## 0818AII Common Core State Standards

1 The solution of $87 e^{0.3 x}=5918$, to the nearest thousandth, is

1) 0.583
2) 1.945
3) 4.220
4) 14.066

2 A researcher randomly divides 50 bean plants into two groups. He puts one group by a window to receive natural light and the second group under artificial light. He records the growth of the plants weekly. Which data collection method is described in this situation?

1) observational study
2) controlled experiment
3) survey
4) systematic sample

3 If $f(x)=x^{2}+9$ and $g(x)=x+3$, which operation would not result in a polynomial expression?

1) $f(x)+g(x)$
2) $f(x)-g(x)$
3) $f(x) \bullet g(x)$
4) $f(x) \div g(x)$

4 Consider the function $p(x)=3 x^{3}+x^{2}-5 x$ and the graph of $y=m(x)$ below.


Which statement is true?

1) $p(x)$ has three real roots and $m(x)$ has two real roots.
2) $\quad p(x)$ has one real root and $m(x)$ has two real roots.
3) $p(x)$ has two real roots and $m(x)$ has three real roots.
4) $\quad p(x)$ has three real roots and $m(x)$ has four real roots.

5 Which expression is equivalent to $\frac{2 x^{4}+8 x^{3}-25 x^{2}-6 x+14}{x+6} ?$

1) $2 x^{3}+4 x^{2}+x-12+\frac{86}{x+6}$
2) $2 x^{3}-4 x^{2}-x+14$
3) $2 x^{3}-4 x^{2}-x+\frac{14}{x+6}$
4) $2 x^{3}-4 x^{2}-x$

6 Given $f(x)=\frac{1}{2} x+8$, which equation represents the inverse, $g(x)$ ?

1) $g(x)=2 x-8$
2) $g(x)=2 x-16$
3) $g(x)=-\frac{1}{2} x+8$
4) $g(x)=-\frac{1}{2} x-16$

7 The value(s) of $x$ that satisfy $\sqrt{x^{2}-4 x-5}=2 x-10$ are

1) $\{5\}$
2) $\{7\}$
3) $\{5,7\}$
4) $\{3,5,7\}$

8 Stephanie found that the number of white-winged cross bills in an area can be represented by the formula $C=550(1.08)^{t}$, where $t$ represents the number of years since 2010. Which equation correctly represents the number of white-winged cross bills in terms of the monthly rate of population growth?

1) $C=550(1.00643)^{t}$
2) $C=550(1.00643)^{12 t}$
3) $C=550(1.00643)^{\frac{t}{12}}$
4) $C=550(1.00643)^{t+12}$

9 The roots of the equation $3 x^{2}+2 x=-7$ are

1) $-2,-\frac{1}{3}$
2) $-\frac{7}{3}, 1$
3) $-\frac{1}{3} \pm \frac{2 i \sqrt{5}}{3}$
4) $-\frac{1}{3} \pm \frac{\sqrt{11}}{3}$

10 The average depreciation rate of a new boat is approximately $8 \%$ per year. If a new boat is purchased at a price of $\$ 75,000$, which model is a recursive formula representing the value of the boat $n$ years after it was purchased?

1) $a_{n}=75,000(0.08)^{n}$
2) $a_{0}=75,000$
$a_{n}=(0.92)^{n}$
3) $a_{n}=75,000(1.08)^{n}$
4) $a_{0}=75,000$

$$
a_{n}=0.92\left(a_{n-1}\right)
$$

11 Given $\cos \theta=\frac{7}{25}$, where $\theta$ is an angle in standard position terminating in quadrant IV, and $\sin ^{2} \theta+\cos ^{2} \theta=1$, what is the value of $\tan \theta$ ?

1) $-\frac{24}{25}$
2) $-\frac{24}{7}$
3) $\frac{24}{25}$
4) $\frac{24}{7}$

12 For $x>0$, which expression is equivalent to
$\frac{\sqrt[3]{x^{2}} \cdot \sqrt{x^{5}}}{\sqrt[6]{x}}$ ?

1) $x$
2) $x^{\frac{3}{2}}$
3) $x^{3}$
4) $x^{10}$

13 Jake wants to buy a car and hopes to save at least $\$ 5000$ for a down payment. The table below summarizes the amount of money he plans to save each week.

| Week | 1 | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Money Saved, in Dollars | 2 | 5 | 12.5 | 31.25 | $\ldots$ |

Based on this plan, which expression should he use to determine how much he has saved in $n$ weeks?

1) $\frac{2-2\left(2.5^{n}\right)}{1-2.5}$
2) $\frac{2-2\left(2.5^{n-1}\right)}{1-2.5}$
3) $\frac{1-2.5^{n}}{1-2.5}$
4) $\frac{1-2.5^{n-1}}{1-2.5}$

14 Which expression is equivalent to
$x^{6} y^{4}\left(x^{4}-16\right)-9\left(x^{4}-16\right)$ ?

1) $x^{10} y^{4}-16 x^{6} y^{4}-9 x^{4}-144$
2) $\left(x^{6} y^{4}-9\right)(x+2)^{3}(x-2)$
3) $\left(x^{3} y^{2}+3\right)\left(x^{3} y^{2}-3\right)(x+2)^{2}(x-2)^{2}$
4) $\left(x^{3} y^{2}+3\right)\left(x^{3} y^{2}-3\right)\left(x^{2}+4\right)\left(x^{2}-4\right)$

15 If $A=-3+5 i, B=4-2 i$, and $C=1+6 i$, where $i$ is the imaginary unit, then $A-B C$ equals

1) $5-17 i$
2) $5+27 i$
3) $-19-17 i$
4) $-19+27 i$

16 Which sketch best represents the graph of $x=3^{y}$ ?

17 The graph below represents national and New York State average gas prices.


If New York State's gas prices are modeled by $G(x)$ and $C>0$, which expression best approximates the national average $x$ months from August 2014?

1) $G(x+C)$
2) $G(x)+C$
3) $G(x-C)$
4) $G(x)-C$

18 Data for the students enrolled in a local high school are shown in the Venn diagram below.


If a student from the high school is selected at random, what is the probability that the student is a sophomore given that the student is enrolled in Algebra II?

1) $\frac{85}{210}$
2) $\frac{85}{295}$
3) $\frac{85}{405}$
4) $\frac{85}{1600}$

19 If $p(x)=2 \ln (x)-1$ and $m(x)=\ln (x+6)$, then what is the solution for $p(x)=m(x)$ ?

1) 1.65
2) 3.14
3) 5.62
4) no solution

20 Which function's graph has a period of 8 and reaches a maximum height of 1 if at least one full period is graphed?

1) $y=-4 \cos \left(\frac{\pi}{4} x\right)-3$
2) $y=-4 \cos \left(\frac{\pi}{4} x\right)+5$
3) $y=-4 \cos (8 x)-3$
4) $y=-4 \cos (8 x)+5$

21 Given $c(m)=m^{3}-2 m^{2}+4 m-8$, the solution of $c(m)=0$ is

1) $\pm 2$
2) 2, only
3) $2 i, 2$
4) $\pm 2 i, 2$

22 The height above ground for a person riding a Ferris wheel after $t$ seconds is modeled by $h(t)=150 \sin \left(\frac{\pi}{45} t+67.5\right)+160$ feet. How many seconds does it take to go from the bottom of the wheel to the top of the wheel?

1) 10
2) 45
3) 90
4) 150

23 The parabola described by the equation $y=\frac{1}{12}(x-2)^{2}+2$ has the directrix at $y=-1$. The focus of the parabola is

1) $(2,-1)$
2) $(2,2)$
3) $(2,3)$
4) $(2,5)$

24 A fast-food restaurant analyzes data to better serve its customers. After its analysis, it discovers that the events $D$, that a customer uses the drive-thru, and $F$, that a customer orders French fries, are independent. The following data are given in a report:

$$
\begin{aligned}
P(F) & =0.8 \\
P(F \cap D) & =0.456
\end{aligned}
$$

Given this information, $P(F \mid D)$ is

1) 0.344
2) 0.3648
3) 0.57
4) 0.8

25 Over the set of integers, factor the expression $x^{4}-4 x^{2}-12$.

26 Express the fraction $\frac{2 x^{\frac{3}{2}}}{(16)^{\frac{1}{4}}}$ in simplest radical

$$
\left(16 x^{4}\right)^{\frac{1}{4}}
$$

form.

27 The world population was 2560 million people in 1950 and 3040 million in 1960 and can be modeled by the function $p(t)=2560 e^{0.017185 t}$, where $t$ is time in years after 1950 and $p(t)$ is the population in millions. Determine the average rate of change of $p(t)$ in millions of people per year, from $4 \leq t \leq 8$. Round your answer to the nearest hundredth.

28 The scores of a recent test taken by 1200 students had an approximately normal distribution with a mean of 225 and a standard deviation of 18 . Determine the number of students who scored between 200 and 245 .

29 Algebraically solve for $x$ : $\frac{-3}{x+3}+\frac{1}{2}=\frac{x}{6}-\frac{1}{2}$

30 Graph $t(x)=3 \sin (2 x)+2$ over the domain $[0,2 \pi]$ on the set of axes below.


31 Solve the following system of equations algebraically. $x^{2}+y^{2}=400$

$$
y=x-28
$$

32 Some smart-phone applications contain "in-app" purchases, which allow users to purchase special content within the application. A random sample of 140 users found that 35 percent made in-app purchases. A simulation was conducted with 200 samples of 140 users assuming 35 percent of the samples make in-app purchases. The approximately normal results are shown below.


Considering the middle $95 \%$ of the data, determine the margin of error, to the nearest hundredth, for the simulated results. In the given context, explain what this value represents.

33 Solve the following system of equations algebraically for all values of $x, y$, and $z$ :

$$
\begin{aligned}
2 x+3 y-4 z & =-1 \\
x-2 y+5 z & =3 \\
-4 x+y+z & =16
\end{aligned}
$$

34 Evaluate $j(-1)$ given $j(x)=2 x^{4}-x^{3}-35 x^{2}+16 x+48$. Explain what your answer tells you about $x+1$ as a factor. Algebraically find the remaining zeros of $j(x)$.

35 Determine, to the nearest tenth of a year, how long it would take an investment to double at a $3 \frac{3}{4} \%$ interest rate, compounded continuously.

36 To determine if the type of music played while taking a quiz has a relationship to results, 16 students were randomly assigned to either a room softly playing classical music or a room softly playing rap music. The results on the quiz were as follows:

Classical: 74, 83, 77, 77, 84, 82, 90, 89
Rap: $\quad 77,80,78,74,69,72,78,69$
John correctly rounded the difference of the means of his experimental groups as 7. How did John obtain this value and what does it represent in the given context? Justify your answer. To determine if there is any significance in this value, John rerandomized the 16 scores into two groups of 8 , calculated the difference of the means, and simulated this process 250 times as shown below.

Classical vs. Rap


Does the simulation support the theory that there may be a significant difference in quiz scores? Explain.

37 A major car company analyzes its revenue, $R(x)$, and costs $C(x)$, in millions of dollars over a fifteen-year period. The company represents its revenue and costs as a function of time, in years, $x$, using the given functions.

$$
\begin{gathered}
R(x)=550 x^{3}-12,000 x^{2}+83,000 x+7000 \\
C(x)=880 x^{3}-21,000 x^{2}+150,000 x-160,000
\end{gathered}
$$

The company's profits can be represented as the difference between its revenue and costs. Write the profit function, $P(x)$, as a polynomial in standard form. Graph $y=P(x)$ on the set of axes below over the domain $2 \leq x \leq 16$.


Over the given domain, state when the company was the least profitable and the most profitable, to the nearest year. Explain how you determined your answer.

## 0818AII Common Core State Standards

Answer Section
1 ANS: 4
$\ln e^{0.3 x}=\ln \frac{5918}{87}$
$x=\frac{\ln \frac{5918}{87}}{0.3}$

PTS: 2 REF: 081801aii NAT: F.LE.A. 4 TOP: Exponential Equations
KEY: without common base
2 ANS: 2 PTS: 2
REF: 081802aii NAT: S.IC.B. 3
TOP: Analysis of Data
3 ANS: $4 \quad$ PTS: 2
TOP: Operations with Functions
4 ANS: 1
PTS: 2
KEY: type
REF: 081803aii NAT: F.BF.A. 1

TOP: Comparing Functions
5 ANS: 3

$$
\begin{gathered}
2 x^{3}-4 x^{2}-x+\frac{14}{x+6} \\
x + 6 \longdiv { 2 x ^ { 4 } + 8 x ^ { 3 } - 2 5 x ^ { 2 } - 6 x + 1 4 } \\
\frac{2 x^{4}+12 x^{3}}{-4 x^{3}-25 x^{2}} \\
\frac{-4 x^{3}-24 x^{2}}{-x^{2}-6 x} \\
-x^{2}-6 x
\end{gathered}
$$ KEY: division

6 ANS: 2
$y=\frac{1}{2} x+8 \quad x=\frac{1}{2} y+8$

$$
\begin{aligned}
2 x & =y+16 \\
y & =2 x-16
\end{aligned}
$$

PTS: 2

PTS: 2 REF: 081805aii NAT: A.APR.D. 6 TOP: Rational Expressions

REF: 081806aii
NAT: F.BF.B. 4 TOP: Inverse of Functions

7 ANS: 3

$$
\begin{aligned}
x^{2}-4 x-5 & =4 x^{2}-40 x+100 \\
3 x^{2}-36 x+105 & =0 \\
x^{2}-12 x+35 & =0 \\
(x-7)(x-5) & =0 \\
x & =5,7
\end{aligned}
$$

PTS: 2 REF: 081807aii NAT: A.REI.A. 2 TOP: Solving Radicals
KEY: extraneous solutions
8 ANS: 2
$1.00643^{12} \approx 1.08$
PTS: 2 REF: 081808aii NAT: A.SSE.B. 3 TOP: Modeling Exponential Functions
9 ANS: 3
$x=\frac{-2 \pm \sqrt{2^{2}-4(3)(7)}}{2(3)}=\frac{-2 \pm \sqrt{-80}}{6}=\frac{-2 \pm i \sqrt{16} \sqrt{5}}{6}=-\frac{1}{3} \pm \frac{2 i \sqrt{5}}{3}$
PTS: 2 REF: 081809aii NAT: A.REI.B. 4 TOP: Solving Quadratics
KEY: complex solutions | quadratic formula
10 ANS: 4 PTS: 2 REF: 081810aii NAT: F.LE.A. 2
TOP: Sequences
11 ANS: 2
If $\cos \theta=\frac{7}{25}, \sin \theta= \pm \frac{24}{25}$, and $\tan \theta=\frac{\sin \theta}{\cos \theta}=\frac{-\frac{24}{25}}{\frac{7}{25}}=-\frac{24}{7}$
PTS: 2 REF: 081811aii NAT: F.TF.C. 8 TOP: Determining Trigonometric Functions
12 ANS: 3
$\frac{x^{\frac{2}{3}} \cdot x^{\frac{5}{2}}}{x^{\frac{1}{6}}}=\frac{x^{\frac{4}{6}} \cdot x^{\frac{15}{6}}}{x^{\frac{1}{6}}}=x^{\frac{18}{6}}=x^{3}$
PTS: 2 REF: 081812aii NAT: N.RN.A. 2 TOP: Operations with Radicals
KEY: with variables, index > 2
13 ANS: 1 PTS: 2
REF: 081813aii NAT: A.SSE.B. 4
TOP: Series

14 ANS: 4

$$
\left(x^{6} y^{4}-9\right)\left(x^{4}-16\right)
$$

$\left(x^{3} y^{2}+3\right)\left(x^{3} y^{2}-3\right)\left(x^{2}+4\right)\left(x^{2}-4\right)$
PTS: 2 REF: 081814aii NAT: A.SSE.A. 2
TOP: Factoring the Difference of Perfect Squares KEY: multivariable AII
15 ANS: 3
$-3+5 i-\left(4+24 i-2 i-12 i^{2}\right)=-3+5 i-(16+22 i)=-19-17 i$
PTS: 2 REF: 081815aii NAT: N.CN.A. 2 TOP: Operations with Complex Numbers
16 ANS: 2
PTS: 2
REF: 081816aii
NAT: F.IF.C. 7
TOP: Graphing Logarithmic Functions KEY: bimodalgraph
17 ANS: 4 PTS: 2 REF: 081817aii NAT: F.BF.B. 3
TOP: Transformations with Functions
18 ANS: 2
$\frac{85}{210+85}$
PTS: 2 REF: 081818aii NAT: S.CP.A. 1 TOP: Venn Diagrams
19 ANS: 3
PTS: 2
REF: 081819aii
NAT: A.REI.D. 11
TOP: Other Systems
20 ANS: 1
$-4(-1)-3=1 \quad 8=\frac{2 \pi}{b}$

$$
b=\frac{\pi}{4}
$$

PTS: 2 REF: 081820aii NAT: F.IF.B. 4 TOP: Graphing Trigonometric Functions
KEY: maximum/minimum
21 ANS: 4

$$
\begin{aligned}
m^{3}-2 m^{2}+4 m-8 & =0 \\
m^{2}(m-2)+4(m-2) & =0 \\
\left(m^{2}+4\right)(m-2) & =0
\end{aligned}
$$

PTS: 2 REF: 081821aii NAT: A.APR.D. 6 TOP: Solving Polynomial Equations
22 ANS: 2
$P=\frac{2 \pi}{\frac{\pi}{45}}=90$
PTS: 2
REF: 081822aii NAT: F.IF.C. 7 TOP: Graphing Trigonometric Functions
KEY: period

23 ANS: 4
The vertex is $(2,2)$ and $p=3.3+2=5$
PTS: 2 REF: 081823aii NAT: G.GPE.A. 2 TOP: Graphing Quadratic Functions
24 ANS: 4 PTS: 2
TOP: Conditional Probability
25 ANS:
$\left(x^{2}-6\right)\left(x^{2}+2\right)$
PTS: 2
REF: 081825aii
NAT: A.SSE.A. 2 TOP: Factoring Polynomials
KEY: higher power
26
$\frac{2 x^{\frac{3}{2}}}{2 x^{\frac{2}{2}}}=x^{\frac{1}{2}}=\sqrt{x}$
PTS: 2
REF: 081826aii
NAT: N.RN.A. 2
TOP: Radicals and Rational Exponents
27
$\frac{p(8)-p(4)}{8-4} \approx 48.78$
PTS: 2
REF: 081827aii
NAT: F.IF.B. 6
TOP: Rate of Change
$1200 \cdot 0.784 \approx 941$
PTS: 2
REF: 081828aii
NAT: S.ID.A. 4
TOP: Normal Distributions
KEY: predict
29
ANS:
$-6(x+3)\left(\frac{-3}{x+3}-\frac{x}{6}+1=0\right)$
$18+x(x+3)-6(x+3)=0$
$18+x^{2}+3 x-6 x-18=0$

$$
\begin{gathered}
x^{2}-3 x=0 \\
x(x-3)=0 \\
x=0,3
\end{gathered}
$$

PTS: 2
REF: 081829aii
NAT: A.REI.A. 2 TOP: Solving Rationals
KEY: rational solutions

30 ANS:


PTS: 2 REF: 081830aii NAT: F.IF.C. 7 TOP: Graphing Trigonometric Functions KEY: graph
31 ANS:

$$
\begin{aligned}
x^{2}+(x-28)^{2} & =400 \quad y=12-28=-16 \quad y=16-28=-12 \\
x^{2}+x^{2}-56 x+784 & =400 \\
2 x^{2}-56 x+384 & =0 \\
x^{2}-28 x+192 & =0 \\
(x-16)(x-12) & =0 \\
x & =12,16
\end{aligned}
$$

PTS: 2 REF: 081831aii NAT: A.REI.C. 7 TOP: Quadratic-Linear Systems
32 ANS:
$2(0.042)=0.084 \approx 0.08$ The percent of users making in-app purchases will be within $8 \%$ of $35 \%$.
PTS: 2 REF: 081832aii NAT: S.IC.B. 4 TOP: Analysis of Data
33 ANS:
$4 x+6 y-8 z=-24 x-8 y+20 z=12 \quad z+2=3 z-4 \quad y=3+2=5-4 x+5+3=16$

| $-4 x+y+z=16$ | $-4 x+y+z=16$ | $6=2 z$ | $-4 x=8$ |
| :---: | :---: | :---: | :---: |
| $7 y-7 z=14$ | $-7 y+21 z=28$ | $z=3$ | $x=-2$ |
| $y-z=2$ | $y-3 z=-4$ |  |  |
| $y=z+2$ | $y=3 z-4$ |  |  |

PTS: 4 REF: 081833aii NAT: A.REI.C. 6 TOP: Solving Linear Systems
KEY: three variables

34 ANS:

$$
\begin{aligned}
j(-1)=2(-1)^{4}-(-1)^{3} & -35(-1)^{2}+16(-1)+48=2+1-35-16+48=0 ; x+1 \text { is a factor of } j(x) ; \\
2 x^{3}-3 x^{2}-32 x+48 & =0 \\
x^{2}(2 x-3)-16(2 x-3) & =0 \\
\left(x^{2}-16\right)(2 x-3) & =0 \\
x & = \pm 4, \frac{3}{2}
\end{aligned}
$$

PTS: 4
REF: 081834aii
NAT: A.APR.B. 2 TOP: Remainder Theorem
35 ANS:
$2=e^{0.0375 t}$
$t \approx 18.5$

PTS: 4
REF: 081835aii
NAT: F.LE.A. 4
TOP: Exponential Growth
36 ANS:
John found the means of the scores of the two rooms and subtracted the means. The mean score for the classical room was 7 higher than the rap room (82-75). Yes, there is less than a $5 \%$ chance this difference occurring due to random chance. It is likely the difference was due to the music.

PTS: 4
REF: 081836aii
NAT: S.IC.B. 5 TOP: Analysis of Data
37
ANS:
$P(x)=R(x)-C(x)=-330 x^{3}+9000 x^{2}-67000 x+167000$


Least profitable at year 5 because there is a minimum in $P(x)$. Most profitable at year 13 because there is a maximum in $P(x)$.

PTS: 6
REF: 081837aii NAT: F.IF.C. 7
TOP: Graphing Polynomial Functions

