

# ADVANCED ALGEBRA

Wednesday, June 20, 1956—9:15 a.m. to 12:15 p.m., only

## Part I

*Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed.*

1. Express  $\frac{2}{1-i}$  in the form  $a + bi$ . 1.....
  2. Write an equation of the line which passes through the origin and is parallel to the line  $2y = 3x - 6$ . 2.....
  3. What is the value of  $\log_b 1$  if  $b$  is a finite positive number not equal to 0 or 1? 3.....
  4. Solve the following set of equations for  $a$ :
 
$$\frac{2}{a} + \frac{3}{b} = -1$$

$$\frac{1}{a} - \frac{3}{b} = 4$$
4.....
  5. If  $x - 1$  is a factor of  $x^{10} - kx^7 - 3$ , find  $k$ . 5.....
  6. Write in simplest form the fifth term in the expansion of  $(e^x + e^{-x})^8$ . 6.....
  7. If the graphs of  $y^2 = 4x$  and  $x^2 = 4y$  are drawn on the same set of axes, in how many points do the graphs intersect? 7.....
  8. Find the sum of the infinite geometric progression 4, -2, 1, . . . . 8.....
  9. A bag contains 6 red and 4 white balls. If 2 balls are drawn at random from the bag, what is the probability that both will be white? 9.....
  10. In how many ways can 5 men— $A, B, C, D$  and  $E$ —be seated on a bench if  $A$  and  $B$  must occupy end positions? 10.....
- Directions (11-14): Indicate the correct completion for each of the following by writing the letter  $a, b$  or  $c$  on the line at the right.*
11. If the complex number  $2x + iy$  is equal to the complex number  $6 + 5i$ , then the value of  $x$  is (a)2 (b)3 (c)6 11.....
  12. The graphs of the equations  $x - 2y = 4$  and  $2x - 4y = 8$  (a)are parallel (b)intersect (c)coincide 12.....
  13. As the value of  $c$  increases, the graph of the equation  $y = x^2 + c$  moves (a)in the positive direction of the  $x$ -axis (b)in the positive direction of the  $y$ -axis (c)in the negative direction of the  $x$ -axis 13.....
  14. The equation  $f(x) = 0$  has no real roots between  $x = 1$  and  $x = 2$  if both  $f(1)$  and  $f(2)$  are positive. This statement is (a)always true (b)sometimes true (c)never true 14.....
  15. Solve for  $x$ :  $2^{x+2} = (\frac{1}{2})^x$  15.....
  16. Find the product of the four fourth roots of 1. 16.....

17. If  $w$  varies directly as  $a$  and inversely as  $d$ , and if  $w = 8$  when  $a = 2$  and  $d = 3$ , find  $w$  when  $a = 1$  and  $d = 6$ . 17.....
18. What is the rational fractional root of the equation  $5x^3 + 6x^2 + 6x + 1 = 0$ ? 18.....
19. Find the sum of the roots of the equation whose roots are the roots of the equation  $x^3 + 6x^2 - 5x + 7 = 0$  each increased by 2. 19.....
20. Write an equation whose roots are the roots of the equation  $x^3 + 6x^2 - 5x + 7 = 0$  each multiplied by 2. 20.....

## Part II

Answer five questions from this part. Show all work.

21. Find to the nearest tenth the positive root of the equation  $x^3 - 4x^2 + 14x - 22 = 0$ . [10]
22. Solve the equation  $x^4 - 3x^3 + x^2 - x - 6 = 0$ . [10]
23. Using logarithms, find to the nearest hundredth  
 a.  $\log_5 1.71$  [4]  
     28.4  
 b.  $\frac{\quad}{\sqrt[3]{-0.648}}$  [6]
24. a. Draw the graph of  $(x - 3)^2 + (y + 4)^2 = 25$ . [4]  
 b. On the same set of axes used in answer to part a, draw the graph of  $3x - y - 8 = 0$ . [2]  
 c. Find the length of the line segment whose end points are the intersections of the graphs drawn in parts a and b. [Answer may be left in radical form.] [4]
25. a. Derive the formula for the roots of the quadratic equation  $ax^2 + bx + c = 0$  in terms of the coefficients  $a$ ,  $b$  and  $c$ . [7]  
 b. Solve for  $x$ :  $rx^2 - 3rx + r + 2 = 0$ . [3]
26. A boat travels in still water at the rate of 5 miles per hour with a load and 15 miles per hour without a load. If the boat travels upstream a distance of 81 miles with a load and makes the return trip without a load, taking 48 hours for the entire trip including 3 hours for unloading, what is the rate of the river current? [10]
27. A man has 20 machines that work automatically when started. Each machine produces 6 yards of material per hour. The man starts the first machine at 8 a.m. and each of the others at 5-minute intervals thereafter. How many yards of material will be completed at 10 a.m.? [6, 4]
- \*28. a. Write equations of the tangents to the graph of  $y = x^2 + 2x - 3$  at the points where the graph cuts the  $x$ -axis. [6]  
 b. Show that these tangents intersect at a point on the axis of symmetry of the graph of  $y = x^2 + 2x - 3$ . [4]
- \*29. a. Find the difference between the amplitudes (angles) of  $-1 + i\sqrt{3}$  and  $1 + i$ . [4]  
 b. Express in polar form  $4 - 3i$ . [Give the amplitude to the nearest degree.] [3]  
 c. Find, in polar form, the fifth root of 32 whose amplitude is between  $90^\circ$  and  $180^\circ$ . [Give the amplitude to the nearest degree.] [3]
- \* These questions are based upon the optional topics in the syllabus.