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Our Students. Their Moment.

Regents Examination in Algebra II (Common Core)

Sample Questions Spring 2015

New York State Common Core Sample Questions: Regents Examination in Algebra II (Common Core)

With the adoption of the New York P-12 Common Core Learning Standards (CCLS) in ELA/Literacy and Mathematics, the Board of Regents signaled a shift in both instruction and assessment. Educators around the state have already begun instituting Common Core instruction in their classrooms. To aid in this transition, we are providing sample Regents Examination in Algebra II (Common Core) questions to help students, parents, and educators better understand the instructional shifts demanded by the Common Core and the rigor required to ensure that all students are on track to college and career readiness.

These Questions Are Teaching Tools

The sample questions emphasize the instructional shifts demanded by the Common Core. For Algebra II (Common Core) we have provided fourteen questions. These questions include multiple-choice and constructed response. The sample questions are teaching tools for educators and can be shared freely with students and parents. They are designed to help clarify the way the Common Core should drive instruction and how students will be assessed on the Regents Examination in Algebra II measuring CCLS beginning in June 2016. NYSED is eager for feedback on these sample questions. Your input will guide us as we develop future exams.

These Questions Are NOT Test Samplers

While educators from around the state have helped craft these sample questions, they have not undergone the same extensive review, vetting, and field testing that occurs with actual questions used on the State exams. The sample questions were designed to help educators think about content, NOT to show how operational exams look exactly or to provide information about how teachers should administer the test.

How to Use the Sample Questions

- Interpret how the standards are conceptualized in each question.
- Note the multiple ways the standards are assessed throughout the sample questions.
- Look for opportunities for mathematical modeling, i.e., connecting mathematics with the real world by conceptualizing, analyzing, interpreting, and validating conclusions in order to make decisions about situations in everyday life, society, or the workplace.
- Consider the instructional changes that will need to occur in your classroom.

- Notice the application of mathematical ways of thinking to real-world issues and challenges.
- Pay attention to the strong distractors in each multiple-choice question.
- **Don't** consider these questions to be the only way the standards will be assessed.
- **Don't** assume that the sample questions represent a mini-version of future State exams.

Understanding Math Sample Questions

Multiple-Choice Questions

Sample multiple-choice math questions are designed to assess CCLS math standards. Math multiple-choice questions assess procedural fluency and conceptual understanding. Unlike questions on past math exams, many require the use of multiple skills and concepts. Within the sample questions, distractors will be based on plausible missteps.

Constructed Response Questions

Math constructed response questions are similar to past questions, asking students to show their work in completing one or more tasks or solving more extensive problems. Constructed response questions allow students to show their understanding of math procedures, conceptual understanding, and application.

Format of the Math Sample Questions Document

The Math Sample Questions document is formatted so that headings follow each question to provide information for teacher use to help interpret the question, understand measurement with the CCLS, and inform instruction. A list of the headings with a brief description of the associated information is shown below.

Key: This is the correct response or, in the case of multiple-choice questions, the correct option.

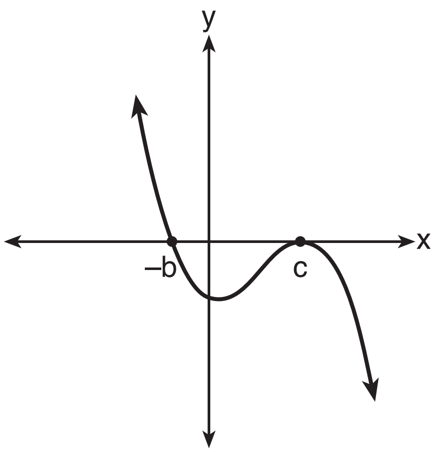
Measures CCLS: This question measures the knowledge, skills, and proficiencies characterized by the standards within the identified cluster.

Mathematical Practices: If applicable, this is a list of mathematical practices associated with the question.

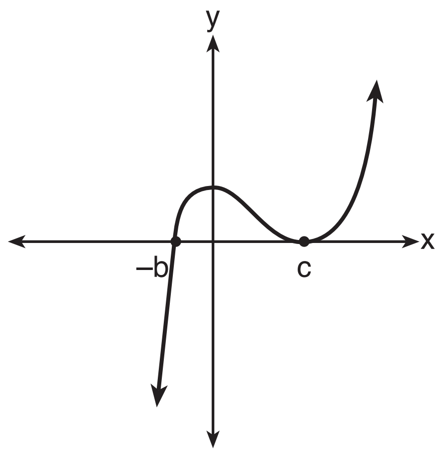
Commentary: This is an explanation of how the question measures the knowledge, skills, and proficiencies characterized by the identified cluster(s).

Rationale: For multiple-choice questions, this section provides the correct option and demonstrates one method for arriving at that response. For constructed response questions, one or more possible approaches to solving the question are shown, followed by the scoring rubric that is specific to the question. Note that there are often multiple approaches to solving each problem. The rationale section provides only one example. The scoring rubrics should be used to evaluate the efficacy of different methods of arriving at a solution.

1 If *a*, *b*, and *c* are all positive real numbers, which graph could represent the sketch of the graph of $p(x) = -a(x+b)(x^2 - 2cx + c^2)$?



(1)



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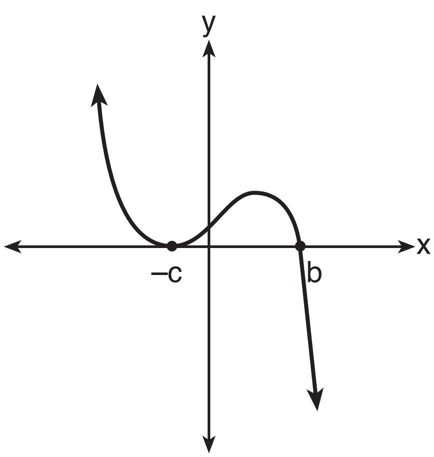

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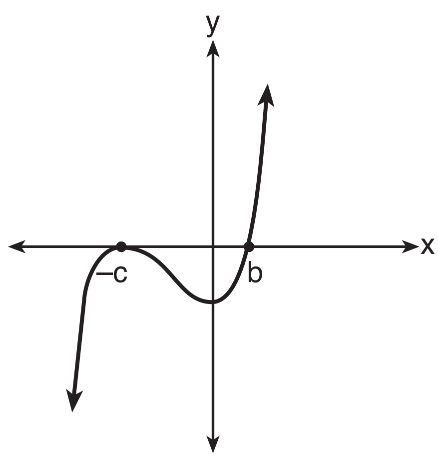


(4)

Key: 1

Measures CCLS Cluster: A-APR.B

Mathematical Practice: 4

Commentary: This question measures A-APR.B because students demonstrate understanding of the relationship between the factors of the polynomial and the zeros, and apply this understanding to constructing a rough graph of the function.

Rationale: Option 1 is correct. The zeros of the polynomial are at $-b$, and c. The sketch of a polynomial of degree 3 with a negative leading coefficient should have end behavior showing as x goes to negative infinity, $f(x)$ goes to positive infinity. The multiplicities of the roots are correctly represented in the graph.

2 Which equation represents a parabola with a focus of (0,4) and a directrix of $y = 2$?

(1) $y = x^2 + 3$

(2) $y = -x^2 + 1$

(3) $y = \frac{x^2}{2} + 3$

(4) $y = \frac{x^2}{4} + 3$

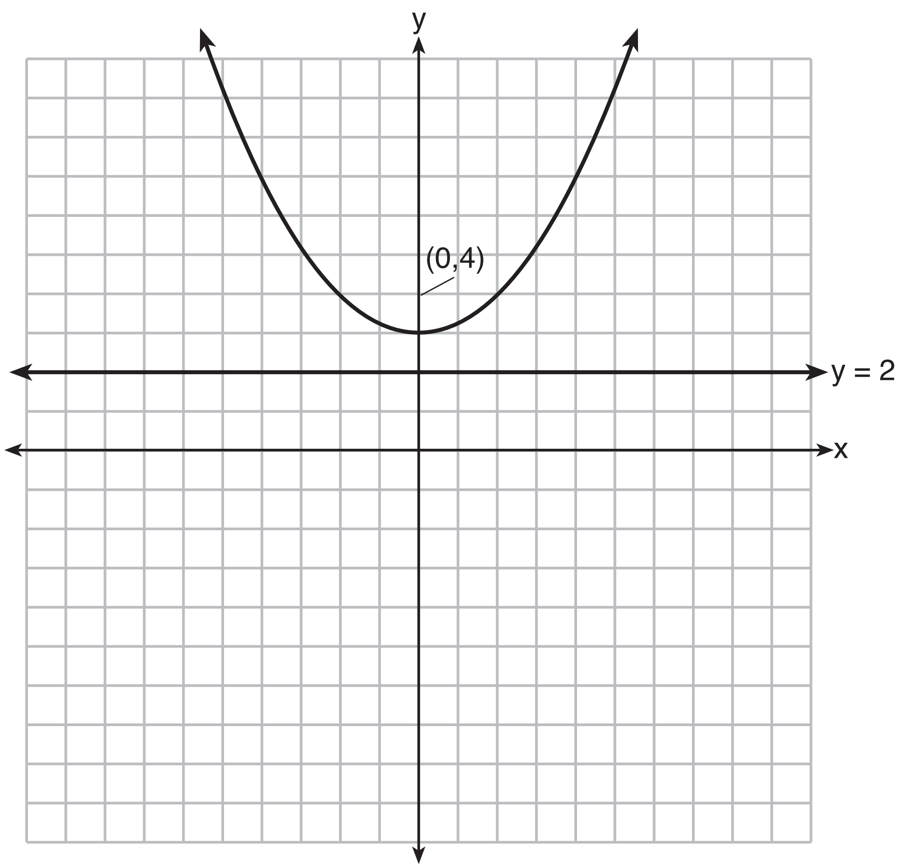
Key: 4

Measures CCLS Cluster: G-GPE.A

Mathematical Practice: 2, 7

Commentary: This question measures G-GPE.A because students need to determine the equation of a parabola given its focus and directrix.

Rationale: Option 4 is correct. A parabola with a focus of (0,4) and a directrix of $y = 2$ is sketched as follows:



By inspection, it is determined that the vertex of the parabola is (0,3). It is also evident that the distance, *p*, between the vertex and the focus is 1. It is possible to use the formula $(x-h)^2 = 4p(y-k)$ to derive the equation of the parabola as follows:

$$
\begin{aligned}
&(x-0)^2 = 4(1)(y-3) \\
&x^2 = 4y - 12 \\
&x^2 + 12 = 4y \\
&\frac{x^2}{4} + 3 = y
\end{aligned}
$$

or

A point (x, y) on the parabola must be the same distance from the focus as it is from the directrix. For any such point (x, y), the distance to the focus is $\sqrt{(x-0)^2 + (y-4)^2}$ and the distance to the directrix is $y-2$. Setting this equal leads to:

$$
\begin{aligned}
&\sqrt{(x-0)^2 + (y-4)^2} = y-2 \\
&x^2 + y^2 - 8y + 16 = y^2 - 4y + 4 \\
&x^2 + 16 = 4y + 4 \\
&\frac{x^2}{4} + 3 = y
\end{aligned}
$$

3 If the terminal side of angle θ, in standard position, passes through point (–4,3), what is the numerical value of sin θ?

(1) $\frac{3}{5}$

(2) $\frac{4}{5}$

(3) $-\frac{3}{5}$

(4) $-\frac{4}{5}$

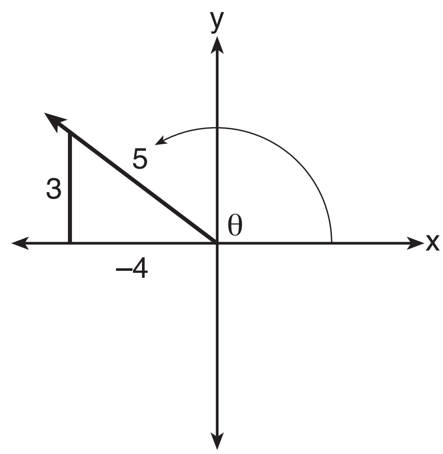
Key: 1

Measures CCLS Cluster: F-TF.A

Mathematical Practice: 5

Commentary: This question measures F-TF.A because students use the unit circle to find the numerical value of a trigonometric function.

Rationale: Option 1 is correct. A reference triangle can be sketched using the coordinates $(-4,3)$ in the second quadrant to find the value of $\sin\theta$.



4 A study of the annual population of the red-winged blackbird in Ft. Mill, South Carolina, shows the population, $B(t)$, can be represented by the function $B(t) = 750(1.16)^t$, where the t represents the number of years since the study began.

In terms of the monthly rate of growth, the population of red-winged blackbirds can be best approximated by the function

(1) $B(t) = 750(1.012)^t$

(2) $B(t) = 750(1.012)^{12t}$

(3) $B(t) = 750(1.16)^{12t}$

(4) $B(t) = 750(1.16)^{\frac{t}{12}}$

Key: 2

Measures CCLS Cluster: A-SSE.B

Mathematical Practice: 4, 6

Commentary: This question measures A-SSE.B because students use properties of exponents to transform an exponential function.

Rationale:

$$B(t) = 750\left(1.16^{\frac{1}{12}}\right)^{12t}$$

$$B(t) = 750(1.012445138)^{12t}$$

$$\boxed{B(t) = 750(1.012)^{12t}}$$

$B(t) = 750\left(1.16^{\frac{1}{12}}\right)^{12t}$ is used, as opposed to $B(t) = 750\left(1 + \frac{0.16}{12}\right)^{12t}$, because the growth is an annual rate that is not compounded monthly.

5 Use the properties of rational exponents to determine the value of y for the equation:

$$\frac{\sqrt[3]{x^8}}{\left(x^4\right)^{\frac{1}{3}}} = x^y,\ x > 1.$$

Key: $\frac{4}{3}$

Measures CCLS Cluster: N-RN.A

Mathematical Practice: 2

Commentary: This question measures N-RN.A because students extend their knowledge of integer exponents to rewriting and working with radicals in terms of rational exponents.

Rationale:

$$\frac{x^{\frac{8}{3}}}{x^{\frac{4}{3}}} = x^y$$

$$x^{\frac{4}{3}} = x^y$$

$$\frac{4}{3} = y$$

Rubric:

[2] $\frac{4}{3}$ or an equivalent value is written, and correct work using rational exponents is shown.

[1] Appropriate work is shown, but one computational error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

or

[1] $\frac{x^{\frac{8}{3}}}{x^{\frac{4}{3}}}$ is written, but no further correct work is shown.

or

[1] $\frac{4}{3}$, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

6 Write $(5+2yi)(4-3i)-(5-2yi)(4-3i)$ in $a+bi$ form, where y is a real number.

Key: $12y + 16yi$

Measures CCLS Cluster: N-CN.A

Mathematical Practice: 1, 7

Commentary: This question measures N-CN.A because students add, subtract, and multiply complex expressions, and apply the concept that $i^2 = -1$. The question rewards seeing structure and can be rewritten efficiently by applying the distributive property. The expression $12y + 16yi$ is correctly written in $a + bi$ form. However, if a student writes $16yi + 12y$, while not in $a + bi$ form, no credit should be deducted.

Rationale:

$$(4-3i)(5+2yi-5+2yi)$$
$$(4-3i)(4yi)$$
$$16yi-12yi^2$$
$$12y+16yi$$

Rubric:

[2] $12y + 16yi$, and correct work is shown.

[1] Appropriate work is shown, but one computational, factoring, or simplification error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

or

[1] $12y + 16yi$, but no work is shown.

[0] A zero response is completely incorrect, irrelevant or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

7 Use an appropriate procedure to show that $x - 4$ is a factor of the function $f\left(x\right) = 2x^3 - 5x^2 - 11x - 4.$ Explain your answer.

Key: See rationale below.

Measures CCLS Cluster: A-APR.B

Mathematical Practice: 3

Commentary: The question measures A-APR.B because an appropriate procedure is used to show 4 is the positive zero for $f(x)$.

Rationale:

$$f(4) = 2(4)^3 - 5(4)^2 - 11(4) - 4$$
$$f(4) = 0$$

or

$$\begin{array}{r|rrrr} 4 & 2 & -5 & -11 & -4 \\ & \downarrow & 8 & 12 & 4 \\ \hline & 2 & 3 & 1 & 0 \end{array}$$

or

$$\begin{array}{r l} & 2x^2 + 3x + 1 \\ x - 4 \,\big) & \overline{2x^3 - 5x^2 - 11x - 4} \\ & \underline{2x^3 - 8x^2} \\ & \quad\quad 3x^2 - 11x \\ & \quad\quad \underline{3x^2 - 12x} \\ & \quad\quad\quad\quad x - 4 \\ & \quad\quad\quad\quad \underline{x - 4} \\ & \quad\quad\quad\quad\quad 0 \end{array}$$

Any method that demonstrates 4 is a zero of $f(x)$ confirms that $x - 4$ is a factor, as suggested by the Remainder Theorem.

Rubric:

[2] Correct work is shown confirming $x - 4$ is a factor, and a correct explanation is written.

[1] Appropriate work is shown, but one computational error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

or

[1] Correct work is shown, but no explanation or an incorrect explanation is written.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

8 Solve algebraically for all values of *x*:

$$\sqrt{x-5}+x=7$$

Key: 6

Measures CCLS Cluster: A-REI.A

Mathematical Practice: 3, 6

Commentary: This question measures A-REI.A because the problem requires students to solve a radical equation and identify extraneous solutions.

Rationale:

$$\sqrt{x-5} = 7 - x$$
$$\left(\sqrt{x-5}\right)^2 = (7-x)^2$$
$$x - 5 = 49 - 14x + x^2$$
$$x^2 - 15x + 54 = 0$$
$$(x-6)(x-9) = 0$$

$$\begin{array}{l|l}
x - 6 = 0 & x - 9 = 0 \\
x = 6 & x = 9 \\
 & \\
\sqrt{6-5} + 6 = 7 & \sqrt{9-5} + 9 = 7 \\
7 = 7 & 11 \neq 7 \\
\textit{Accept} & \textit{Reject}
\end{array}$$

Rubric:

[2] 6 and correct algebraic work is shown.

[1] Appropriate work is shown, but one computational or simplification error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

or

[1] 6, but a method other than algebraic is used.

or

[1] Appropriate work is shown, but 9 is not rejected.

[0] 6, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

9 Monthly mortgage payments can be found using the formula below:

$$M = \frac{P\left(\frac{r}{12}\right)\left(1+\frac{r}{12}\right)^n}{\left(1+\frac{r}{12}\right)^n - 1}$$

M = monthly payment
P = amount borrowed
r = annual interest rate
n = number of monthly payments

The Banks family would like to borrow \$120,000 to purchase a home. They qualified for an annual interest rate of 4.8%. Algebraically determine the *fewest* number of whole years the Banks family would need to include in the mortgage agreement in order to have a monthly payment of no more than \$720.

Key: 23

Measures CCLS Cluster: A-SSE.B

Mathematical Practice: 1, 6

Commentary: This question measures A-SSE.B because students work with the sum of a finite geometric series through a related mortgage formula.

Rationale: A correct equation or inequality solved algebraically should receive full credit.

$$720 = \frac{120{,}000(0.004)(1.004)^n}{(1.004)^n - 1}$$

$$720(1.004)^n - 720 = 480(1.004)^n$$

$$240(1.004)^n = 720$$

$$1.004^n = 3$$

$$\frac{n \log 1.004}{\log 1.004} = \frac{\log 3}{\log 1.004}$$

$$n = 275.2020128 \qquad \frac{275.2020128}{12} \approx 23 \text{ years}$$

Rubric:

[4] 23, and correct algebraic work is shown.

[3] Appropriate work is shown, but one computational or rounding error is made.

or

[3] Appropriate work is shown to find n, but no further correct work is shown.

[2] Appropriate work is shown, but two or more computational or rounding errors are made.

or

[2] Appropriate work is shown, but one conceptual error is made.

or

[2] Appropriate work is shown to find $1.004^n = 3$, but no further correct work is shown.

or

[2] 23, but a method other than algebraic is used.

[1] Appropriate work is shown, but one conceptual error and one computational or rounding error are made.

or

[1] A correct substitution is made into the monthly mortgage formula but no further correct word is shown.

or

[1] 23, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

10 Solve the following system of equations algebraically for all values of x, y, and z:

$$\begin{aligned} x + 3y + 5z &= 45 \\ 6x - 3y + 2z &= -10 \\ -2x + 3y + 8z &= 72 \end{aligned}$$

Key: $x = -2$, $y = 4$, and $z = 7$

Measures CCLS Cluster: A-REI.C

Mathematical Practice: 1, 6

Commentary: This question measures A-REI.C because students are required to solve a 3×3 system of equations.

Rationale: Correct algebraic work is shown below.

$$\begin{aligned} x + 3y + 5z &= 45 && (1) \\ 6x - 3y + 2z &= -10 && (2) \\ -2x + 3y + 8z &= 72 && (3) \end{aligned}$$

$$\begin{aligned}
&7x + 7z = 35 && (1) + (2) \\
&\begin{bmatrix} x + z = 5 \\ 4x + 10z = 62 \end{bmatrix} && (2) + (3) \\
&-4(x + z = 5) \\
&\underline{+4x + 10z = 62} \\
&\frac{6z}{6} = \frac{42}{6} \\
&\boxed{z = 7} \\
&x + 7 = 5 \\
&\boxed{x = -2} \\
&-2 + 3y + 5(7) = 45 && (1) \\
&-2 + 3y = 10 \\
&3y = 12 \\
&\boxed{y = 4}
\end{aligned}$$

Rubric:

[4] $x = -2,\ y = 4$, and $z = 7$, and correct algebraic work is shown.

[3] Appropriate work is shown, but one computational error is made.

or

[3] Appropriate work is shown to find two of the solutions, but no further correct work is shown.

[2] Appropriate work is shown, but two or more computational errors are made.

or

[2] Appropriate work is shown, but one conceptual error is made.

or

[2] Appropriate work is shown to find one of the solutions, but no further correct work is shown.

or

[2] $x = -2,\ y = 4$, and $z = 7$, but a method other than algebraic is used.

[1] Appropriate work is shown, but one conceptual error and one computational error are made.

or

[1] Appropriate work is shown to eliminate y to create a system of two equations, but no further correct work is shown.

or

[1] $x = -2,\ y = 4$, and $z = 7$, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

11 Write an explicit formula for a_n, the nth term of the recursively defined sequence below.

$$a_1 = x + 1$$
$$a_n = x\left(a_{n-1}\right)$$

For what values of x would $a_n = 0$ when $n > 1$?

Key: $a_n = x^{n-1}(x+1)$ or $a_n = x^n + x^{n-1}$, and $x = 0$ and $x = -1$

Measures CCLS Cluster: F-BF.A

Mathematical Practice: 2, 8

Commentary: This question measures F-BF.A because students are required to translate a sequence from its recursive form to an explicit form.

Rationale:

$$\begin{aligned} a_1 &= x+1 \\ a_2 &= x(x+1) \\ a_3 &= x^2(x+1) \\ &\boxed{a_n = x^{n-1}(x+1)} \end{aligned} \qquad \textit{or} \qquad \begin{aligned} a_1 &= x+1 \\ a_2 &= x^2 + x \\ a_3 &= x^3 + x^2 \\ &\boxed{a_n = x^n + x^{n-1}} \end{aligned}$$

$$\begin{array}{c|c} x^{n-1} = 0 & (x+1) = 0 \\ \boxed{x=0} & \boxed{x=-1} \end{array}$$

Note: Students are not required to show work to solve $x^{n-1} = 0$. It is expected that *x* raised to a positive value set equal to 0 yields the solution $x = 0$.

Rubric:

[4] $a_n = x^{n-1}(x+1)$ or equivalent and $x = 0$ and $x = -1$, and correct work is shown.

[3] Appropriate work is shown, but one computational or simplification error is made.

[2] Appropriate work is shown, but two or more computational or simplification errors are made.

or

[2] Appropriate work is shown, but one conceptual error is made.

or

[2] Appropriate work is shown to find $a_n = x^{n-1}(x+1)$ or $x = 0$ and $x = -1$.

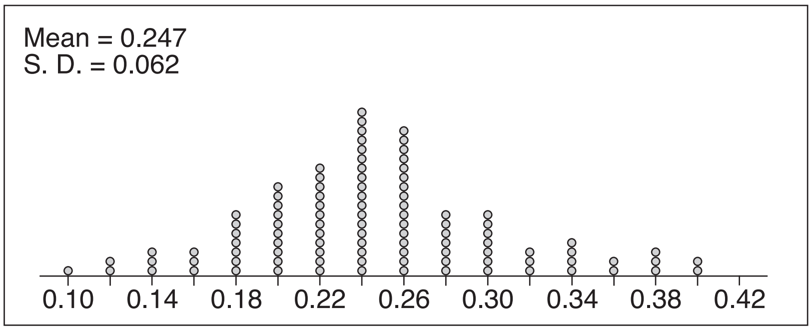
[1] Appropriate work is shown, but one conceptual error and one computational or simplification error are made.

or

[1] $a_n = x^{n-1}(x+1)$ and $x = 0$ and $x = -1,$ but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

12 Stephen's Beverage Company is considering whether to produce a new brand of cola. The company will launch the product if at least 25% of cola drinkers will buy the product. Fifty cola drinkers are randomly selected to take a blind taste-test of products *A*, *B*, and the new product. Nine out of fifty participants preferred Stephen's new cola to products *A* and *B*. The company then devised a simulation based on the requirement that 25% of cola drinkers will buy the product. Each dot in the graph shown below represents the proportion of people who preferred Stephen's new product, each of sample size 50, simulated 100 times.



Proportion Preferring Stephen's Product

Assume the set of data is approximately normal and the company wants to be 95% confident of its results. Does the sample proportion obtained from the blind taste-test, nine out of fifty, fall within the margin of error developed from the simulation? Justify your answer.

The company decides to continue developing the product even though only nine out of fifty participants preferred its brand of cola in the taste-test. Describe how the simulation data could be used to support this decision.

Key: See rationale below.

Measures CCLS Cluster: S-IC.B

Mathematical Practice: 2, 4

Commentary: This question measures S-IC.B since students estimate the margin of error based on a simulation model and use the data from the simulation to evaluate a company's decision.

Rationale: Yes. The margin of error from this simulation indicates that 95% of the observations fall within ± 0.12 of the simulated proportion, 0.25. The margin of error can be estimated by multiplying the standard deviation, shown to be 0.06 in the dotplot, by 2, or applying the estimated standard error formula, $\left(\sqrt{\frac{p(1-p)}{n}}\right) or \left(\sqrt{\frac{(0.25)(0.75)}{50}}\right)$ and multiplying by 2.

The interval 0.25 ± 0.12 includes plausible values for the true proportion of people who prefer Stephen's new product.

The company has evidence that the population proportion could be at least 25%. As seen in the dotplot, it can be expected to obtain a sample proportion of 0.18 (9 out of 50) or less several times, even when the population proportion is 0.25, due to sampling variability. Given this information, the results of the survey do not provide enough evidence to suggest that the true proportion is not at least 0.25, so the development of the product should continue at this time.

[4] Yes, and a correct justification is given, and a correct description is given.

[3] Appropriate work is shown, but one computational error is made.

[2] Appropriate work is shown, but two or more computational errors are made.

or

[2] Appropriate work is shown, but one conceptual error is made.

or

[2] Yes, and a correct justification is given, but no further correct work is shown.

or

[2] A correct description is given, but no further correct work is shown.

[1] Appropriate work is shown, but one conceptual error and one computational error are made.

or

[1] A correct margin of error is stated and yes, but no further correct work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

13 In contract negotiations between a local government agency and its workers, it is estimated that there is a 50% chance that an agreement will be reached on the salaries of the workers. It is estimated that there is a 70% chance that there will be an agreement on the insurance benefits. There is a 20% chance that no agreement will be reached on either issue.

Find the probability that an agreement will be reached on *both* issues.

Based on this answer, determine whether the agreement on salaries and the agreement on insurance are independent events. Justify your answer.

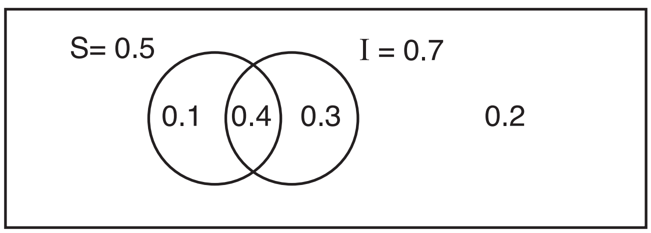
Key: 0.4 or an equivalent answer, and “no” with a correct justification.

Measures CCLS Cluster: S-CP.A

Mathematical Practice: 2, 4

Commentary: This question measures S-CP.A because the student must understand events as subsets of the sample space, including the concepts of union, intersection, and complement. Additionally, the student must reason using properties of probability about whether or not events are independent.

Rationale: This scenario can be modeled with a Venn Diagram:



Since $P(S \cup I)_c = 0.2, P(S \cup I) = 0.8$

Then, $P(S \cap I) = P(S) + P(I) - P(\mathrm{S} \cup \mathrm{I})$

$= 0.5 + 0.7 - 0.8$

$= 0.4$

If S and I are independent, then the Product Rule must be satisfied. However,

$$(0.5)(0.7) \neq 0.4$$

Therefore, salary and insurance have not been treated independently.

or

If S and I are independent, the conditional probability of $P(S / I) = P(S)$. However, $\frac{0.4}{0.7} \neq 0.5.$

Therefore, salary and insurance have not been treated independently.

Rubric:

[4] 0.4 or an equivalent answer and “no,” and correct work is shown, and a correct justification is given.

[3] Appropriate work is shown, but one computational, simplification, or rounding error is made.

or

[3] Appropriate work is shown to find 0.4 and “no,” but no justification is given.

[2] Appropriate work is shown, but two or more computational, simplification, or rounding errors are made.

or

[2] Appropriate work is shown, but one conceptual error is made.

or

[2] 0.4 or an equivalent answer, and correct work is shown, but no further correct work is shown.

[1] Appropriate work is shown, but one conceptual error and one computational, simplification, or rounding error are made.

or

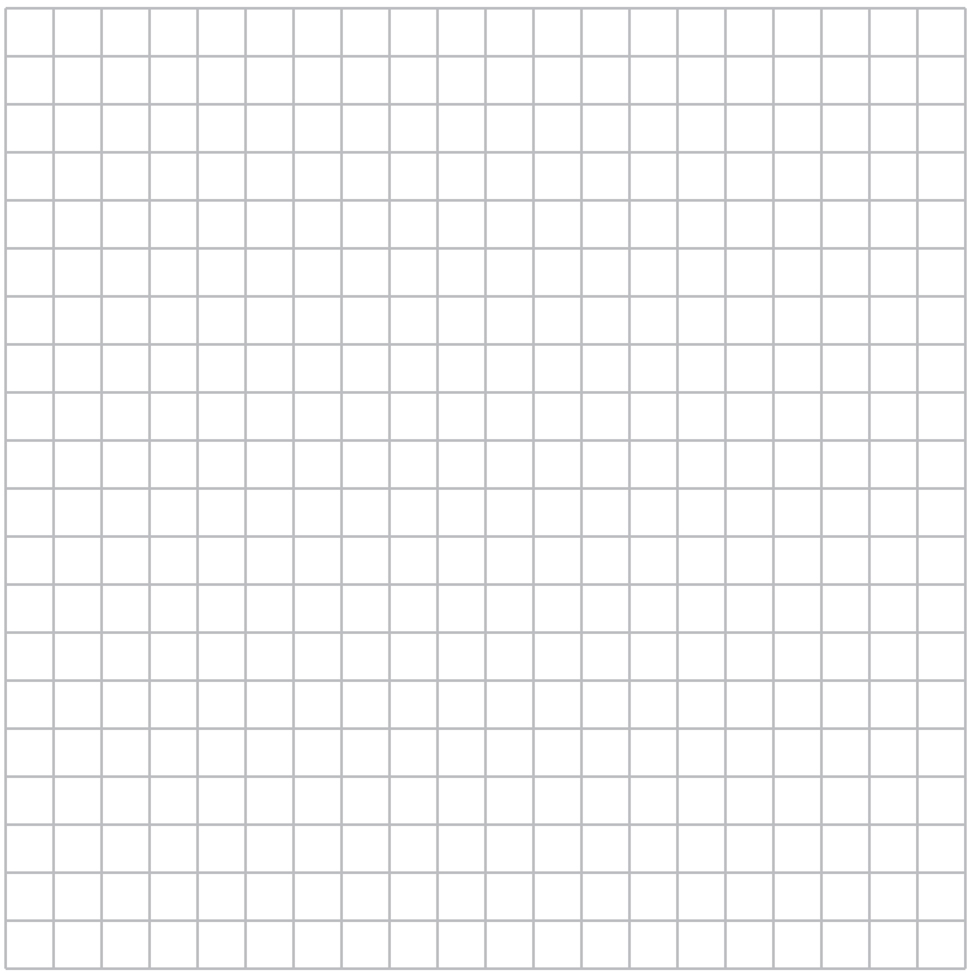
[1] 0.4 and “no,” but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

14 The ocean tides near Carter Beach follow a repeating pattern over time, with the amount of time between each low and high tide remaining relatively constant. On a certain day, low tide occurred at 8:30 a.m. and high tide occurred at 3:00 p.m. At high tide, the water level was 12 inches above the average local sea level; at low tide it was 12 inches below the average local sea level. Assume that high tide and low tide are the maximum and minimum water levels each day, respectively.

Write a cosine function of the form $f(t) = A\cos(Bt)$, where A and B are real numbers, that models the water level, $f(t)$, in inches above or below the average Carter Beach sea level, as a function of the time measured in t hours since 8:30 a.m.

On the grid below, graph one cycle of this function.



People who fish in Carter Beach know that a certain species of fish is most plentiful when the water level is increasing. Explain whether you would recommend fishing for this species at 7:30 p.m. or 10:30 p.m. using evidence from the given context.

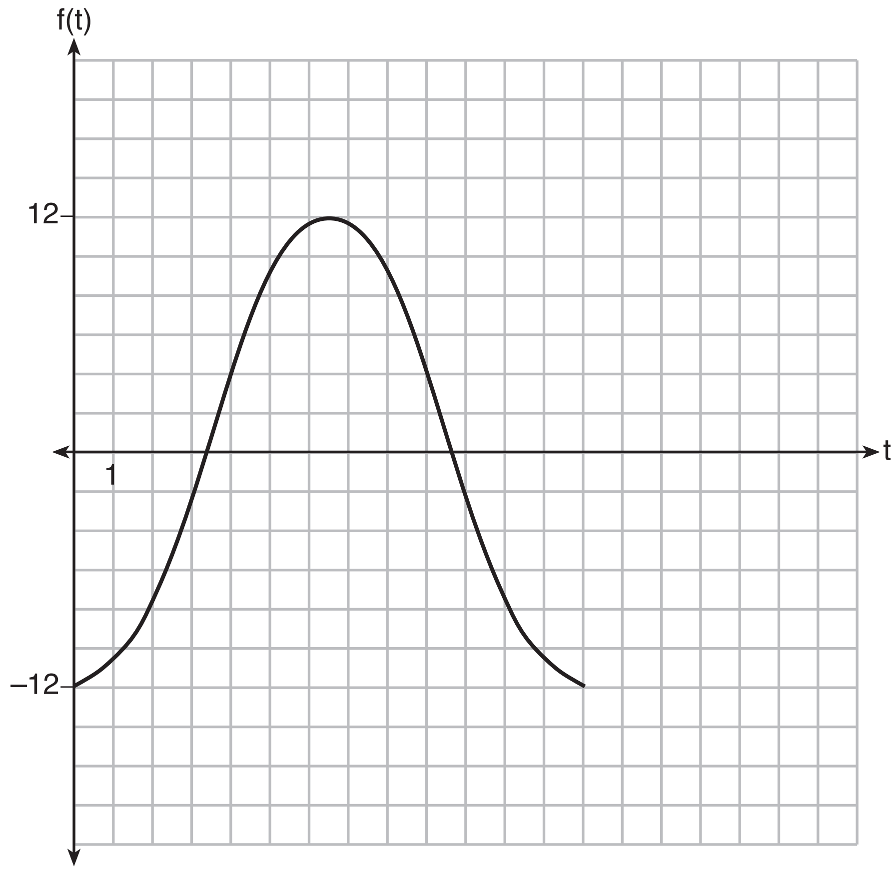
Key: $f(t) = -12\cos\left(\frac{2\pi}{13}t\right)$ or equivalent function, a correct graph is drawn, and 10:30 pm with appropriate evidence from the context.

Measures CCLS Cluster: F-IF.B

Mathematical Practice: 2,4

Commentary: The question measures F-IF.B because the student must interpret a key feature of the graph of a function, and intervals on which the function is increasing. This question also measures F-TF.B and F-IF.C because the student must choose and graph a trigonometric function to model a periodic phenomenon, the level of water near Carter Beach over time.

Rationale: The amplitude, 12, can be interpreted from the situation, since the water level has a minimum of −12 and a maximum of 12. The value of *A* is −12 since at 8:30 it is low tide. The period of the function is 13 hours, and is expressed in the function through the parameter *B*. By experimentation with technology or using the relation $P = \frac{2\pi}{B}$ (where *P* is the period), it is determined that $B = \frac{2\pi}{13}$.



In order to answer the question about when to fish, the student must interpret the function and determine which choice, 7:30 pm or 10:30 pm, is on an increasing interval. Since the function is increasing from t = 13 to t = 19.5 (which corresponds to 9:30 pm to 4:00 am), 10:30 is the appropriate choice.

Rubric:

[6] $f(t) = -12\cos\left(\frac{2\pi}{13}t\right)$ or equivalent function, a correct graph is drawn, and 10:30 pm with appropriate evidence from the context.

[5] Appropriate work is shown, but one computational or graphing error is made.

or

[5] Appropriate work is shown, but the explanation is incomplete.

or

[5] Appropriate work is shown, but an error is made in determining *A* or *B* in the function.

[4] Appropriate work is shown, but two computational or graphing errors are made.

or

[4] Appropriate work is shown, but one conceptual error is made.

or

[4] $f(t) = -12\cos\left(\dfrac{2\pi}{13}t\right)$ and a correct graph is drawn, but no further correct work is shown.

[3] Appropriate work is shown, but three or more computational errors are made.

or

[3] Appropriate work is shown, but one conceptual and one computational error are made.

[2] Appropriate work is shown, but two conceptual errors are made.

or

[2] Appropriate work is shown, but one conceptual and two or more computational errors are made.

or

[2] 10:30 pm and a correct explanation with evidence from the context is written, but no further correct work is shown.

[1] Appropriate work is shown, but two conceptual and one computational errors are made.

or

[1] $-12\cos\left(\dfrac{2\pi}{13}t\right)$ but no further correct work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by and obviously incorrect procedure.

Regents Examination in Algebra II (Common Core)

Sample Questions Fall 2015

New York State Common Core Sample Questions: Regents Examination in Algebra II (Common Core)

With the adoption of the New York P-12 Common Core Learning Standards (CCLS) in ELA/Literacy and Mathematics, the Board of Regents signaled a shift in both instruction and assessment. Educators around the state have already begun instituting Common Core instruction in their classrooms. To aid in this transition, we are providing sample Regents Examination in Algebra II (Common Core) questions to help students, parents, and educators better understand the instructional shifts demanded by the Common Core and the rigor required to ensure that all students are on track to college and career readiness.

These Questions Are Teaching Tools

The sample questions emphasize the instructional shifts demanded by the Common Core. For Algebra II (Common Core) we have provided seventeen questions. These questions include multiple-choice and constructed response. The sample questions are teaching tools for educators and can be shared freely with students and parents. They are designed to help clarify the way the Common Core should drive instruction and how students will be assessed on the Regents Examination in Algebra II measuring CCLS beginning in June 2016. NYSED is eager for feedback on these sample questions. Your input will guide us as we develop future exams.

These Questions Are NOT Test Samplers

While educators from around the state have helped craft these sample questions, they have not undergone the same extensive review, vetting, and field testing that occurs with actual questions used on the State exams. The sample questions were designed to help educators think about content, NOT to show how operational exams look exactly or to provide information about how teachers should administer the test.

How to Use the Sample Questions

- Interpret how the standards are conceptualized in each question.
- Note the multiple ways the standards are assessed throughout the sample questions.
- Look for opportunities for mathematical modeling, i.e., connecting mathematics with the real world by conceptualizing, analyzing, interpreting, and validating conclusions in order to make decisions about situations in everyday life, society, or the workplace.
- Consider the instructional changes that will need to occur in your classroom.
- Notice the application of mathematical ways of thinking to real-world issues and challenges.

- Pay attention to the strong distractors in each multiple-choice question.
- **Don't** consider these questions to be the only way the standards will be assessed.
- **Don't** assume that the sample questions represent a mini-version of future State exams.

Understanding Math Sample Questions

Multiple-Choice Questions

Sample multiple-choice math questions are designed to assess CCLS math standards. Math multiple-choice questions assess procedural fluency and conceptual understanding. Unlike questions on past math exams, many require the use of multiple skills and concepts. Within the sample questions, distractors will be based on plausible missteps.

Constructed Response Questions

Math constructed response questions are similar to past questions, asking students to show their work in completing one or more tasks or solving more extensive problems. Constructed response questions allow students to show their understanding of math procedures, conceptual understanding, and application.

Format of the Math Sample Questions Document

The Math Sample Questions document is formatted so that headings follow each question to provide information for teacher use to help interpret the question, understand measurement with the CCLS, and inform instruction. A list of the headings with a brief description of the associated information is shown below.

Key: This is the correct response or, in the case of multiple-choice questions, the correct option.

Measures CCLS: This question measures the knowledge, skills, and proficiencies characterized by the standards within the identified cluster.

Mathematical Practices: If applicable, this is a list of mathematical practices associated with the question.

Commentary: This is an explanation of how the question measures the knowledge, skills, and proficiencies characterized by the identified cluster(s).

Rationale: For multiple-choice questions, this section provides the correct option and demonstrates one method for arriving at that response. For constructed response questions, one or more possible approaches to solving the question are shown, followed by the scoring rubric that is specific to the question. Note that there are often multiple approaches to solving each problem. The rationale section provides only one example. The scoring rubrics should be used to evaluate the efficacy of different methods of arriving at a solution.

1 What is the solution set of the equation $\dfrac{3x + 25}{x + 7} - 5 = \dfrac{3}{x}$?

(1) $\left\{\dfrac{3}{2}, 7\right\}$

(2) $\left\{\dfrac{7}{2}, -3\right\}$

(3) $\left\{-\dfrac{3}{2}, 7\right\}$

(4) $\left\{-\dfrac{7}{2}, -3\right\}$

Key: 4

Measures CCLS Cluster: A-REI.A

Mathematical Practice: 2, 7

Commentary: This question measures A-REI.A because students must solve a rational equation.

Rationale: Option 4 is correct.

$$x(x+7)\left[\frac{3x+25}{x+7}-5=\frac{3}{x}\right];\ x\neq -7,\ x\neq 0$$

$$x(3x+25)-5x(x+7)=3(x+7)$$

$$3x^2+25x-5x^2-35x=3x+21$$

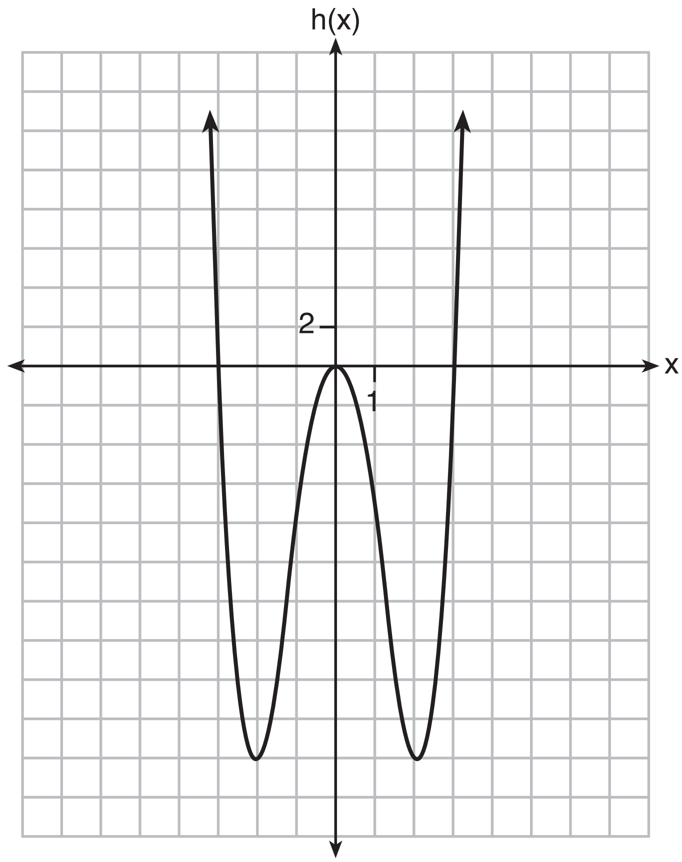
$$2x^2+13x+21=0$$

$$(2x+7)(x+3)=0$$

$2x+7=0$	$x+3=0$
$x=-\frac{7}{2}$	$x=-3$

2 Functions f, g, and h are given below.

$$f(x) = \sin(2x)$$
$$g(x) = f(x) + 1$$



Which statement is true about functions f, g, and h?

(1) $f(x)$ and $g(x)$ are odd, $h(x)$ is even.

(2) $f(x)$ and $g(x)$ are even, $h(x)$ is odd.

(3) $f(x)$ is odd, $g(x)$ is neither, $h(x)$ is even.

(4) $f(x)$ is even, $g(x)$ is neither, $h(x)$ is odd.

Key: 3

Measures CCLS Cluster: F-BF.B

Mathematical Practice: 2, 5

Commentary: This question measures F-BF.B because students must be able to recognize even and odd functions.

Rationale: Option 3 is correct.

$f(-x) = -f(x)$ or f is symmetric about the origin
$f(x) \rightarrow$ odd

$h(-x) = h(x)$ or h is symmetric about the y-axis
$h(x) \rightarrow$ even

For example, consider $x = 1$

$g(1) = 1.9093$
$g(-1) = .0907$
$g(-x) \neq g(x) \rightarrow$ not even
$g(-x) \neq -g(x) \rightarrow$ not odd
$g(x) \rightarrow$ neither

3 The expression $\dfrac{6x^3+17x^2+10x+2}{2x+3}$ equals

(1) $3x^2+4x-1+\dfrac{5}{2x+3}$

(2) $6x^2+8x-2+\dfrac{5}{2x+3}$

(3) $6x^2-x+13-\dfrac{37}{2x+3}$

(4) $3x^2+13x+\dfrac{49}{2}+\dfrac{151}{2x+3}$

Key: 1

Measures CCLS Cluster: A-APR.D

Mathematical Practice: 8

Commentary: This question measures A-APR.D because students must rewrite a simple rational expression in quotient-remainder form.

Rationale: Option 1 is correct.

$$\begin{array}{r}
3x^2+4x-1 \\
2x+3\overline{)6x^3+17x^2+10x+2} \\
\underline{6x^3+9x^2} \qquad\qquad \\
8x^2+10x \qquad \\
\underline{8x^2+12x} \qquad \\
-2x+2 \\
\underline{-2x-3} \\
5
\end{array}$$

$$3x^2+4x-1+\frac{5}{2x+3}$$

4 The solutions to the equation $-\frac{1}{2}x^2 = -6x + 20$ are

(1) $-6 \pm 2i$

(2) $-6 \pm 2\sqrt{19}$

(3) $6 \pm 2i$

(4) $6 \pm 2\sqrt{19}$

Key: 3

Measures CCLS Cluster: A-REI.B

Mathematical Practice: 7

Commentary: This question measures A-REI.B because students must solve a quadratic equation with complex solutions.

Rationale: Option 3 is correct.

Method 1:

$$-\frac{1}{2}x^2 = -6x + 20$$

$$-\frac{1}{2}x^2 + 6x - 20 = 0$$

$$x = \frac{-6 \pm \sqrt{36 - 4\left(-\frac{1}{2}\right)(-20)}}{2\left(-\frac{1}{2}\right)}$$

$$x = \frac{-6 \pm \sqrt{-4}}{-1}$$

$$x = 6 \pm 2i$$

Method 2:

$$-2\left(-\frac{1}{2}x^2 = -6x + 20\right)$$

$$x^2 = 12x - 40$$

$$x^2 - 12x + 40 = 0$$

$$x^2 - 12x + 36 = -40 + 36$$

$$(x - 6)^2 = -4$$

$$x - 6 = \pm\, 2i$$

$$x = 6 \pm 2i$$

5 What is the completely factored form of $k^4 - 4k^2 + 8k^3 - 32k + 12k^2 - 48$?

(1) $(k-2)(k-2)(k+3)(k+4)$

(2) $(k-2)(k-2)(k+6)(k+2)$

(3) $(k+2)(k-2)(k+3)(k+4)$

(4) $(k+2)(k-2)(k+6)(k+2)$

Key: 4

Measures CCLS Cluster: A-SSE.A

Mathematical Practice: 5, 7

Commentary: This question measures A-SSE.A because students use the structure of an expression to identify ways to rewrite it.

Rationale: Option 4 is correct.

$$k^4 - 4k^2 + 8k^3 - 32k + 12k^2 - 48$$

$$\left(k^4 - 4k^2\right) + \left(8k^3 - 32k\right) + \left(12k^2 - 48\right)$$

$$k^2\left(k^2 - 4\right) + 8k\left(k^2 - 4\right) + 12\left(k^2 - 4\right)$$

$$\left(k^2 - 4\right)\left(k^2 + 8k + 12\right)$$

$$(k+2)(k-2)(k+6)(k+2)$$

6 Which statement is *incorrect* for the graph of the function $y = -3\cos\left[\frac{\pi}{3}(x-4)\right] + 7$?

(1) The period is 6.

(2) The amplitude is 3.

(3) The range is [4,10].

(4) The midline is $y = -4$.

Key: 4

Measures CCLS Cluster: F.IF.C

Mathematical Practice: 5, 7

Commentary: This question measures F-IF.C because students must determine key features of the graph of a given trigonometric function.

Rationale: Option 4 states an incorrect midline.

The midline is $y = 7$ since 7 is the average of the endpoints of the range.

7 Algebraically determine the values of x that satisfy the system of equations below.

$$y = -2x + 1$$
$$y = -2x^2 + 3x + 1$$

Key: $0, \frac{5}{2}$

Measures CCLS Cluster: A-REI.C

Mathematical Practice: 2, 7

Commentary: This question measures A-REI.C because students must be able to solve a linear-quadratic system in two variables.

Rationale: $-2x+1=-2x^2+3x+1$

$$2x^2-5x=0$$

$$x(2x-5)=0$$

$$x=0 \quad \Big| \quad 2x-5=0$$

$$x=\frac{5}{2}$$

Rubric:

[2] $0, \frac{5}{2}$ and correct algebraic work is shown.

[1] Appropriate work is shown, but one computational or factoring error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

or

[1] $0, \frac{5}{2}$ but a method other than algebraic is used.

or

[1] $0, \frac{5}{2}$ but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

8 The results of a poll of 200 students are shown in the table below:

	Preferred Music Style		
	Techno	**Rap**	**Country**
Female	54	25	27
Male	36	40	18

For this group of students, do these data suggest that gender and preferred music styles are independent of each other? Justify your answer.

Key: See rationale below

Measures CCLS Cluster: S-CP.A

Mathematical Practice: 1, 2, 3

Commentary: This question measures S-CP.A because students must demonstrate an understanding of conditional probability and interpret independence of events.

Rationale: Based on these data, the two events do not appear to be independent. The probability that a student is female given that she prefers techno music is $\frac{54}{90} = 0.6$ while the probability that a student is female is $\frac{106}{200} = 0.53$. These probabilities are not the same. This suggests that the events are not independent.

Other music styles can be used such as

$$P(\text{Female} \mid \text{Rap}) = \frac{25}{65} = 0.385; P(\text{Female} \mid \text{Country}) = \frac{27}{45} = 0.6$$

Rubric:

[2] The events are not independent and correct work is shown.

[1] Appropriate work is shown, but one computational error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

[0] Not independent, but no work is shown.

or

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

9 For the function $f(x) = (x - 3)^3 + 1$, find $f^{-1}(x)$.

Key: $(x-1)^{\frac{1}{3}}+3$

Measures CCLS Cluster: F-BF.B

Mathematical Practice: 7

Commentary: This question measures F-BF.B because students must write the inverse of a given function.

Rationale:

$$x=(y-3)^3+1$$
$$x-1=(y-3)^3$$
$$(x-1)^{\frac{1}{3}}=\left[(y-3)^3\right]^{\frac{1}{3}}$$
$$(x-1)^{\frac{1}{3}}=y-3$$
$$(x-1)^{\frac{1}{3}}+3=y$$
$$f^{-1}(x)=(x-1)^{\frac{1}{3}}+3$$

Rubric:

[2] $(x-1)^{\frac{1}{3}}+3$ or an equivalent expression and correct work is shown.

[1] Appropriate work is shown, but one computational error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

or

[1] $x=(y-3)^3+1$ is written, but no further correct work is shown.

or

[1] $(x-1)^{\frac{1}{3}}+3,$ but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

10 Given: $h(x) = \frac{2}{9}x^3 + \frac{8}{9}x^2 - \frac{16}{13}x + 2$

$k(x) = -|0.7x| + 5$

State the solutions to the equation $h(x) = k(x)$, rounded to the *nearest hundredth*.

Key: –5.17, –1.13, and 1.75.

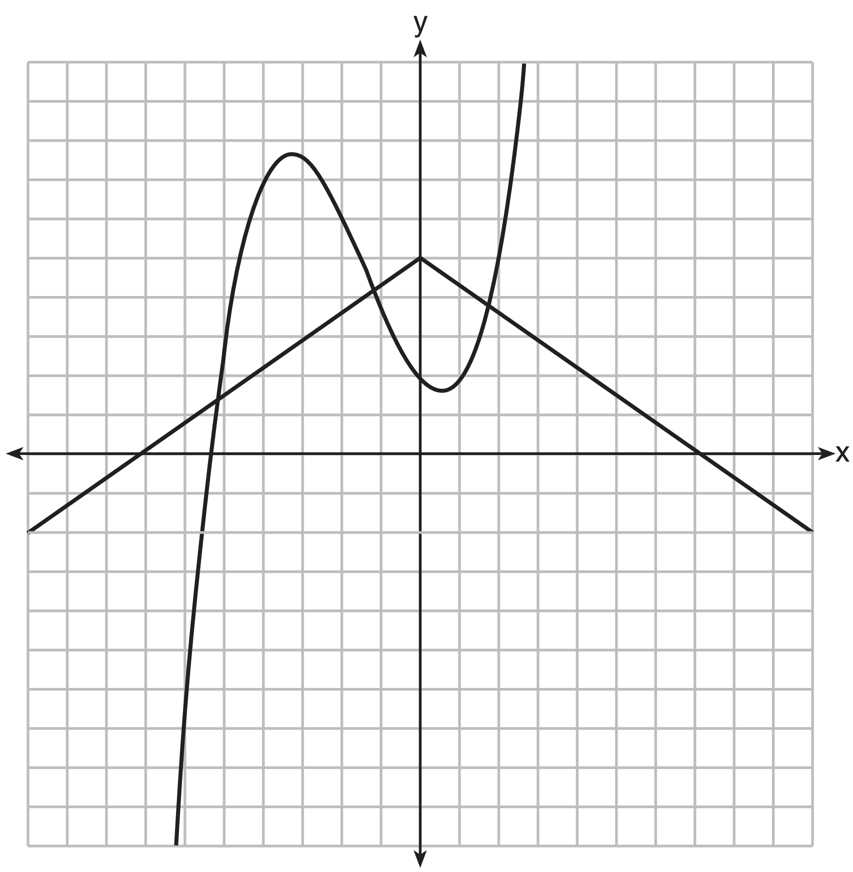
Measures CCLS Cluster: A-REI.D

Mathematical Practice: 5, 6

Commentary: This question measures A-REI.D because students are required to find the approximate solutions to $h(x) = k(x)$.

Rationale: Using technology and $y_1 = h(x)$ and $y_2 = k(x)$, the intersect function is used to determine all values of x for which $y_1 = y_2$.

On their calculator screens, students should see an image similar to the one below.



Rubric:

[2] −5.17, −1.13, and 1.75.

[1] One computational or rounding error is made.

or

[1] One conceptual error is made.

or

[1] Only two correct values are found.

or

[1] (−5.17, 1.38), (−1.13, 4.21), and (1.75, 3.77) are written.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

11 Algebraically prove that the difference of the squares of any two consecutive integers is an odd integer.

Key: See rationale below.

Measures CCLS Cluster: A-APR.C

Mathematical Practice: 1, 8

Commentary: This question measures A-APR.C because students must prove a polynomial identity.

Rationale: Let $x =$ the first integer

$x+1 =$ the next integer

The difference of their squares is

$(x+1)^2 - x^2 = x^2 + 2x + 1 - x^2 = 2x + 1$

$2x$ is an even integer, therefore $2x+1$ is an odd integer.

or

$x^2 - (x+1)^2 = x^2 - \left(x^2 + 2x + 1\right) = -2x - 1$

$-2x$ is an even integer, therefore $-2x-1$ is an odd integer.

Rubric:

[2] A correct algebraic proof is written.

[1] Appropriate work is shown, but one computational error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

or

[1] Appropriate work is shown to find $2x+1$ or $-2x-1$, but no concluding statement is written.

or

[1] An incomplete algebraic proof is written.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

12 Rewrite the expression $\left(4x^2 + 5x\right)^2 - 5\left(4x^2 + 5x\right) - 6$ as a product of four linear factors.

Key: See rationale below.

Measures CCLS Cluster: A-SSE.A

Mathematical Practice: 1, 2, 7

Commentary: This question measures A-SSE.A because students produce an equivalent form of an expression.

Rationale: The problem is of the form $y^2 - 5y - 6,$ which factors to $(y-6)(y+1)$. Therefore:

$$\left(4x^2+5x\right)^2 - 5\left(4x^2+5x\right) - 6$$
$$\left(4x^2+5x-6\right)\left(4x^2+5x+1\right)$$
$$(4x-3)(x+2)(4x+1)(x+1)$$

Rubric:

[2] $(4x-3)(x+2)(4x+1)(x+1)$ and correct work is shown.

[1] Appropriate work is shown, but one computational or factoring error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

or

[1] $\left(4x^2+5x-6\right)\left(4x^2+5x+1\right)$ is written, but no further correct work is shown.

or

[1] $(4x-3)(x+2)(4x+1)(x+1)$, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

13 After sitting out of the refrigerator for a while, a turkey at room temperature (68°F) is placed into an oven at 8 a.m., when the oven temperature is 325°F. Newton's Law of Heating explains that the temperature of the turkey will increase proportionally to the difference between the temperature of the turkey and the temperature of the oven, as given by the formula below:

$$T = T_a + \left(T_o - T_a\right)e^{-kt}$$

$T_a =$ the temperature surrounding the object
$T_o =$ the initial temperature of the object
$t =$ the time in hours
$T =$ the temperature of the object after t hours
$k =$ decay constant

The turkey reaches the temperature of approximately 100° F after 2 hours. Find the value of k, to the *nearest thousandth*, and write an equation to determine the temperature of the turkey after t hours.

Determine the Fahrenheit temperature of the turkey, to the *nearest degree*, at 3 p.m.

Key: $k = 0.066, T = 325 - 257e^{-0.066t}, 163$

Measures CCLS Cluster: A-CED.A

Mathematical Practice: 1, 4

Commentary: This question measures A-CED.A because students must create an exponential equation and use it to solve problems.

Rationale: $100 = 325 + (68 - 325)e^{-2k}$

$$-225 = -257e^{-2k}$$

$$k = \frac{ln\left(\frac{-225}{-257}\right)}{-2}$$

$$k \approx 0.066$$

$$T = 325 - 257e^{-0.066t}$$

At 3 pm, $t = 7$.

$$T = 325 - 257e^{-0.066(7)}$$

$$T \approx 163$$

Rubric:

[4] $k = 0.066, T = 325 - 257e^{-0.066t}$, 163, and correct work is shown.

[3] Appropriate work is shown, but one computational or rounding error is made.

or

[3] Appropriate work is shown, $T = 325 - 257e^{-0.066t}$ is written, but no further correct work is shown.

or

[3] Appropriate work is shown, but the equation is written without *T* or *t*.

[2] Appropriate work is shown, but two or more computational or rounding errors are made.

or

[2] Appropriate work is shown, but one conceptual error is made.

or

[2] Appropriate work is shown to find *k* = 0.066, but no further correct work is shown.

or

[2] The expression $325 - 257e^{-0.066t}$ is written, but no further correct work is shown.

[1] Appropriate work is shown, but one conceptual and one computational or rounding error is made.

or

[1] 0.066, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

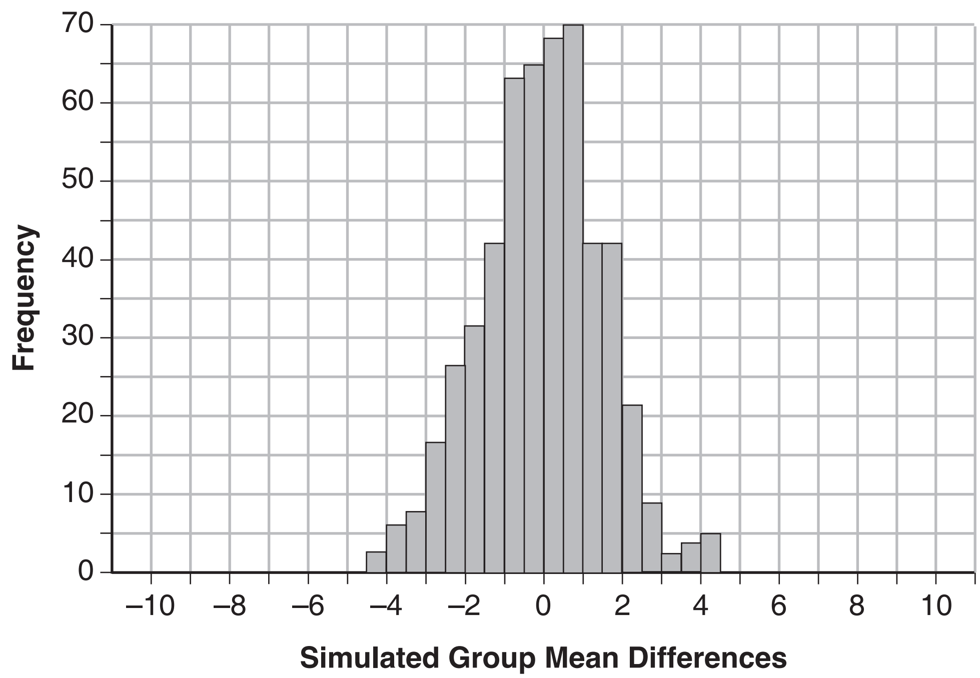
14 Seventy-two students are randomly divided into two equally-sized study groups. Each member of the first group (group 1) is to meet with a tutor after school twice each week for one hour. The second group (group 2), is given an online subscription to a tutorial account that they can access for a maximum of two hours each week. Students in both groups are given the same tests during the year.

A summary of the two groups' final grades is shown below:

	Group 1	**Group 2**
$\bar{x}$	80.16	83.8
S_x	6.9	5.2

Calculate the mean difference in the final grades (group 1 – group 2) and explain its meaning in the context of the problem.

A simulation was conducted in which the students' final grades were rerandomized 500 times. The results are shown below.



Use the simulation to determine if there is a significant difference in the final grades. Explain your answer.

Key: See rationale below.

Measures CCLS Cluster: S-IC.B

Mathematical Practice: 1, 3, 6

Commentary: This question measures S-IC.B because students use a simulation to determine if a difference in sample means in an experiment is significant.

Rationale: The mean difference between the students' final grades in group 1 and group 2 is –3.64. This value indicates that students who met with a tutor had a mean final grade of 3.64 points less than students who used an on-line subscription.

One can infer whether this difference is due to the differences in intervention or due to which students were assigned to each group by using a simulation to rerandomize the students' final grades many (500) times. If the observed difference –3.64 is the result of the assignment of students to groups alone, then a difference of –3.64 or less should be observed fairly regularly in the simulation output. However, a difference of –3 or less occurs in only about 2% of the rerandomizations. Therefore, it is quite unlikely that the assignment to groups alone accounts for the difference; rather, it is likely that the difference between the interventions themselves accounts for the difference between the two groups' mean final grades.

The rerandomization process always involves the following steps:

1. Shuffle all observations.
2. Divide the observations into 2 equal groups.
3. Find the mean difference between the groups.
4. Repeat steps 1 through 3 many times.

Rubric:

[4] −3.64 and a correct explanation, and yes and a correct explanation is given.

[3] Appropriate work is shown, but one computational error is made.

or

[3] Appropriate work is shown, but the mean difference is calculated incorrectly.

[2] Appropriate work is shown, but one conceptual error is made.

or

[2] Appropriate work is shown, but two or more computational errors are made.

or

[2] −3.64 and a correct explanation, but no further correct work is shown.

[1] −3.64, but no work is shown.

[0] Yes, but no explanation is given.

or

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

15 Given $z(x) = 6x^3 + bx^2 - 52x + 15$, $z(2) = 35$, and $z(-5) = 0$, algebraically determine all the zeros of $z(x)$.

Key: $\frac{3}{2}, \frac{1}{3}$ and -5

Measures CCLS Cluster: A-APR.B

Mathematical Practice: 1, 7

Commentary: This question measures A-APR.B because students must apply the Remainder Theorem and then identify the zeros of a polynomial when a suitable factorization is available.

Rationale: Find b:

$$35 = 6(2)^3 + b(2)^2 - 52(2) + 15$$
$$35 = 48 + 4b - 104 + 15$$
$$35 = -41 + 46$$
$$76 = 4b$$
$$19 = b$$

or

$$0 = 6(-5)^3 + b(-5)^2 - 5\!$$
$$0 = -475 + 25b$$
$$475 = 25b$$
$$19 = b$$

$z(x) = 6x^3 + 19x^2 - 52x + 15$

$z(-5) = 0$, by the Remainder Theorem;

$$\begin{array}{r|rrrr} -5 & 6 & 19 & -52 & 15 \\ & & -30 & 55 & -15 \\ \hline & 6 & 11 & 3 & 0 \end{array}$$

$$6x^2 - 11x + 3 = 0$$

$$6x^2 - 9x - 2x + 3 = 0$$
$$3x(2x-3) - 1(2x-3) = 0$$
$$(2x-3)(3x-1) = 0 \qquad z(-5) = 0$$

$2x - 3 = 0$	$3x - 1 = 0$	$z(-5) = 0$
$x = \frac{3}{2}$	$x = \frac{1}{3}$	$x = -5$

Rubric:

[4] $\frac{3}{2}, \frac{1}{3}$, and -5, and correct algebraic work is shown.

[3] Appropriate work is shown to find $\frac{3}{2}$ and $\frac{1}{3}$**,** only.

or

[3] Appropriate work is shown, but one computational error is made.

[2] Appropriate work is shown, but two or more computational errors are made.

or

[2] Appropriate work is shown, but one conceptual error is made.

or

[2] Appropriate work is shown, but a method other than algebraic is used.

[1] Appropriate work is shown, but one conceptual and one computational error are made.

or

[1] Appropriate work is shown to find $b = 19$, but no further correct work is shown.

or

[1] $\frac{3}{2}, \frac{1}{3}$, and -5 but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

16 Two versions of a standardized test are given, an April version and a May version. The statistics for the April version show a mean score of 480 and a standard deviation of 24. The statistics for the May version show a mean score of 510 and a standard deviation of 20. Assume the scores are normally distributed.

Joanne took the April version and scored in the interval 510-540. What is the probability, to the *nearest ten thousandth*, that a test paper selected at random from the April version scored in the same interval?

Maria took the May version. In what interval must Maria score to claim she scored as well as Joanne?

Key: See rationale below.

Measures CCLS Cluster: S-ID.A

Mathematical Practice: 1, 3, 5

Commentary: This question measures S-ID.A because students must be able to use their calculators to estimate the area under the curve.

Rationale: The probability of a score being between 510 and 540 on the April exam can be found using the normal probability cumulative density function,
normcdf(510, 540, 480, 24) = 0.0994.

Use z-scores to compare the two sets of data. Joanne's scores correspond to $z = \frac{510 - 480}{24} = 1.25$ to $z = \frac{540 - 480}{24} = 2.5$.

Calculating equivalent scores,

$1.25 = \frac{x - 510}{20}$	$2.5 = \frac{x - 510}{20}$
$x = 535$	$x = 560$

Maria must score in the interval 535–560.

Rubric:

[4] 0.0994 and [535,560] and correct work is shown.

[3] Appropriate work is shown, but one computational or rounding error is made.

[2] Appropriate work is shown, but two or more computational or rounding errors are made.

or

[2] Appropriate work is shown, but one conceptual error is made.

or

[2] Appropriate work is shown to find 0.0994, but no further correct work is shown.

or

[2] Appropriate work is shown to find [535,560], but no further correct work is shown.

or

[2] 0.0994 and [535,560], but no work is shown.

[1] Appropriate work is shown, but one conceptual and one computational or rounding error are made.

or

[1] 0.0994 or [535,560], but no work is shown.

[0] A zero response if completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

17 Titanium-44 is a radioactive isotope such that every 63 years, its mass decreases by half. For a sample of titanium-44 with an initial mass of 100 grams, write a function that will give the mass of the sample remaining after any amount of time. Define all variables.

Scientists sometimes use the average yearly decrease in mass for estimation purposes. Use the average yearly decrease in mass of the sample between year 0 and year 10 to predict the amount of the sample remaining after 40 years. Round your answer to the *nearest tenth*.

Is the actual mass of the sample or the estimated mass greater after 40 years? Justify your answer.

Key: See rationale below.

Measures CCLS Cluster: F-BF.A

Mathematical Practice: 2, 4

Commentary: This question measures F-BF.A because students must write a function that describes a relationship between two quantities. This question also measures F-IF.B because students must calculate and interpret the average rate of change of a function.

Rationale: Method 1:

$A(t)=100(0.5)^{\frac{t}{63}}$, where t = time in years and
$A(t)=$ amount of titanium-44 remaining after t years.

$$\frac{A(10)-A(0)}{10-0}=\frac{89.58132-100}{10}=-1.041868$$

At $t=40,$ the estimated mass is

$$100-40(1.041868)$$
$$58.32528\approx 58.3 \text{ g}$$

The actual mass is

$$A(40)=100(0.5)^{\frac{40}{63}}=64.3976$$

The estimation is less than the actual.

Method 2:

$$y = 100e^{-kt}$$

$$\frac{1}{2}(100) = 100e^{-63k}$$

$$\frac{1}{2} = e^{-63k}$$

$$\ln\frac{1}{2} = -63k$$

$$0.011002 = k$$

$y = 100e^{-0.011002t}$, where $y =$ amount of titanium-44 remaining after t years and t = time in years.

$$\frac{y(10) - y(0)}{10 - 0} = \frac{89.58132 - 100}{10} = -1.041868$$

At $t = 40,$ the estimated mass is

$100 - 40(1.041868) \approx 58.3$ grams

The actual mass is

$$y = 100e^{-0.011002(40)} = 64.3976$$

The estimation is less than the actual.

Rubric:

[6] A correct function with defined variables is written, 58.3, actual, and a correct justification is given.

[5] Appropriate work is shown, but one computational error is made.

or

[5] Appropriate work is shown, but the function's variables are not defined.

[4] Appropriate work is shown, but two computational errors are made.

or

[4] Appropriate work is shown, but one conceptual error is made.

or

[4] Appropriate work is shown to find 58.3, actual, and a correct explanation are stated, but no further correct work is shown.

[3] Appropriate work is shown, but three or more computational errors are made.

or

[3] Appropriate work is shown, but one conceptual and one computational error is made.

or

[3] A correct function and 58.3 are stated, but no further correct work is shown.

[2] Appropriate work is shown, but two conceptual errors are made.

or

[2] Appropriate work is shown, but one conceptual and two or more computational errors are made.

or

[2] A correct function is written with defined variables, but no further correct work is shown.

[1] Appropriate work is shown, but two conceptual and one computational errors are made.

or

[1] 58.3, but no work is shown.

[0] Actual, but no work is shown.

or

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.