Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. 

1 Which statement is not always true?
(1) The product of two irrational numbers is irrational.
(2) The product of two rational numbers is rational.
(3) The sum of two rational numbers is rational.
(4) The sum of a rational number and an irrational number is irrational.

2 A satellite television company charges a one-time installation fee and a monthly service charge. The total cost is modeled by the function \( y = 40 + 90x \). Which statement represents the meaning of each part of the function?
(1) \( y \) is the total cost, \( x \) is the number of months of service, $90 is the installation fee, and $40 is the service charge per month.
(2) \( y \) is the total cost, \( x \) is the number of months of service, $40 is the installation fee, and $90 is the service charge per month.
(3) \( x \) is the total cost, \( y \) is the number of months of service, $40 is the installation fee, and $90 is the service charge per month.
(4) \( x \) is the total cost, \( y \) is the number of months of service, $90 is the installation fee, and $40 is the service charge per month.

3 If \( 4x^2 - 100 = 0 \), the roots of the equation are
(1) \(-25\) and \(25\)
(2) \(-25\), only
(3) \(-5\) and \(5\)
(4) \(-5\), only

\[ y(x^2 - 25) = 0 \]
\[ x^2 - 25 = 0 \]
\[ x = \pm 5 \]
4 Isaiah collects data from two different companies, each with four employees. The results of the study, based on each worker's age and salary, are listed in the tables below.

<table>
<thead>
<tr>
<th>Company 1</th>
<th>Company 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Worker's Age in Years</strong></td>
<td><strong>Salary in Dollars</strong></td>
</tr>
<tr>
<td>25</td>
<td>30,000</td>
</tr>
<tr>
<td>27</td>
<td>32,000</td>
</tr>
<tr>
<td>28</td>
<td>35,000</td>
</tr>
<tr>
<td>33</td>
<td>38,000</td>
</tr>
</tbody>
</table>

Which statement is true about these data?

1. The median salaries in both companies are greater than $37,000.
2. The mean salary in company 1 is greater than the mean salary in company 2.
3. The salary range in company 2 is greater than the salary range in company 1.
4. The mean age of workers at company 1 is greater than the mean age of workers at company 2.

5 Which point is not on the graph represented by \( y = x^2 + 3x - 6 \)?

1. \((-6, 12)\)  
2. \((-4, -2)\)  
3. \((2, 4)\)  
4. \((3, -6)\)  
5. \((3, -2)\)
6 A company produces $x$ units of a product per month, where $C(x)$ represents the total cost and $R(x)$ represents the total revenue for the month. The functions are modeled by $C(x) = 300x + 250$ and $R(x) = -0.5x^2 + 800x - 100$. The profit is the difference between revenue and cost where $P(x) = R(x) - C(x)$. What is the total profit, $P(x)$, for the month?

(1) $P(x) = -0.5x^2 + 500x - 150$
(2) $P(x) = -0.5x^2 + 500x - 350$
(3) $P(x) = -0.5x^2 - 500x + 350$
(4) $P(x) = -0.5x^2 + 500x + 350$

7 What is one point that lies in the solution set of the system of inequalities graphed below?

(1) (7,0)  (3) (0,7)
(2) (3,0)  (4) (-3,5)
8. The value of the x-intercept for the graph of $4x - 5y = 40$ is

(1) 10
(2) $\frac{4}{5}$

(3) $-\frac{4}{5}$
(4) $-8$

$4x - 5(0) = 40$

$4x = 40$
$x = 10$

9. Sam and Jeremy have ages that are consecutive odd integers. The product of their ages is 783. Which equation could be used to find Jeremy's age, $j$, if he is the younger man?

(1) $j^2 + 2 = 783$
(2) $j^2 - 2 = 783$

(3) $j^2 + 2j = 783$
(4) $j^2 - 2j = 783$

$\frac{j(j + 2)}{2} = 783$

10. A population that initially has 20 birds approximately doubles every 10 years. Which graph represents this population growth?

(1) 
(2) 
(3) 
(4)
11 Let \( f \) be a function such that \( f(x) = 2x - 4 \) is defined on the domain \( 2 \leq x \leq 6 \). The range of this function is

(1) \( 0 \leq y \leq 8 \)  
(2) \( 0 \leq y < \infty \)  
(3) \( 2 \leq y \leq 6 \)  
(4) \( -\infty < y < \infty \)

\[ 2(2)-4 = 4-4 = 0 \]
\[ 2(6)-4 = 12-4 = 8 \]

12 Which situation could be modeled by using a linear function?

(1) a bank account balance that grows at a rate of 5% per year, compounded annually
(2) a population of bacteria that doubles every 4.5 hours
(3) the cost of cell phone service that charges a base amount plus 20 cents per minute
(4) the concentration of medicine in a person's body that decays by a factor of one-third every hour

13 Which graph shows a line where each value of \( y \) is three more than half of \( x \)?

(1) \( y = \frac{1}{2}x + 3 \)  
(2) \( y = \frac{1}{3}x \)  
(3) \( y = x + 3 \)  
(4) \( y = 2x + 3 \)
14 The table below shows the average diameter of a pupil in a person's eye as he or she grows older.

<table>
<thead>
<tr>
<th>Age (years)</th>
<th>Average Pupil Diameter (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>4.7</td>
</tr>
<tr>
<td>30</td>
<td>4.3</td>
</tr>
<tr>
<td>40</td>
<td>3.9</td>
</tr>
<tr>
<td>50</td>
<td>3.5</td>
</tr>
<tr>
<td>60</td>
<td>3.1</td>
</tr>
<tr>
<td>70</td>
<td>2.7</td>
</tr>
<tr>
<td>80</td>
<td>2.3</td>
</tr>
</tbody>
</table>

What is the average rate of change, in millimeters per year, of a person's pupil diameter from age 20 to age 80?

(1) 2.4
(2) 0.04
(3) –2.4
(4) –0.04

15 Which expression is equivalent to $x^4 - 12x^2 + 36$?

(1) $(x^2 - 6)(x^2 - 6)$
(2) $(x^2 + 6)(x^2 + 6)$
(3) $(6 - x^2)(6 + x^2)$
(4) $(x^2 + 6)(x^2 - 6)$

16 The third term in an arithmetic sequence is 10 and the fifth term is 26. If the first term is $a_1$, which is an equation for the $n$th term of this sequence?

(1) $a_n = 8n + 10$
(2) $a_n = 8n - 14$
(3) $a_n = 16n + 10$
(4) $a_n = 16n - 38$
17 The graph of the equation $y = ax^2$ is shown below.

If $a$ is multiplied by $-\frac{1}{2}$, the graph of the new equation is

(1) wider and opens downward
(2) wider and opens upward
(3) narrower and opens downward
(4) narrower and opens upward

18 The zeros of the function $f(x) = (x + 2)^2 - 25$ are

(1) $-2$ and $5$
(2) $-3$ and $7$
(3) $-5$ and $2$
(4) $-7$ and $3$
19 During the 2010 season, football player McGee’s earnings, \( m \), were 0.005 million dollars more than those of his teammate Fitzpatrick’s earnings, \( f \). The two players earned a total of 3.95 million dollars. Which system of equations could be used to determine the amount each player earned, in millions of dollars?

\[
\begin{align*}
(1) \quad m + f &= 3.95 \\
(2) \quad m &= f + 3.95 \\
(3) \quad f &= m + 3.95 \\
(4) \quad m + f &= 3.95 \\
\end{align*}
\]

20 What is the value of \( x \) in the equation \( \frac{x - 2}{3} + \frac{1}{6} = \frac{5}{6} \)?

(1) 4 \hspace{1cm} (3) 8
(2) 6 \hspace{1cm} (4) 11

21 The table below shows the number of grams of carbohydrates, \( x \), and the number of Calories, \( y \), of six different foods.

<table>
<thead>
<tr>
<th>Carbohydrates (( x ))</th>
<th>Calories (( y ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>120</td>
</tr>
<tr>
<td>9.5</td>
<td>138</td>
</tr>
<tr>
<td>10</td>
<td>147</td>
</tr>
<tr>
<td>6</td>
<td>88</td>
</tr>
<tr>
<td>7</td>
<td>108</td>
</tr>
<tr>
<td>4</td>
<td>62</td>
</tr>
</tbody>
</table>

Which equation best represents the line of best fit for this set of data?

(1) \( y = 15x \) \hspace{1cm} (3) \( y = 0.1x - 0.4 \)
(2) \( y = 0.07x \) \hspace{1cm} (4) \( y = 14.1x + 5.8 \)
22 A function is graphed on the set of axes below.

Which function is related to the graph?

(1) \( f(x) = \begin{cases} x^2, & x < 1 \\ x - 2, & x > 1 \end{cases} \)

(2) \( f(x) = \begin{cases} x^2, & x < 1 \\ \frac{1}{2}x + \frac{1}{2}, & x > 1 \end{cases} \)

(3) \( f(x) = \begin{cases} x^2, & x < 1 \\ 2x - 7, & x > 1 \end{cases} \)

(4) \( f(x) = \begin{cases} x^2, & x < 1 \\ \frac{3}{2}x - \frac{9}{2}, & x > 1 \end{cases} \)

23 The function \( h(t) = -16t^2 + 144 \) represents the height, \( h(t) \), in feet, of an object from the ground at \( t \) seconds after it is dropped. A realistic domain for this function is

(1) \(-3 \leq t \leq 3\)  
(2) \(0 \leq t \leq 3\)  
(3) \(0 \leq h(t) \leq 144\)  
(4) all real numbers

24 If \( f(1) = 3 \) and \( f(n) = -2f(n - 1) + 1 \), then \( f(5) = \)

(1) \(-5\)  
(2) \(11\)  
(3) \(21\)  
(4) \(43\)
Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

25 In the equation $x^2 + 10x + 24 = (x + a)(x + b)$, $b$ is an integer. Find algebraically all possible values of $b$.

$$= (x + 6)(x + 4)$$
$$= (x + 4)(x + 6)$$

6 4 4

26 Rhonda deposited $3000 in an account in the Merrick National Bank, earning 4.2% interest, compounded annually. She made no deposits or withdrawals. Write an equation that can be used to find $B$, her account balance after $t$ years.

$$B = 3000 (1.042)^t$$
Guy and Jim work at a furniture store. Guy is paid $185 per week plus 3% of his total sales in dollars, $x$, which can be represented by $g(x) = 185 + 0.03x$. Jim is paid $275 per week plus 2.5% of his total sales in dollars, $x$, which can be represented by $f(x) = 275 + 0.025x$. Determine the value of $x$, in dollars, that will make their weekly pay the same.

\[ 185 + 0.03x = 275 + 0.025x \]

\[ 0.005x = 90 \]

\[ x = 18,000 \]
28 Express the product of $2x^2 + 7x - 10$ and $x + 5$ in standard form.

\[2x^3 + 7x^2 - 10x + 10x^2 + 35x - 50\]

\[2x^3 + 17x^2 + 25x - 50\]
29 Let $f$ be the function represented by the graph below.

Let $g$ be a function such that $g(x) = -\frac{1}{2}x^2 + 4x + 3$.

Determine which function has the larger maximum value. Justify your answer.

\[ x = \frac{-b}{2a}, \quad \frac{-4}{2(-\frac{1}{2})} = \frac{-4}{-1} = 4 \]
\[ y = \frac{-1}{2} \left(4\right)^2 + 4(4) + 3 \]
\[ = -8 + 16 + 3 \]
\[ = 11 \text{ is the Max} \]
\[ g(x) \text{ has the larger maximum} \]
30 Solve the inequality below to determine and state the smallest possible value for $x$ in the solution set.

$$3(x + 3) \leq 5x - 3$$

$$3x + 9 \leq 5x - 3$$

$$12 \leq 2x$$

$$6 \leq x$$

6 is smallest possible value
31 The table below represents the residuals for a line of best fit.

<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>3</th>
<th>3</th>
<th>4</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>Residual</td>
<td>2</td>
<td>1</td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
<td>-2</td>
<td>-1</td>
<td>2</td>
<td>0</td>
<td>3</td>
</tr>
</tbody>
</table>

Plot these residuals on the set of axes below.

Using the plot, assess the fit of the line for these residuals and justify your answer.

The line is a poor fit because the residuals form a pattern.
32 A student was given the equation \( x^2 + 6x - 13 = 0 \) to solve by completing the square. The first step that was written is shown below.

\[
x^2 + 6x = 13
\]

The next step in the student’s process was \( x^2 + 6x + c = 13 + c \).

State the value of \( c \) that creates a perfect square trinomial.

\[
0
\]

Explain how the value of \( c \) is determined.

\[
\left( \frac{6}{2} \right)^2
\]
Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33 On the axes below, graph \( f(x) = |3x| \).

If \( g(x) = f(x) - 2 \), how is the graph of \( f(x) \) translated to form the graph of \( g(x) \)?

\[ \text{Shift down 2 units} \]

If \( h(x) = f(x - 4) \), how is the graph of \( f(x) \) translated to form the graph of \( h(x) \)?

\[ \text{Shift right 4 units} \]
The formula for the area of a trapezoid is \( A = \frac{1}{2}h(b_1 + b_2) \). Express \( b_1 \) in terms of \( A \), \( h \), and \( b_2 \).

\[
2A = h(b_1 + b_2) \\
\frac{2A}{h} = b_1 + b_2 \\
\frac{2A}{h} - b_2 = b_1
\]

The area of a trapezoid is 60 square feet, its height is 6 ft, and one base is 12 ft. Find the number of feet in the other base.

\[
\frac{2(60)}{6} - 12 \\
20 - 12 \\
8
\]
35 Let \( f(x) = -2x^2 \) and \( g(x) = 2x - 4 \). On the set of axes below, draw the graphs of \( y = f(x) \) and \( y = g(x) \).

Using this graph, determine and state all values of \( x \) for which \( f(x) = g(x) \).

\[-2, 1\]
A school is building a rectangular soccer field that has an area of 6000 square yards. The soccer field must be 40 yards longer than its width. Determine algebraically the dimensions of the soccer field, in yards.

\[ W(W + 40) = 6000 \]

\[ W^2 + 40W - 6000 = 0 \]

\[ (W + 100)(W - 60) = 0 \]

\[ W = 60 \]

\[ L = 100 \]
37 Edith babysits for $x$ hours a week after school at a job that pays $4$ an hour. She has accepted a job that pays $8$ an hour as a library assistant working $y$ hours a week. She will work both jobs. She is able to work no more than 15 hours a week, due to school commitments. Edith wants to earn at least $80$ a week, working a combination of both jobs.

Write a system of inequalities that can be used to represent the situation.

\[ x + y \leq 15 \quad \text{and} \quad 4x + 8y \geq 80 \]

Graph these inequalities on the set of axes below.

Question 37 is continued on the next page.
Question 37 continued

Determine and state one combination of hours that will allow Edith to earn at least $80 per week while working no more than 15 hours.

\[(1, 10)\]