

# ADVANCED ALGEBRA.

Tuesday, January 22, 1957—9:15 a.m. to 12:15 p.m., only

## Part I

*Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed.*

1. Write an equation of the line that passes through the point (1, -2) and that is parallel to the line whose equation is  $3x + 5y = 17$ . 1.....
2. In *simplest form*, express  $\frac{2 + \sqrt{-3}}{1 + 2\sqrt{-3}}$  as an equivalent fraction with a real denominator. 2.....
3. Solve the equation  $8x^8 = (\frac{1}{4})^3$  for  $x$ . 3.....
4. For what value of  $k$  is  $(x + 1)$  a factor of  $3x^4 - 5x^3 + kx^2 - 6x - 10$ ? 4.....
5. Find the cube root of 0.0753 to the *nearest thousandth*. 5.....
6. Find  $\log_3 7.2$  to the *nearest tenth*. 6.....
7. Write in *simplest form* the sixth term in the expansion of  $(2 + \frac{x}{2})^8$ . 7.....
8. An equation with real coefficients has  $1 - i$  and  $i$  among its roots. What is the lowest possible degree of the equation? 8.....
9. If the graphs of  $x^2 - y^2 = 1$  and  $4x^2 + y^2 = 16$  are drawn on the same set of axes, how many points will they have in common? 9.....
10. How many different committees of 3 can be chosen from 10 people if a certain person must always be included in each committee? 10.....
11. If an event may happen in 4 ways and fail to happen in 7 ways, each equally probable, what is the probability that the event will happen? 11.....
12. If  $\log_a 2 = x$  and  $\log_a 5 = y$ , express  $\log_a 40$  in terms of  $x$  and  $y$ . 12.....
13. The sum of two roots of the equation  $2x^3 - 6x^2 - 8x + q = 0$  is zero. Find the third root. 13.....
14. Find the rational fractional root of  $2x^3 + x^2 + 2x - 12 = 0$ . 14.....
15. If  $x$  varies directly as  $y$  and inversely as the square of  $z$ , and if  $x = 72$  when  $y = 3$  and  $z = 1/2$ , find  $x$  when  $y = 1/4$  and  $z = 1/6$ . 15.....
16. If  $f(x) = \frac{2}{x^3 + x^{-1}}$ , find in *simplest form* the value of  $f(8)$ . 16.....

17. For what positive value of  $k$  is the graph of  $y = kx^2 + 6x + k + 8$  tangent to the  $x$ -axis? 17.....
18. Find the number of real roots of the equation  $x^5 + x^3 + 1 = 0$ . 18.....
19. Transform the equation  $x^4 - 27x^2 - 27x - 162 = 0$  into an equation whose roots are those of the original equation each divided by 3. 19.....
20. Transform the equation  $2x^3 - x^2 + 2x + 7 = 0$  into an equation whose roots are those of the original equation each decreased by 2. 20.....

Part II

Answer five questions from this part. Show all work.

21. Solve the equation  $2x^4 + x^3 - 3x^2 + 5x - 2 = 0$ . [10]
22. Find to the nearest tenth the real root of  $2x^3 - 5x^2 - 2 = 0$ . [10]
23. a. Draw the graph of  $y = 2^x$  for values of  $x$  from  $-2$  to  $4$ , inclusive. [6]  
 b. On the same set of axes used in part a, draw the graph of  $y = x^2 - 4x + 2$  for values of  $x$  from  $-1$  to  $5$ , inclusive. [3]  
 c. From the graphs made in answer to parts a and b, estimate to tenths the real root of the equation  $2^x = x^2 - 4x + 2$ . [1]
24. Given  $A^{0.3} = \frac{\quad}{h}$ . If  $A = 4.91$  and  $h = 2.86$ , find  $T$  to the nearest hundredth. [10]
25. [The proof of this formula is not a required topic in the syllabus for Twelfth Year Mathematics.]  
 a. Prove that  ${}_nC_r$  equals  ${}_nC_{n-r}$ . [8]  
 b. Find the value of  ${}_{40}C_{37}$ . [2]
26. A man traveled 60 miles by bus and 600 miles by plane, taking 6 hours for the trip. On the return trip the average speed of the plane was reduced by 50 miles an hour, but the speed of the bus was increased by 10 miles an hour so that the return trip also took 6 hours. Find the average speed of the plane and the average speed of the bus. [5, 5]
27. Bruce and Roy working together can do a job in 6 hours, Bruce and John working together can do the job in 8 hours and Roy and John together can do the job in 12 hours. How many hours will it take each boy working alone to do the job? [4, 6]
- \*28. a. Using derivatives, find the coordinates of the maximum point, the minimum point and the point of inflection of the equation  $y = 2x^3 + 3x^2 - 12x + 1$ . [3, 2, 3]  
 b. Sketch the graph of the equation  $y = 2x^3 + 3x^2 - 12x + 1$ . [2]
- \*29. a. Express  $-3i$  in the form  $r(\cos \theta + i \sin \theta)$ . [2]  
 b. Express  $-1 + i\sqrt{3}$  in the form  $r(\cos \theta + i \sin \theta)$ . [2]  
 c. Express  $10(\cos 234^\circ + i \sin 234^\circ)$  in  $a + bi$  form. Express the values of  $a$  and  $b$  to the nearest tenth. [3]  
 d. Express in the form  $r(\cos \theta + i \sin \theta)$  the root of  $x^5 - 1 = 0$  that, when represented graphically, lies in the fourth quadrant. [3]
- \*These questions are based upon optional topics in the syllabus.