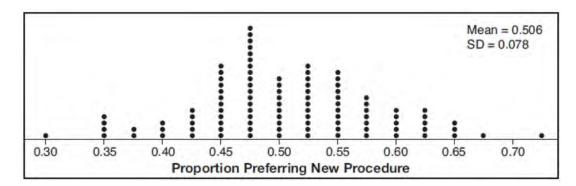
Use the simulation to determine if there is a significant difference in the final grades. Explain your answer.

530 Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band. A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.



Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth*. Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% - 50% split. Explain what statistical evidence supports this concern.

531 Charlie's Automotive Dealership is considering implementing a new check-in procedure for customers who are bringing their vehicles for routine maintenance. The dealership will launch the procedure if 50% or more of the customers give the new procedure a favorable rating when compared to the current procedure. The dealership devises a simulation based on the minimal requirement that 50% of the customers prefer the new procedure. Each dot on the graph below represents the proportion of the customers who preferred the new check-in procedure, each of sample size 40, simulated 100 times.



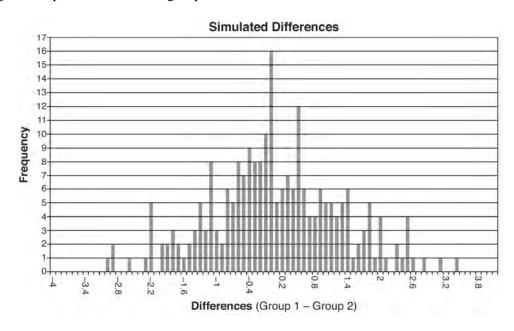
Assume the set of data is approximately normal and the dealership wants to be 95% confident of its results. Determine an interval containing the plausible sample values for which the dealership will launch the new procedure. Round your answer to the *nearest hundredth*. Forty customers are selected randomly to undergo the new check-in procedure and the proportion of customers who prefer the new procedure is 32.5%. The dealership decides *not* to implement the new check-in procedure based on the results of the study. Use statistical evidence to explain this decision.

### Algebra II Regents Exam Questions by State Standard: Topic www.jmap.org

Ayva designed an experiment to determine the effect of a new energy drink on a group of 20 volunteer students. Ten students were randomly selected to form group 1 while the remaining 10 made up group 2. Each student in group 1 drank one energy drink, and each student in group 2 drank one cola drink. Ten minutes later, their times were recorded for reading the same paragraph of a novel. The results of the experiment are shown below.

| Group 1     | Group 2     |
|-------------|-------------|
| (seconds)   | (seconds)   |
| 17.4        | 23.3        |
| 18.1        | 18.8        |
| 18.2        | 22.1        |
| 19.6        | 12.7        |
| 18.6        | 16.9        |
| 16.2        | 24.4        |
| 16.1        | 21.2        |
| 15.3        | 21.2        |
| 17.8        | 16.3        |
| 19.7        | 14.5        |
| Mean = 17.7 | Mean = 19.1 |

Ayva thinks drinking energy drinks makes students read faster. Using information from the experimental design or the results, explain why Ayva's hypothesis may be *incorrect*. Using the given results, Ayva randomly mixes the 20 reading times, splits them into two groups of 10, and simulates the difference of the means 232 times.

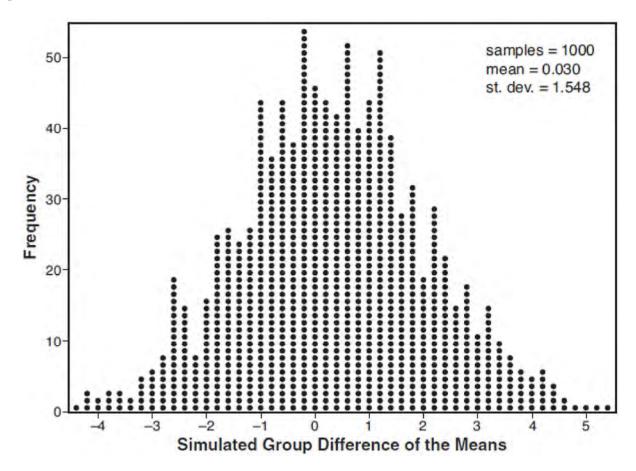


Ayva has decided that the difference in mean reading times is not an unusual occurrence. Support her decision using the results of the simulation. Explain your reasoning.

533 Joseph was curious to determine if scent improves memory. A test was created where better memory is indicated by higher test scores. A controlled experiment was performed where one group was given the test on scented paper and the other group was given the test on unscented paper. The summary statistics from the experiment are given below.

|                | Scented Paper | <b>Unscented Paper</b> |
|----------------|---------------|------------------------|
| $\bar{x}$      | 23            | 18                     |
| S <sub>x</sub> | 2.898         | 2.408                  |

Calculate the difference in means in the experimental test grades (scented -unscented). A simulation was conducted in which the subjects' scores were rerandomized into two groups 1000 times. The differences of the group means were calculated each time. The results are shown below.



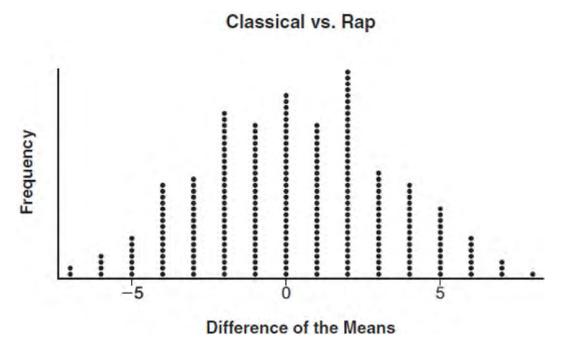
Use the simulation results to determine the interval representing the middle 95% of the difference in means, to the *nearest hundredth*. Is the difference in means in Joseph's experiment statistically significant based on the simulation? Explain.

### Algebra II Regents Exam Questions by State Standard: Topic www.jmap.org

To determine if the type of music played while taking a quiz has a relationship to results, 16 students were randomly assigned to either a room softly playing classical music or a room softly playing rap music. The results on the quiz were as follows:

Classical: 74, 83, 77, 77, 84, 82, 90, 89 Rap: 77, 80, 78, 74, 69, 72, 78, 69

John correctly rounded the difference of the means of his experimental groups as 7. How did John obtain this value and what does it represent in the given context? Justify your answer. To determine if there is any significance in this value, John rerandomized the 16 scores into two groups of 8, calculated the difference of the means, and simulated this process 250 times as shown below.



Does the simulation support the theory that there may be a significant difference in quiz scores? Explain.

### S.IC.B.6: ANALYSIS OF DATA

- 535 A public opinion poll was conducted on behalf of Mayor Ortega's reelection campaign shortly before the election. 264 out of 550 likely voters said they would vote for Mayor Ortega; the rest said they would vote for his opponent. Which statement is *least* appropriate to make, according to the results of the poll?
  - 1 There is a 48% chance that Mayor Ortega will win the election.
  - 2 The point estimate ( $\hat{p}$ ) of voters who will vote for Mayor Ortega is 48%.
  - 3 It is most likely that between 44% and 52% of voters will vote for Mayor Ortega.
  - 4 Due to the margin of error, an inference cannot be made regarding whether Mayor Ortega or his opponent is most likely to win the election.

her favorite fast-food restaurant the last time she visited. She was upset about having to wait that long and notified the manager. The manager assured her that her experience was very unusual and that it would not happen again. A study of customers commissioned by this restaurant found an approximately normal distribution of results. The mean wait time was 226 seconds and the standard deviation was 38 seconds. Given these data, and using a 95% level of confidence, was Elizabeth's wait time unusual? Justify your answer.

#### S.ID.B.6: REGRESSION

537 Consider the data in the table below.

| X | 1   | 2 | 3  | 4    | 5  | 6    |
|---|-----|---|----|------|----|------|
| y | 3.9 | 6 | 11 | 18.1 | 28 | 40.3 |

State an exponential regression equation to model these data, rounding all values to the *nearest thousandth*.

A runner is using a nine-week training app to prepare for a "fun run." The table below represents the amount of the program completed, *A*, and the distance covered in a session, *D*, in miles.

| A | $\frac{4}{9}$ | <u>5</u><br>9 | <u>6</u><br>9 | <u>8</u><br>9 | 1    |
|---|---------------|---------------|---------------|---------------|------|
| D | 2             | 2             | 2.25          | 3             | 3.25 |

Based on these data, write an exponential regression equation, rounded to the *nearest thousandth*, to model the distance the runner is able to complete in a session as she continues through the nine-week program.

539 A cup of coffee is left out on a countertop to cool. The table below represents the temperature, F(t), in degrees Fahrenheit, of the coffee after it is left out for t minutes.

|   | t    | 0   | 5   | 10  | 15  | 20   | 25   |
|---|------|-----|-----|-----|-----|------|------|
| Ī | F(t) | 180 | 144 | 120 | 104 | 93.3 | 86.2 |

Based on these data, write an exponential regression equation, F(t), to model the temperature of the coffee. Round all values to the *nearest thousandth*.

540 The table below gives air pressures in kPa at selected altitudes above sea level measured in kilometers.

| X | Altitude (km)      | 0   | 1  | 2  | 3  | 4  | 5  |
|---|--------------------|-----|----|----|----|----|----|
| y | Air Pressure (kPa) | 101 | 90 | 79 | 70 | 62 | 54 |

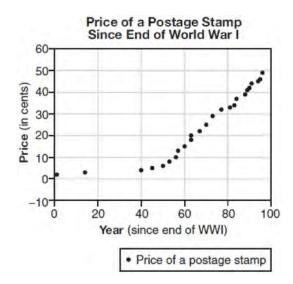
Write an exponential regression equation that models these data rounding all values to the *nearest thousandth*. Use this equation to algebraically determine the altitude, to the *nearest hundredth* of a kilometer, when the air pressure is 29 kPa.

541 Using a microscope, a researcher observed and recorded the number of bacteria spores on a large sample of uniformly sized pieces of meat kept at room temperature. A summary of the data she recorded is shown in the table below.

| Hours (x) | Average Number<br>of Spores (y) |
|-----------|---------------------------------|
| 0         | 4                               |
| 0.5       | 10                              |
| 1         | 15                              |
| 2         | 60                              |
| 3         | 260                             |
| 4         | 1130                            |
| 6         | 16,380                          |

Using these data, write an exponential regression equation, rounding all values to the *nearest thousandth*. The researcher knows that people are likely to suffer from food-borne illness if the number of spores exceeds 100. Using the exponential regression equation, determine the maximum amount of time, to the *nearest quarter hour*, that the meat can be kept at room temperature safely.

542 The price of a postage stamp in the years since the end of World War I is shown in the scatterplot below.



The equation that best models the price, in cents, of a postage stamp based on these data is

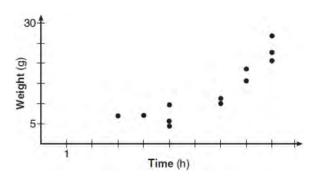
$$1 \qquad y = 0.59x - 14.82$$

$$y = 1.04(1.43)^x$$

$$3 \quad y = 1.43(1.04)^x$$

4 
$$y = 24\sin(14x) + 25$$

A scatterplot showing the weight, w, in grams, of each crystal after growing t hours is shown below.



The relationship between weight, w, and time, t, is best modeled by

1 
$$w = 4^t + 5$$

$$2 \quad w = (1.4)^t + 2$$

$$3 \quad w = 5(2.1)^t$$

$$4 \quad w = 8(.75)^t$$

### S.ID.A.4: NORMAL DISTRIBUTIONS

- Suppose two sets of test scores have the same mean, but different standard deviations,  $\sigma_1$  and  $\sigma_2$ , with  $\sigma_2 > \sigma_1$ . Which statement best describes the variability of these data sets?
  - 1 Data set one has the greater variability.
  - 2 Data set two has the greater variability.
  - 3 The variability will be the same for each data set
  - 4 No conclusion can be made regarding the variability of either set.
- 545 The heights of women in the United States are normally distributed with a mean of 64 inches and a standard deviation of 2.75 inches. The percent of women whose heights are between 64 and 69.5 inches, to the *nearest whole percent*, is

- 2 48
- 3 68
- 4 95

# Algebra II Regents Exam Questions by State Standard: Topic www.jmap.org

- 546 The distribution of the diameters of ball bearings made under a given manufacturing process is normally distributed with a mean of 4 cm and a standard deviation of 0.2 cm. What proportion of the ball bearings will have a diameter less than 3.7 cm?
  - 1 0.0668
  - 2 0.4332
  - 3 0.8664
  - 4 0.9500
- 547 The mean intelligence quotient (IQ) score is 100, with a standard deviation of 15, and the scores are normally distributed. Given this information, the approximate percentage of the population with an IQ greater than 130 is closest to
  - 1 2%
  - 2 31%
  - 3 48%
  - 4 95%
- 548 The weight of a bag of pears at the local market averages 8 pounds with a standard deviation of 0.5 pound. The weights of all the bags of pears at the market closely follow a normal distribution.

  Determine what percentage of bags, to the *nearest integer*, weighed *less* than 8.25 pounds.
- 549 The scores on a collegiate mathematics readiness assessment are approximately normally distributed with a mean of 680 and a standard deviation of 120. Determine the percentage of scores between 690 and 900, to the *nearest percent*.

- 550 In 2013, approximately 1.6 million students took the Critical Reading portion of the SAT exam. The mean score, the modal score, and the standard deviation were calculated to be 496, 430, and 115, respectively. Which interval reflects 95% of the Critical Reading scores?
  - 1  $430 \pm 115$
  - $2 430 \pm 230$
  - $3496 \pm 115$
  - 4  $496 \pm 230$
- 551 There are 440 students at Thomas Paine High School enrolled in U.S. History. On the April report card, the students' grades are approximately normally distributed with a mean of 79 and a standard deviation of 7. Students who earn a grade less than or equal to 64.9 must attend summer school. The number of students who must attend summer school for U.S. History is closest to
  - 1 3
  - 2 5
  - 3 10
  - 4 22
- 552 The scores on a mathematics college-entry exam are normally distributed with a mean of 68 and standard deviation 7.2. Students scoring higher than one standard deviation above the mean will not be enrolled in the mathematics tutoring program. How many of the 750 incoming students can be expected to be enrolled in the tutoring program?
  - 1 631
  - 2 512
  - 3 238
  - 4 119

- 553 There are 400 students in the senior class at Oak Creek High School. All of these students took the SAT. The distribution of their SAT scores is approximately normal. The number of students who scored within 2 standard deviations of the mean is approximately
  - 1 75
  - 2 95
  - 3 300
  - 4 380
- 554 The heights of the 3300 students at Oceanview High School are approximately normally distributed with a mean of 65.5 inches and a standard deviation of 2.9 inches. The number of students at Oceanview who are between 64 and 68 inches tall is closest to
  - 1 1660
  - 2 1070
  - 3 2244
  - 4 1640
- 555 The scores of a recent test taken by 1200 students had an approximately normal distribution with a mean of 225 and a standard deviation of 18.

  Determine the number of students who scored between 200 and 245.
- 556 According to a study done at a hospital, the average weight of a newborn baby is 3.39 kg, with a standard deviation of 0.55 kg. The weights of all the newborns in this hospital closely follow a normal distribution. Last year, 9256 babies were born at this hospital. Determine, to the *nearest integer*, approximately how many babies weighed more than 4 kg.

- 557 The lifespan of a 60-watt lightbulb produced by a company is normally distributed with a mean of 1450 hours and a standard deviation of 8.5 hours. If a 60-watt lightbulb produced by this company is selected at random, what is the probability that its lifespan will be between 1440 and 1465 hours?
  - 1 0.3803
  - 2 0.4612
  - 3 0.8415
  - 4 0.9612
- The weights of bags of Graseck's Chocolate
  Candies are normally distributed with a mean of
  4.3 ounces and a standard deviation of 0.05 ounces.
  What is the probability that a bag of these
  chocolate candies weighs less than 4.27 ounces?
  - 1 0.2257
  - 2 0.2743
  - 3 0.7257
  - 4 0.7757
- April versions of a standardized test are given, an April version and a May version. The statistics for the April version show a mean score of 480 and a standard deviation of 24. The statistics for the May version show a mean score of 510 and a standard deviation of 20. Assume the scores are normally distributed. Joanne took the April version and scored in the interval 510-540. What is the probability, to the *nearest ten thousandth*, that a test paper selected at random from the April version scored in the same interval? Maria took the May version. In what interval must Maria score to claim she scored as well as Joanne?

### **PROBABILITY**

### S.CP.B.7: THEORETICAL PROBABILITY

560 The probability that Gary and Jane have a child with blue eyes is 0.25, and the probability that they have a child with blond hair is 0.5. The probability that they have a child with both blue eyes and blond hair is 0.125. Given this information, the events blue eyes and blond hair are

I: dependent

II: independent

III: mutually exclusive

1 I, only

2 II, only

3 I and III

4 II and III

561 A suburban high school has a population of 1376 students. The number of students who participate in sports is 649. The number of students who participate in music is 433. If the probability that a student participates in either sports or music is  $\frac{974}{1376}$ , what is the probability that a student participates in both sports and music?

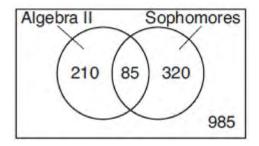
# S.CP.A.2: PROBABILITY OF COMPOUND EVENTS

- On a given school day, the probability that Nick oversleeps is 48% and the probability he has a pop quiz is 25%. Assuming these two events are independent, what is the probability that Nick oversleeps and has a pop quiz on the same day?
  - 1 73%
  - 2 36%
  - 3 23%
  - 4 12%

- 563 Given  $P(A) = \frac{1}{3}$  and  $P(B) = \frac{5}{12}$ , where A and B are independent events, determine  $P(A \cap B)$ .
- 564 Given events A and B, such that P(A) = 0.6, P(B) = 0.5, and  $P(A \cup B) = 0.8$ , determine whether A and B are independent or dependent.
- 565 In contract negotiations between a local government agency and its workers, it is estimated that there is a 50% chance that an agreement will be reached on the salaries of the workers. It is estimated that there is a 70% chance that there will be an agreement on the insurance benefits. There is a 20% chance that no agreement will be reached on either issue. Find the probability that an agreement will be reached on *both* issues. Based on this answer, determine whether the agreement on salaries and the agreement on insurance are independent events. Justify your answer.

### S.CP.A.1: VENN DIAGRAMS

Data for the students enrolled in a local high school are shown in the Venn diagram below.



If a student from the high school is selected at random, what is the probability that the student is a sophomore given that the student is enrolled in Algebra II?

- $1 \frac{85}{210}$
- $2 \frac{85}{295}$
- $3 \frac{85}{405}$
- $4 \frac{85}{1600}$

567 In a group of 40 people, 20 have brown hair, 22 have blue eyes, and 15 have both brown hair and blue eyes. How many people have neither brown hair nor blue eyes?

- 1 0
- 2 13
- 3 27
- 4 32

### S.CP.A.3: CONDITIONAL PROBABILITY

568 Which situation best describes conditional probability?

- 1 finding the probability of an event occurring two or more times
- 2 finding the probability of an event occurring only once
- 3 finding the probability of two independent events occurring at the same time
- 4 finding the probability of an event occurring given another event had already occurred

569 Sean's team has a baseball game tomorrow. He pitches 50% of the games. There is a 40% chance of rain during the game tomorrow. If the probability that it rains given that Sean pitches is 40%, it can be concluded that these two events are

- 1 independent
- 2 dependent
- 3 mutually exclusive
- 4 complements

570 A fast-food restaurant analyzes data to better serve its customers. After its analysis, it discovers that the events *D*, that a customer uses the drive-thru, and *F*, that a customer orders French fries, are independent. The following data are given in a report:

$$P(F) = 0.8$$
$$P(F \cap D) = 0.456$$

Given this information, P(F|D) is

- 1 0.344
- 2 0.3648
- 3 0.57
- 4 0.8

571 Consider the probability statements regarding events *A* and *B* below.

$$P(A \text{ or } B) = 0.3;$$
  
 $P(A \text{ and } B) = 0.2;$  and  
 $P(A|B) = 0.8$ 

What is P(B)?

- 1 0.1
- 2 0.25
- 3 0.375
- 4 0.667
- 572 Suppose events A and B are independent and P(A and B) is 0.2. Which statement could be true?

1 
$$P(A) = 0.4, P(B) = 0.3, P(A \text{ or } B) = 0.5$$

- P(A) = 0.8, P(B) = 0.25
- P(A|B) = 0.2, P(B) = 0.2
- 4 P(A) = 0.15, P(B) = 0.05

- 573 The probability that a resident of a housing community opposes spending money for community improvement on plumbing issues is 0.8. The probability that a resident favors spending money on improving walkways given that the resident opposes spending money on plumbing issues is 0.85. Determine the probability that a randomly selected resident opposes spending money on plumbing issues and favors spending money on walkways.
- 574 A student is chosen at random from the student body at a given high school. The probability that the student selects Math as the favorite subject is  $\frac{1}{4}$ . The probability that the student chosen is a junior is  $\frac{116}{459}$ . If the probability that the student selected is a junior or that the student chooses Math as the favorite subject is  $\frac{47}{108}$ , what is the exact probability that the student selected is a junior whose favorite subject is Math? Are the events "the student is a junior" and "the student's favorite subject is Math" independent of each other? Explain your answer.

#### S.CP.A.4: CONDITIONAL PROBABILITY

575 Consider the data in the table below.

|        | Right Handed | Left Handed |
|--------|--------------|-------------|
| Male   | 87           | 13          |
| Female | 89           | 11          |

What is the probability that a randomly selected person is male given the person is left handed?

 $1 \qquad \frac{13}{200}$ 

 $\frac{13}{50}$ 

 $2 \frac{13}{100}$ 

 $4 \frac{13}{24}$ 

576 The set of data in the table below shows the results of a survey on the number of messages that people of different ages text on their cell phones each month.

| Text Messages per Month             |    |    |     |  |  |  |
|-------------------------------------|----|----|-----|--|--|--|
| <b>Age Group</b> 0-10 11-50 Over 50 |    |    |     |  |  |  |
| 15-18                               | 4  | 37 | 68  |  |  |  |
| 19-22                               | 6  | 25 | 87  |  |  |  |
| 23-60                               | 25 | 47 | 157 |  |  |  |

If a person from this survey is selected at random, what is the probability that the person texts over 50 messages per month given that the person is between the ages of 23 and 60?

 $1 \frac{157}{229}$ 

 $3 \frac{157}{384}$ 

 $2 \frac{157}{312}$ 

 $4 \frac{157}{456}$ 

577 The results of a poll of 200 students are shown in the table below:

|        | Preferred Music Style |    |    |  |  |
|--------|-----------------------|----|----|--|--|
|        | Techno Rap Country    |    |    |  |  |
| Female | 54                    | 25 | 27 |  |  |
| Male   | 36                    | 40 | 18 |  |  |

For this group of students, do these data suggest that gender and preferred music styles are independent of each other? Justify your answer.

578 The results of a survey of the student body at Central High School about television viewing preferences are shown below.

|         | Comedy Series | Drama Series | Reality Series | Total |
|---------|---------------|--------------|----------------|-------|
| Males   | 95            | 65           | 70             | 230   |
| Females | 80            | 70           | 110            | 260   |
| Total   | 175           | 135          | 180            | 490   |

Are the events "student is a male" and "student prefers reality series" independent of each other? Justify your answer.

579 Data collected about jogging from students with two older siblings are shown in the table below.

|                         | Neither Sibling<br>Jogs | One Sibling<br>Jogs | Both Siblings<br>Jog |
|-------------------------|-------------------------|---------------------|----------------------|
| Student Does<br>Not Jog | 1168                    | 1823                | 1380                 |
| Student Jogs            | 188                     | 416                 | 400                  |

Using these data, determine whether a student with two older siblings is more likely to jog if one sibling jogs or if both siblings jog. Justify your answer.

A survey about television-viewing preferences was given to randomly selected freshmen and seniors at Fairport High School. The results are shown in the table below.

| Favorite Type of Program |        |                     |                      |  |
|--------------------------|--------|---------------------|----------------------|--|
|                          | Sports | <b>Reality Show</b> | <b>Comedy Series</b> |  |
| Senior                   | 83     | 110                 | 67                   |  |
| Freshmen                 | 119    | 103                 | 54                   |  |

A student response is selected at random from the results. State the *exact* probability the student response is from a freshman, given the student prefers to watch reality shows on television.

Juan and Filipe practice at the driving range before playing golf. The number of wins and corresponding practice times for each player are shown in the table below.

|                     | Juan Wins | Felipe Wins |
|---------------------|-----------|-------------|
| Short Practice Time | 8         | 10          |
| Long Practice Time  | 15        | 12          |

Given that the practice time was long, determine the exact probability that Filipe wins the next match. Determine whether or not the two events "Filipe wins" and "long practice time" are independent. Justify your answer.

The table below shows the results of gender and music preference. Based on these data, determine if the events "the person is female" and "the person prefers classic rock" are independent of each other. Justify your answer.

|        | Rap | Techno | Classic Rock | Classical |
|--------|-----|--------|--------------|-----------|
| Male   | 39  | 17     | 42           | 12        |
| Female | 17  | 37     | 36           | 15        |

The relative frequency table shows the proportion of a population who have a given eye color and the proportion of the same population who wear glasses.

|                   | Wear<br>Glasses | Don't Wear<br>Glasses |
|-------------------|-----------------|-----------------------|
| Blue Eyes         | 0.14            | 0.26                  |
| <b>Brown Eyes</b> | 0.11            | 0.24                  |
| Green Eyes        | 0.10            | 0.15                  |

Given the data, are the events of having blue eyes and wearing glasses independent? Justify your answer.

### S.CP.B.6: CONDITIONAL PROBABILITY

- 584 The guidance department has reported that of the senior class, 2.3% are members of key club, *K*, 8.6% are enrolled in AP Physics, *P*, and 1.9% are in both. Determine the probability of *P* given *K*, to the *nearest tenth of a percent*. The principal would like a basic interpretation of these results. Write a statement relating your calculated probabilities to student enrollment in the given situation.
- A study was designed to test the effectiveness of a new drug. Half of the volunteers received the drug. The other half received a sugar pill. The probability of a volunteer receiving the drug and getting well was 40%. What is the probability of a volunteer getting well, given that the volunteer received the drug?
- At Andrew Jackson High School, students are only allowed to enroll in AP U.S. History if they have already taken AP World History or AP European History. Out of 825 incoming seniors, 165 took AP World History, 66 took AP European History, and 33 took both. Given this information, determine the probability a randomly selected incoming senior is allowed to enroll in AP U.S. History.

### Algebra II Regents Exam Questions by State Standard: Topic **Answer Section**

1 ANS: 4

$$(1) \ \frac{B(60) - B(10)}{60 - 10} \approx 28\% \ \ (2) \ \frac{B(69) - B(19)}{69 - 19} \approx 33\% \ \ (3) \ \frac{B(72) - B(36)}{72 - 36} \approx 38\% \ \ (4) \ \frac{B(73) - B(60)}{73 - 60} \approx 46\%$$

PTS: 2

REF: 011721aii

NAT: F.IF.B.6

TOP: Rate of Change

2 ANS: 1

$$(1)\frac{9-0}{2-1} = 9 (2)\frac{17-0}{3.5-1} = 6.8 (3)\frac{0-0}{5-1} = 0 (4)\frac{17--5}{3.5-1} \approx 6.3$$

PTS: 2

REF: 011724aii

NAT: F.IF.B.6 TOP: Rate of Change REF: 061904aii NAT: F.IF.B.6

3 ANS: 1

PTS: 2

TOP: Rate of Change

4 ANS: 1

$$\frac{N(10) - N(1)}{10 - 1} \approx -2.03, \frac{N(20) - N(10)}{20 - 10} \approx -1.63, \frac{N(25) - N(15)}{25 - 15} \approx -1.46, \frac{N(30) - N(1)}{30 - 1} \approx -1.64$$

PTS: 2

REF: 061807aii

NAT: F.IF.B.6

TOP: Rate of Change

5 ANS: 1

$$\frac{N(6) - N(0)}{6 - 0} \approx -8.93$$

PTS: 2

REF: 012012aii NAT: F.IF.B.6

TOP: Rate of Change

6 ANS: 3

$$\log_{0.8}\left(\frac{V}{17000}\right) = t$$

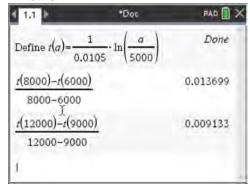
$$\log_{0.8} \left( \frac{V}{17000} \right) = t \qquad \frac{17,000(0.8)^3 - 17,000(0.8)^1}{3 - 1} \approx -2450$$

$$0.8^{t} = \frac{V}{17000}$$

$$V = 17000(0.8)^{t}$$

PTS: 2

REF: 081709aii NAT: F.IF.B.6 TOP: Rate of Change



PTS: 2

REF: 081922aii

NAT: F.IF.B.6

TOP: Rate of Change

8 ANS: 3

$$\frac{f(7) - f(-7)}{7 - -7} = \frac{2^{-0.25(7)} \cdot \sin\left(\frac{\pi}{2}(7)\right) - 2^{-0.25(-7)} \cdot \sin\left(\frac{\pi}{2}(-7)\right)}{14} \approx -0.26$$

PTS: 2

REF: 061721aii

NAT: F.IF.B.6

TOP: Rate of Change

9 ANS:

 $\frac{306.25 - 156.25}{70 - 50} = \frac{150}{20} = 7.5$  Between 50-70 mph, each additional mph in speed requires 7.5 more feet to stop.

PTS: 2

REF: 081631aii

NAT: F.IF.B.6

TOP: Rate of Change

10 ANS:

 $\frac{13.9-9.4}{4-1}$  = 1.5 The average rate of change in the number of hours of daylight from January 1-April 1 is 1.5.

PTS: 2

REF: 061925aii

NAT: F.IF.B.6

TOP: Rate of Change

11 ANS:

$$\frac{60-20}{4-2} = \frac{40}{2} = 20$$

PTS: 2

REF: 082225aii

NAT: F.IF.B.6

TOP: Rate of Change

12 ANS:

$$\frac{p(8) - p(4)}{8 - 4} \approx 48.78$$

PTS: 2

REF: 081827aii

NAT: F.IF.B.6

TOP: Rate of Change

13 ANS:

 $\frac{B(11) - B(8)}{11 - 8} \approx -10.1$  The average monthly high temperature decreases 10.1° each month from August to November.

PTS: 2

REF: 011930aii

NAT: F.IF.B.6

TOP: Rate of Change

 $\frac{B(10) - B(6)}{10 - 6} \approx -3.88$ . The average monthly high temperature decreases about 4° each month from June and October.

PTS: 4

REF: 012336aii

NAT: F.IF.B.6

TOP: Rate of Change

15 ANS: 4

$$wx^2 + w = 0$$

$$w(x^2+1)=0$$

$$x^2 = -1$$

$$x = \pm i$$

PTS: 2

REF: 061912aii

NAT: A.REI.B.4

**TOP:** Solving Quadratics

KEY: complex solutions | taking square roots

16 ANS: 4

$$4x^2 = -98$$

$$x^2 = -\frac{98}{4}$$

$$x^2 = -\frac{49}{2}$$

$$x = \pm \sqrt{-\frac{49}{2}} = \pm \frac{7i}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \pm \frac{7i\sqrt{2}}{2}$$

REF: 061707aii

NAT: A.REI.B.4

**TOP:** Solving Quadratics

KEY: complex solutions | taking square roots

17 ANS: 3

$$x^2 + 2x + 1 = -5 + 1$$

$$(x+1)^2 = -4$$

$$x + 1 = \pm 2i$$

$$x = -1 \pm 2i$$

PTS: 2

REF: 081703aii

NAT: A.REI.B.4 TOP: Solving Quadratics

KEY: complex solutions | completing the square

$$-2\left(-\frac{1}{2}x^2 = -6x + 20\right)$$

$$x^2 - 12x = -40$$

$$x^2 - 12x + 36 = -40 + 36$$

$$(x-6)^2 = -4$$

$$x-6=\pm 2i$$

$$x = 6 \pm 2i$$

PTS: 2 REF: fall1504aii NAT: A.REI.B.4 TOP: Solving Quadratics

KEY: complex solutions | completing the square

#### 19 ANS: 1

$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(2)}}{2(2)} = \frac{-3 \pm \sqrt{-7}}{4} = -\frac{3}{4} \pm \frac{i\sqrt{7}}{4}$$

PTS: 2 REF: 061612aii NAT: A.REI.B.4 TOP: Solving Quadratics

KEY: complex solutions | quadratic formula

#### 20 ANS: 3

$$x = \frac{-2 \pm \sqrt{2^2 - 4(3)(7)}}{2(3)} = \frac{-2 \pm \sqrt{-80}}{6} = \frac{-2 \pm i\sqrt{16}\sqrt{5}}{6} = -\frac{1}{3} \pm \frac{2i\sqrt{5}}{3}$$

PTS: 2 REF: 081809aii NAT: A.REI.B.4 TOP: Solving Quadratics

KEY: complex solutions | quadratic formula

#### 21 ANS: 2

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(5)(4)}}{2(5)} = \frac{2 \pm \sqrt{-76}}{10} = \frac{2 \pm i\sqrt{4}\sqrt{19}}{10} = \frac{1}{5} \pm \frac{i\sqrt{19}}{5}$$

PTS: 2 REF: 011905aii NAT: A.REI.B.4 TOP: Solving Quadratics

KEY: complex solutions | quadratic formula

#### 22 ANS: 4

$$x = \frac{8 \pm \sqrt{(-8)^2 - 4(6)(29)}}{2(6)} = \frac{8 \pm \sqrt{-632}}{12} = \frac{8 \pm i\sqrt{4}\sqrt{158}}{12} = \frac{2}{3} \pm \frac{1}{6}i\sqrt{158}$$

PTS: 2 REF: 011711aii NAT: A.REI.B.4 TOP: Solving Quadratics

KEY: complex solutions | quadratic formula

$$5x^{2} - 4x + 2 = 0 \quad \frac{4 \pm \sqrt{(-4)^{2} - 4(5)(2)}}{2(5)} = \frac{4 \pm \sqrt{-24}}{10} = \frac{4 \pm 2i\sqrt{6}}{10} = \frac{2 \pm i\sqrt{6}}{5}$$

PTS: 2 REF: 012020aii NAT: A.REI.B.4 TOP: Solving Quadratics

KEY: complex solutions | quadratic formula

24 ANS:

$$x = \frac{-5 \pm \sqrt{5^2 - 4(2)(8)}}{2(2)} = -\frac{5}{4} \pm \frac{i\sqrt{39}}{4}$$

PTS: 2 REF: 061827aii NAT: A.REI.B.4 TOP: Solving Quadratics

KEY: complex solutions | quadratic formula

25 ANS:

 $x^2 - 6x = -17$  The solution is imaginary because the parabola and line do not intersect.

$$x^2 - 6x + 9 = -17 + 9$$

$$(x-3)^2 = -8$$

$$x - 3 = \pm 2i\sqrt{2}$$

$$x = 3 \pm 2i\sqrt{2}$$

PTS: 4 REF: 081936aii NAT: A.REI.B.4 TOP: Solving Quadratics

KEY: complex solutions | completing the square

26 ANS: 4

(1) quadratic has two roots and both are real (-2,0) and (-0.5,0), (2)  $x = \pm \sqrt{32} - 3$ , (3) the real root is 3, with a multiplicity of 2, (4)  $x = \pm 4i$ 

PTS: 2 REF: 011909aii NAT: A.REI.B.4 TOP: Using the Discriminant

KEY: determine nature of roots given equation, graph, table

27 ANS:

$$b^2 - 4ac = (-4)^2 - 4(1)(13) = 16 - 52 = -36$$
 imaginary

PTS: 2 REF: 062225aii NAT: A.REI.B.4 TOP: Using the Discriminant

KEY: determine nature of roots given equation, graph, table

The product of the roots equals  $(3+i)(3-i) = 9-i^2 = 10 = \frac{c}{a}$ . OR

$$(x - (3+i))(x - (3-i)) = 0$$

$$(x-3-i)(x-3+i) = 0$$

$$((x-3)-i)((x-3)+i)=0$$

$$(x-3)^2 - i^2 = 0$$

$$x^2 - 6x + 9 + 1 = 0$$

$$x^2 - 6x + 10 = 0$$

PTS: 2 REF: 082208aii NAT: A.REI.B.4 TOP: Complex Conjugate Root Theorem

29 ANS: 4

If 1-i is one solution, the other is 1+i. (x-(1-i))(x-(1+i))=0

$$x^{2}-x-ix-x+ix+(1-i^{2})=0$$

$$x^2 - 2x + 2 = 0$$

PTS: 2 REF: 081601aii NAT: A.REI.B.4 TOP: Complex Conjugate Root Theorem

30 ANS: 1

The vertical distance from the directrix to the vertex, p, is 2. The vertical distance from the vertex to the focus must also be 2.

PTS: 2 REF: 062213aii NAT: G.GPE.A.2 TOP: Graphing Quadratic Functions

31 ANS: 4

The vertex is (2,2) and p = 3. 3 + 2 = 5

PTS: 2 REF: 081823aii NAT: G.GPE.A.2 TOP: Graphing Quadratic Functions

32 ANS: 1

In vertex form, the parabola is  $y = -\frac{1}{4(2)}(x+4)^2 + 3$ . The vertex is (-4,3) and p=2. 3+2=5

PTS: 2 REF: 011816aii NAT: G.GPE.A.2 TOP: Graphing Quadratic Functions

33 ANS: 4

The vertex is (2,-1) and p = 2.  $y = -\frac{1}{4(2)}(x-2)^2 - 1$ 

PTS: 2 REF: 081619aii NAT: G.GPE.A.2 TOP: Graphing Quadratic Functions

The vertex of the parabola is (0,0). The distance, p, between the vertex and the focus or the vertex and the directrix is 1.  $y = \frac{-1}{4p}(x-h)^2 + k$ 

$$y = \frac{-1}{4(1)} (x - 0)^2 + 0$$

$$y = -\frac{1}{4}x^2$$

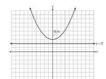
PTS: 2

REF: 081706aii

NAT: G.GPE.A.2

TOP: Graphing Quadratic Functions

35 ANS: 4



A parabola with a focus of (0,4) and a directrix of y=2 is sketched as follows: By inspection, it is determined that the vertex of the parabola is (0,3). It is also evident that the distance, p, between the vertex and the focus is 1. It is possible to use the formula  $(x-h)^2 = 4p(y-k)$  to derive the equation of the parabola as follows:  $(x-0)^2 = 4(1)(y-3)$ 

$$x^2 = 4y - 12$$

$$x^2 + 12 = 4y$$

$$\frac{x^2}{4} + 3 = y$$

or A point (x,y) on the parabola must be the same distance from the focus as it is from the directrix. For any such point (x,y), the distance to the focus is  $\sqrt{(x-0)^2 + (y-4)^2}$  and the distance to the directrix is y-2. Setting this equal leads to:  $x^2 + y^2 - 8y + 16 = y^2 - 4y + 4$ 

$$x^2 + 16 = 4y + 4$$

$$\frac{x^2}{4} + 3 = y$$

PTS: 2

REF: spr1502aii

NAT: G.GPE.A.2

**TOP:** Graphing Quadratic Functions

36 ANS: 4

The vertex is (1,0) and p = 2.  $y = \frac{1}{4(2)}(x-1)^2 + 0$ 

PTS: 2

REF: 061717aii

NAT: G.GPE.A.2

**TOP:** Graphing Quadratic Functions

37 ANS: 3

The vertex is (-3,5) and p = 2.  $y = \frac{-1}{4(2)}(x+3)^2 + 5$ 

PTS: 2

REF: 011914aii

NAT: G.GPE.A.2

**TOP:** Graphing Quadratic Functions

The distance from the vertex to the focus, p, is 4. Since the focus is below the vertex, p is negative.

$$y = -\frac{1}{4(4)} (x - 2)^2 + 1$$

PTS: 2

REF: 082212aii

NAT: G.GPE.A.2

**TOP:** Graphing Quadratic Functions

39 ANS: 4

$$\frac{5+9}{2}$$
 = 7, vertex: (-2,7);  $p = 7-9 = -2$ ,  $y = \frac{1}{4(-2)}(x+2)^2 + 7$ 

$$y - 7 = \frac{1}{-8} (x+2)^2$$

$$-8(y-7) = (x+2)^2$$

PTS: 2

REF: 061821aii

NAT: G.GPE.A.2

TOP: Graphing Quadratic Functions

40 ANS: 4

The distance between the focus and directrix is 1-3=4. p is half this distance, or 2. The vertex of the parabola is (4,-1). Since the directrix is above the focus, the parabola faces downward.  $y = -\frac{1}{4p}(x-h)^2 + k$ 

$$y = -\frac{1}{4(2)} (x - 4)^2 - 1$$

$$y + 1 = -\frac{1}{8}(x - 4)^2$$

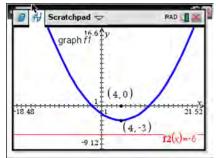
PTS: 2

REF: 012322aii

NAT: G.GPE.A.2

TOP: Graphing Quadratic Functions

41 ANS:



The vertex of the parabola is (4,-3). The x-coordinate of the focus and the

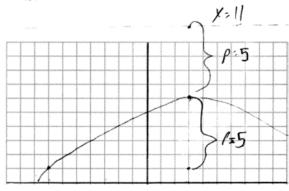
vertex is the same. Since the distance from the vertex to the directrix is 3, the distance from the vertex to the focus is 3, so the *y*-coordinate of the focus is 0. The coordinates of the focus are (4,0).

PTS: 2

REF: 061630aii

NAT: G.GPE.A.2

**TOP:** Graphing Quadratic Functions



vertex (3,6), focus (3,1), p = 5, directrix y = 6 + 5 = 11

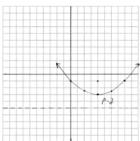
PTS: 2

REF: 012028aii

NAT: G.GPE.A.2

TOP: Graphing Quadratic Functions

43 ANS:



$$y = \frac{-1 + -5}{2} = -3$$
. The vertex is (4,-3) and  $p = 2$ .

PTS: 4

REF: 061935aii

NAT: G.GPE.A.2 TOP: Graphing Quadratic Functions

44 ANS: 2

$$B(t) = 750 \left( 1.16^{\frac{1}{12}} \right)^{12t} \approx 750(1.012)^{12t} \quad B(t) = 750 \left( 1 + \frac{0.16}{12} \right)^{12t}$$
 is wrong, because the growth is an annual rate

that is not compounded monthly.

PTS: 2

REF: spr1504aii

NAT: A.SSE.B.3

TOP: Modeling Exponential Functions

45 ANS: 3

$$0.75^{\frac{1}{10}} \approx .9716$$

PTS: 2

REF: 061713aii

NAT: A.SSE.B.3

**TOP:** Modeling Exponential Functions

46 ANS: 3

$$\left(\frac{1}{2}\right)^{\frac{1}{73.83}} \approx 0.990656$$

PTS: 2

REF: 081710aii

NAT: A.SSE.B.3

TOP: Modeling Exponential Functions

47 ANS: 4

1 year = 365 days

PTS: 2

REF: 061823aii

NAT: A.SSE.B.3

TOP: Modeling Exponential Functions

48 ANS: 2  $1.00643^{12} \approx 1.08$ PTS: 2 REF: 081808aii NAT: A.SSE.B.3 **TOP:** Modeling Exponential Functions 49 ANS: 3  $1.04^{\frac{1}{12}} \approx 1.0032737$ PTS: 2 REF: 011906aii NAT: A.SSE.B.3 **TOP:** Modeling Exponential Functions 50 ANS: 4  $1 + \frac{.009}{12} = 1.00075$ PTS: 2 REF: 011918aii NAT: A.SSE.B.3 TOP: Modeling Exponential Functions 51 ANS: 1  $1.025^{\frac{1}{12}} \approx 1.00206$ PTS: 2 REF: 081924aii NAT: A.SSE.B.3 TOP: Modeling Exponential Functions 52 ANS: 1  $\approx 1.00247^{12t}$ PTS: 2 REF: 062224aii NAT: A.SSE.B.3 **TOP:** Modeling Exponential Functions 53 ANS: 1  $0.5^{\frac{1}{0.0803}} \approx 0.000178$ PTS: 2 REF: 082224aii NAT: A.SSE.B.3 TOP: Modeling Exponential Functions 54 ANS: 1  $1.0325^{\frac{1}{12}} \approx 1.0027$ PTS: 2 REF: 012323aii NAT: A.SSE.B.3 TOP: Modeling Exponential Functions 55 ANS: 4 PTS: 2 REF: 011808aii NAT: A.SSE.B.3 **TOP:** Modeling Exponential Functions 56 ANS: 3  $1.0525^{\frac{1}{12}} \approx 1.00427$ PTS: 2 NAT: F.BF.A.1 REF: 061621aii **TOP:** Modeling Exponential Functions

REF: 081622aii

NAT: F.BF.A.1

57 ANS: 4

PTS: 2

TOP: Modeling Exponential Functions

PTS: 2 REF: 061924aii NAT: F.BF.A.1 TOP: Modeling Exponential Functions

59 ANS: 1

 $2000 \left(1 + \frac{.032}{12}\right)^{12t} \approx 2000(1.003)^{12t}$ 

PTS: 2 REF: 012004aii NAT: F.BF.A.1 TOP: Modeling Exponential Functions

60 ANS: 1

$$\frac{A}{P} = e^{rt}$$

$$0.42 = e^{rt}$$

$$\ln 0.42 = \ln e^{rt}$$

$$-0.87 \approx rt$$

PTS: 2 REF: 011723aii NAT: F.BF.A.1 TOP: Modeling Exponential Functions

61 ANS:

$$B(t) = 100(2)^{\frac{t}{30}}$$

PTS: 2 REF: 012031aii NAT: F.BF.A.1 TOP: Modeling Exponential Functions

62 ANS: 1

$$P(28) = 5(2)^{\frac{98}{28}} \approx 56$$

PTS: 2 REF: 011702aii NAT: F.LE.A.2 TOP: Modeling Exponential Functions

63 ANS: 4

$$5000 \left( 1 + \frac{.035}{12} \right)^{12 \cdot 6} \approx 6166.50$$

PTS: 2 REF: 081917aii NAT: F.LE.A.2 TOP: Modeling Exponential Functions

64 ANS: 3

$$y = 278(0.5)^{\frac{18}{1.8}} \approx 0.271$$

PTS: 2 REF: 011920aii NAT: F.LE.A.2 TOP: Modeling Exponential Functions

65 ANS:

$$N(t) = 950e^{0.0475t}$$
 The base is e because growth is continuous.  $N\left(\frac{36}{24}\right) \approx 1020$ 

PTS: 4 REF: 081933aii NAT: F.LE.A.2 TOP: Modeling Exponential Functions

 $A(t) = 100(0.5)^{\frac{t}{63}}$ , where t is time in years, and A(t) is the amount of titanium-44 left after t years.

$$\frac{A(10) - A(0)}{10 - 0} = \frac{89.58132 - 100}{10} = -1.041868$$
 The estimated mass at  $t = 40$  is  $100 - 40(-1.041868) \approx 58.3$ . The

actual mass is  $A(40) = 100(0.5)^{\frac{40}{63}} \approx 64.3976$ . The estimated mass is less than the actual mass.

PTS: 6

REF: fall1517aii

NAT: F.LE.A.2

TOP: Modeling Exponential Functions

67 ANS: 4

PTS: 2

REF: 012303aii

NAT: F.LE.B.5

TOP: Modeling Exponential Functions

68 ANS: 1

The car lost approximately 19% of its value each year.

PTS: 2

REF: 081613aii

NAT: F.LE.B.5

TOP: Modeling Exponential Functions

69 ANS: 4

PTS: 2

REF: 011805aii

NAT: F.LE.B.5

**TOP:** Modeling Exponential Functions

70 ANS: 1

1) 
$$A(20) > 0$$
; 2)  $.5 \times .5 = .25$ ; 3) true; 4)  $A(7) \approx 9.9$ 

PTS: 2

REF: 082211aii

NAT: F.LE.B.5

TOP: Modeling Exponential Functions

71 ANS: 2

PTS: 2

REF: 061917aii

NAT: F.LE.B.5

TOP: Modeling Exponential Functions

72 ANS: 2

The 2010 population is 110 million.

PTS: 2

REF: 061718aii

NAT: F.LE.B.5

**TOP:** Modeling Exponential Functions

73 ANS: 2

The mass of the carbon-14 is decreasing by half every 5715 years.

PTS: 2

REF: 062211aii

NAT: F.LE.B.5

**TOP:** Modeling Exponential Functions

74 ANS: 3

$$M = \frac{240000 \left(\frac{4.5\%}{12}\right) \left(1 + \frac{4.5\%}{12}\right)^{15 \times 12}}{\left(1 + \frac{4.5\%}{12}\right)^{15 \times 12}} \approx 1835.98$$

PTS: 2

REF: 062209aii

NAT: F.IF.B.4

**TOP:** Evaluating Exponential Expressions

75 ANS:

$$20000 = PMT \left( \frac{1 - (1 + .00625)^{-60}}{0.00625} \right) 21000 - x = 300 \left( \frac{1 - (1 + .00625)^{-60}}{0.00625} \right)$$

*PMT* ≈ 400.76

 $x \approx 6028$ 

PTS: 4

REF: 011736aii

NAT: F.IF.B.4

**TOP:** Evaluating Exponential Expressions

$$M = 172600 \bullet \frac{0.00305(1 + 0.00305)^{12 \cdot 15}}{(1 + 0.00305)^{12 \cdot 15} - 1} \approx 1247 \qquad 1100 = (172600 - x) \bullet \frac{0.00305(1 + 0.00305)^{12 \cdot 15}}{(1 + 0.00305)^{12 \cdot 15} - 1}$$
$$1100 \approx (172600 - x) \bullet (0.007228)$$
$$152193 \approx 172600 - x$$
$$20407 \approx x$$

PTS: 4

REF: 061734aii

NAT: F.IF.B.4

TOP: Evaluating Exponential Expressions

77 ANS:

$$M = \frac{(152500 - 15250) \left(\frac{.036}{12}\right) \left(1 + \frac{.036}{12}\right)^{360}}{\left(1 + \frac{.036}{12}\right)^{360} - 1} \approx 624$$

PTS: 2

REF: 061831aii

NAT: F.IF.B.4

**TOP:** Evaluating Exponential Expressions

78 ANS: 3

$$d = 10\log\frac{6.3 \times 10^{-3}}{1.0 \times 10^{-12}} \approx 98$$

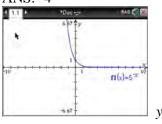
PTS: 2

REF: 011715aii

NAT: F.IF.B.4

TOP: Evaluating Logarithmic Expressions

79 ANS: 4



 $y = 5^{-t} = \left(\frac{1}{5}\right)^t$ 

PTS: 2

REF: 061615aii

NAT: F.IF.C.7

**TOP:** Graphing Exponential Functions

80 ANS: 2

$$p(x) = 4^x$$
,  $q(x) = \left(\frac{5}{9}\right)^x$ ,  $r(x) = 5.29^x$ ,  $s(x) = 2^x$ 

PTS: 2

REF: 012304aii

NAT: F.IF.C.7

**TOP:** Graphing Exponential Functions

81 ANS: 4

There is no *x*-intercept.

PTS: 2

REF: 011823aii

NAT: F.IF.C.7

TOP: Graphing Exponential Functions

82 ANS: 2

PTS: 2

REF: 061802aii

NAT: F.IF.C.7

**TOP:** Graphing Exponential Functions

83 ANS: 3

PTS: 2

REF: 082214aii

NAT: F.IF.C.7

**TOP:** Graphing Exponential Functions

 $\frac{\left(\ln\frac{1}{2}\right)}{1590}$  is negative, so M(t) represents decay.

PTS: 2

REF: 011728aii

NAT: F.IF.C.7

**TOP:** Graphing Exponential Functions

85 ANS:

Translation 3 units right and 4 units up

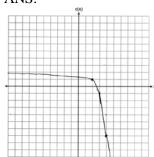
PTS: 2

REF: 012027aii

NAT: F.IF.C.7

**TOP:** Graphing Exponential Functions

86 ANS:



As  $x \to \infty$ ,  $c(x) \to -\infty$ . As  $x \to -\infty$ ,  $c(x) \to 2$ .

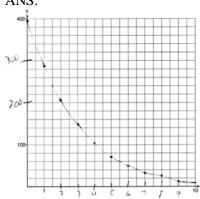
PTS: 4

REF: 012335aii

NAT: F.IF.C.7

**TOP:** Graphing Exponential Functions

87 ANS:



PTS: 2

REF: 061729aii

NAT: F.IF.C.7

**TOP:** Graphing Exponential Functions

88 ANS: 4

Translate the parent log function 2 to the right and reflect over the *x*-axis.

PTS: 2

REF: 082207aii

NAT: F.IF.C.7

TOP: Graphing Logarithmic Functions

89 ANS: 2

PTS: 2

REF: 081816aii

NAT: F.IF.C.7

TOP: Graphing Logarithmic Functions

KEY: bimodalgraph

$$\log_2(x - 1) - 1 = 0$$

$$\log_2(x-1) = 1$$

$$x - 1 = 2^1$$

$$x = 3$$

PTS: 2

REF: 061819aii

NAT: F.IF.C.7

TOP: Graphing Logarithmic Functions

91 ANS: 1

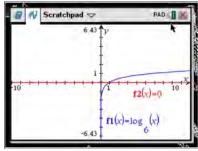
PTS: 2

REF: 011902aii

NAT: F.IF.C.7

TOP: Graphing Logarithmic Functions

92 ANS: 1



PTS: 2

REF: 061618aii

NAT: F.IF.C.7

TOP: Graphing Logarithmic Functions

93 ANS: 4

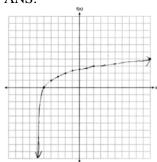
PTS: 2

REF: 062215aii

NAT: F.IF.C.7

TOP: Graphing Logarithmic Functions

94 ANS:

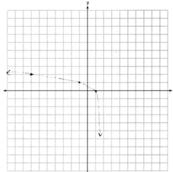


PTS: 2

REF: 061927aii

NAT: F.IF.C.7

TOP: Graphing Logarithmic Functions



Domain: x < 2, Asymptote x = 2

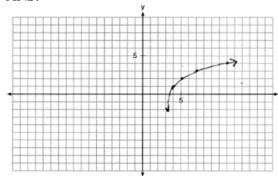
PTS: 4

REF: 012034aii

NAT: F.IF.C.7

TOP: Graphing Logarithmic Functions

96 ANS:



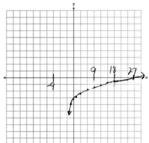
PTS: 2

REF: 011932aii

NAT: F.IF.C.7

TOP: Graphing Logarithmic Functions

97 ANS:



As  $x \to -3$ ,  $y \to -\infty$ . As  $x \to \infty$ ,  $y \to \infty$ .

PTS: 4

REF: 061735aii

NAT: F.IF.C.7

TOP: Graphing Logarithmic Functions

98 ANS:

$$a^{x+1} = a^{\frac{2}{3}}$$

$$x+1=\frac{2}{3}$$

$$x = -\frac{1}{3}$$

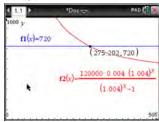
PTS: 2

REF: 012326aii

NAT: A.CED.A.1

TOP: Exponential Equations

KEY: common base shown



$$720 = \frac{120000 \left(\frac{.048}{12}\right) \left(1 + \frac{.048}{12}\right)^n}{\left(1 + \frac{.048}{12}\right)^n - 1} \quad \frac{275.2}{12} \approx 23 \text{ years}$$

$$720(1.004)^{n} - 720 = 480(1.004)^{n}$$
$$240(1.004)^{n} = 720$$
$$1.004^{n} = 3$$
$$n \log 1.004 = \log 3$$
$$n \approx 275.2 \text{ months}$$

PTS: 4 REF: spr1509aii NAT: A.CED.A.1 TOP: Exponential Growth

100 ANS:

ANS:  

$$A = 5000(1.045)^{n} \qquad 5000 \left(1 + \frac{.046}{4}\right)^{4(6)} - 5000(1.045)^{6} \approx 6578.87 - 6511.30 \approx 67.57 \quad 10000 = 5000 \left(1 + \frac{.046}{4}\right)^{4n}$$

$$2 = 1.0115^{4n}$$

$$\log 2 = 4n \cdot \log 1.0115$$

$$n = \frac{\log 2}{4 \log 1.0115}$$

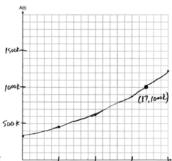
$$n \approx 15.2$$

PTS: 6 REF: 081637aii NAT: A.CED.A.1 TOP: Exponential Growth

101 ANS:

$$C(t) = 63000 \left( 1 + \frac{0.0255}{12} \right)^{12t} \quad 63000 \left( 1 + \frac{0.0255}{12} \right)^{12t} = 100000$$
$$12t \log(1.002125) = \log \frac{100}{63}$$
$$t \approx 18.14$$

PTS: 4 REF: 061835aii NAT: A.CED.A.1 TOP: Exponential Growth



 $A(t) = 318000(1.07)^t$ 

 $318000(1.07)^{t} = 1000000$  The graph of A(t) nearly intersects

$$1.07^t = \frac{1000}{318}$$

$$t \log 1.07 = \log \frac{1000}{318}$$

$$t = \frac{\log \frac{1000}{318}}{\log 1.07}$$

$$t \approx 17$$

the point (17, 1000000).

PTS: 6 REF: 011937aii

NAT: A.CED.A.1 TOP: Exponential Growth

103 ANS:

$$T = (400 - 75)e^{-0.0735t} + 75, \ 325e^{-0.0735(5)} + 75 \approx 300, \ 270 = (450 - 75)e^{-8r} + 75, \ 325e^{-0.0735t} + 75 = 375e^{-0.0817t} + 75$$
$$r \approx 0.0817$$
$$t \approx 17$$

PTS: 6

REF: 012337aii NAT: A.CED.A.1 TOP: Exponential Decay

104 ANS: 3

$$e^{bt} = \frac{c}{a}$$

$$\ln e^{bt} = \ln \frac{c}{a}$$

$$bt\ln e = \ln\frac{c}{a}$$

$$t = \frac{\ln \frac{c}{a}}{b}$$

PTS: 2

REF: 011813aii

NAT: F.LE.A.4 TOP: Exponential Equations

KEY: without common base

$$8(2^{x+3}) = 48$$

$$2^{x+3}=6$$

$$(x+3)\ln 2 = \ln 6$$

$$x + 3 = \frac{\ln 6}{\ln 2}$$

$$x = \frac{\ln 6}{\ln 2} - 3$$

PTS: 2

REF: 061702aii

NAT: F.LE.A.4

TOP: Exponential Equations

KEY: without common base

$$\ln e^{x+2} = \ln \frac{7}{5}$$

$$(x+2)\ln e = \ln\frac{7}{5}$$

$$x = -2 + \ln \frac{7}{5}$$

PTS: 2

REF: 062207aii

NAT: F.LE.A.4

TOP: Exponential Equations

KEY: without common base

$$\ln e^{0.3x} = \ln \frac{5918}{87}$$

$$x = \frac{\ln \frac{5918}{87}}{0.3}$$

PTS: 2

REF: 081801aii

NAT: F.LE.A.4

**TOP:** Exponential Equations

KEY: without common base

$$4300e^{0.07x} = 5123$$

$$\ln e^{0.07x} = \ln \frac{5123}{4300}$$

$$0.07x = \ln \frac{5123}{4300}$$

$$x = \frac{\ln \frac{5123}{4300}}{0.07}$$

$$x \approx 2.5$$

PTS: 2

REF: 012302aii

NAT: F.LE.A.4

**TOP:** Exponential Equations

KEY: without common base

$$\log 2^t = \log \sqrt{10} \quad 2) \frac{\log \sqrt{10}}{\log 2} = \log_2 \sqrt{10}, \quad 1) \log_2 \sqrt{10} = \log_2 10^{\frac{1}{2}} = \frac{1}{2} \log_2 10, \quad 3) \log_4 10 = \frac{\log_2 10}{\log_2 4} = \frac{1}{2} \log_2 10$$

$$t\log 2 = \log \sqrt{10}$$

$$t = \frac{\log \sqrt{10}}{\log 2}$$

PTS: 2

REF: 012009aii

NAT: F.LE.A.4

TOP: Exponential Equations

KEY: without common base

$$\frac{15000}{12000} = \frac{12000e^{.025t}}{12000}$$

$$1.25 = e^{.025t}$$

$$\ln 1.25 = \ln e^{.025t}$$

$$\ln 1.25 = .025t$$

$$\frac{\ln 1.25}{.025} = t$$

PTS: 2

REF: 082209aii

NAT: F.LE.A.4

TOP: Exponential Growth

$$9110 = 5000e^{30r}$$

$$\ln \frac{911}{500} = \ln e^{30r}$$

$$\frac{\ln\frac{911}{500}}{30} = r$$

$$r \approx .02$$

PTS: 2

REF: 011810aii NAT: F.LE.A.4 TOP: Exponential Growth

112 ANS:

$$A = Pe^{rt}$$

 $135000 = 100000e^{5r}$ 

$$1.35 = e^{5r}$$

$$\ln 1.35 = \ln e^{5r}$$

$$\ln 1.35 = 5r$$

$$.06 \approx r \text{ or } 6\%$$

PTS: 2

REF: 061632aii NAT: F.LE.A.4 TOP: Exponential Growth

113 ANS:

4% 
$$8.75 = 1.25(1+r)^{49}$$
 or  $8.75 = 1.25e^{49r}$ 

$$7 = (1+r)^{49} \qquad \ln 7 = \ln e^{49r}$$

$$\ln 7 = \ln e^{49r}$$

$$r + 1 = \sqrt[49]{7}$$

$$\ln 7 = 49r$$

$$r \approx .04$$

$$r = \frac{\ln 7}{49}$$

$$r \approx .04$$

PTS: 2

REF: 081730aii NAT: F.LE.A.4 TOP: Exponential Growth

114 ANS:

$$2 = e^{0.0375t}$$

$$t \approx 18.5$$

PTS: 4

REF: 081835aii NAT: F.LE.A.4

TOP: Exponential Growth

a) 
$$p(t) = 11000(2)^{\frac{t}{20}}$$
; b)  $\frac{1000000}{11000} = \frac{11000(2)^{\frac{t}{20}}}{11000}$ 

$$\log \frac{1000}{11} = \log 2^{\frac{t}{20}}$$

$$\log \frac{1000}{11} = \frac{t \cdot \log 2}{20}$$

$$\frac{20 \log \frac{1000}{11}}{\log 2} = t$$

$$t \approx 130.13$$

PTS: 4

REF: 082233aii

NAT: F.LE.A.4

TOP: Exponential Growth

116 ANS:



$$100 = 325 + (68 - 325)e^{-2k} \quad T = 325 - 257e^{-0.066t}$$

$$-225 = -257e^{-2k}$$

$$T = 325 - 257e^{-0.066(7)} \approx 163$$

$$k = \frac{\ln\left(\frac{-225}{-257}\right)}{-2}$$

$$k \approx 0.066$$

PTS: 4

REF: fall1513aii NAT: F.LE.A.4 TOP: Exponential Growth

117 ANS: 1

$$100 \left(\frac{1}{2}\right)^{\frac{d}{8}} = 100e^{kd}$$

$$\left(\frac{1}{2}\right)^{\frac{1}{8}} = e^k$$

$$k \approx -0.087$$

PTS: 2

REF: 061818aii NAT: F.LE.A.4 TOP: Exponential Decay

$$120 = 68 + (195 - 68)e^{-0.05t}$$

$$52 = 127e^{-0.05t}$$

$$\ln \frac{52}{127} = \ln e^{-0.05t}$$

$$\ln \frac{52}{127} = -0.05t$$

$$\frac{\ln \frac{52}{127}}{-0.05} = t$$

$$18 \approx t$$

PTS: 2

REF: 081918aii

NAT: F.LE.A.4

TOP: Exponential Decay

119 ANS:

$$7 = 20(0.5)^{\frac{t}{8.02}}$$

$$\log 0.35 = \log 0.5^{\frac{t}{8.02}}$$

$$\log 0.35 = \frac{t \log 0.5}{8.02}$$

$$\frac{8.02\log 0.35}{\log 0.5} = t$$

$$t \approx 12$$

PTS: 4

REF: 081634aii

NAT: F.LE.A.4

TOP: Exponential Decay

120 ANS:

$$s(t) = 200(0.5)^{\frac{t}{15}}$$
  $\frac{1}{10} = (0.5)^{\frac{t}{15}}$ 

$$\log \frac{1}{10} = \log(0.5)^{\frac{t}{15}}$$

$$-1 = \frac{t \cdot \log(0.5)}{15}$$

$$t = \frac{-15}{\log(0.5)} \approx 50$$

PTS: 4

REF: 061934aii

NAT: F.LE.A.4

TOP: Exponential Decay

$$100 = 140 \left(\frac{1}{2}\right)^{\frac{5}{h}} \log \frac{100}{140} = \log \left(\frac{1}{2}\right)^{\frac{5}{h}} \qquad 40 = 140 \left(\frac{1}{2}\right)^{\frac{t}{10.3002}}$$

$$\log \frac{5}{7} = \frac{5}{h} \log \frac{1}{2} \qquad \log \frac{2}{7} = \log \left(\frac{1}{2}\right)^{\frac{t}{10.3002}}$$

$$h = \frac{5 \log \frac{1}{2}}{\log \frac{5}{7}} \approx 10.3002 \qquad \log \frac{2}{7} = \frac{t \log \left(\frac{1}{2}\right)}{10.3002}$$

$$t = \frac{10.3002 \log \frac{2}{7}}{\log \frac{1}{2}} \approx 18.6$$

PTS: 6 REF: 061737aii NAT: F.LE.A.4 TOP: Exponential Decay

122 ANS: 2

$$u = x + 2 u2 + 4u + 3$$

$$(u+3)(u+1)$$

$$(x+2+3)(x+2+1)$$

$$(x+5)(x+3)$$

PTS: 2 REF: 081901aii NAT: A.SSE.A.2 TOP: Factoring Polynomials

KEY: higher power

123 ANS: 1

$$u = x + 2 u^{2} - 5u + 6$$

$$(u - 3)(u - 2)$$

$$(x + 2 - 3)(x + 2 - 2)$$

$$(x - 1)x$$

PTS: 2 REF: 012301aii NAT: A.SSE.A.2 TOP: Factoring Polynomials

KEY: higher power

124 ANS: 3

$$(x+a)^{2} + 5(x+a) + 4 \text{ let } u = x+a$$

$$u^{2} + 5u + 4$$

$$(u+4)(u+1)$$

$$(x+a+4)(x+a+1)$$

PTS: 2 REF: 012006aii NAT: A.SSE.A.2 TOP: Factoring Polynomials

KEY: multivariable

$$m^5 + m^3 - 6m = m(m^4 + m^2 - 6) = m(m^2 + 3)(m^2 - 2)$$

PTS: 2 REF: 011703aii NAT: A.SSE.A.2 TOP: Factoring Polynomials

KEY: higher power

126 ANS: 2 PTS: 2 REF: 081904aii NAT: A.SSE.A.2

TOP: Factoring Polynomials KEY: higher power

127 ANS: 3

$$2d(d^3 + 3d^2 - 9d - 27)$$

$$2d(d^2(d+3)-9(d+3))$$

$$2d(d^2-9)(d+3)$$

$$2d(d+3)(d-3)(d+3)$$

$$2d(d+3)^2(d-3)$$

PTS: 2 REF: 081615aii NAT: A.SSE.A.2 TOP: Factoring Polynomials

KEY: factoring by grouping

128 ANS: 4

$$k^4 - 4k^2 + 8k^3 - 32k + 12k^2 - 48$$

$$k^{2}(k^{2}-4)+8k(k^{2}-4)+12(k^{2}-4)$$

$$(k^2-4)(k^2+8k+12)$$

$$(k+2)(k-2)(k+6)(k+2)$$

PTS: 2 REF: fall1505aii NAT: A.SSE.A.2 TOP: Factoring Polynomials

KEY: factoring by grouping

129 ANS: 2

$$n^{2}(n^{2}-9)+4n(n^{2}-9)-12(n^{2}-9)$$

$$(n^2 + 4n - 12)(n^2 - 9)$$

$$(n+6)(n-2)(n+3)(n-3)$$

PTS: 2 REF: 061911aii NAT: A.SSE.A.2 TOP: Factoring Polynomials

KEY: factoring by grouping

130 ANS: 4

$$(x^6y^4 - 9)(x^4 - 16)$$

$$(x^3y^2+3)(x^3y^2-3)(x^2+4)(x^2-4)$$

PTS: 2 REF: 081814aii NAT: A.SSE.A.2 TOP: Factoring Polynomials

KEY: factoring by grouping

$$(m-2)^2(m+3) = (m^2 - 4m + 4)(m+3) = m^3 + 3m^2 - 4m^2 - 12m + 4m + 12 = m^3 - m^2 - 8m + 12$$

PTS: 2 REF: 081605aii NAT: A.SSE.A.2 TOP: Factoring Polynomials

KEY: factoring by grouping

132 ANS: 1

1) let 
$$y = x + 2$$
, then  $y^2 + 2y - 8$   
 $(y + 4)(y - 2)$   
 $(x + 2 + 4)(x + 2 - 2)$ 

$$(x+6)x$$

PTS: 2 REF: 081715aii NAT: A.SSE.A.2 TOP: Factoring Polynomials

KEY: multivariable

133 ANS:

$$(x^2-6)(x^2+2)$$

PTS: 2 REF: 081825aii NAT: A.SSE.A.2 TOP: Factoring Polynomials

KEY: higher power

134 ANS:

$$x^4 - 5x^2 + 4$$

$$(x^2-4)(x^2-1)$$

$$(x+2)(x-2)(x+1)(x-1)$$

PTS: 2 REF: 012331aii NAT: A.SSE.A.2 TOP: Factoring Polynomials

135 ANS:

$$x^{3} - 2x^{2} - 9x + 18 = x^{2}(x - 2) - 9(x - 2) = (x^{2} - 9)(x - 2) = (x + 3)(x - 3)(x - 2)$$

PTS: 2 REF: 082226aii NAT: A.SSE.A.2 TOP: Factoring Polynomials

KEY: factoring by grouping

136 ANS:

$$-x(2x^3-x^2-18x+9)$$

$$-x(x^2(2x-1)-9(2x-1))$$

$$-x(x^2-9)(2x-1)$$

$$-x(x+3)(x-3)(2x-1)$$

PTS: 2 REF: 062228aii NAT: A.SSE.A.2 TOP: Factoring Polynomials

KEY: factoring by grouping

$$x^{2}(4x-1) + 4(4x-1) = (x^{2}+4)(4x-1)$$

REF: 061727aii

NAT: A.SSE.A.2

**TOP:** Factoring Polynomials

KEY: factoring by grouping

138 ANS:

$$3x^3 + x^2 + 3xy + y = x^2(3x+1) + y(3x+1) = (x^2 + y)(3x+1)$$

PTS: 2

REF: 011828aii

NAT: A.SSE.A.2

**TOP:** Factoring Polynomials

KEY: factoring by grouping

139 ANS:

The expression is of the form  $y^2 - 5y - 6$  or (y - 6)(y + 1). Let  $y = 4x^2 + 5x$ :

$$(4x^2 + 5x - 6)(4x^2 + 5x + 1)$$

$$(4x-3)(x+2)(4x+1)(x+1)$$

PTS: 2

REF: fall1512aii

NAT: A.SSE.A.2

**TOP:** Factoring Polynomials

KEY: a>1

140 ANS: 4

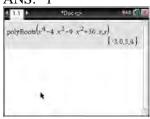
PTS: 2

REF: 081708aii

NAT: A.APR.B.3

**TOP:** Solving Polynomial Equations

141 ANS: 1



$$x^4 - 4x^3 - 9x^2 + 36x = 0$$

$$x^{3}(x-4) - 9x(x-4) = 0$$

$$(x^3 - 9x)(x - 4) = 0$$

$$x(x^2 - 9)(x - 4) = 0$$

$$x(x+3)(x-3)(x-4) = 0$$

$$x = 0, \pm 3, 4$$

PTS: 2

REF: 061606aii

NAT: A.APR.B.3 TOP: Solving Polynomial Equations

142 ANS: 4

$$m^3 - 2m^2 + 4m - 8 = 0$$

$$m^2(m-2) + 4(m-2) = 0$$

$$\left(m^2+4\right)(m-2)=0$$

PTS: 2

REF: 081821aii NAT: A.APR.B.3 TOP: Solving Polynomial Equations

143 ANS: 1 
$$x^3 + 2x^2 = 0x$$

$$x^3 + 2x^2 - 9x - 18 = 0$$
  $x^3 - 9x + 2x^2 - 18 = 0$   $x^3 - 9x + 2x^2 - 18 = 0$ 

$$x^{2}(x+2) - 9(x+2) = 0$$
  $x(x^{2}-9) + 2(x^{2}-9) = 0$   $x(x^{2}-9) + 2(x^{2}-9) = 0$ 

$$(x+2)(x^2-9)=0$$

PTS: 2 REF: 011903aii NAT: A.APR.B.3 TOP: Solving Polynomial Equations

144 ANS: 4

1) -1 is also a zero. 2)  $x^2(x-a) + 16(x-a) = (x^2 + 16)(x-a) a$  is the only zero. 3) -a is the only zero. 4)  $x^2(x-a) - 9(x-a) = (x^2 - 9)(x-a)$ .

PTS: 2 REF: 012019aii NAT: A.APR.B.3 TOP: Solving Polynomial Equations

145 ANS:

$$3(x^3 + 4x^2 - x - 4) = 0$$

$$(x^{2}(x+4)-(x+4))=0$$

$$(x^2-1)(x+4)=0$$

$$x = \pm 1, -4$$

PTS: 2 REF: 012325aii NAT: A.APR.B.3 TOP: Solving Polynomial Equations

146 ANS: 1

$$x^2 + 2x + 1 = (x+1)^2$$

PTS: 2 REF: 011919aii NAT: A.APR.B.3 TOP: Graphing Polynomial Functions

147 ANS: 1 PTS: 2 REF: 061701aii NAT: A.APR.B.3

TOP: Graphing Polynomial Functions

148 ANS: 4 PTS: 2 REF: 061921aii NAT: A.APR.B.3

**TOP:** Graphing Polynomial Functions

149 ANS: 4

$$f(x) = (x+1)(x-1)(x-2) = (x^2-1)(x-2) = x^3 - 2x^2 - x + 2$$

PTS: 2 REF: 081921aii NAT: A.APR.B.3 TOP: Graphing Polynomial Functions

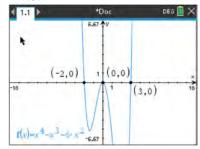
150 ANS: 2 1)  $x \to \infty$ ,  $f(x) \to \infty$ ; 3) quartic polynomial; 4) three real roots

PTS: 2 REF: 012318aii NAT: A.APR.B.3 TOP: Graphing Polynomial Functions

151 ANS:

$$f(x) = x^{2}(x+4)(x-3); g(x) = (x+2)^{2}(x+6)(x-1)$$

PTS: 4 REF: 011836aii NAT: A.APR.B.3 TOP: Graphing Polynomial Functions



PTS: 2

REF: 012316aii

NAT: F.IF.B.4

**TOP:** Graphing Polynomial Functions

153 ANS: 2

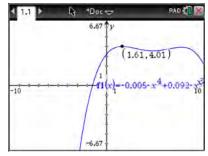
PTS: 2

REF: 081908aii

NAT: F.IF.B.4

**TOP:** Graphing Polynomial Functions

154 ANS: 3



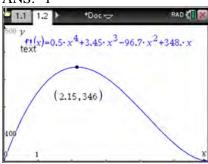
PTS: 2

REF: 011817aii

NAT: F.IF.B.4

**TOP:** Graphing Polynomial Functions

155 ANS: 1



PTS: 2

REF: 011908aii

NAT: F.IF.B.4

**TOP:** Graphing Polynomial Functions

156 ANS: 4

The maximum volume of p(x) = -(x+2)(x-10)(x-14) is about 56, at x = 12.1

PTS: 2

REF: 081712aii

NAT: F.IF.B.4

**TOP:** Graphing Polynomial Functions

157 ANS: 2

PTS: 2

REF: 061620aii

NAT: F.IF.B.4

**TOP:** Graphing Polynomial Functions 158 ANS: 3

PTS: 2

REF: 012005aii

NAT: F.IF.B.4

$$16x^4 - 81 = \left(4x^2 + 9\right)\left(4x^2 - 9\right) = \left(4x^2 + 9\right)(2x + 3)(2x - 3).$$
 No, because  $\pm \frac{3i}{2}$  are roots.

PTS: 4

REF: 061933aii

NAT: F.IF.B.4

**TOP:** Graphing Polynomial Functions

160 ANS: 2

1) 1 real, mult. 2; 3) not a quadratic; 4) not a function.

PTS: 2

REF: 012324aii

NAT: F.IF.C.7

**TOP:** Graphing Polynomial Functions

161 ANS: 1

The zeros of the polynomial are at -b, and c. The sketch of a polynomial of degree 3 with a negative leading coefficient should have end behavior showing as x goes to negative infinity, f(x) goes to positive infinity. The multiplicities of the roots are correctly represented in the graph.

PTS: 2

REF: spr1501aii

NAT: F.IF.C.7

**TOP:** Graphing Polynomial Functions

KEY: bimodalgraph

162 ANS: 3

The graph shows three real zeros, and has end behavior matching the given end behavior.

PTS: 2

REF: 061604aii

NAT: F.IF.C.7

**TOP:** Graphing Polynomial Functions

KEY: bimodalgraph

163 ANS: 2

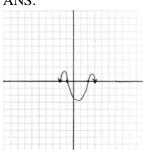
PTS: 2

REF: 061816aii

NAT: F.IF.C.7

TOP: Graphing Polynomial Functions KEY: bimodalgraph

164 ANS:



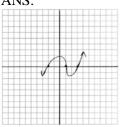
PTS: 2

REF: 011831aii

NAT: F.IF.C.7

**TOP:** Graphing Polynomial Functions

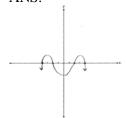
165 ANS:



PTS: 2

REF: 011729aii

NAT: F.IF.C.7



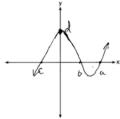
PTS: 2

REF: 011926aii

NAT: F.IF.C.7

TOP: Graphing Polynomial Functions

167 ANS:



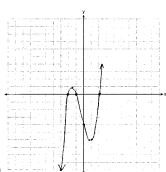
PTS: 2

REF: 081732aii

NAT: F.IF.C.7

**TOP:** Graphing Polynomial Functions

168 ANS:



$$0 = x^2(x+1) - 4(x+1)$$

$$0 = (x^2 - 4)(x + 1)$$

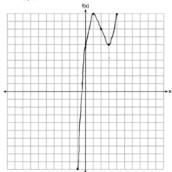
$$0 = (x+2)(x-2)(x+1)$$

$$x = -2, -1, 2$$

PTS: 4

REF: 081633aii

NAT: F.IF.C.7



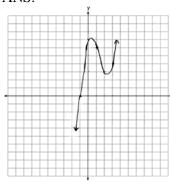
PTS: 2

REF: 061826aii

NAT: F.IF.C.7

TOP: Graphing Polynomial Functions

170 ANS:



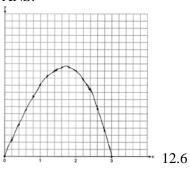
PTS: 2

REF: 012032aii

NAT: F.IF.C.7

TOP: Graphing Polynomial Functions

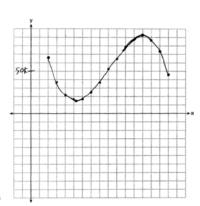
171 ANS:



PTS: 4

REF: 082234aii

NAT: F.IF.C.7



 $P(x) = R(x) - C(x) = -330x^3 + 9000x^2 - 67000x + 167000$ 

Least profitable at year

5 because there is a minimum in P(x). Most profitable at year 13 because there is a maximum in P(x).

PTS: 6

REF: 081837aii

NAT: F.IF.C.7

**TOP:** Graphing Polynomial Functions

173 ANS: 3

Since x + 4 is a factor of p(x), there is no remainder.

PTS: 2

REF: 081621aii

NAT: A.APR.B.2

TOP: Remainder and Factor Theorems

174 ANS: 2

Since there is a remainder when the cubic is divided by x + 4, this binomial is not a factor.

PTS: 2

REF: 081720aii

NAT: A.APR.B.2

TOP: Remainder and Factor Theorems

175 ANS: 4

PTS: 2

REF: 061907aii

NAT: A.APR.B.2

TOP: Remainder and Factor Theorems

176 ANS: 1

Since there is no remainder when the quartic is divided by x-2, this binomial is a factor.

PTS: 2

REF: 061711aii

NAT: A.APR.B.2

TOP: Remainder and Factor Theorems

177 ANS: 4

$$p(5) = 2(5)^3 - 3(5) + 5 = 240$$

PTS: 2

REF: 011819aii

NAT: A.APR.B.2

TOP: Remainder and Factor Theorems

178 ANS: 3

$$1^3 - k(1)^2 + 2(1) = 0$$

$$k = 3$$

PTS: 2

REF: 061812aii

NAT: A.APR.B.2

TOP: Remainder and Factor Theorems

179 ANS: 2

PTS: 2

REF: 011720aii

NAT: A.APR.B.2

TOP: Remainder and Factor Theorems

PTS: 2

REF: 062206aii

NAT: A.APR.B.2

TOP: Remainder and Factor Theorems

181 ANS: 2

$$2x^3 + x^2 - 18x - 9$$

$$x^{2}(2x+1)-9(2x+1)$$

$$(x^2-9)(2x+1)$$

$$(x+3)(x-3)(2x+1)$$

PTS: 2

REF: 082206aii

NAT: A.APR.B.2 TOP: Remainder and Factor Theorems

182 ANS: 2

$$2x^4 - x^3 - 16x + 8 = 0$$

$$x^{3}(2x-1)-8(2x-1)=0$$

$$(x^3 - 8)(2x - 1) = 0$$

$$x = 2, \frac{1}{2}$$

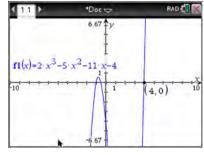
PTS: 2

REF: 012307aii

NAT: A.APR.B.2 TOP: Remainder and Factor Theorems

183 ANS:

 $f(4) = 2(4)^3 - 5(4)^2 - 11(4) - 4 = 128 - 80 - 44 - 4 = 0$  Any method that demonstrates 4 is a zero of f(x) confirms



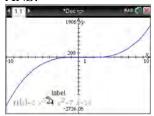
that x - 4 is a factor, as suggested by the Remainder Theorem.

PTS: 2

REF: spr1507aii

NAT: A.APR.B.2

TOP: Remainder and Factor Theorems



$$\frac{2x^2 + 6x + 23}{x - 5 \int 2x^3 - 4x^2 - 7x - 10}$$
 Since there is a remainder,  $x - 5$  is not a factor.

$$\frac{2x^{3} - 10x^{2}}{6x^{2} - 7x}$$

$$\frac{6x^{2} - 30x}{23x - 10}$$

$$\frac{23x - 115}{105}$$

PTS: 2 REF: 061627aii NAT: A.APR.B.2 TOP: Remainder and Factor Theorems

185 ANS:

r(2) = -6. Since there is a remainder when the cubic is divided by x - 2, this binomial is not a factor.

PTS: 2 REF: 061725aii NAT: A.APR.B.2 TOP: Remainder and Factor Theorems

186 ANS:

P(-2) = 60 Q(-2) = 0 (x + 2) is a factor of Q(x) since Q(-2) = 0.

PTS: 2 REF: 081929aii NAT: A.APR.B.2 TOP: Remainder and Factor Theorems

187 ANS:

 $m(3) = 3^3 - 3^2 - 5(3) - 3 = 27 - 9 - 15 - 3 = 0$  Since m(3) = 0, there is no remainder when m(x) is divided by x - 3, and so x - 3 is a factor.

PTS: 2 NAT: A.APR.B.2 REF: 012026aii TOP: Remainder and Factor Theorems

188 ANS:

$$j(-1) = 2(-1)^4 - (-1)^3 - 35(-1)^2 + 16(-1) + 48 = 2 + 1 - 35 - 16 + 48 = 0$$
;  $x + 1$  is a factor of  $j(x)$ ;  $2x^3 - 3x^2 - 32x + 48 = 0$ 

$$x^2(2x-3) - 16(2x-3) = 0$$

$$(x^2 - 16)(2x - 3) = 0$$

$$x = \pm 4, \frac{3}{2}$$

PTS: 4 REF: 081834aii NAT: A.APR.B.2 TOP: Remainder and Factor Theorems

$$0 = 6(-5)^3 + b(-5)^2 - 52(-5) + 15 \quad z(x) = 6x^3 + 19x^2 - 52x + 15$$
$$0 = -750 + 25b + 260 + 15$$

$$475 = 25b$$

$$6x^2 - 11x + 3 = 0$$

$$(2x-3)(3x-1) = 0$$

$$x = \frac{3}{2}, \frac{1}{3}, -5$$

PTS: 4 REF: fall1515aii NAT: A.APR.B.2 TOP: Remainder and Factor Theorems

190 ANS: 3 PTS: 2 REF: 012003aii NAT: A.APR.C.4

**TOP:** Polynomial Identities

191 ANS: 2 PTS: 2 REF: 011806aii NAT: A.APR.C.4

**TOP:** Polynomial Identities

192 ANS: 1  $(x+7)(x-1) = x^2 + 6x - 7 = x^2 + 6x + 9 - 7 - 9 = (x+3)^2 - 16$ 

PTS: 2 REF: 061808aii NAT: A.APR.C.4 TOP: Polynomial Identities

193 ANS: 2 PTS: 2 REF: 012311aii NAT: A.APR.C.4

**TOP:** Polynomial Identities

194 ANS: 1

2) 
$$(x^4 - x^2y^2 + y^4) \neq (x^2 - y^2)(x^2 - y^2)$$
; 3)  $x^6 + y^6 \neq (x^3 + y^3)^2$ ; 4)  $\frac{x^6 + y^6}{x^2 + y^2} \neq x^6 + y^6 - (x^2 + y^2)$ 

PTS: 2 REF: 082219aii NAT: A.APR.C.4 TOP: Polynomial Identities

195 ANS: 4  $(x-y)^2 = x^2 - 2xy + y^2 (x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$ 

PTS: 2 REF: 061902aii NAT: A.APR.C.4 TOP: Polynomial Identities

196 ANS: 4  $(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3 \neq x^3 + 3xy + y^3$ 

PTS: 2 REF: 081620aii NAT: A.APR.C.4 TOP: Polynomial Identities

197 ANS: 4
$$(a+b+c)^{2} = a^{2} + ab + ac + ab + b^{2} + bc + ac + ab + c^{2}$$

$$x = a^{2} + b^{2} + c^{2} + 2(ab + bc + ac)$$

$$x = y + 2z$$

PTS: 2 REF: 061822aii NAT: A.APR.C.4 TOP: Polynomial Identities

198 ANS:

$$(a+b)^3 = a^3 + b^3$$
 No. Erin's shortcut only works if  $a = 0$ ,  $b = 0$  or  $a = -b$ .

$$a^3 + 3a^2b + 3ab^2 + b^3 = a^3 + b^3$$

$$3ab^2 + 3a^2b = 0$$

$$3ab(b+a) = 0$$

$$a = 0, b = 0, a = -b$$

PTS: 2 REF: 011927aii NAT: A.APR.C.4 TOP: Polynomial Identities

199 ANS:

$$\frac{x^3+9}{x^3+8} = \frac{x^3+8}{x^3+8} + \frac{1}{x^3+8}$$

$$\frac{x^3+9}{x^3+8} = \frac{x^3+9}{x^3+8}$$

PTS: 2 REF: 061631aii NAT: A.APR.C.4 TOP: Polynomial Identities

200 ANS:

$$(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$$

$$x^4 + 2x^2y^2 + y^4 = x^4 - 2x^2y^2 + y^4 + 4x^2y^2$$

$$x^4 + 2x^2y^2 + y^4 = x^4 + 2x^2y^2 + y^4$$

PTS: 2 REF: 081727aii NAT: A.APR.C.4 TOP: Polynomial Identities

201 ANS:

$$2x^{3} - 10x^{2} + 11x - 7 = 2x^{3} + hx^{2} + 3x - 8x^{2} - 4hx - 12 + k \quad h = -2$$
$$-2x^{2} + 8x + 5 = hx^{2} - 4hx + k \qquad k = 5$$

PTS: 4 REF: 011733aii NAT: A.APR.C.4 TOP: Polynomial Identities

202 ANS:

Let x equal the first integer and x + 1 equal the next.  $(x + 1)^2 - x^2 = x^2 + 2x + 1 - x^2 = 2x + 1$ . 2x + 1 is an odd integer.

PTS: 2 REF: fall1511aii NAT: A.APR.C.4 TOP: Polynomial Identities

203 ANS: 1 
$$\left(a\sqrt[3]{2b^2}\right)\left(\sqrt[3]{4a^2b}\right) = a\sqrt[3]{8a^2b^3} = 2ab\sqrt[3]{a^2}$$

PTS: 2 REF: 082213aii NAT: N.RN.A.2 TOP: Operations with Radicals

KEY: with variables, index > 2

204 ANS: 4

$$\sqrt{3x^2y} \bullet \sqrt[3]{27x^3y^2} = 3^{\frac{1}{2}}xy^{\frac{1}{2}} \bullet 3^{\frac{2}{2}}xy^{\frac{2}{3}} = 3^{\frac{3}{2}}x^2y^{\frac{7}{6}}$$

PTS: 2 REF: 081914aii NAT: N.RN.A.2 TOP: Operations with Radicals

KEY: with variables, index > 2

# Algebra II Regents Exam Questions by State Standard: Topic **Answer Section**

205 ANS: 2

$$4x \bullet x^{\frac{2}{3}} + 2x^{\frac{5}{3}} = 4x^{\frac{5}{3}} + 2x^{\frac{5}{3}} = 6x^{\frac{5}{3}} = 6\sqrt[3]{x^5}$$

PTS: 2

REF: 061820aii

NAT: N.RN.A.2 TOP: Operations with Radicals

KEY: with variables, index > 2

206 ANS: 3

$$\frac{x^{\frac{2}{3}} \cdot x^{\frac{5}{2}}}{\frac{1}{x^{\frac{1}{6}}}} = \frac{x^{\frac{4}{6}} \cdot x^{\frac{15}{6}}}{\frac{1}{x^{\frac{1}{6}}}} = x^{\frac{18}{6}} = x^{3}$$

PTS: 2

REF: 081812aii

NAT: N.RN.A.2 TOP: Operations with Radicals

KEY: with variables, index > 2

207 ANS:

$$\sqrt[3]{x} \cdot \sqrt{x} = x^{\frac{1}{3}} \cdot x^{\frac{1}{2}} = x^{\frac{3}{6}} \cdot x^{\frac{3}{6}} = x^{\frac{5}{6}}$$

PTS: 2

REF: 061731aii

NAT: N.RN.A.2

**TOP:** Operations with Radicals

KEY: with variables, index > 2

208 ANS: 2

$$x^2 = 3x + 40$$
.  $x = -5$  is an extraneous solution.

$$x^2 - 3x - 40 = 0$$

$$(x-8)(x+5)=0$$

$$x = 8, -5$$

PTS: 2

REF: 012010aii NAT: A.REI.A.2 TOP: Solving Radicals

KEY: extraneous solutions

209 ANS: 3

$$\sqrt{56-x} = x$$
 —8 is extraneous.

$$56 - x = x^2$$

$$0 = x^2 + x - 56$$

$$0 = (x + 8)(x - 7)$$

$$x = 7$$

PTS: 2

REF: 061605aii NAT: A.REI.A.2 TOP: Solving Radicals

KEY: extraneous solutions

$$b^2 = 2b^2 - 64 - 8$$
 is extraneous.

$$-b^2 = -64$$

$$b = \pm 8$$

PTS: 2 REF: 061919aii NAT: A.REI.A.2 TOP: Solving Radicals

KEY: extraneous solutions

## 211 ANS: 3

$$\sqrt{x+1} = x+1$$

$$x+1=x^2+2x+1$$

$$0 = x^2 + x$$

$$0 = x(x+1)$$

$$x = -1,0$$

PTS: 2 REF: 011802aii NAT: A.REI.A.2 TOP: Solving Radicals

KEY: extraneous solutions

### 212 ANS: 3

$$x^2 - 4x - 5 = 4x^2 - 40x + 100$$

$$3x^2 - 36x + 105 = 0$$

$$x^2 - 12x + 35 = 0$$

$$(x-7)(x-5) = 0$$

$$x = 5.7$$

PTS: 2 REF: 081807aii NAT: A.REI.A.2 TOP: Solving Radicals

KEY: extraneous solutions

$$\sqrt{x+14} = \sqrt{2x+5} + 1$$
  $\sqrt{22+14} - \sqrt{2(22)+5} = 1$ 

$$x + 14 = 2x + 5 + 2\sqrt{2x + 5} + 1$$

$$6-7 \neq 1$$

$$-x + 8 = 2\sqrt{2x + 5}$$

$$x^2 - 16x + 64 = 8x + 20$$

$$x^2 - 24x + 44 = 0$$

$$(x-22)(x-2)=0$$

$$x = 2,22$$

PTS: 2 REF: 081704aii NAT: A.REI.A.2 TOP: Solving Radicals

KEY: advanced

$$3\sqrt{x} - 2x = -5$$
 1 is extraneous.

$$3\sqrt{x} = 2x - 5$$

$$9x = 4x^2 - 20x + 25$$

$$4x^2 - 29x + 25 = 0$$

$$(4x - 25)(x - 1) = 0$$

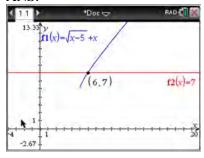
$$x = \frac{25}{4}, 1$$

PTS: 4

REF: 011936aii NAT: A.REI.A.2 TOP: Solving Radicals

KEY: extraneous solutions

215 ANS:



 $\sqrt{x-5} = -x+7$   $\sqrt{x-5} = -9+7 = -2$  is extraneous.

$$x - 5 = x^2 - 14x + 49$$

$$0 = x^2 - 15x + 54$$

$$0 = (x-6)(x-9)$$

$$x = 6, 9$$

PTS: 2

REF: spr1508aii NAT: A.REI.A.2 TOP: Solving Radicals

KEY: extraneous solutions

216 ANS:

$$\sqrt{x-4} = -x+6$$
  $\sqrt{x-4} = -8+6 = -2$  is extraneous.

$$x - 4 = x^2 - 12x + 36$$

$$0 = x^2 - 13x + 40$$

$$0 = (x - 8)(x - 5)$$

$$x = 5, 8$$

PTS: 2

REF: 061730aii NAT: A.REI.A.2

**TOP:** Solving Radicals

KEY: extraneous solutions

$$\left(\sqrt{2x-7}\right)^2 = (5-x)^2 \qquad \sqrt{2(4)-7} + 4 = 5 \quad \sqrt{2(8)-7} + 8 = 5$$

$$2x-7 = 25 - 10x + x^2 \qquad \sqrt{1} = 1 \qquad \sqrt{9} \neq -3$$

$$0 = x^2 - 12x + 32$$

$$0 = (x-8)(x-4)$$

$$x = 4.8$$

PTS: 4 REF: 081635aii NAT: A.REI.A.2 TOP: Solving Radicals

KEY: extraneous solutions

218 ANS:

$$\sqrt{4x+1} = 11-x$$
 20 is extraneous.  
 $4x+1 = 121-22x+x^2$   
 $0 = x^2 - 26x + 120$   
 $0 = (x-6)(x-20)$ 

x = 6,20

PTS: 2 REF: 082227aii NAT: A.REI.A.2 TOP: Solving Radicals

KEY: extraneous solutions

219 ANS:

$$\sqrt{49 - 10x} = 2x - 5 \qquad -\frac{3}{2} \text{ is extraneous.}$$

$$49 - 10x = 4x^2 - 20x + 25$$

$$0 = 4x^2 - 10x - 24$$

$$0 = 2x^2 - 5x - 12$$

$$0 = (2x + 3)(x - 4)$$

$$x = -\frac{3}{2}, 4$$

PTS: 4 REF: 012333aii NAT: A.REI.A.2 TOP: Solving Radicals

KEY: extraneous solutions

$$\sqrt{6-2x} + x = 2x + 30 - 9$$
  $\sqrt{6-2(-29)} \neq -29 + 21$ , so  $-29$  is extraneous.  
 $\sqrt{6-2x} = x + 21$   $\sqrt{64} \neq -8$ 

$$6 - 2x = x^2 + 42x + 441$$

$$x^2 + 44x + 435 = 0$$

$$(x+29)(x+15) = 0$$

$$x = -29, -15$$

PTS: 4 REF: 061833aii NAT: A.REI.A.2 TOP: Solving Radicals

KEY: extraneous solutions

### 221 ANS:

$$0 = \sqrt{t} - 2t + 6 \ 2\left(\frac{9}{4}\right) - 6 < 0, \text{ so } \frac{9}{4} \text{ is extraneous.}$$
$$2t - 6 = \sqrt{t}$$

$$4t^2 - 24t + 36 = t$$

$$4t^2 - 25t + 36 = 0$$

$$(4t-9)(t-4)=0$$

$$t = \frac{9}{4}, 4$$

$$(\sqrt{1} - 2(1) + 6) - (\sqrt{3} - 2(3) + 6) = 5 - \sqrt{3} \approx 3.268$$
 327 mph

•

PTS: 6 REF: 011737aii NAT: A.REI.A.2 TOP: Solving Radicals

KEY: context

#### 222 ANS:

$$t = 2\pi \sqrt{\frac{67}{9.81}} \approx 16.4 \ 9.6 = 2\pi \sqrt{\frac{L}{9.81}}$$

$$L \approx 22.9$$

PTS: 4 REF: 062234aii NAT: A.REI.A.2 TOP: Solving Radicals

KEY: context

$$B = 1.69\sqrt{30 + 4.45} - 3.49 \approx 6$$
, which is a steady breeze.

$$15 = 1.69\sqrt{s + 4.45} - 3.49$$

$$18.49 = 1.69\sqrt{s + 4.45}$$

$$\frac{18.49}{1.69} = \sqrt{s + 4.45}$$

$$\left(\frac{18.49}{1.69}\right)^2 = s + 4.45$$

$$s = \left(\frac{18.49}{1.69}\right)^2 - 4.45$$

$$s \approx 115$$

$$9.5 = 1.69\sqrt{s + 4.45} - 3.49$$

$$9.5 = 1.69\sqrt{s + 4.45} - 3.49$$
  $10.49 = 1.69\sqrt{s + 4.45} - 3.49$  55-64

$$12.99 = 1.69\sqrt{s + 4.45}$$

$$13.98 = 1.69\sqrt{s + 4.45}$$

$$\frac{12.99}{1.69} = \sqrt{s + 4.45} \qquad \frac{13.98}{1.69} = \sqrt{s + 4.45}$$

$$\frac{13.98}{1.69} = \sqrt{s + 4.45}$$

$$\left(\frac{12.99}{1.69}\right)^2 = s + 4.45$$
  $\left(\frac{13.98}{1.69}\right)^2 = s + 4.45$ 

$$\left(\frac{13.98}{1.69}\right)^2 = s + 4.45$$

$$s = \left(\frac{12.99}{1.69}\right)^2 - 4.45$$
 
$$s = \left(\frac{13.98}{1.69}\right)^2 - 4.45$$

$$s = \left(\frac{13.98}{1.69}\right)^2 - 4.45$$

$$s \approx 55$$

PTS: 6

REF: 081937aii

NAT: A.REI.A.2

**TOP:** Solving Radicals

KEY: context

224 ANS:

> The denominator of the rational exponent represents the index of a root, and the numerator of the rational exponent represents the power of the base.  $\left(\sqrt{9}\right)^5 = 243$

PTS: 2

REF: 081926aii

NAT: N.RN.A.1

TOP: Radicals and Rational Exponents

225 ANS:

The denominator of the rational exponent represents the index of a root, and the 4th root of 81 is 3 and 3<sup>3</sup> is 27.

PTS: 2

REF: 011832aii

NAT: N.RN.A.1

**TOP:** Radicals and Rational Exponents

226 ANS:

Rewrite  $\frac{4}{3}$  as  $\frac{1}{3} \cdot \frac{4}{1}$ , using the power of a power rule.

PTS: 2

REF: 081725aii

NAT: N.RN.A.1

TOP: Radicals and Rational Exponents

Applying the commutative property,  $\left(3^{\frac{1}{5}}\right)^2$  can be rewritten as  $\left(3^2\right)^{\frac{1}{5}}$  or  $9^{\frac{1}{5}}$ . A fractional exponent can be

rewritten as a radical with the denominator as the index, or  $9^{\frac{1}{5}} = \sqrt[5]{9}$ .

PTS: 2 REF: 081626aii

NAT: N.RN.A.1

TOP: Radicals and Rational Exponents

228 ANS: 1 PTS: 2 REF: 062201aii NAT: N.RN.A.2

TOP: Radicals and Rational Exponents

229 ANS: 1  $\sqrt[4]{81x^8y^6} = 81^{\frac{1}{4}} x^{\frac{8}{4}} y^{\frac{6}{4}} = 3x^2 y^{\frac{3}{2}}$ 

PTS: 2 REF: 012001aii NAT: N.RN.A.2 TOP: Radicals and Rational Exponents

KEY: variables

230 ANS: 1

$$(x^{\frac{3}{2}})^2 = x^3$$

PTS: 2 REF: 061908aii NAT: N.RN.A.2 TOP: Radicals and Rational Exponents

KEY: variables

231 ANS: 4 PTS: 2 REF: 061601aii NAT: N.RN.A.2

TOP: Radicals and Rational Exponents KEY: variables

232 ANS: 4

$$\frac{n}{m} = \frac{\sqrt{a^5}}{a} = \frac{a^{\frac{5}{2}}}{a^{\frac{2}{2}}} = a^{\frac{3}{2}} = \sqrt{a^3}$$

PTS: 2 REF: 011811aii NAT: N.RN.A.2 TOP: Radicals and Rational Exponents

KEY: variables

233 ANS: 3

$$\frac{x^{\frac{1}{5}}}{\frac{1}{x^2}} = x^{\frac{1}{5} - \frac{1}{2}} = x^{-\frac{3}{10}} = \frac{1}{\frac{3}{10}} = \frac{1}{\sqrt[10]{x^3}}$$

PTS: 2 REF: 012312aii NAT: N.RN.A.2 TOP: Radicals and Rational Exponents

$$\left(m^{\frac{5}{3}}\right)^{-\frac{1}{2}} = m^{-\frac{5}{6}} = \frac{1}{\sqrt[6]{m^5}}$$

PTS: 2 REF: 011707aii NAT: N.RN.A.2 TOP: Radicals and Rational Exponents

KEY: variables

235 ANS: 4

$$\left(\frac{-54x^9}{y^4}\right)^{\frac{2}{3}} = \frac{(2 \cdot -27)^{\frac{2}{3}} x^{\frac{18}{3}}}{y^{\frac{8}{3}}} = \frac{2^{\frac{2}{3}} \cdot 9x^6}{y^2 \cdot y^{\frac{2}{3}}} = \frac{9x^6 \sqrt[3]{4}}{y^2 \sqrt[3]{y^2}}$$

PTS: 2 REF: 081723aii NAT: N.RN.A.2 TOP: Radicals and Rational Exponents

KEY: variables

236 ANS: 4 PTS: 2 REF: 061716aii NAT: N.RN.A.2

TOP: Radicals and Rational Exponents KEY: variables

237 ANS:

$$\left(x^{\frac{5}{3}}\right)^{\frac{6}{5}} = \left(y^{\frac{5}{6}}\right)^{\frac{6}{5}}$$
$$x^{2} = y$$

PTS: 2 REF: 011730aii NAT: N.RN.A.2 TOP: Radicals and Rational Exponents

KEY: variables

238 ANS:

No. 
$$\left(\sqrt[7]{x^2}\right)\left(\sqrt[5]{x^3}\right) = x^{\frac{2}{7}} \cdot x^{\frac{3}{5}} = x^{\frac{31}{35}} = \sqrt[35]{x^{31}}$$

PTS: 2 REF: 061929aii NAT: N.RN.A.2 TOP: Radicals and Rational Exponents

KEY: variables

239 ANS:

$$\left(p^{2}n^{\frac{1}{2}}\right)^{8}\sqrt{p^{5}n^{4}} = \left(p^{16}n^{4}\right)p^{2}n^{2}\sqrt{p} = p^{18}n^{6}\sqrt{p}$$

PTS: 2 REF: 012025aii NAT: N.RN.A.2 TOP: Radicals and Rational Exponents

$$\sqrt[3]{81} = \sqrt[3]{3^4} = 3^{\frac{4}{3}}$$
  $a = \frac{4}{3}$ 

PTS: 2

KEY: variables

REF: 062230aii

NAT: N.RN.A.2

**TOP:** Radicals and Rational Exponents

241 ANS:

$$\frac{x^{\frac{8}{3}}}{x^{\frac{4}{3}}} = x^y$$

$$x^{\frac{4}{3}} = x^{3}$$

$$\frac{4}{3} = y$$

PTS: 2

REF: spr1505aii

NAT: N.RN.A.2 TOP: Radicals and Rational Exponents

KEY: numbers

242 ANS:

$$\frac{\sqrt[3]{x^2y^5}}{\sqrt[4]{x^3y^4}} = \frac{x^{\frac{2}{3}}y^{\frac{5}{3}}}{\sqrt[3]{\frac{3}{4}}} = \frac{x^{\frac{8}{12}}y^{\frac{20}{12}}}{\sqrt[9]{\frac{12}{12}}} = x^{-\frac{1}{12}}y^{\frac{2}{3}}$$

PTS: 2

REF: 011925aii

NAT: N.RN.A.2

TOP: Radicals and Rational Exponents

KEY: variables 243 ANS:

$$\frac{2x^{\frac{3}{2}}}{\frac{2}{3}} = x^{\frac{1}{2}} = \sqrt{x}$$

PTS: 2

REF: 081826aii

NAT: N.RN.A.2

TOP: Radicals and Rational Exponents

KEY: variables

244 ANS: 3

$$-3+5i-\left(4+24i-2i-12i^2\right)=-3+5i-(16+22i)=-19-17i$$

PTS: 2

REF: 081815aii

NAT: N.CN.A.2

TOP: Operations with Complex Numbers

245 ANS: 2

$$6xi^{3}(-4xi+5) = -24x^{2}i^{4} + 30xi^{3} = -24x^{2}(1) + 30x(-1) = -24x^{2} - 30xi$$

PTS: 2

REF: 061704aii

NAT: N.CN.A.2

TOP: Operations with Complex Numbers

246 ANS: 2 
$$(2-yi)(2-yi) = 4-4yi+y^2i^2 = -y^2-4yi+4$$

247 ANS: 4 
$$(x-2i)(x-2i) = x^2 - 4xi + 4i^2 = x^2 - 4xi - 4$$

248 ANS: 3  

$$(3k-2i)^2 = 9k^2 - 12ki + 4i^2 = 9k^2 - 12ki - 4$$

249 ANS: 1
$$6 - (3x - 2i)(3x - 2i) = 6 - \left(9x^2 - 12xi + 4i^2\right) = 6 - 9x^2 + 12xi + 4 = -9x^2 + 12xi + 10$$

250 ANS: 1
$$7 - 3i + x^2 - 4xi + 4i^2 - 4i - 2x^2 = 7 - 7i - x^2 - 4xi - 4 = 3 - x^2 - 4xi - 7i = (3 - x^2) - (4x + 7)i$$

251 ANS: 3
$$(x+3i)^2 - (2x-3i)^2 = x^2 + 6xi + 9i^2 - \left(4x^2 - 12xi + 9i^2\right) = -3x^2 + 18xi$$

252 ANS: 1  

$$(2x-i)^{2} - (2x-i)(2x+3i)$$

$$(2x-i)[(2x-i) - (2x+3i)]$$

$$(2x-i)(-4i)$$

$$-8xi + 4i^{2}$$

$$-8xi - 4$$

253 ANS: 4  

$$x^{3} - x^{2}yi - xy^{2} + x^{2}yi - xy^{2}i^{2} - y^{3}i = x^{3} - xy^{2} - xy^{2}(-1) - y^{3}i = x^{3} - y^{3}i$$

$$(6-ki)^2 = 27-36i$$

$$36 - 12ki + k^2i^2 = 27 - 36i$$

$$9 - k^2 - 12ki = -36i$$

Set real part equal to real part:  $9 - k^2 = 0$  Set imaginary part equal to imaginary part: -12ki = -36i

$$\frac{-12ki}{-12i} = \frac{-36i}{-12i}$$

$$k = 3$$

PTS: 2

REF: 012308aii

NAT: N.CN.A.2

TOP: Operations with Complex Numbers

255 ANS:

$$xi(-6i)^2 = xi(36i^2) = 36xi^3 = -36xi$$

PTS: 2

REF: 081627aii NAT: N.CN.A.2 TOP: Operations with Complex Numbers

256 ANS:

$$(1-i)(1-i)(1-i) = (1-2i+i^2)(1-i) = -2i(1-i) = -2i+2i^2 = -2-2i$$

PTS: 2

REF: 011725aii

NAT: N.CN.A.2 TOP: Operations with Complex Numbers

257 ANS:

$$(4-3i)(5+2yi-5+2yi)$$

$$(4 - 3i)(4yi)$$

$$16vi - 12vi^2$$

$$12y + 16yi$$

PTS: 2

REF: spr1506aii NAT: N.CN.A.2

TOP: Operations with Complex Numbers

258 ANS:

$$-\frac{1}{2}i^{3}(3i-4) - 3i^{2} = -\frac{3}{2}i^{4} + 2i^{3} - 3i^{2} = -\frac{3}{2} - 2i + 3 = \frac{3}{2} - 2i$$

PTS: 2

REF: 081927aii

NAT: N.CN.A.2

**TOP:** Operations with Complex Numbers

259 ANS:

$$i^2 = -1$$
, and not 1;  $10 + 10i$ 

PTS: 2

REF: 011825aii

NAT: N.CN.A.2

**TOP:** Operations with Complex Numbers

260 ANS: 1

$$x^2 + 2x - 8 = 0$$

$$(x+4)(x-2)=0$$

$$x = -4.2$$

PTS: 2

REF: 081701aii

NAT: A.APR.D.6 TOP: Undefined Rationals

$$\frac{-3x^2 - 5x + 2}{x^3 + 2x^2} = \frac{(-3x + 1)(x + 2)}{x^2(x + 2)} = \frac{-3x}{x^2} + \frac{1}{x^2} = -3x^{-1} + x^{-2}$$

PTS: 2

REF: 061723aii

NAT: A.APR.D.6 TOP: Expressions with Negative Exponents

KEY: variables

262 ANS:

$$\left(\frac{\frac{17}{y}}{\frac{10}{y}}\right)^{-4} = y^{n} \quad n = -\frac{7}{2}$$

$$\left(y^{\frac{7}{8}}\right)^{-4} = y^n$$

$$y^{-\frac{7}{2}} = y^n$$

PTS: 2

REF: 082228aii

NAT: A.APR.D.6 TOP: Expressions with Negative Exponents

KEY: variables

263 ANS: 4

$$\frac{x^2 - 4x}{2x} = \frac{x(x - 4)}{2x} = \frac{x - 4}{2} = \frac{x}{2} - 2 \quad \frac{x - 1}{2} - \frac{3}{2} = \frac{x - 1 - 3}{2} = \frac{x - 4}{2}$$

PTS: 2

REF: 011921aii

NAT: A.APR.D.6 TOP: Rational Expressions

KEY: factoring

264 ANS: 2

$$\frac{x^2 + 3x}{x^2 + 5x + 6} = \frac{x(x+3)}{(x+2)(x+3)}$$

PTS: 2

REF: 082215aii NAT: A.APR.D.6 TOP: Rational Expressions

KEY: factoring

265 ANS: 1

$$\frac{x(x^2-9)}{-(x^2-9)} = -x$$

PTS: 2

REF: 012023aii

NAT: A.APR.D.6 TOP: Rational Expressions

KEY: factoring

266 ANS: 3

$$\frac{c^2 - d^2}{d^2 + cd - 2c^2} = \frac{(c+d)(c-d)}{(d+2c)(d-c)} = \frac{-(c+d)}{d+2c} = \frac{-c-d}{d+2c}$$

PTS: 2

REF: 011818aii

NAT: A.APR.D.6 TOP: Rational Expressions

KEY: factoring

$$\frac{x^2(x+2)-9(x+2)}{x(x^2-x-6)} = \frac{(x^2-9)(x+2)}{x(x-3)(x+2)} = \frac{(x+3)(x-3)}{x(x-3)} = \frac{x+3}{x}$$

PTS: 2

REF: 061803aii

NAT: A.APR.D.6 TOP: Rational Expressions

KEY: factoring 268 ANS: 3

$$\frac{2x+1}{x+2)2x^2+5x+8}$$

$$2x^2 + 4x$$

$$x + 8$$

$$\underline{x+2}$$

PTS: 2

REF: 012007aii

NAT: A.APR.D.6 TOP: Rational Expressions

KEY: division

269 ANS: 2

$$x^{2} + 0x + 1$$

$$x + 2 ) x^{3} + 2x^{2} + x + 6$$

$$x^3 + 2x^2$$

$$0x^2 + x$$

$$0x^2 + 0x$$

$$x + 6$$

$$\underline{x+2}$$

PTS: 2

KEY: division

REF: 081611aii

NAT: A.APR.D.6 TOP: Rational Expressions

$$\begin{array}{r}
 2x^2 + x - 6 \\
 x + 3 \overline{\smash)2x^3 + 7x^2 - 3x - 25} \\
 \underline{2x^3 + 6x^2} \\
 x^2 - 3x \\
 \underline{x^2 + 3x} \\
 -6x - 25 \\
 \underline{-6x - 18}
 \end{array}$$

PTS: 2

-7

REF: 062203aii NAT: A.APR.D.6 TOP: Rational Expressions

KEY: division

$$2x^{3} - 4x^{2} - x + \frac{14}{x+6}$$

$$x+6) 2x^{4} + 8x^{3} - 25x^{2} - 6x + 14$$

$$2x^{4} + 12x^{3}$$

$$-4x^{3} - 25x^{2}$$

$$-4x^{3} - 24x^{2}$$

$$-x^{2} - 6x$$

$$-x^{2} - 6x$$

PTS: 2

REF: 081805aii

NAT: A.APR.D.6 TOP: Rational Expressions

KEY: division 272 ANS: 4

$$\begin{array}{c|c}
5x^2 + x - 3 \\
2x - 1 ) 10x^3 - 3x^2 - 7x + 3
\end{array}$$

$$\frac{10x^3 - 5x^2}{2x^2 - 7x}$$

$$\frac{2x^2 - x}{-6x + 3}$$

PTS: 2

REF: 011809aii

NAT: A.APR.D.6 TOP: Rational Expressions

KEY: division

$$\begin{array}{r}
3x^2 + 4x - 1 \\
2x + 3 \overline{\smash{\big)}\ 6x^3 + 17x^2 + 10x + 2} \\
\underline{6x^3 + 9x^2} \\
8x^2 + 10x \\
\underline{-2x + 2} \\
-2x - 3
\end{array}$$

PTS: 2

REF: fall1503aii NAT: A.APR.D.6 TOP: Rational Expressions

KEY: division

274 ANS: 2

$$x^2 + 3 ) x^2 + 0x + 12$$

$$\underline{x^2 + 0x + 3}$$

PTS: 2

REF: 062218aii NAT: A.APR.D.6 TOP: Rational Expressions

KEY: division

275 ANS: 2

$$x^{2} + 2x + 4$$

$$x - 2 ) x^{3} - 0x^{2} + 0x - 2$$

$$\underline{x^3 - 2x^2}$$

$$2x^2 + 0x$$

$$2x^2-4x$$

$$4x - 2$$

$$4x-8$$

6

PTS: 2

KEY: division

REF: 082217aii NAT: A.APR.D.6 TOP: Rational Expressions

$$3x + 1 ) 9x^2 + 0x - 2$$

$$9x^2 + 3x$$
$$-3x - 2$$
$$-3x - 1$$

- 1

PTS: 2

REF: 081910aii

NAT: A.APR.D.6 TOP: Rational Expressions

KEY: division

277 ANS: 2

$$4x^3 + 6x^2$$

$$-6x^2 + 5x$$

$$-6x^2 - 9x$$

$$14x + 10$$

$$\underline{14x + 21}$$

PTS: 2

REF: 061614aii

NAT: A.APR.D.6 TOP: Rational Expressions

KEY: division 278 ANS: 1

$$2x^2 + x + 5$$
$$2x - 1 ) 4x^3 + 0x^2 + 9x - 5$$

$$4x^3 - 2x^2$$

$$2x^2 + 9x$$

$$2x^2-x$$

$$10x - 5$$

$$10x - 5$$

PTS: 2

REF: 081713aii

NAT: A.APR.D.6 TOP: Rational Expressions

KEY: division

$$\frac{x^3 - 2x^2 - x + 6}{x + 2 \int x^4 + 0x^3 - 5x^2 + 4x + 14}$$

$$\frac{x^{4} + 2x^{3}}{-2x^{3} - 5x^{2}}$$

$$-2x^{3} - 5x^{2}$$

$$-x^{2} + 4x$$

$$-x^{2} - 2x$$

$$6x + 14$$

$$6x + 12$$

PTS: 2

KEY: division

REF: 012305aii NAT: A.APR.D.6 TOP: Rational Expressions

280 ANS:

$$\begin{array}{r}
3x+13 \\
x-2 \overline{\smash)3x^2 + 7x - 20} \\
3x+13 + \frac{6}{x-2} \\
\underline{3x^2 - 6x} \\
13x-20 \\
\underline{13x-26}
\end{array}$$

PTS: 2

REF: 011732aii NAT: A.APR.D.6 TOP: Rational Expressions

KEY: division

$$\frac{3x^{2} + 8x + 34}{x - 4}$$

$$x - 4 \overline{\smash)3x^{3} - 4x^{2} + 2x - 1}$$

$$3x^{2} + 8x + 34 + \frac{135}{x - 4}$$

$$x = 4 \text{ is not a root of } f(x) \text{ because } \frac{f(x)}{g(x)} \text{ has a remainder.}$$

$$\frac{3x^{3} - 12x^{2}}{8x^{2} + 2x}$$

$$\frac{8x^{2} - 32x}{34x - 1}$$

$$\frac{34x - 136}{135}$$

PTS: 4 REF: 082235aii NAT: A.APR.D.6 TOP: Rational Expressions

KEY: division 282 ANS:

$$\frac{2a^{2} + 5a + 2}{3a - 2) 6a^{3} + 11a^{2} - 4a - 9} 2a^{2} + 5a + 2 - \frac{5}{3a - 2}$$

$$\underline{6a^{3} - 4a^{2}}$$

$$15a^{2} - 4a$$

$$\underline{15a^{2} - 10a}$$

$$6a - 9$$

<u>6a – 4</u>

PTS: 2 REF: 061829aii NAT: A.APR.D.6 TOP: Rational Expressions

**KEY**: division

283 ANS:

$$(x+2)$$
 $\frac{x^3}{x^4+2x^3+4x-10}$  $x^3+4-\frac{18}{x+2}$ . No, because there is a remainder.

$$\frac{x^4 + 2x^3}{4x - 10}$$

$$\frac{4x + 8}{-18}$$

PTS: 4 REF: 011934aii NAT: A.APR.D.6 TOP: Rational Expressions

KEY: division

$$\frac{p(x)}{x-1} = x^2 + 7 + \frac{5}{x-1}$$

$$p(x) = x^3 - x^2 + 7x - 7 + 5$$

$$p(x) = x^3 - x^2 + 7x - 2$$

PTS: 2

REF: 061930aii

NAT: A.APR.D.6

**TOP:** Rational Expressions

KEY: division

$$2 - \frac{x-1}{x+2}$$

$$1 + \frac{x+2}{x+2} - \frac{x-1}{x+2}$$

$$1 + \frac{x + 2 - (x - 1)}{x + 2}$$

$$1 + \frac{3}{x+2}$$

PTS: 2

REF: 081907aii

NAT: A.APR.D.7 TOP: Addition and Subtraction of Rationals

286 ANS: 1

$$x - \frac{20}{x} = 8$$

$$x^2 - 8x - 20 = 0$$

$$(x-10)(x+2)=0$$

$$x = 10, -2$$

PTS: 2 REF: 061916aii NAT: A.CED.A.1 TOP: Modeling Rationals

287 ANS: 3 PTS: 2 REF: 061602aii NAT: A.CED.A.1

**TOP:** Modeling Rationals

REF: 061824aii NAT: A.CED.A.1 288 ANS: 3 PTS:

TOP: Modeling Rationals

289 ANS: 2 REF: 082222aii NAT: A.CED.A.1 PTS:

**TOP:** Modeling Rationals

290 ANS: 3 NAT: A.CED.A.1 PTS: REF: 061722aii

**TOP:** Modeling Rationals

$$\frac{1}{J} = \frac{1}{F} - \frac{1}{W}$$

$$\frac{1}{J} = \frac{W - F}{FW}$$

$$J = \frac{FW}{W - F}$$

PTS: 2

REF: 081617aii

NAT: A.REI.A.2

TOP: Solving Rationals

KEY: rational solutions

292 ANS: 4

$$\frac{2}{x} = \frac{4x}{x+3}$$

$$2x + 6 = 4x^2$$

$$4x^2 - 2x - 6 = 0$$

$$2\left(2x^2 - x - 3\right) = 0$$

$$(2x - 3)(x + 1) = 0$$

$$x = \frac{3}{2}, -1$$

PTS: 2

REF: 061809aii

NAT: A.REI.A.2

**TOP:** Solving Rationals

293 ANS: 3

$$\frac{2}{3x+1} = \frac{1}{x} - \frac{6x}{3x+1} - \frac{1}{3}$$
 is extraneous.

$$\frac{6x+2}{3x+1} = \frac{1}{x}$$

$$6x^2 + 2x = 3x + 1$$

$$6x^2 - x - 1 = 0$$

$$(2x-1)(3x+1) = 0$$

$$x = \frac{1}{2}, -\frac{1}{3}$$

PTS: 2

REF: 011915aii

NAT: A.REI.A.2

**TOP:** Solving Rationals

$$\frac{x+2}{x} + \frac{x}{3} = \frac{2x^2 + 6}{3x}$$
 0 is extraneous.

$$\frac{x^2 + 3x + 6}{3x} = \frac{2x^2 + 6}{3x}$$

$$x^2 + 3x + 6 = 2x^2 + 6$$

$$x^2 - 3x = 0$$

$$x(x-3)=0$$

$$x = 0.3$$

PTS: 2

REF: 012309aii NAT: A.REI.A.2 TOP: Solving Rationals

295 ANS: 4

$$x(x-2)\left(\frac{10}{x^2-2x} + \frac{4}{x} = \frac{5}{x-2}\right)$$
 2 is extraneous.

$$10 + 4(x - 2) = 5x$$

$$10 + 4x - 8 = 5x$$

$$2 = x$$

PTS: 2

REF: 081915aii NAT: A.REI.A.2

**TOP:** Solving Rationals

KEY: rational solutions

296 ANS: 3

$$\frac{4}{k^2 - 8k + 12} = \frac{k(k - 6) + (k - 2)}{k^2 - 8k + 12} \quad k = 6 \text{ is extraneous}$$
$$4 = k^2 - 6k + k - 2$$
$$0 = k^2 - 5k - 6$$

$$0 = (k - 6)(k + 1)$$

$$k = 6, -1$$

PTS: 2

REF: 082218aii NAT: A.REI.A.2 TOP: Solving Rationals

297 ANS: 1

$$\frac{2(x-4)}{(x+3)(x-4)} + \frac{3(x+3)}{(x-4)(x+3)} = \frac{2x-2}{x^2 - x - 12}$$
$$2x - 8 + 3x + 9 = 2x - 2$$

$$3x = -3$$

$$x = -1$$

PTS: 2

REF: 011717aii NAT: A.REI.A.2

**TOP:** Solving Rationals

KEY: rational solutions

$$x - \frac{4}{x - 1} = 2$$
  $x = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-2)}}{2(1)} = \frac{3 \pm \sqrt{17}}{2}$ 

$$x(x-1)-4=2(x-1)$$

$$x^2 - x - 4 = 2x - 2$$

$$x^2 - 3x - 2 = 0$$

PTS: 2 REF: 011812aii NAT: A.REI.A.2 TOP: Solving Rationals

KEY: rational solutions

299 ANS: 4

$$x(x+7)\left[\frac{3x+25}{x+7} - 5 = \frac{3}{x}\right]$$

$$x(3x+25) - 5x(x+7) = 3(x+7)$$

$$3x^2 + 25x - 5x^2 - 35x = 3x + 21$$

$$2x^2 + 13x + 21 = 0$$

$$(2x+7)(x+3) = 0$$

$$x = -\frac{7}{2}, -3$$

PTS: 2 REF: fall1501aii NAT: A.REI.A.2 TOP: Solving Rationals

KEY: rational solutions

300 ANS: 1

$$\frac{2x}{x-2} \left( \frac{x}{x} \right) - \frac{11}{x} \left( \frac{x-2}{x-2} \right) = \frac{8}{x^2 - 2x}$$

$$2x^2 - 11x + 22 = 8$$

$$2x^2 - 11x + 14 = 0$$

$$(2x - 7)(x - 2) = 0$$

$$x = \frac{7}{2}, 2$$

PTS: 2 REF: 061719aii NAT: A.REI.A.2 TOP: Solving Rationals

$$\frac{3}{n} = \frac{2}{n^2}$$
 0 is an extraneous solution.

$$3n^2 = 2n$$

$$3n^2 - 2n = 0$$

$$n(3n-2)=0$$

$$n = 0, \frac{2}{3}$$

PTS: 2

REF: 062227aii NAT: A.REI.A.2

**TOP:** Solving Rationals

302 ANS:

$$\frac{3p}{p-5} = \frac{p+2}{p+3}$$

$$3p^2 + 9p = p^2 - 3p - 10$$

$$2p^2 + 12p + 10 = 0$$

$$p^2 + 6p + 5 = 0$$

$$(p+5)(p+1) = 0$$

$$p = -5, -1$$

PTS: 4

REF: 081733aii

NAT: A.REI.A.2

**TOP:** Solving Rationals

KEY: rational solutions

303 ANS:

$$\frac{1}{x} - \frac{1}{3} = -\frac{1}{3x}$$

$$\frac{3-x}{3x} = -\frac{1}{3x}$$

$$3 - x = -1$$

$$x = 4$$

PTS: 2

REF: 061625aii

NAT: A.REI.A.2

**TOP:** Solving Rationals

KEY: rational solutions

$$-6(x+3)\left(\frac{-3}{x+3} - \frac{x}{6} + 1 = 0\right)$$

$$18 + x(x+3) - 6(x+3) = 0$$

$$18 + x^2 + 3x - 6x - 18 = 0$$

$$x^2 - 3x = 0$$

$$x(x-3)=0$$

$$x = 0.3$$

PTS: 2

REF: 081829aii

NAT: A.REI.A.2

**TOP:** Solving Rationals

KEY: rational solutions

305 ANS:

$$\frac{7}{2x} - \frac{2}{x+1} = \frac{1}{4}$$

$$\frac{7x + 7 - 4x}{2x^2 + 2x} = \frac{1}{4}$$

$$2x^2 + 2x = 12x + 28$$

$$x^2 - 5x - 14 = 0$$

$$(x-7)(x+2) = 0$$

$$x = 7, -2$$

PTS: 2

REF: 061926aii

NAT: A.REI.A.2

**TOP:** Solving Rationals

KEY: rational solutions

306 ANS:

$$\frac{1}{8} + \frac{1}{6} = \frac{1}{t_b}$$
;  $\frac{24t_b}{8} + \frac{24t_b}{6} = \frac{24t_b}{t_b}$ 

$$3t_b + 4t_b = 24$$

$$t_b = \frac{24}{7} \approx 3.4$$

PTS: 2

REF: 011827aii

NAT: A.REI.A.2

**TOP:** Solving Rationals

antibiotic 
$$n(0) = \frac{0+1}{0+5} + \frac{18}{0^2 + 8(0) + 15} = \frac{3}{15} + \frac{18}{15} = \frac{21}{15}$$
  $\frac{t+1}{t+5} + \frac{18}{t^2 + 8t + 15} = \frac{9}{t+3}$ 

$$a(0) = \frac{9}{0+3} = 3$$
  $\frac{(t+1)(t+3)}{(t+5)(t+3)} + \frac{18}{(t+3)(t+5)} = \frac{9(t+5)}{(t+3)(t+5)}$ 

$$t^2 + 4t + 3 + 18 = 9t + 45$$

$$t^2 - 5t - 24 = 0$$

$$(t-8)(t+3) = 0$$

$$t = 8$$

PTS: 6 REF: 012037aii NAT: A.REI.A.2 TOP: Solving Rationals

KEY: rational solutions

308 ANS: 2

$$2x + 4y - 2z = 2 - x - 3y + 2z = 0 x + y = 2 3 + 2y - z = 1 2y - z = -2$$

$$-x - 3y + 2z = 0 4x - 8y + 2z = 20 x - y = 4 6 - 4y + z = 10 2(-1) - z = -2$$

$$x + y = 2 5x - 5y = 20 2x = 6 2y - z = -2 z = 0$$

$$x - y = 4 x = 3 -4y + z = 4$$

$$-2y = 2$$

$$y = -1$$

PTS: 2 REF: 062208aii NAT: A.REI.C.6 TOP: Solving Linear Systems

KEY: three variables

309 ANS: 4

$$3x - (-2x + 14) = 16$$
  $3(6) - 4z = 2$   
 $5x = 30$   $-4z = -16$   
 $x = 6$   $z = 4$ 

PTS: 2 REF: 011803aii NAT: A.REI.C.6 TOP: Solving Linear Systems

KEY: three variables

310 ANS: 2

Combining (1) and (3): 
$$-6c = -18$$
 Combining (1) and (2):  $5a + 3c = -1$  Using (3):  $-(-2) - 5b - 5(3) = 2$ 

$$c = 3$$

$$5a + 3(3) = -1$$

$$2 - 5b - 15 = 2$$

$$5a = -10$$

$$b = -3$$

$$a = -2$$

PTS: 2 REF: 081623aii NAT: A.REI.C.6 TOP: Solving Linear Systems

KEY: three variables

$$x+y-z=6 2x+2y-2z=12 5y-4z=31 5y-2(-4)=23 x+3-(-4)=6$$

$$-x+4y-z=17 2x-3y+2z=-19 5y-2z=23 5y=15 x=-1$$

$$5y-2z=23 5y-4z=31 -2z=8 y=3$$

$$z=-4$$

PTS: 2 REF: 061923aii NAT: A.REI.C.6 TOP: Solving Linear Systems

KEY: three variables

## 312 ANS: 1

$$x+y+z=9 4-y-z=-1 4-6+z=9$$

$$x-y-z=-1 4-y+z=21 z=11$$

$$2x=8 -y-z=-5$$

$$x=4 -y+z=17$$

$$-2y=12$$

$$y=-6$$

PTS: 2 REF: 012018aii NAT: A.REI.C.6 TOP: Solving Linear Systems

KEY: three variables

#### 313 ANS:

$$6x - 3y + 2z = -10 \quad x + 3y + 5z = 45 \quad 4x + 10z = 62 \quad 4x + 4(7) = 20 \quad 6(-2) - 3y + 2(7) = -10$$

$$-2x + 3y + 8z = 72 \quad 6x - 3y + 2z = -10 \quad 4x + 4z = 20 \quad 4x = -8 \quad -3y = -12$$

$$4x + 10z = 62 \quad 7x + 7z = 35 \quad 6z = 42 \quad x = -2 \quad y = 4$$

$$4x + 4z = 20 \quad z = 7$$

PTS: 4 REF: spr1510aii NAT: A.REI.C.6 TOP: Solving Linear Systems

KEY: three variables

#### 314 ANS:

$$x + y + z = 1$$
  $x + y + z = 1$   $x + y + z = 1$   $-2z - z = 3$   $y - (-1) = 3$   $x + 2 - 1 = 1$   
 $x + 2y + 3z = 1$   $x + 2y + 3z = 1$   $-x + 3y - 5z = 11$   $-3z = 3$   $y = 2$   $x = 0$   
 $-x + 3y - 5z = 11$   $y + 2z = 0$   $4y - 4z = 12$   $z = -1$   
 $y = -2z$   $y - z = 3$ 

PTS: 4 REF: 061733aii NAT: A.REI.C.6 TOP: Solving Linear Systems

KEY: three variables

$$4x + 6y - 8z = -2 \quad 4x + 6y - 8z = -2 \quad 4x - 8y + 20z = 12 \quad z + 2 = 3z - 4 \quad y = 3 + 2 \quad -4x + 5 + 3 = 16$$

$$4x - 8y + 20z = 12 \quad \underline{-4x + y + z = 16} \quad \underline{-4x + y + z = 16} \quad 6 = 2z \qquad = 5 \qquad -4x = 8$$

$$-4x + y + z = 16 \quad 7y - 7z = 14 \quad -7y + 21z = 28 \quad z = 3 \quad x = -2$$

$$y - z = 2 \quad y - 3z = -4$$

$$y = z + 2 \quad y = 3z - 4$$

PTS: 4 REF: 081833aii NAT: A.REI.C.6 TOP: Solving Linear Systems

KEY: three variables

316 ANS:

$$a + 4b + 6c = 23 \qquad a + 2b + c = 2 \qquad 8b + 3c = 16 \quad 2b + 5(4) = 21 \quad a + 4\left(\frac{1}{2}\right) + 6(4) = 23$$

$$a + 2b + c = 2 \qquad -a + 6b + 2c = 14 \qquad 8b + 20c = 84 \qquad 2b = 1$$

$$2b + 5c = 21 \qquad 8b + 3c = 16 \qquad 17c = 68$$

$$c = 4 \qquad b = \frac{1}{2} \qquad a = -3$$

PTS: 4 REF: 011933aii NAT: A.REI.C.6 TOP: Solving Linear Systems

KEY: three variables

317 ANS: 4

$$y = g(x) = (x-2)^{2} (x-2)^{2} = 3x-2 y = 3(6)-2 = 16$$

$$x^{2}-4x+4 = 3x-2 y = 3(1)-2 = 1$$

$$x^{2}-7x+6 = 0$$

$$(x-6)(x-1) = 0$$

$$x = 6, 1$$

PTS: 2 REF: 011705aii NAT: A.REI.C.7 TOP: Quadratic-Linear Systems

318 ANS: 4

$$\frac{1}{2}x^{2} + 2x = \frac{1}{4}x - 8 \qquad b^{2} - 4ac$$

$$2x^{2} + 8x = x - 32 \qquad 7^{2} - 4(2)(32) < 0$$

$$2x^{2} + 7x + 32 = 0$$

PTS: 2 REF: 012310aii NAT: A.REI.C.7 TOP: Quadratic-Linear Systems

$$x^2 + 4x - 1 = x - 3 \quad y + 3 = -1$$

$$x^2 + 3x + 2 = 0 y = -4$$

$$(x+2)(x+1) = 0$$

$$x = -2, -1$$

PTS: 2

REF: 061801aii NAT: A.REI.C.7

TOP: Quadratic-Linear Systems

## 320 ANS: 3

$$(x+4)^2 - 10 = 3x + 6$$
  $y = 3(-5) + 6 = -9$ 

$$x^{2} + 8x + 16 - 10 = 3x + 6$$
  $y = 3(0) + 6 = 6$ 

$$x^2 + 5x = 0$$

$$x(x+5)=0$$

$$x = -5,0$$

PTS: 2

REF: 061903aii NAT: A.REI.C.7 TOP: Quadratic-Linear Systems

## 321 ANS: 3

$$x^2 + (2x)^2 = 5$$
  $y = 2x = \pm 2$ 

$$x^2 + 4x^2 = 5$$

$$5x^2 = 5$$

$$x = \pm 1$$

PTS: 2

REF: 081916aii NAT: A.REI.C.7 TOP: Quadratic-Linear Systems

### 322 ANS: 1

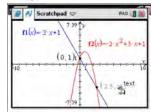
$$(x+3)^2 + (2x-4)^2 = 8$$
  $b^2 - 4ac$ 

$$x^2 + 6x + 9 + 4x^2 - 16x + 16 = 8 \quad 100 - 4(5)(17) < 0$$

$$5x^2 - 10x + 17 = 0$$

PTS: 2

REF: 081719aii NAT: A.REI.C.7 TOP: Quadratic-Linear Systems



$$-2x + 1 = -2x^2 + 3x + 1$$

$$2x^2 - 5x = 0$$

$$x(2x-5)=0$$

$$x = 0, \frac{5}{2}$$

PTS: 2 REF: fall1507aii NAT: A.REI.C.7 TOP: Quadratic-Linear Systems

324 ANS:

$$2x^{2} - 7x + 4 = 11 - 2x \quad y = 11 - 2\left(\frac{7}{2}\right) = 4 \quad \left\{ \left(\frac{7}{2}, 4\right), (-1, 13) \right\}$$
$$2x^{2} - 5x - 7 = 0$$
$$y = 11 - 2(-1) = 13$$
$$(2x - 7)(x + 1) = 0$$
$$x = \frac{7}{2}, -1$$

PTS: 2 REF: 082232aii NAT: A.REI.C.7 TOP: Quadratic-Linear Systems

325 ANS:

$$x^{2} + (x - 28)^{2} = 400$$
  $y = 12 - 28 = -16$   $y = 16 - 28 = -12$ 

$$x^2 + x^2 - 56x + 784 = 400$$

$$2x^2 - 56x + 384 = 0$$

$$x^2 - 28x + 192 = 0$$

$$(x - 16)(x - 12) = 0$$

$$x = 12,16$$

PTS: 2 REF: 081831aii NAT: A.REI.C.7 TOP: Quadratic-Linear Systems

326 ANS:

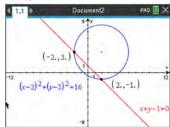
$$x^{2} + (2x - 5)^{2} = 25$$
  $y + 5 = 2(0)$   $y + 5 = 2(4)$   $(0, -5), (4, 3)$   
 $x^{2} + 4x^{2} - 20x + 25 = 25$   $y = -5$   $y = 3$ 

$$5x^2 - 20x = 0$$

$$5x(x-4) = 0$$

$$x = 0.4$$

PTS: 4 REF: 062236aii NAT: A.REI.C.7 TOP: Quadratic-Linear Systems



$$y = -x + 1$$
  $y = -2 + 1 = -1$  (2,-1)

$$(x-2)^2 + (-x+1-3)^2 = 16$$
  $y = 2+1=3$  (-2,3)

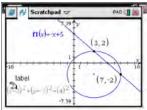
$$x^2 - 4x + 4 + x^2 + 4x + 4 = 16$$

$$2x^2 = 8$$

$$x = -2.2$$

PTS: 4 REF: 012035aii NAT: A.REI.C.7 TOP: Quadratic-Linear Systems

328 ANS:



$$y = -x + 5$$
  $y = -7 + 5 = -2$ 

$$(x-3)^2 + (-x+5+2)^2 = 16$$
  $y = -3+5=2$ 

$$x^2 - 6x + 9 + x^2 - 14x + 49 = 16$$

$$2x^2 - 20x + 42 = 0$$

$$x^2 - 10x + 21 = 0$$

$$(x-7)(x-3)=0$$

$$x = 7,3$$

PTS: 4 REF: 061633aii NAT: A.REI.C.7 TOP: Quadratic-Linear Systems

329 ANS: 3

$$-33t^2 + 360t = 700 + 5t$$

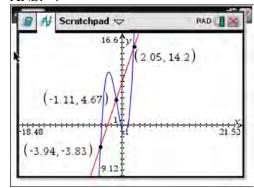
$$-33t^2 + 355t - 700 = 0$$

$$t = \frac{-355 \pm \sqrt{355^2 - 4(-33)(-700)}}{2(-33)} \approx 3.8$$

PTS: 2 REF: 081606aii NAT: A.REI.D.11 TOP: Quadratic-Linear Systems

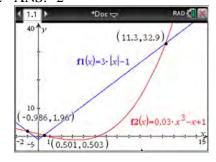
330 ANS: 4 PTS: 2 REF: 061914aii NAT: A.REI.D.11

TOP: Other Systems



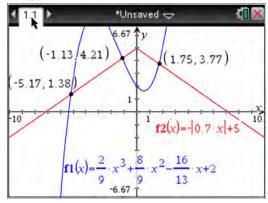
PTS: 2 REF: 061622aii NAT: A.REI.D.11 TOP: Other Systems

332 ANS: 2



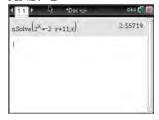
PTS: 2 REF: 061705aii NAT: A.REI.D.11 TOP: Other Systems

333 ANS:

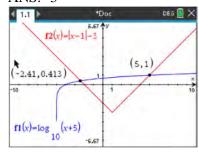


PTS: 2 REF: fall1510aii NAT: A.REI.D.11 TOP: Other Systems

334 ANS: 2

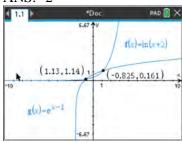


PTS: 2 REF: 081603aii NAT: A.REI.D.11 TOP: Other Systems



PTS: 2 REF: 012317aii NAT: A.REI.D.11 TOP: Other Systems

336 ANS: 2



PTS: 2 REF: 081920aii NAT: A.REI.D.11 TOP: Other Systems

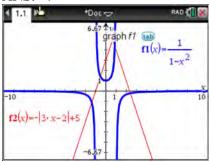
337 ANS: 3 PTS: 2 REF: 081819aii NAT: A.REI.D.11

TOP: Other Systems

338 ANS: 1 PTS: 2 REF: 011814aii NAT: A.REI.D.11

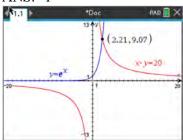
TOP: Other Systems

339 ANS: 4

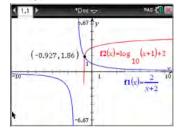


PTS: 2 REF: 011924aii NAT: A.REI.D.11 TOP: Other Systems

340 ANS: 1



PTS: 2 REF: 082210aii NAT: A.REI.D.11 TOP: Other Systems

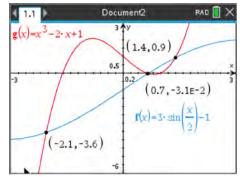


PTS: 2

REF: 011712aii

NAT: A.REI.D.11 TOP: Other Systems

342 ANS: 2



PTS: 2

REF: 012021aii

NAT: A.REI.D.11 TOP: Other Systems

343 ANS: 1

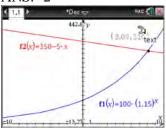
 $1240(1.06)^x = 890(1.11)^x$ 

 $x \approx 7$ 

PTS: 2

REF: 061814aii NAT: A.REI.D.11 TOP: Other Systems

344 ANS: 2



PTS: 2

REF: 011716aii

NAT: A.REI.D.11 TOP: Other Systems

$$20e^{.05t} = 30e^{.03t}$$

$$\frac{\frac{2}{3}e^{.05t}}{e^{.05t}} = \frac{e^{.03t}}{e^{.05t}}$$

$$\ln\frac{2}{3} = \ln e^{-.02t}$$

$$\ln\frac{2}{3} = -.02t \ln e$$

$$\frac{\ln\frac{2}{3}}{-.02} = 1$$

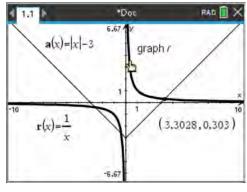
$$20.3 \approx t$$

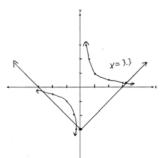
PTS: 2

REF: 011829aii

NAT: A.REI.D.11 TOP: Other Systems

346 ANS:





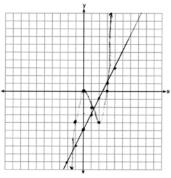
PTS: 2

REF: 081932aii

NAT: A.REI.D.11

TOP: Other Systems

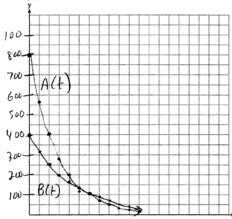
347 ANS:



PTS: 4

REF: 062233aii

NAT: A.REI.D.11 TOP: Other Systems



$$A(t) = 800e^{-0.347t}$$

$$B(t) = 400e^{-0.231t}$$

$$800e^{-0.347t} = 400e^{-0.231t} \qquad 0.15 = e^{-0.347t}$$

$$\ln 2e^{-0.347t} = \ln e^{-0.231t} \quad \ln$$

$$\ln 0.15 = \ln e^{-0.347t}$$

$$\ln 2 + \ln e^{-0.347t} = \ln e^{-0.231t}$$

$$\ln 0.15 = -0.347t \cdot \ln e$$

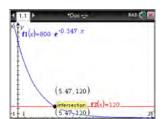
$$\ln 2 - 0.347t = -0.231t$$

$$5.5\approx t$$

$$\ln 2 = 0.116t$$

$$6 \approx t$$

(5.98, 101) r2(x)=400 e<sup>-0</sup> 231



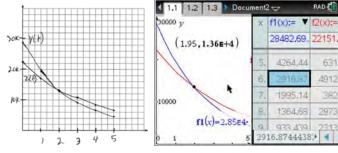
PTS: 6

REF: 061637aii

NAT: A.REI.D.11

6311 4912. TOP: Other Systems

349 ANS:



At 1.95 years, the value of the car equals the loan

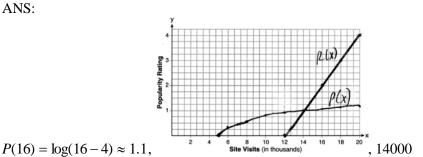
balance. Zach can cancel the policy after 6 years.

PTS: 4

REF: 081737aii

NAT: A.REI.D.11

TOP: Other Systems



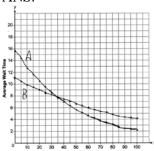
PTS: 6

REF: 061837aii

NAT: A.REI.D.11

TOP: Other Systems

351 ANS:



humber of CSRs between the plans. 35;  $B(100) - A(100) \approx 2$ , which represents the difference of the average wait time when there are 100 CSRs between the plans.

PTS: 6

REF: 082237aii

NAT: A.REI.D.11

TOP: Other Systems

352 ANS: 3

PTS: 2

REF: 011710aii

NAT: F.BF.A.1

TOP: Operations with Functions

353 ANS: 4

PTS: 2

REF: 081803aii

NAT: F.BF.A.1

**TOP:** Operations with Functions

354 ANS: 4

$$\frac{m(c)}{g(c)} = \frac{c+1}{1-c^2} = \frac{c+1}{(1+c)(1-c)} = \frac{1}{1-c}$$

PTS: 2

REF: 061608aii

NAT: F.BF.A.1

**TOP:** Operations with Functions

355 ANS: 3

$$x^{2} - 6x + 9 - \left(x^{2} + 6x + 9\right) = -12x$$

PTS: 2

REF: 062210aii

NAT: F.BF.A.1

**TOP:** Operations with Functions

356 ANS: 2

$$x(30 - 0.01x) - (0.15x^3 + 0.01x^2 + 2x + 120) = 30x - 0.01x^2 - 0.15x^3 - 0.01x^2 - 2x - 120$$

$$= -0.15x^3 - 0.02x^2 + 28x - 120$$

PTS: 2

REF: 061709aii

NAT: F.BF.A.1

TOP: Operations with Functions

$$p(x) = r(x) - c(x)$$

$$-0.5x^{2} + 250x - 300 = -0.3x^{2} + 150x - c(x)$$

$$c(x) = 0.2x^{2} - 100x + 300$$

PTS: 2

REF: 061813aii

NAT: F.BF.A.1

TOP: Operations with Functions

358 ANS: 3

PTS: 2

REF: 012002aii

NAT: F.BF.A.1

TOP: Operations with Functions

$$(2x^{2} + x - 3) \bullet (x - 1) - [(2x^{2} + x - 3) + (x - 1)]$$
$$(2x^{3} - 2x^{2} + x^{2} - x - 3x + 3) - (2x^{2} + 2x - 4)$$
$$2x^{3} - 3x^{2} - 6x + 7$$

PTS: 4

REF: 011833aii

NAT: F.BF.A.1

**TOP:** Operations with Functions

360 ANS:

$$(x^3 + 2x - 1)(x^2 + 7) - 3(x^4 - 5x)$$

$$x^5 + 7x^3 + 2x^3 + 14x - x^2 - 7 - 3x^4 + 15x$$

$$x^5 - 3x^4 + 9x^3 - x^2 + 29x - 7$$

PTS: 2

REF: 012330aii

NAT: F.BF.A.1

TOP: Operations with Functions

361 ANS: 3

PTS: 2

REF: 061906aii

NAT: F.LE.A.2

TOP: Families of Functions

362 ANS: 1

2) linear, 3) quadratic, 4) cubic

PTS: 2

REF: 061920aii

NAT: F.LE.A.2

**TOP:** Families of Functions

363 ANS: 1

PTS: 2

REF: 081903aii

NAT: F.LE.A.2

TOP: Families of Functions

364 ANS: 4

The maximum of p is 5. The minimum of f is  $-\frac{21}{4}$   $(x = \frac{-6}{2(4)} = -\frac{3}{4})$ 

$$f\left(-\frac{3}{4}\right) = 4\left(-\frac{3}{4}\right)^2 + 6\left(-\frac{3}{4}\right) - 3 = 4\left(\frac{9}{16}\right) - \frac{18}{4} - \frac{12}{4} = -\frac{21}{4}$$
).  $\frac{20}{4} - \left(-\frac{21}{4}\right) = \frac{41}{4} = 10.25$ 

PTS: 2

REF: 011922aii

NAT: F.IF.C.9

**TOP:** Comparing Functions

365 ANS: 1

PTS: 2

REF: 081804aii

NAT: F.IF.C.9

**TOP:** Comparing Functions

$$g(x)$$
:  $\frac{10-6}{4-2} = 2$   $t(x)$ :  $\frac{3--5}{4-2} = 4$ 

PTS: 2

REF: 062212ai

NAT: F.IF.C.9

**TOP:** Comparing Functions

367 ANS: 2

PTS: 2

REF: 062222aii

NAT: F.IF.C.9

TOP: Comparing Functions 368 ANS: 2

h(x) does not have a y-intercept.

PTS: 2

REF: 011719aii

NAT: F.IF.C.9

**TOP:** Comparing Functions

369 ANS:

$$\frac{f(4)-f(-2)}{4--2} = \frac{80-1.25}{6} = 13.125$$
 g(x) has a greater rate of change

$$\frac{g(4) - g(-2)}{4 - -2} = \frac{179 - -49}{6} = 38$$

PTS: 4

REF: 061636aii

NAT: F.IF.C.9

**TOP:** Comparing Functions

370 ANS:

 $0 = \log_{10}(x - 4)$  The x-intercept of h is (2,0). f has the larger value.

$$10^0 = x - 4$$

$$1 = x - 4$$

$$x = 5$$

PTS: 2

REF: 081630aii

NAT: F.IF.C.9

**TOP:** Comparing Functions

371 ANS:

q has the smaller minimum value for the domain [-2,2]. h's minimum is -1(2(-1)+1) and q's minimum is -8.

PTS: 2

REF: 011830aii

NAT: F.IF.C.9

**TOP:** Comparing Functions

372 ANS: 4

PTS: 2

REF: 081817aii

NAT: F.BF.B.3

TOP: Transformations with Functions

373 ANS: 3

PTS: 2

REF: 062205aii

NAT: F.BF.B.3

TOP: Transformations with Functions

374 ANS: 2

PTS: 2

REF: 081911aii

NAT: F.BF.B.3

TOP: Even and Odd Functions

375 ANS: 3

f(x) = -f(x), so f(x) is odd.  $g(-x) \neq g(x)$ , so g(x) is not even.  $g(-x) \neq -g(x)$ , so g(x) is not odd. h(-x) = h(x), so h(x) is even.

PTS: 2

REF: fall1502aii

NAT: F.BF.B.3

TOP: Even and Odd Functions

376 ANS: 1

The graph of  $y = \sin x$  is unchanged when rotated 180° about the origin.

PTS: 2

REF: 081614aii

NAT: F.BF.B.3

TOP: Even and Odd Functions

$$f(x) = f(-x)$$

$$x^2 - 4 = (-x)^2 - 4$$

$$x^2 - 4 = x^2 - 4$$

PTS: 2

REF: 061806aii NAT: F.BF.B.3 TOP: Even and Odd Functions

$$j(-x) = (-x)^4 - 3(-x)^2 - 4 = x^2 - 3x^2 - 4$$
 Since  $j(x) = j(-x)$ , the function is even.

PTS: 2

REF: 081731aii

NAT: F.BF.B.3

TOP: Even and Odd Functions

$$x = -6(y - 2)$$

$$-\frac{x}{6} = y - 2$$

$$-\frac{x}{6} + 2 = y$$

PTS: 2

REF: 011821aii NAT: F.BF.B.4 TOP: Inverse of Functions

KEY: linear

$$y = \frac{1}{2}x + 8$$
  $x = \frac{1}{2}y + 8$ 

$$2x = y + 16$$

$$y = 2x - 16$$

PTS: 2

REF: 081806aii NAT: F.BF.B.4 TOP: Inverse of Functions

KEY: linear

$$x = 4y + 5$$

$$x - 5 = 4y$$

$$\frac{1}{4}x - \frac{5}{4} = y$$

PTS: 2

KEY: linear

REF: 061909aii

NAT: F.BF.B.4

**TOP:** Inverse of Functions

$$y = -6x + \frac{1}{2}$$

$$x = -6y + \frac{1}{2}$$

$$x - \frac{1}{2} = -6y$$

$$-\frac{1}{6}\left(x-\frac{1}{2}\right) = y$$

PTS: 2

REF: 062217aii

NAT: F.BF.B.4

**TOP:** Inverse of Functions

KEY: linear

$$x = \frac{1}{2}y + 2$$

$$2x = y + 4$$

$$y = 2x - 4$$

PTS: 2

REF: 012315aii NAT: F.BF.B.4

**TOP:** Inverse of Functions

KEY: linear

384 ANS: 2

$$x = -\frac{3}{4}y + 2$$

$$-4x = 3y - 8$$

$$-4x + 8 = 3y$$

$$-\frac{4}{3}x + \frac{8}{3} = y$$

PTS: 2

REF: 061616aii NAT: F.BF.B.4

**TOP:** Inverse of Functions

KEY: linear

$$x = -\frac{2y}{5} + 4$$
  $y = -\frac{5}{2}(6) + 10 = -5$ 

$$5x = -2y + 20$$

$$2y = -5x + 20$$

$$y = -\frac{5}{2}x + 10$$

KEY: linear

PTS: 2

REF: 082223aii

NAT: F.BF.B.4

**TOP:** Inverse of Functions

386 ANS: 3  

$$y = x^{3} - 2$$

$$x = y^{3} - 2$$

$$x + 2 = y^{3}$$

$$\sqrt[3]{x+2} = y$$

PTS: 2 REF: 061815aii NAT: F.BF.B.4 TOP: Inverse of Functions

KEY: other

387 ANS: 3 PTS: 2 REF: 011917aii NAT: F.BF.B.4

TOP: Inverse of Functions KEY: other

388 ANS: 3 PTS: 2 REF: 011708aii NAT: F.BF.B.4

TOP: Inverse of Functions KEY: other

# Algebra II Regents Exam Questions by State Standard: Topic Answer Section

$$x = \frac{y+1}{y-2}$$

$$xy - 2x = y + 1$$

$$xy - y = 2x + 1$$

$$y(x-1) = 2x + 1$$

$$y = \frac{2x+1}{x-1}$$

PTS: 2

REF: 081714aii

NAT: F.BF.B.4

**TOP:** Inverse of Functions

KEY: other

390 ANS: 2

$$x = \frac{y}{y+2}$$

$$xy + 2x = y$$

$$xy - y = -2x$$

$$y(x-1) = -2x$$

$$y = \frac{-2x}{x - 1}$$

PTS: 2

KEY: other

REF: 081924aii

NAT: F.BF.B.4

TOP: Inverse of Functions

391 ANS:

$$x = \left(y - 3\right)^3 + 1$$

$$x - 1 = \left(y - 3\right)^3$$

$$\sqrt[3]{x-1} = y-3$$

$$\sqrt[3]{x-1} + 3 = y$$

$$f^{-1}(x) = \sqrt[3]{x-1} + 3$$

PTS: 2

REF: fall1509aii

NAT: F.BF.B.4

TOP: Inverse of Functions

KEY: other

392 ANS: 3

PTS: 2

REF: 061720aii

NAT: F.LE.A.2

TOP: Sequences KEY: explicit

(2) is not recursive

PTS: 2 REF: 081608aii NAT: F.LE.A.2 TOP: Sequences

KEY: recursive

394 ANS: 3 PTS: 2 REF: 081618aii NAT: F.LE.A.2

TOP: Sequences KEY: recursive

395 ANS: 4

(1) and (3) are not recursive

PTS: 2 REF: 012013aii NAT: F.LE.A.2 TOP: Sequences

KEY: recursive

396 ANS: 4

1) is a correct formula, but not recursive

PTS: 2 REF: 082216aii NAT: F.LE.A.2 TOP: Sequences

KEY: recursive

397 ANS:

$$a_1 = 4$$

$$a_n = 3a_{n-1}$$

PTS: 2 REF: 081931aii NAT: F.LE.A.2 TOP: Sequences

KEY: recursive

398 ANS:

$$\frac{9}{6} = 1.5 \ a_1 = 6$$

$$a_n = 1.5 \cdot a_{n-1}$$

PTS: 2 REF: 061931aii NAT: F.LE.A.2 TOP: Sequences

KEY: recursive

399 ANS:

$$a_1 = 4$$
  $a_8 = 639$ 

$$a_n = 2a_{n-1} + 1$$

PTS: 2 REF: 081729aii NAT: F.LE.A.2 TOP: Sequences

KEY: recursive

400 ANS:

$$a_n = x^{n-1}(x+1) \ x^{n-1} = 0 \ x+1 = 0$$

$$x = 0$$
  $x = -1$ 

PTS: 4 REF: spr1511aii NAT: F.LE.A.2 TOP: Sequences

KEY: recursive

1.5%; 
$$P(t) = 92.2(1.015)^{t}$$
;  $\frac{300}{92.2} = (1.015)^{t}$   

$$\log \frac{300}{92.2} = t \log(1.015)$$

$$\frac{\log \frac{300}{92.2}}{\log(1.015)} = t$$

$$t \approx 79$$

PTS: 6 REF: 062237aii NAT: F.LE.A.2 TOP: Sequences

KEY: recursive

402 ANS: 2  $a_2 = 8 + \log_{2+1} 1 = 8 + 0 = 8$ 

$$a_3 = 8 + \log_{3+1} 2 = 8 + \frac{1}{2} = 8.5$$

PTS: 2 REF: 062221aii NAT: F.IF.A.3 TOP: Sequences

403 ANS: 2

$$121(b)^{2} = 64 \quad 64 \left(\frac{8}{11}\right)^{2} \approx 34$$
$$b = \frac{8}{11}$$

PTS: 2 REF: 011904aii NAT: F.IF.A.3 TOP: Sequences

KEY: explicit

404 ANS: 1  $\frac{-12}{16} = \frac{9}{-12} = \frac{-6.75}{9}$ 

PTS: 2 REF: 012017aii NAT: F.IF.A.3 TOP: Sequences

KEY: difference or ratio

405 ANS: 1

406 ANS:

$$d = 18; \ r = \pm \frac{5}{4}$$

PTS: 2 REF: 011714aii NAT: F.IF.A.3 TOP: Sequences

KEY: explicit

 $a_1 = 3$   $a_2 = 7$   $a_3 = 15$   $a_4 = 31$ ; No, because there is no common ratio:  $\frac{7}{3} \neq \frac{15}{7}$ 

PTS: 2 REF: 061830aii NAT: F.IF.A.3 TOP: Sequences

KEY: recursive

| 407                      | ANS: 3 TOP: Sequences   | PTS:<br>KEY: |           | REF:  | 081909aii | NAT:      | F.BF.A.2     |
|--------------------------|---|--------------|-----------|-------|-----------|-----------|--------------|
| 408                      | _   | PTS:         |           | REF:  | 011824aii | NAT:      | F.BF.A.2     |
| 409                      | ANS: 3 TOP: Sequences   | PTS:         | 2         | REF:  | 081724aii | NAT:      | F.BF.A.2     |
| 410                      | -   | PTS:         | 2         | REF:  | 081624aii | NAT:      | F.BF.A.2     |
| 411                      | ANS: 3 TOP: Sequences   | PTS:         | 2         | REF:  | 061623aii | NAT:      | F.BF.A.2     |
| 412                      | ANS: 4 TOP: Sequences   | PTS:         | 2         | REF:  | 081810aii | NAT:      | F.BF.A.2     |
| 413                      | ANS: 4<br>$a_1 = 2.5 + 0.5(1) = 3$  |              |           |       |           |           |              |
| $a_1 = 2.5 + 0.5(1) = 5$ |   |              |           |       |           |           |              |
|                          | PTS: 2  | DEE.         | 0110160;; | NIAT. | EDE A 2   | TOD:      | Saguenaes    |
| 414                      | ANS: 2  |              |           |       |           |           | _            |
| 414                      | TOP: Sequences  | Г15.         | 2         | KLI.  | 012321411 | NAI.      | r.br.A.2     |
| 415                      | ANS: 4  |              |           |       |           |           |              |
| 413                      | The scenario represents a decreasing geometric sequence with a common ratio of 0.80.  |              |           |       |           |           |              |
|                          | 60  |              |           |       |           |           |              |
|                          | PTS: 2  | REF:         | 061610aii | NAT:  | F.BF.A.2  | TOP:      | Sequences    |
| 416                      | ANS: 3  |              |           |       |           |           | F.BF.A.2     |
|                          | TOP: Sequences  |              |           |       |           |           |              |
| 417                      | ANS:  |              |           |       |           |           |              |
|                          | $\frac{6.25 - 2.25}{21 - 5} = \frac{4}{16} = \$.25 \text{ fine per day. } 2.25 - 5(.25) = \$1 \text{ replacement fee. } a_n = 1.25 + (n - 1)(.25). \ a_{60} = \$16$ |              |           |       |           |           |              |
| /11 <b>Q</b>             | PTS: 4<br>ANS:  | REF:         | 081734aii | NAT:  | F.BF.A.2  | TOP:      | Sequences    |
| 710                      | Jillian's plan, because distance increases by one mile each week. $a_1 = 10$ $a_n = n + 12$   |              |           |       |           |           |              |
|                          | similar 5 plans, occases distance increases by one nine each week. $u_1 = 10$ $u_n = n + 12$  |              |           |       |           |           |              |
|                          |   |              |           |       |           | $a_n = a$ | $_{n-1} + 1$ |
|                          | PTS: 4  | REF:         | 011734aii | NAT:  | F.BF.A.2  | TOP:      | Sequences    |
|                          | KEY: recursive  |              |           |       |           |           |              |
| 419                      | ANS: 1  | PTS:         | 2         | REF:  | 082221aii | NAT:      | F.BF.B.6     |
|                          | TOP: Sigma Notation   | on           |           | KEY:  | represent |           |              |
| 420                      | ANS: 1  | PTS:         | 2         |       | 081609aii | NAT:      | F.BF.B.6     |
|                          | TOP: Sigma Notation   | on           |           | KEY:  | represent |           |              |
| 421                      | ANS: 4  |              |           |       |           |           |              |
|                          | $d = 32(.8)^{b-1} S_n = \frac{32 - 32(.8)^{12}}{18} \approx 149$  |              |           |       |           |           |              |
|                          | PTS: 2  | REF:         | 081721aii | NAT:  | A.SSE.B.4 | TOP:      | Series       |
|                          |   |              |           |       |           |           |              |

KEY: geometric

$$8r^3 = 216 \ S_{12} = \frac{8 - 8(3)^{12}}{1 - 3} = 2125760$$
  
 $r^3 = 27$ 

r = 3

PTS: 2 REF: 081902aii NAT: A.SSE.B.4 TOP: Series

KEY: geometric

423 ANS: 3

$$S_{20} = \frac{-2 - (-2)(-3)^{20}}{1 - (-3)} = 1,743,392,200$$

PTS: 2 REF: 012306aii NAT: A.SSE.B.4 TOP: Series

KEY: geometric

424 ANS: 2

$$S_{20} = \frac{.01 - .01(3)^{20}}{1 - 3} = 17,433,922$$

PTS: 2 REF: 011822aii NAT: A.SSE.B.4 TOP: Series

KEY: geometric

425 ANS: 4

$$S_7 = \frac{85000 - 85000(1.06)^7}{1 - 1.06} \approx 713476.20$$

PTS: 2 REF: 061905aii NAT: A.SSE.B.4 TOP: Series

KEY: geometric

426 ANS: 2 PTS: 2 REF: 061724aii NAT: A.SSE.B.4

TOP: Series KEY: geometric

427 ANS: 1 PTS: 2 REF: 081813aii NAT: A.SSE.B.4

TOP: Series KEY: geometric

428 ANS:

$$r = \frac{360}{300} = 1.2 \ S_n = \frac{300 - 300(1.2)^n}{1 - 1.2} \ S_{10} = \frac{300 - 300(1.2)^{10}}{1 - 1.2} \approx 7787.6$$

PTS: 2 REF: 012029aii NAT: A.SSE.B.4 TOP: Series

KEY: geometric

429 ANS:

$$S_n = \frac{33000 - 33000(1.04)^n}{1 - 1.04} \quad S_{15} = \frac{33000 - 33000(1.04)^{15}}{1 - 1.04} \approx 660778.39$$

PTS: 4 REF: 061634aii NAT: A.SSE.B.4 TOP: Series

KEY: geometric

$$S_{10} = \frac{15 - 15(1.03)^{10}}{1 - 1.03} \approx 171.958$$

PTS: 2

REF: 011929aii NAT: A.SSE.B.4

TOP: Series

KEY: geometric

431 ANS:

$$a_n = 100(.8)^{n-1}$$
  $S_{20} = \frac{100 - 100(.8)^{20}}{1 - .8} \approx 494$  No, because  $494 > 40 \times 12$ .

PTS: 4

REF: 012033aii

NAT: A.SSE.B.4

TOP: Series

KEY: geometric

432 ANS:

$$S_5 = \frac{6 - 6(.8)^5}{1 - .8} \approx 20.17$$

PTS: 2

REF: 062226aii

NAT: A.SSE.B.4

TOP: Series

KEY: geometric

433 ANS: 1

PTS: 2

REF: 081616aii

NAT: F.TF.A.1

KEY: bimodalgraph TOP: Unit Circle

434 ANS: 2

PTS: 2

REF: 062219aii

NAT: F.TF.A.1

TOP: Unit Circle

435 ANS: 1

PTS: 2

REF: 011815aii

NAT: F.TF.A.2

TOP: Unit Circle

TOP: Unit Circle

436 ANS: 4

PTS: 2

REF: 082205aii

NAT: F.TF.A.2

437 ANS:

$$t^2 + \left(\frac{4}{7}\right)^2 = 1 \qquad -\frac{\sqrt{33}}{7}$$

$$t^2 + \frac{16}{49} = \frac{49}{49}$$

$$t^2 = \frac{33}{49}$$

$$t = \frac{\pm\sqrt{33}}{7}$$

PTS: 2

REF: 011931aii

NAT: F.TF.A.2 TOP: Unit Circle

438 ANS:

 $\csc \theta = \frac{1}{\sin \theta}$ , and  $\sin \theta$  on a unit circle represents the y value of a point on the unit circle. Since  $y = \sin \theta$ ,  $\csc \theta = \frac{1}{v}$ .

PTS: 2

REF: 011727aii

NAT: F.TF.A.2

TOP: Reciprocal Trigonometric Relationships

 $\pi < \theta < 2\pi \rightarrow$  Quadrant III or IV  $\theta$  must be in Quadrant IV, where  $\tan \theta$  is negative.

$$\cos \theta = \frac{\sqrt{3}}{4} \rightarrow \text{Quadrant I or IV}$$

PTS: 2

REF: 012332aii

NAT: F.TF.A.2

TOP: Finding the Terminal Side of an Angle

440 ANS: 4

PTS: 2

REF: 081707aii

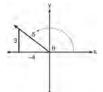
NAT: F.TF.A.2

TOP: Reference Angles

KEY: bimodalgraph

441 ANS: 1

A reference triangle can be sketched using the coordinates (-4,3) in the second quadrant to find the value of  $\sin \theta$ .



PTS: 2

REF: spr1503aii

NAT: F.TF.A.2

TOP: Determining Trigonometric Functions

KEY: extension to reals

442 ANS: 2

If 
$$\cos \theta = \frac{7}{25}$$
,  $\sin \theta = \pm \frac{24}{25}$ , and  $\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{-\frac{24}{25}}{\frac{7}{25}} = -\frac{24}{7}$ 

PTS: 2

REF: 081811aii

NAT: F.TF.C.8

TOP: Determining Trigonometric Functions

443 ANS: 2

PTS: 2

REF: 011804aii

NAT: F.TF.A.2

TOP: Determining Trigonometric Functions

KEY: radians

444 ANS: 3

$$T(19) = 8\sin(0.3(19) - 3) + 74 \approx 77$$

PTS: 2

KEY: radians

REF: 061922aii

NAT: F.TF.A.2

TOP: Determining Trigonometric Functions

445 ANS:

$$\frac{-1}{\sqrt{2^2 + (-1)^2}} = -\frac{1}{\sqrt{5}}$$

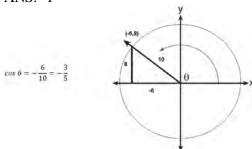
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REF: 061832aii

NAT: F.TF.A.2

TOP: Determining Trigonometric Functions

KEY: extension to reals



PTS: 2

REF: 061617aii

NAT: F.TF.A.2

TOP: Determining Trigonometric Functions

KEY: extension to reals

447 ANS: 1

$$-\sqrt{1 - \left(-\frac{3}{4}\right)^2} = -\sqrt{\frac{16}{16} - \frac{9}{16}} = -\sqrt{\frac{7}{16}} = -\frac{\sqrt{7}}{4}$$

PTS: 2

REF: 081905aii

NAT: F.TF.C.8

**TOP:** Determining Trigonometric Functions

448 ANS: 3

$$\sin^2 A + \left(\frac{\sqrt{5}}{3}\right)^2 = 1 \quad \text{Since } \tan A < 0, \sin A = -\frac{2}{3}$$

$$\sin^2 A + \frac{5}{9} = \frac{9}{9}$$

$$\sin^2 A = \frac{4}{9}$$

$$\sin A = \pm \frac{2}{3}$$

PTS: 2

REF: 012320aii

NAT: F.TF.C.8

TOP: Determining Trigonometric Functions

449 ANS: 2

$$\cos \theta = \pm \sqrt{1 - \left(\frac{-\sqrt{2}}{5}\right)^2} = \pm \sqrt{\frac{25}{25} - \frac{2}{25}} = \pm \frac{\sqrt{23}}{5}$$

PTS: 2

REF: 061712aii

NAT: F.TF.C.8

TOP: Determining Trigonometric Functions

450 ANS:

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\frac{-7}{25}}{\frac{-24}{25}} \cos \theta = \frac{-24}{25}$$

PTS: 2

REF: 061928aii

NAT: F.TF.C.8

TOP: Determining Trigonometric Functions

$$\sin^2 \theta + (-0.7)^2 = 1$$
 Since  $\theta$  is in Quadrant II,  $\sin \theta = \sqrt{.51}$  and  $\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\sqrt{.51}}{-0.7} \approx -1.02$   
 $\sin^2 \theta = .51$   
 $\sin \theta = \pm \sqrt{.51}$ 

PTS: 2

REF: 081628aii

NAT: F.TF.C.8

TOP: Determining Trigonometric Functions

452 ANS:

$$\cos A = \frac{\cos A}{\sin A}$$

$$-3 = \frac{\frac{3}{\sqrt{10}}}{\sin A}$$

$$\sin A = \frac{3}{-3\sqrt{10}} = -\frac{1}{\sqrt{10}}$$

PTS: 2

REF: 082229aii

NAT: F.TF.C.8

TOP: Determining Trigonometric Functions

453 ANS: 1

PTS: 2

REF: 011704aii

NAT: F.TF.C.8

TOP: Simplifying Trigonometric Expressions

454 ANS: 4

period = 
$$\frac{2\pi}{B}$$

$$\frac{1}{60} = \frac{2\pi}{B}$$

$$B = 120\pi$$

PTS: 2

REF: 061624aii

NAT: F.TF.B.5

TOP: Modeling Trigonometric Functions

455 ANS: 1

PTS: 2

REF: 061708aii

NAT: F.TF.B.5

**TOP:** Modeling Trigonometric Functions

456 ANS: 2

$$1 = \frac{2\pi}{k}$$

$$k = 2\pi$$

PTS: 2

REF: 012313aii

NAT: F.TF.B.5

**TOP:** Modeling Trigonometric Functions

457 ANS: 1

The cosine function has been translated +3. Since the maximum is 5 and the minimum is 1, the amplitude is 2.  $\frac{\pi}{3} = \frac{2\pi}{b}.$ 

$$b = 6$$

PTS: 2

REF: 011913aii

NAT: F.TF.B.5

TOP: Modeling Trigonometric Functions

$$a = \frac{14-4}{2} = 5, d = \frac{14+4}{2} = 9$$

PTS: 2

REF: 061810aii

NAT: F.TF.B.5

TOP: Modeling Trigonometric Functions

459 ANS: 4

PTS: 2

REF: 061706aii

NAT: F.IF.B.4

TOP: Graphing Trigonometric Functions 460 ANS: 2

PTS: 2

REF: 011701aii

NAT: F.IF.B.4

TOP: Graphing Trigonometric Functions

461 ANS: 1

$$-4(-1) - 3 = 1 \quad 8 = \frac{2\pi}{b}$$

$$b = \frac{\pi}{4}$$

PTS: 2

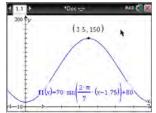
REF: 081820aii

NAT: F.IF.B.4

**TOP:** Graphing Trigonometric Functions

KEY: maximum/minimum

462 ANS: 3



H(t) is at a minimum at 70(-1) + 80 = 10

PTS: 2

REF: 061613aii

NAT: F.IF.B.4

**TOP:** Graphing Trigonometric Functions

KEY: maximum/minimum

463 ANS: 2

PTS: 2

REF: 081610aii

NAT: F.IF.B.4

TOP: Graphing Trigonometric Functions KEY: increasing/decreasing

464 ANS: 4

PTS: 2

REF: 012016aii

NAT: F.IF.B.4

TOP: Graphing Trigonometric Functions KEY: increasing/decreasing

465 ANS: 4

PTS: 2

REF: 082220aii

NAT: F.IF.B.4

**TOP:** Graphing Trigonometric Functions

466 ANS: 3

PTS: 2

REF: 081705aii

NAT: F.IF.B.4

TOP: Graphing Trigonometric Functions KEY: increasing/decreasing

467 ANS: 4

1) 
$$d(2) = 2$$
; 2)  $d(1) = 12$ ; 3)  $d(9) \approx 11$ ; 4)  $d(-1) = 2$ 

PTS: 2

REF: 062220aii

NAT: F.IF.B.4

**TOP:** Graphing Trigonometric Functions

|         | Bar Harbor | Phoenix |
|---------|------------|---------|
| Minimum | 31.386     | 66.491  |
| Midline | 55.3       | 86.729  |
| Maximum | 79.214     | 106.967 |
| Range   | 47.828     | 40.476  |

PTS: 2

REF: 061715aii

NAT: F.IF.B.4

**TOP:** Graphing Trigonometric Functions

KEY: maximum/minimum

469 ANS:

250(1) + 2450 = 2700 The maximum lung capacity of a person is 2700 mL.

PTS: 2

REF: 081928aii

NAT: F.IF.B.4

**TOP:** Graphing Trigonometric Functions

470 ANS:

 $\frac{h(2) - h(1)}{2 - 1} = -12, \ h(t) = 0 \text{ at } t \approx 2.2, 3.8, \text{ using a graphing calculator to find where } h(t) = 0.$ 

PTS: 4

REF: 061836aii

NAT: F.IF.B.4

**TOP:** Graphing Trigonometric Functions

471 ANS: 4

PTS: 2

REF: 081718aii

NAT: F.IF.C.7

NAT: F.IF.C.7

TOP: Graphing Trigonometric Functions KEY: amplitude

PTS: 2 REF: 082203aii

TOP: Graphing Trigonometric Functions KEY: amplitude

473 ANS:

$$\frac{10.1 - -2}{2} - \frac{2.5 - -0.1}{2} = 6.05 - 1.3 = 4.75$$

PTS: 2

REF: 081930aii

NAT: F.IF.C.7

**TOP:** Graphing Trigonometric Functions

KEY: amplitude

474 ANS: 2

$$P = \frac{2\pi}{\frac{\pi}{45}} = 90$$

PTS: 2

REF: 081822aii

NAT: F.IF.C.7

TOP: Graphing Trigonometric Functions

KEY: period

475 ANS:

period is  $\frac{2}{3}$ . The wheel rotates once every  $\frac{2}{3}$  second.

PTS: 2

REF: 061728aii

NAT: F.IF.C.7

**TOP:** Graphing Trigonometric Functions

KEY: period

476 ANS:

Light wave C. The periods for A, B, and C are 280, 220 and 320.

PTS: 2

REF: 012030aii

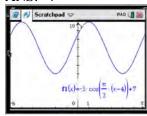
NAT: F.IF.C.7

**TOP:** Graphing Trigonometric Functions

KEY: period

TOP: Graphing Trigonometric Functions

477 ANS: 4



As the range is [4,10], the midline is  $y = \frac{4+10}{2} = 7$ .

NAT: F.IF.C.7

REF: fall1506aii

KEY: mixed

PTS: 2

478 ANS: 1 The time of the next high tide will be the midpoint of consecutive low tides.

PTS: 2 REF: 011907aii NAT: F.IF.C.7 TOP: Graphing Trigonometric Functions

KEY: mixed

479 ANS: 4 PTS: 2 NAT: F.IF.C.7 REF: 081912aii

TOP: Graphing Trigonometric Functions KEY: mixed

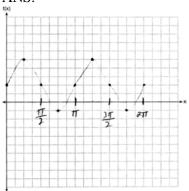
480 ANS:

Amplitude, because the height of the graph shows the volume of the air.

PTS: 2 REF: 081625aii NAT: F.IF.C.7 TOP: Graphing Trigonometric Functions

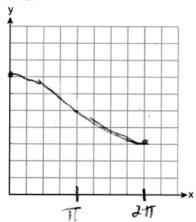
KEY: mixed

481 ANS:



PTS: 2 REF: 081830aii NAT: F.IF.C.7 TOP: Graphing Trigonometric Functions

KEY: graph

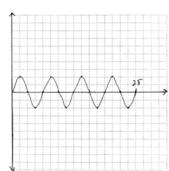


PTS: 2 KEY: graph REF: 062231aii

NAT: F.IF.C.7

TOP: Graphing Trigonometric Functions

483 ANS:



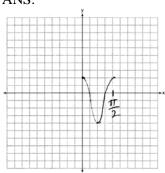
 $y = 2\sin 4x$ 

PTS: 4 KEY: graph REF: 081934aii

NAT: F.IF.C.7

TOP: Graphing Trigonometric Functions

484 ANS:



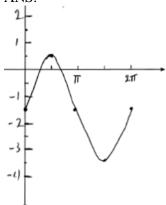
PTS: 2

KEY: graph

REF: 061628aii

NAT: F.IF.C.7

TOP: Graphing Trigonometric Functions



Part a sketch is shifted  $\frac{\pi}{3}$  units right.

PTS: 4

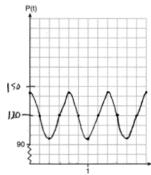
REF: 081735aii

NAT: F.IF.C.7

TOP: Graphing Trigonometric Functions

KEY: graph

486 ANS:



The period of P is  $\frac{2}{3}$ , which means the patient's blood pressure reaches a high every  $\frac{2}{3}$ 

second and a low every  $\frac{2}{3}$  second. The patient's blood pressure is high because 144 over 96 is greater than 120 over 80.

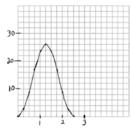
PTS: 6

REF: 011837aii

NAT: F.IF.C.7

TOP: Graphing Trigonometric Functions

KEY: graph 487 ANS:



period =  $\frac{2\pi}{0.8\pi}$  = 2.5. The wheel rotates once every 2.5 seconds.

No, because the maximum

of f(t) = 26.

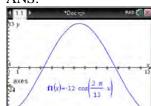
PTS: 6

REF: 061937aii

NAT: F.IF.C.7

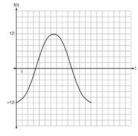
**TOP:** Graphing Trigonometric Functions

KEY: graph



The amplitude, 12, can be interpreted from the situation, since the water level has a minimum of -12 and a maximum of 12. The value of A is -12 since at 8:30 it is low tide. The period of the function is 13 hours, and is expressed in the function through the parameter B. By experimentation with

technology or using the relation  $P = \frac{2\pi}{B}$  (where P is the period), it is determined that  $B = \frac{2\pi}{13}$ .



 $f(t) = -12\cos\left(\frac{2\pi}{13}t\right)$ 

In order to answer the question about when to fish, the student must interpret the function and determine which choice, 7:30 pm or 10:30 pm, is on an increasing interval. Since the function is increasing from t = 13 to t = 19.5 (which corresponds to 9:30 pm to 4:00 am), 10:30 is the appropriate choice.

PTS: 6 REF: spr1514aii

NAT: F.IF.C.7

TOP: Graphing Trigonometric Functions

KEY: graph

489 ANS: 3

(3) repeats 3 times over  $2\pi$ .

PTS: 2

REF: 011722aii

NAT: F.IF.C.7

TOP: Graphing Trigonometric Functions

KEY: recognize | bimodalgraph

490 ANS: 4

$$4(x^2 - 6x + 9) + 4(y^2 + 18y + 81) = 76 + 36 + 324$$

$$4(x-3)^2 + 4(y+9)^2 = 436$$

PTS: 2

REF: 061619aii

NAT: G.GPE.A.1

**TOP:** Equations of Circles

KEY: completing the square

491 ANS: 3

PTS: 2

REF: 061607aii

NAT: S.IC.A.2

TOP: Analysis of Data

492 ANS: 3

PTS: 2

REF: 061710aii

NAT: S.IC.A.2

TOP: Analysis of Data

TOP: Analysis of Data

493 ANS: 2

PTS: 2

REF: 011820aii

NAT: S.IC.A.2

494 ANS:

sample: pails of oranges; population: truckload of oranges. It is likely that about 5% of all the oranges are unsatisfactory.

PTS: 2

REF: 011726aii

NAT: S.IC.A.2

TOP: Analysis of Data

495 ANS: Since there are six flavors, each flavor can be assigned a number, 1-6. Use the simulation to see the number of times the same number is rolled 4 times in a row. TOP: Analysis of Data PTS: 2 REF: 081728aii NAT: S.IC.A.2 496 ANS: No.  $0.499 \pm 2(0.049) \rightarrow 0.401 - 0.597$ . Since 0.43 falls within this interval, Robin's coin is likely not unfair. PTS: 2 REF: 061932aii NAT: S.IC.A.2 TOP: Analysis of Data 497 ANS:  $\frac{1}{10}$ ,  $\frac{1}{5}$ , and no, since 0.10 clearly falls within 95% of 0.20. PTS: 4 REF: 012334aii NAT: S.IC.A.2 TOP: Analysis of Data 498 ANS:  $138.905 \pm 2.7.95 = 123 - 155$ . No, since 125 (50% of 250) falls within the 95% interval. PTS: 4 REF: 011835aii NAT: S.IC.A.2 TOP: Analysis of Data 499 ANS:  $.651 \pm 2 \cdot .034 = .58 - .72$ . No, since .61 (122/200) falls within the 95% interval. PTS: 4 REF: 062235aii NAT: S.IC.A.2 TOP: Analysis of Data 500 ANS:  $29.101 \pm 2 \cdot 0.934 = 27.23 - 30.97$ . Yes, since 30 falls within the 95% interval. PTS: 4 REF: 011935aii NAT: S.IC.A.2 TOP: Analysis of Data 501 ANS:  $.819 \pm 2 \cdot .053 = .713 - .925$ . Since .70 does not fall within the 95% interval. PTS: 4 REF: 082236aii TOP: Analysis of Data NAT: S.IC.A.2 502 ANS: 3 PTS: 2 REF: 011706aii NAT: S.IC.B.3 TOP: Analysis of Data KEY: type 503 ANS: 4 NAT: S.IC.B.3 PTS: 2 REF: 012314aii TOP: Analysis of Data KEY: type 504 ANS: 2 NAT: S.IC.B.3 PTS: 2 REF: 081802aii TOP: Analysis of Data KEY: type 505 ANS: 3 REF: 012015aii NAT: S.IC.B.3 TOP: Analysis of Data KEY: type 506 ANS: 4 NAT: S.IC.B.3 PTS: 2 REF: 062216aii TOP: Analysis of Data KEY: type 507 ANS: 3 NAT: S.IC.B.3 PTS: 2 REF: 061901aii TOP: Analysis of Data KEY: type 508 ANS: 4 REF: 081906aii NAT: S.IC.B.3

TOP: Analysis of Data

TOP: Analysis of Data

PTS: 2

509 ANS: 2

KEY: type

KEY: type

REF: 082204aii

NAT: S.IC.B.3

between 000 and 449, inclusive  $\rightarrow \frac{450}{1000} = 45\%$ 

PTS: 2 REF: 012024aii NAT: S.IC.B.3 TOP: Analysis of Data

KEY: type

511 ANS: 3 PTS: 2 REF: 082201aii NAT: S.IC.B.3

TOP: Analysis of Data KEY: type

512 ANS: 2 PTS: 2 REF: 081717aii NAT: S.IC.B.3

TOP: Analysis of Data KEY: type

513 ANS:

Randomly assign participants to two groups. One group uses the toothpaste with ingredient X and the other group uses the toothpaste without ingredient X.

PTS: 2 REF: 061626aii NAT: S.IC.B.3 TOP: Analysis of Data

KEY: type

514 ANS: 2 PTS: 2 REF: 011910aii NAT: S.IC.B.3

TOP: Analysis of Data KEY: bias

515 ANS: 3

To determine student opinion, survey the widest range of students.

PTS: 2 REF: 062202aii NAT: S.IC.B.3 TOP: Analysis of Data

KEY: bias

516 ANS: 3

Self selection causes bias.

PTS: 2 REF: 061703aii NAT: S.IC.B.3 TOP: Analysis of Data

KEY: bias

517 ANS: 4 PTS: 2 REF: 011801aii NAT: S.IC.B.3

TOP: Analysis of Data KEY: bias

518 ANS: 1

II. Ninth graders drive to school less often; III. Students know little about adults; IV. Calculus students love math!

PTS: 2 REF: 081602aii NAT: S.IC.B.3 TOP: Analysis of Data

KEY: bias

519 ANS:

Self selection is a cause of bias because people with more free time are more likely to respond.

PTS: 2 REF: 061828aii NAT: S.IC.B.3 TOP: Analysis of Data

KEY: bias

520 ANS: 2

$$ME = \left(z\sqrt{\frac{p(1-p)}{n}}\right) = \left(1.96\sqrt{\frac{(0.55)(0.45)}{900}}\right) \approx 0.03 \text{ or } \frac{1}{\sqrt{900}} \approx 0.03$$

PTS: 2 REF: 081612aii NAT: S.IC.B.4 TOP: Analysis of Data

$$\frac{212}{1334} \approx .16 \ ME = \left(z\sqrt{\frac{p(1-p)}{n}}\right) = \left(1.96\sqrt{\frac{(0.16)(0.84)}{1334}}\right) \approx 0.02 \text{ or } \frac{1}{\sqrt{1334}} \approx .027$$

PTS: 2

REF: 081716aii

NAT: S.IC.B.4

TOP: Analysis of Data

522 ANS: 4

$$2 \times 0.035 = 0.07 \text{ or } ME = \left(z\sqrt{\frac{p(1-p)}{n}}\right) = \left(1.96\sqrt{\frac{(0.65)(0.35)}{200}}\right) \approx 0.07$$

PTS: 2

REF: 012319aii

NAT: S.IC.B.4

TOP: Analysis of Data

523 ANS:

Yes. The margin of error from this simulation indicates that 95% of the observations fall within  $\pm$  0.12 of the simulated proportion, 0.25. The margin of error can be estimated by multiplying the standard deviation, shown to

be 0.06 in the dotplot, by 2, or applying the estimated standard error formula,  $\left(\sqrt{\frac{p(1-p)}{n}}\right)$  or  $\left(\sqrt{\frac{(0.25)(0.75)}{50}}\right)$ 

and multiplying by 2. The interval  $0.25 \pm 0.12$  includes plausible values for the true proportion of people who prefer Stephen's new product. The company has evidence that the population proportion could be at least 25%. As seen in the dotplot, it can be expected to obtain a sample proportion of 0.18 (9 out of 50) or less several times, even when the population proportion is 0.25, due to sampling variability. Given this information, the results of the survey do not provide enough evidence to suggest that the true proportion is not at least 0.25, so the development of the product should continue at this time.

PTS: 4

REF: spr1512aii

NAT: S.IC.B.4

TOP: Analysis of Data

524 ANS:

 $2(0.042) = 0.084 \approx 0.08$  The percent of users making in-app purchases will be within 8% of 35%.

PTS: 2

REF: 081832aii

NAT: S.IC.B.4

TOP: Analysis of Data

525 ANS: 2

 $0.254 \pm 2(0.060) \rightarrow (0.134, 0.374)$ 

PTS: 2

REF: 061913aii

NAT: S.IC.B.5

TOP: Analysis of Data

526 ANS: 4

PTS: 2

REF: 012014aii

NAT: S.IC.B.5

TOP: Analysis of Data

527 ANS: 2

528 ANS:

PTS: 2

REF: 011709aii

NAT: S.IC.B.5

TOP: Analysis of Data

 $0.301 \pm 2(0.058) \rightarrow 0.185 - 0.417$   $\frac{14}{60} \approx 0.23$ . It is not unusual because 0.23 falls within this interval.

PTS: 4

REF: 081935aii

NAT: S.IC.B.5

TOP: Analysis of Data

The mean difference between the students' final grades in group 1 and group 2 is -3.64. This value indicates that students who met with a tutor had a mean final grade of 3.64 points less than students who used an on-line subscription. One can infer whether this difference is due to the differences in intervention or due to which students were assigned to each group by using a simulation to rerandomize the students' final grades many (500) times. If the observed difference -3.64 is the result of the assignment of students to groups alone, then a difference of -3.64 or less should be observed fairly regularly in the simulation output. However, a difference of -3 or less occurs in only about 2% of the rerandomizations. Therefore, it is quite unlikely that the assignment to groups alone accounts for the difference; rather, it is likely that the difference between the interventions themselves accounts for the difference between the two groups' mean final grades.

PTS: 4 REF: fall1514aii NAT: S.IC.B.5 TOP: Analysis of Data

530 ANS:

 $0.602 \pm 2 \cdot 0.066 = 0.47 - 0.73$ . Since 0.50 falls within the 95% interval, this supports the concern there may be an even split.

PTS: 4 REF: 061635aii NAT: S.IC.B.5 TOP: Analysis of Data

531 ANS:

 $0.506 \pm 2 \cdot 0.078 = 0.35 - 0.66$ . The 32.5% value falls below the 95% confidence level.

PTS: 4 REF: 061736aii NAT: S.IC.B.5 TOP: Analysis of Data

532 ANS:

Some of the students who did not drink energy drinks read faster than those who did drink energy drinks.

17.7 - 19.1 = -1.4 Differences of -1.4 and less occur  $\frac{25}{232}$  or about 10% of the time, so the difference is not unusual.

PTS: 4 REF: 081636aii NAT: S.IC.B.5 TOP: Analysis of Data

533 ANS:

23 - 18 = 5,  $x \pm 2\sigma = -3.07 - 3.13$ , Yes, a difference of 5 or more occurred three times out of a thousand, which is statistically significant.

PTS: 4 REF: 061834aii NAT: S.IC.B.5 TOP: Analysis of Data

534 ANS:

John found the means of the scores of the two rooms and subtracted the means. The mean score for the classical room was 7 higher than the rap room (82-75). Yes, there is less than a 5% chance this difference occurring due to random chance. It is likely the difference was due to the music.

PTS: 4 REF: 081836aii NAT: S.IC.B.5 TOP: Analysis of Data

535 ANS: 1 PTS: 2 REF: 081722aii NAT: S.IC.B.6

TOP: Analysis of Data

536 ANS:

Using a 95% level of confidence,  $x \pm 2$  standard deviations sets the usual wait time as 150-302 seconds. 360 seconds is unusual.

PTS: 2 REF: 081629aii NAT: S.IC.B.6 TOP: Analysis of Data

$$y = 2.459(1.616)^x$$

PTS: 2 REF: 012329aii NAT: S.ID.B.6 TOP: Regression

KEY: exponential

538 ANS:

$$D = 1.223(2.652)^A$$

PTS: 2 REF: 011826aii NAT: S.ID.B.6 TOP: Regression

KEY: exponential

539 ANS:

$$F(t) = 169.136(.971)^{t}$$

PTS: 2 REF: 062232aii NAT: S.ID.B.6 TOP: Regression

KEY: exponential

540 ANS:

$$y = 101.523(.883)^{x}$$
 29 = 101.523(.883)<sup>x</sup>

$$\frac{29}{101.523} = (.883)^x$$

$$\log \frac{29}{101.523} = x \log(.883)$$

$$\frac{\log \frac{29}{101.523}}{\log(.883)} = x$$

$$x$$
 ≈ 10.07

PTS: 4 REF: 012036aii NAT: S.ID.B.6 TOP: Regression

KEY: exponential

541 ANS:

$$y = 4.168(3.981)^x$$
.  $100 = 4.168(3.981)^x$ 

$$\log \frac{100}{4.168} = \log(3.981)^x$$

$$\log \frac{100}{4.168} = x \log(3.981)$$

$$\frac{\log \frac{100}{4.168}}{\log(3.981)} = x$$

$$x \approx 2.25$$

PTS: 4 REF: 081736aii NAT: S.ID.B.6 TOP: Regression

KEY: exponential

The pattern suggests an exponential pattern, not linear or sinusoidal. A 4% growth rate is accurate, while a 43% growth rate is not.

PTS: 2 REF: 011713aii NAT: S.ID.B.6 TOP: Regression

KEY: choose model

543 ANS: 2 PTS: 2 REF: 061804aii NAT: S.ID.B.6

TOP: Regression KEY: choose model

544 ANS: 2 PTS: 2 REF: 011901aii NAT: S.ID.A.4

TOP: Normal Distributions KEY: mean and standard deviation

545 ANS: 2



 $x + 2\sigma$  represents approximately 48% of the data.

PTS: 2 REF: 061609aii NAT: S.ID.A.4 TOP: Normal Distributions

KEY: percent

546 ANS: 1



PTS: 2 REF: 081711aii NAT: S.ID.A.4 TOP: Normal Distributions

KEY: percent

547 ANS: 1



PTS: 2 REF: 081919aii NAT: S.ID.A.4 TOP: Normal Distributions

KEY: percent



PTS: 2 REF: 061726aii NAT: S.ID.A.4 TOP: Normal Distributions

KEY: percent

549 ANS:



PTS: 2 REF: 012328aii NAT: S.ID.A.4 TOP: Normal Distributions

KEY: percent

550 ANS: 4  $496 \pm 2(115)$ 

PTS: 2 REF: 011718aii NAT: S.ID.A.4 TOP: Normal Distributions

KEY: interval

551 ANS: 3

 $440 \times 2.3\% \approx 10$ 

PTS: 2 REF: 011807aii NAT: S.ID.A.4 TOP: Normal Distributions

KEY: predict

552 ANS: 1

 $84.1\% \times 750 \approx 631$ 

PTS: 2 REF: 011923aii NAT: S.ID.A.4 TOP: Normal Distributions

KEY: predict

553 ANS: 4

 $400.954 \approx 380$ 

PTS: 2 REF: 061918aii NAT: S.ID.A.4 TOP: Normal Distributions

KEY: predict

554 ANS: 1 PTS: 2 REF: 062214aii NAT: S.ID.A.4

TOP: Normal Distributions KEY: predict

 $1200 \cdot 0.784 \approx 941$ 

PTS: 2 REF: 081828aii NAT: S.ID.A.4 TOP: Normal Distributions

KEY: predict

556 ANS:

 $0.133696 \times 9256 \approx 1237$ 

PTS: 2 REF: 082230aii NAT: S.ID.A.4 TOP: Normal Distributions

KEY: predict

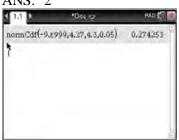
557 ANS: 3



PTS: 2 REF: 081604aii NAT: S.ID.A.4 TOP: Normal Distributions

KEY: probability

558 ANS: 2



PTS: 2 REF: 061817aii NAT: S.ID.A.4 TOP: Normal Distributions

KEY: probability

559 ANS:

normcdf(510, 540, 480, 24) = 0.0994 
$$z = \frac{510 - 480}{24} = 1.25$$
  $1.25 = \frac{x - 510}{20}$   $2.5 = \frac{x - 510}{20}$  535-560  $z = \frac{540 - 480}{24} = 2.5$   $x = 535$   $x = 560$ 

PTS: 4 REF: fall1516aii NAT: S.ID.A.4 TOP: Normal Distributions

KEY: probability

The events are independent because  $P(A \text{ and } B) = P(A) \cdot P(B)$ .

$$0.125 = 0.5 \cdot 0.25$$

If P(A or B) = P(A) + P(B) - P(A and B) = 0.25 + 0.5 - .125 = 0.625, then the events are not mutually exclusive because P(A or B) = P(A) + P(B)

$$0.625 \neq 0.5 + 0.25$$

PTS: 2

REF: 061714aii

NAT: S.CP.B.7

TOP: Theoretical Probability

561 ANS:

$$P(S \cap M) = P(S) + P(M) - P(S \cup M) = \frac{649}{1376} + \frac{433}{1376} - \frac{974}{1376} = \frac{108}{1376}$$

PTS: 2

REF: 061629aii

NAT: S.CP.B.7

TOP: Theoretical Probability

562 ANS: 4

 $0.48 \cdot 0.25 = 0.12$ 

PTS: 1

REF: 061811aii

NAT: S.CP.A.2

TOP: Probability of Compound Events

KEY: probability

563 ANS:

$$\frac{1}{3} \times \frac{5}{12} = \frac{5}{36}$$

PTS: 2

REF: 012327aii

NAT: S.CP.A.2

TOP: Probability of Compound Events

KEY: probability

564 ANS:

 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$  A and B are independent since  $P(A \cap B) = P(A) \cdot P(B)$ 

$$0.8 = 0.6 + 0.5 - P(A \cap B)$$

$$0.3 = 0.6 \cdot 0.5$$

$$P(A \cap B) = 0.3$$

$$0.3 = 0.3$$

PTS: 2

REF: 081632aii

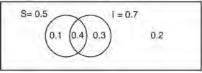
NAT: S.CP.A.2

**TOP:** Probability of Compound Events

Since

KEY: independence

565 ANS:



This scenario can be modeled with a Venn Diagram:

This section of modeled with a veini Diagram.

 $P(S \cup I)_c = 0.2$ ,  $P(S \cup I) = 0.8$ . Then,  $P(S \cap I) = P(S) + P(I) - P(S \cup I)$  If S and I are independent, then the

$$= 0.5 + 0.7 - 0.8$$

$$= 0.4$$

Product Rule must be satisfied. However,  $(0.5)(0.7) \neq 0.4$ . Therefore, salary and insurance have not been treated independently.

PTS: 4

REF: spr1513aii

NAT: S.CP.A.2

TOP: Probability of Compound Events

KEY: independence

566 ANS: 2  $\frac{85}{210+85}$ 

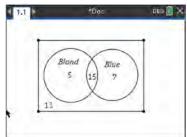
PTS: 2

REF: 081818aii

NAT: S.CP.A.1

TOP: Venn Diagrams

567 ANS: 2



40 - (20 + 22 - 15) = 13

PTS: 2

REF: 062204aii

NAT: S.CP.A.1

TOP: Venn Diagrams

568 ANS: 4

PTS: 2

REF: 012008aii

NAT: S.CP.A.3

**TOP:** Conditional Probability

569 ANS: 1

The probability of rain equals the probability of rain, given that Sean pitches.

PTS: 2

REF: 061611aii

NAT: S.CP.A.3

TOP: Conditional Probability

570 ANS: 4

PTS: 2

REF: 081824aii

NAT: S.CP.A.3

TOP: Conditional Probability

571 ANS: 2

 $P(B) \cdot P(A|B) = P(A \text{ and } B)$ 

$$P(B) \cdot 0.8 = 0.2$$

$$P(B) = 0.25$$

PTS: 2

REF: 081913aii

NAT: S.CP.A.3

TOP: Conditional Probability

572 ANS: 2

(1) 
$$0.4 \cdot 0.3 \neq 0.2$$
, (2)  $0.8 \cdot 0.25 = 0.2$ , (3)  $P(A|B) = P(A) = 0.2$ , (4)  $0.2 \neq 0.15 \cdot 0.05$   
 $0.2 \neq 0.2 \cdot 0.2$ 

PTS: 2

REF: 011912aii

NAT: S.CP.A.3

**TOP:** Conditional Probability

573 ANS:

$$P(A + B) = P(A) \cdot P(B|A) = 0.8 \cdot 0.85 = 0.68$$

PTS: 2

REF: 011928aii

NAT: S.CP.A.3

**TOP:** Conditional Probability

574 ANS:

$$\frac{47}{108} = \frac{1}{4} + \frac{116}{459} - P(M \text{ and } J); \text{ No, because } \frac{31}{459} \neq \frac{1}{4} \cdot \frac{116}{459}$$

$$P(M \text{ and } J) = \frac{31}{459}$$

PTS: 4

REF: 011834aii

NAT: S.CP.A.3

TOP: Conditional Probability

575 ANS: 4
$$\frac{13}{13+11} = \frac{13}{24}$$

PTS: 2 REF: 012011aii NAT: S.CP.A.4 TOP: Conditional Probability 576 ANS: 1

157
25+47+157

PTS: 2 REF: 081607aii NAT: S.CP.A.4 TOP: Conditional Probability 577 ANS:

Based on these data, the two events do not appear to be independent.  $P(F) = \frac{106}{200} = 0.53$ , while  $P(F|T) = \frac{54}{90} = 0.6$ ,  $P(F|R) = \frac{25}{65} = 0.39$ , and  $P(F|C) = \frac{27}{45} = 0.6$ . The probability of being female are not the same as the conditional probabilities. This suggests that the events are not independent.

PTS: 2 REF: fall1508aii NAT: S.CP.A.4 TOP: Conditional Probability

578 ANS: No, because  $P(M/R) \neq P(M)$ 

$$\frac{70}{180} \neq \frac{230}{490}$$

 $0.38 \neq 0.47$ 

PTS: 2 REF: 011731aii NAT: S.CP.A.4 TOP: Conditional Probability

579 ANS:
A student is more likely to jog if both siblings jog. 1 jogs:  $\frac{416}{2239} \approx 0.19$ . both jog:  $\frac{400}{1780} \approx 0.22$ 

PTS: 2 REF: 061732aii NAT: S.CP.A.4 TOP: Conditional Probability 580 ANS:

 $\frac{103}{110+103} = \frac{103}{213}$ 

PTS: 2 REF: 061825aii NAT: S.CP.A.4 TOP: Conditional Probability

581 ANS:  $P(F|L) = \frac{12}{27} P(F) = \frac{22}{45} \text{ Since } P(F|L) \neq P(F), \text{ the events are not independent.}$ 

PTS: 4 REF: 061936aii NAT: S.CP.A.4 TOP: Conditional Probability

No, because  $P(F/CR) \neq P(F)$ 

$$\frac{36}{42+36} \neq \frac{17+37+36+15}{39+17+42+12+17+37+36+15}$$
$$\frac{36}{78} \neq \frac{105}{215}$$
$$\frac{6}{13} \neq \frac{21}{43}$$

PTS: 2

REF: 082231aii

NAT: S.CP.A.4 TOP: Conditional Probability

583 ANS:

Yes. P(Bl) = P(Bl|Gl)

$$0.14 + 0.26 = \frac{.14}{.35}$$

.4 = .4

PTS: 2

REF: 062229aii

NAT: S.CP.A.4 TOP: Conditional Probability

584 ANS:

 $P(P/K) = \frac{P(P^{\wedge}K)}{P(K)} = \frac{1.9}{2.3} \approx 82.6\%$  A key club member has an 82.6% probability of being enrolled in AP Physics.

PTS: 4

REF: 011735aii

NAT: S.CP.B.6

**TOP:** Conditional Probability

585 ANS:

$$P(W/D) = \frac{P(W^{\wedge}D)}{P(D)} = \frac{.4}{.5} = .8$$

PTS: 2

REF: 081726aii NAT: S.CP.B.6 TOP: Conditional Probability

586 ANS:

$$\frac{165 + 66 - 33}{825} = \frac{198}{825}$$

PTS: 2

REF: 081925aii

NAT: S.CP.B.6 TOP: Conditional Probability