ALGEBRA

II

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II

Thursday, January 24, 2019 — 1:15 to 4:15 p.m., only

Student Name: Mr. Sibol

School Name: JMAP

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 37 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III,** and **IV** directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will not be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice…

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 Suppose two sets of test scores have the same mean, but different standard deviations, σ_1 and σ_2, with $\sigma_2 > \sigma_1$. Which statement best describes the variability of these data sets?

(1) Data set one has the greater variability.

(2) Data set two has the greater variability.

(3) The variability will be the same for each data set.

(4) No conclusion can be made regarding the variability of either set.

2 If $f(x) = \log_3 x$ and $g(x)$ is the image of $f(x)$ after a translation five units to the left, which equation represents $g(x)$?

(1) $g(x) = \log_3 (x + 5)$
(2) $g(x) = \log_3 x + 5$
(3) $g(x) = \log_3 (x - 5)$
(4) $g(x) = \log_3 x - 5$

3 When factoring to reveal the roots of the equation $x^3 + 2x^2 - 9x - 18 = 0$, which equations can be used?

I. $x^2(x + 2) - 9(x + 2) = 0$

II. $x(x^2 - 9) + 2(x^2 - 9) = 0$

III. $(x - 2)(x^2 - 9) = 0$

(1) I and II, only
(2) I and III, only
(3) II and III, only
(4) I, II, and III

$x^3 + 2x^2 - 9x - 18 = 0$
$x^2(x+2) - 9(x+2)$ ✓

$x^3 - 9x + 2x^2 - 18 = 0$
$x(x^2-9) + 2(x^2-9) = 0$ ✓
$(x+2)(x^2-9) = 0$

Use this space for computations.

4 When a ball bounces, the heights of consecutive bounces form a geometric sequence. The height of the first bounce is 121 centimeters and the height of the third bounce is 64 centimeters. To the *nearest centimeter*, what is the height of the fifth bounce?

(1) 25 (3) 36

(2) 34 (4) 42

$121b^2 = 64$

$b = \frac{8}{11}$

$64\left(\frac{8}{11}\right)^2 \approx 34$

5 The solutions to the equation $5x^2 - 2x + \overset{4}{\cancel{13}} = \overset{0}{\cancel{9}}$ are

(1) $\frac{1}{5} \pm \frac{\sqrt{21}}{5}$ (3) $\frac{1}{5} \pm \frac{\sqrt{66}}{5}i$

(2) $\frac{1}{5} \pm \frac{\sqrt{19}}{5}i$ (4) $\frac{1}{5} \pm \frac{\sqrt{66}}{5}$

$x = \frac{2 \pm \sqrt{(-2)^2 - 4(5)(4)}}{2(5)}$

$\frac{2 \pm \sqrt{-76}}{10} = \frac{2 \pm 2i\sqrt{19}}{10}$

6 Julia deposits \$2000 into a savings account that earns 4% interest per year. The exponential function that models this savings account is $y = 2000(1.04)^t$, where t is the time in years. Which equation correctly represents the amount of money in her savings account in terms of the monthly growth rate?

(1) $y = 166.67(1.04)^{0.12t}$ (3) $y = 2000(1.0032737)^{12t}$

(2) $y = 2000(1.01)^t$ (4) $y = 166.67(1.0032737)^t$

$(1.04)^{1/12} \approx 1.0032737$

7 Tides are a periodic rise and fall of ocean water. On a typical day at a seaport, to predict the time of the next high tide, the most important value to have would be the

(1) time between consecutive low tides

(2) time when the tide height is 20 feet

(3) average depth of water over a 24-hour period

(4) difference between the water heights at low and high tide

The time of the next high tide will be the midpoint of consecutive low tides

Use this space for computations.

8 An estimate of the number of milligrams of a medication in the bloodstream t hours after 400 mg has been taken can be modeled by the function below.

$$I(t) = 0.5t^4 + 3.45t^3 - 96.65t^2 + 347.7t, \text{ where } 0 \le t \le 6$$

Over what time interval does the amount of medication in the bloodstream strictly increase?

(1) 0 to 2 hours (3) 2 to 6 hours
(2) 0 to 3 hours (4) 3 to 6 hours

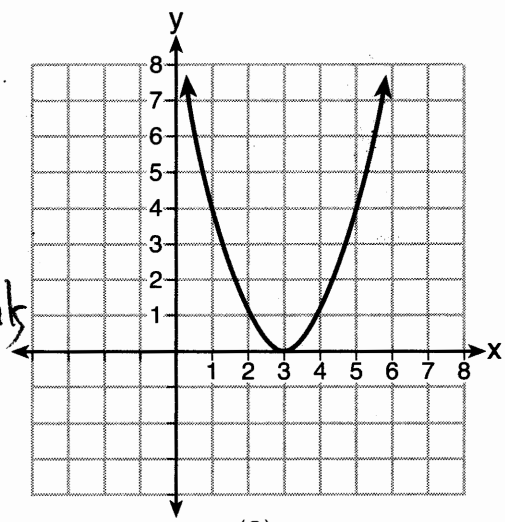
The maximum is at about $x = 2.15$

9 Which representation of a quadratic has imaginary roots?

x	y
–2.5	2
–2.0	0
–1.5	–1
–1.0	–1
–0.5	0
0.0	2

(1)

2 reals



(3)

$(x+3)^2 = 0$

$2(x + 3)^2 = 64$

(2)

$(x+3)^2 = 32$

$x+3 = \pm\sqrt{32}$

$x = -3 \pm \sqrt{32}$

$2x^2 + 32 = 0$

(4)

$2x^2 = -32$

$x^2 = -16$

$x = \pm 4i$

Use this space for computations.

10 A random sample of 100 people that would best estimate the proportion of all registered voters in a district who support improvements to the high school football field should be drawn from registered voters in the district at a

(1) football game (3) school fund-raiser

(2) supermarket (4) high school band concert

most random population

11 Which expression is equivalent to $(2x - i)^2 - (2x - i)(2x + 3i)$ where i is the imaginary unit and x is a real number?

(1) $-4 - 8xi$ (3) 2

(2) $-4 - 4xi$ (4) $8x - 4i$

$(2x-i)((2x-i)-(2x+3i))$

$(2x-i)(-4i)$

$-8xi+4i^2 = -4-8xi$

12 Suppose events A and B are independent and $P(A \text{ and } B)$ is 0.2. Which statement could be true?

(1) $P(A) = 0.4, P(B) = 0.3, P(A \text{ or } B) = 0.5$

(2) $P(A) = 0.8, P(B) = 0.25$

(3) $P(A|B) = 0.2, P(B) = 0.2$

(4) $P(A) = 0.15, P(B) = 0.05$

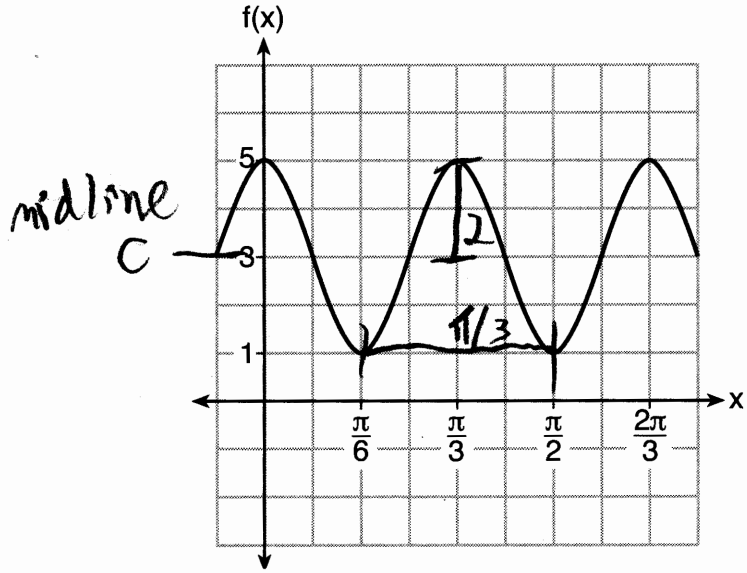
$(.4)(.3) \neq 0.2$

$(.8)(.25) = 0.2$

$P(A|B) = P(A) = 0.2$ $(0.2)(0.2) \neq 0.2$

$(0.2)(0.15) \neq 0.05$

13 The function $f(x) = a\cos bx + c$ is plotted on the graph shown below.



$$\frac{2\pi}{\frac{\pi}{3}} = 6$$

What are the values of a, b, and c?

(1) $a = 2, b = 6, c = 3$ (3) $a = 4, b = 6, c = 5$

(2) $a = 2, b = 3, c = 1$ (4) $a = 4, b = \frac{\pi}{3}, c = 3$

14 Which equation represents the equation of the parabola with focus $(-3,3)$ and directrix $y = 7$?

(1) $y = \frac{1}{8}(x + 3)^2 - 5$ (3) $y = -\frac{1}{8}(x + 3)^2 + 5$

(2) $y = \frac{1}{8}(x - 3)^2 + 5$ (4) $y = -\frac{1}{8}(x - 3)^2 + 5$

$p = \frac{7-3}{2} = 2$ Vertex $(-3, 5)$

$$y = \frac{-1}{4(2)}(x+3)^2 + 5$$

Use this space for computations.

15 What is the solution set of the equation $\frac{2}{3x+1} = \frac{1}{x} - \frac{6x}{3x+1}$?

(1) $\left\{-\frac{1}{3}, \frac{1}{2}\right\}$

(2) $\left\{-\frac{1}{3}\right\}$

(3) $\left\{\frac{1}{2}\right\}$

(4) $\left\{\frac{1}{3}, -2\right\}$

$\frac{6x+2}{3x+1} = \frac{1}{x}$

$6x^2+2x = 3x+1$

$6x^2-x-1=0$

$(2x-1)(3x+1)=0$

$x = \frac{1}{2}, \cancel{-\frac{1}{3}}$

16 Savannah just got contact lenses. Her doctor said she can wear them 2 hours the first day, and can then increase the length of time by 30 minutes each day. If this pattern continues, which formula would *not* be appropriate to determine the length of time, in either minutes or hours, she could wear her contact lenses on the nth day?

(1) $a_1 = 120$
$a_n = a_{n-1} + 30$

(2) $a_n = 90 + 30n$

(3) $a_1 = 2$
$a_n = a_{n-1} + 0.5$

(4) $a_n = 2.5 + 0.5n$

$a_1 = 2.5 + 0.5(1) = 3 \neq 2$

17 If $f(x) = a^x$ where $a > 1$, then the inverse of the function is

(1) $f^{-1}(x) = \log_x a$

(2) $f^{-1}(x) = a\log x$

(3) $f^{-1}(x) = \log_a x$

(4) $f^{-1}(x) = x\log a$

Use this space for computations.

18 Kelly-Ann has \$20,000 to invest. She puts half of the money into an account that grows at an annual rate of 0.9% compounded monthly. At the same time, she puts the other half of the money into an account that grows continuously at an annual rate of 0.8%. Which function represents the value of Kelly-Ann's investments after t years?

(1) $f(t) = 10{,}000(1.9)^t + 10{,}000e^{0.8t}$

(2) $f(t) = 10{,}000(1.009)^t + 10{,}000e^{0.008t}$

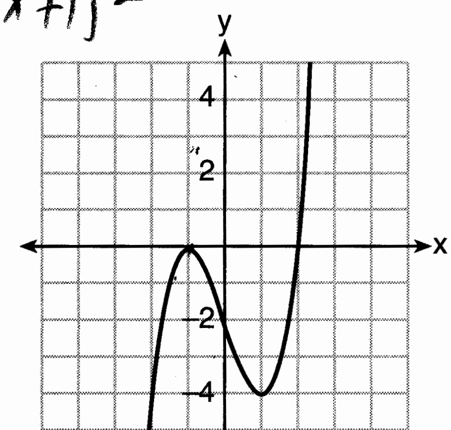
(3) $f(t) = 10{,}000(1.075)^{12t} + 10{,}000e^{0.8t}$

(4) $f(t) = 10{,}000(1.00075)^{12t} + 10{,}000e^{0.008t}$

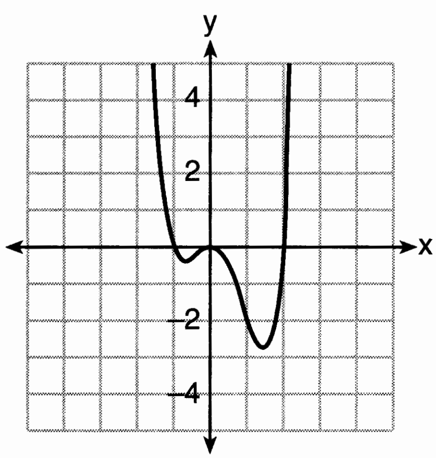
$1 + \frac{.009}{.12} : 1.00075$

19 Which graph represents a polynomial function that contains $x^2 + 2x + 1$ as a factor?

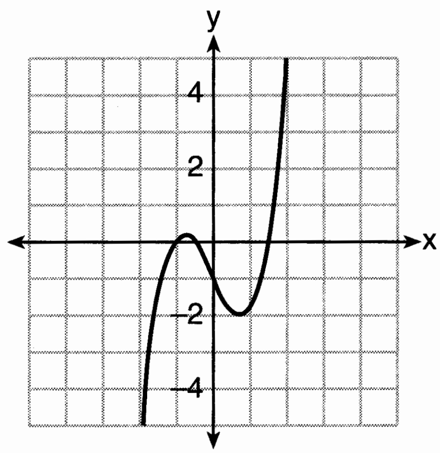
$(x+1)^2$



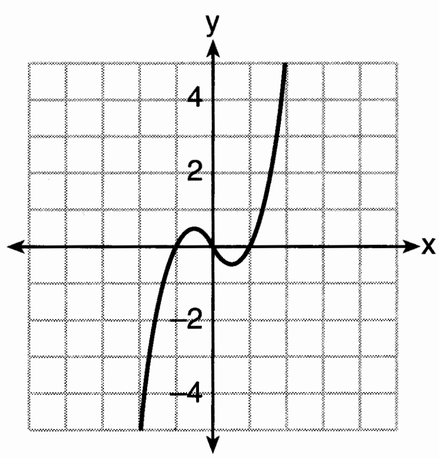
(1)



(2)



(3)



(4)

Use this space for computations.

20 Sodium iodide-131, used to treat certain medical conditions, has a half-life of 1.8 hours. The data table below shows the amount of sodium iodide-131, rounded to the nearest thousandth, as the dose fades over time.

0	Number of Half Lives	1	2	3	4	5
278	Amount of Sodium Iodide-131	139.000	69.500	34.750	17.375	8.688

What approximate amount of sodium iodide-131 will remain in the body after 18 hours?

(1) 0.001

(2) 0.136

(3) 0.271

(4) 0.543

$278(0.5)^{\frac{18}{1.8}} \approx 0.271$

21 Which expression(s) are equivalent to $\frac{x^2 - 4x}{2x}$, where $x \neq 0$?

I. $\frac{x}{2} - 2$ II. $\frac{x-4}{2}$ III. $\frac{x-1}{2} - \frac{3}{2}$

(1) II, only

(2) I and II

(3) II and III

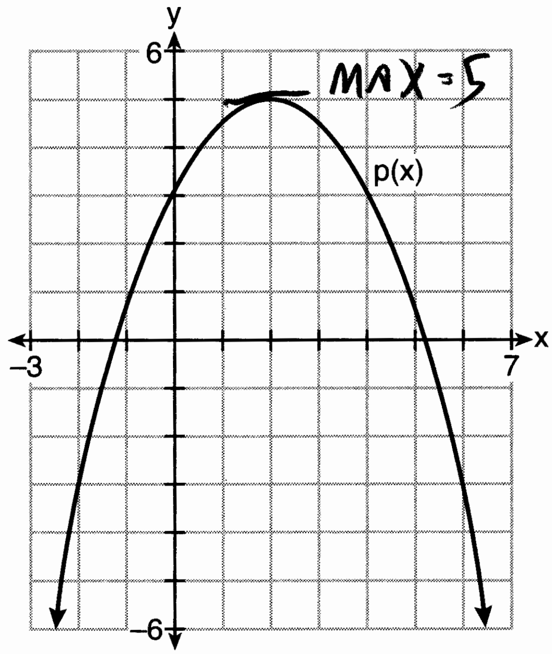
(4) I, II, and III

$\frac{\cancel{x}(x-4)}{2\cancel{x}} = \frac{x-4}{2} = \frac{x}{2} - \frac{\cancel{4}^{2}}{\cancel{2}}$

$\frac{x-1-3}{2} = \frac{x-4}{2}$

Use this space for computations.

22 Consider $f(x) = 4x^2 + 6x - 3$, and $p(x)$ defined by the graph below.



$F(x)$

$x = \frac{-6}{2(4)} = -\frac{3}{4}$

$F\left(-\frac{3}{4}\right) = -\frac{21}{4}$

$\frac{20}{4} - \left(-\frac{21}{4}\right) = \frac{41}{4}$

The difference between the values of the maximum of p and minimum of f is

(1) 0.25 (3) 3.25

(2) 1.25 (4) 10.25

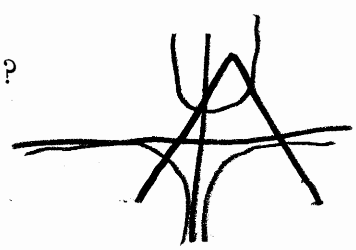
23 The scores on a mathematics college-entry exam are normally distributed with a mean of 68 and standard deviation 7.2. Students scoring higher than one standard deviation above the mean will not be enrolled in the mathematics tutoring program. How many of the 750 incoming students can be expected to be enrolled in the tutoring program?

(1) 631 (3) 238

(2) 512 (4) 119

$84.1\% \times 750 \approx 631$

24 How many solutions exist for $\frac{1}{1 - x^2} = -|3x - 2| + 5$?



(1) 1 (3) 3

(2) 2 (4) 4

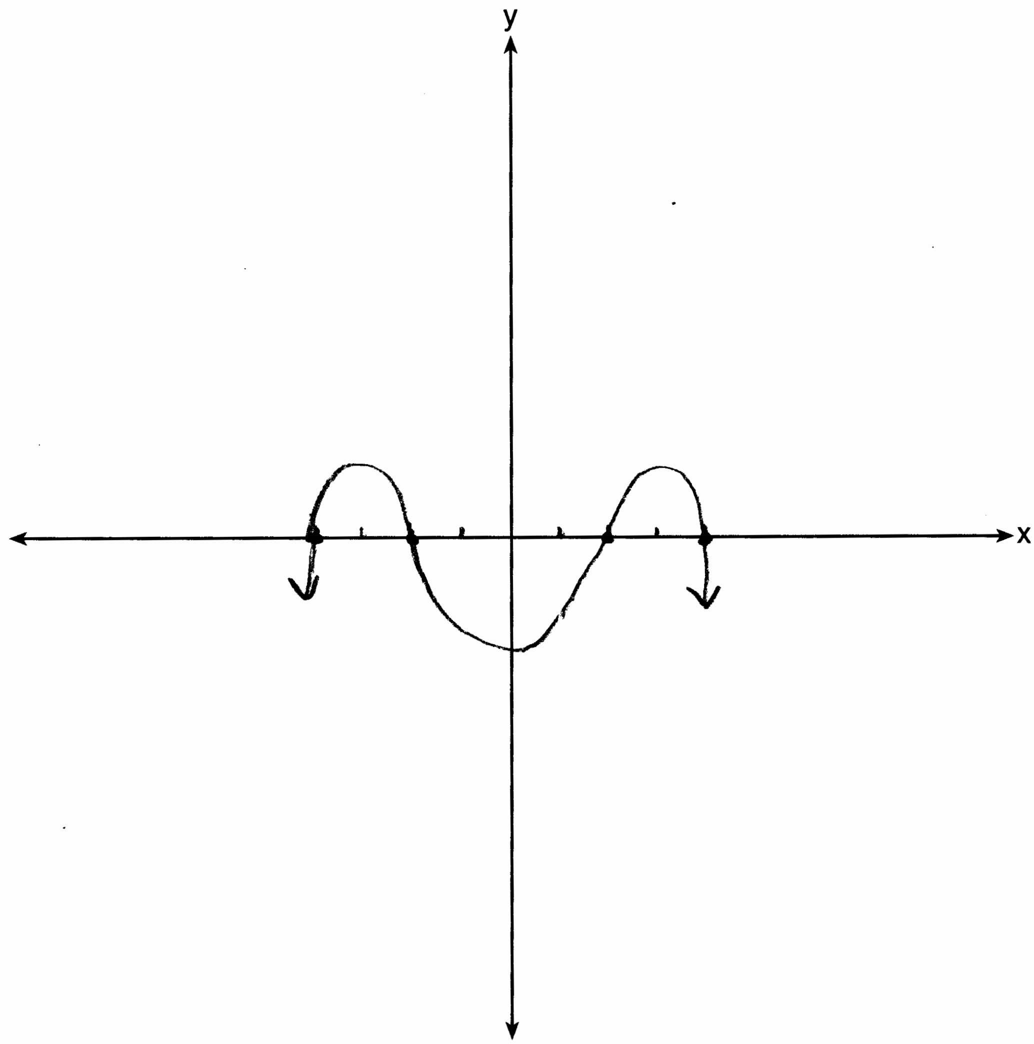
Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

25 Justify why $\frac{\sqrt[3]{x^2y^5}}{\sqrt[4]{x^3y^4}}$ is equivalent to $x^{\frac{-1}{12}}y^{\frac{2}{3}}$ using properties of rational exponents, where $x \neq 0$ and $y \neq 0$.

$$\frac{x^{2/3}y^{5/3}}{x^{3/4}y} = x^{-1/12}y^{2/3}$$

26 The zeros of a quartic polynomial function are 2, −2, 4, and −4. Use the zeros to construct a possible sketch of the function, on the set of axes below.



27 Erin and Christa were working on cubing binomials for math homework. Erin believed they could save time with a shortcut. She wrote down the rule below for Christa to follow.

$$(a + b)^3 = a^3 + b^3$$

Does Erin's shortcut always work? Justify your result algebraically.

$(a+b)^3 = a^3 + b^3$

$\cancel{a^3} + 3ab^2 + 3a^2b + \cancel{b^3} = \cancel{a^3} + \cancel{b^3}$

$3ab^2 + 3a^2b = 0$

$3ab(b+a) = 0$

$a = 0, b = 0, a = -b$

No only when

28 The probability that a resident of a housing community opposes spending money for community improvement on plumbing issues is 0.8. The probability that a resident favors spending money on improving walkways given that the resident opposes spending money on plumbing issues is 0.85. Determine the probability that a randomly selected resident opposes spending money on plumbing issues and favors spending money on walkways.

$$P(A+B) = P(A) \cdot P(B|A)$$

$$= .8 \cdot .85$$

$$= .68$$

29 Rowan is training to run in a race. He runs 15 miles in the first week, and each week following, he runs 3% more than the week before. Using a geometric series formula, find the total number of miles Rowan runs over the first ten weeks of training, rounded to the *nearest thousandth*.

$$S_{10} = \frac{15 - 15(1.03)^{10}}{1 - 1.03} \approx 171.958$$

30 The average monthly high temperature in Buffalo, in degrees Fahrenheit, can be modeled by the function $B(t) = 25.29\sin(0.4895t - 1.9752) + 55.2877$, where t is the month number (January = 1). State, to the *nearest tenth*, the average monthly rate of temperature change between August and November.

$$\frac{B(11) - B(8)}{11 - 8} \approx -10.1$$

Explain its meaning in the given context.

The average monthly high temperature decreases 10.1° each month from August to November

31 Point $M\left(t, \frac{4}{7}\right)$ is located in the second quadrant on the unit circle. Determine the exact value of t.

$$t^2 + \left(\frac{4}{7}\right)^2 = 1$$

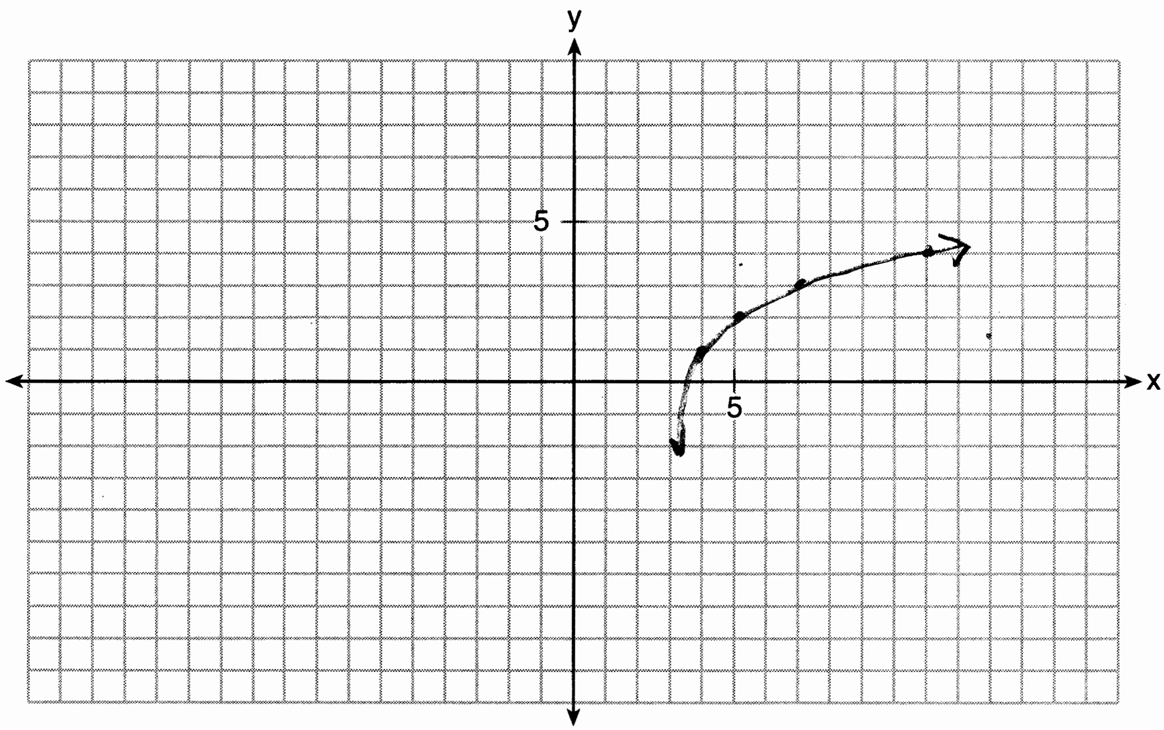
$$t^2 + \frac{16}{49} = \frac{49}{49}$$

$$t^2 = \frac{33}{49}$$

$$t = \frac{\pm\sqrt{33}}{7}$$

$$t = \frac{-\sqrt{33}}{7}$$

32 On the grid below, graph the function $y = \log_2(x - 3) + 1$



Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33 Solve the following system of equations algebraically for all values of a, b, and c.

$$a + 4b + 6c = 23$$
$$a + 2b + c = 2$$
$$6b + 2c = a + 14$$

$a+4b+6c=23$
$a+2b+c=2$
$2b+5c=21$

$a+2b+c=2$
$-a+6b+2c=14$
$8b+3c=16$

$\times 4 = \quad 8b+20c=84$

$17c=68$

$c=4$

$2b+5(4)=21$

$2b=1$

$b=\frac{1}{2}$

$a+4(\frac{1}{2})+6(4)=23$

$a+2+24=23$

$a=-3$

34 Given $a(x) = x^4 + 2x^3 + 4x - 10$ and $b(x) = x + 2$, determine $\frac{a(x)}{b(x)}$ in the form $q(x) + \frac{r(x)}{b(x)}$.

$$
\begin{array}{r|l}
 & x^3 \quad +4 \\
\hline
x+2 & x^4+2x^3+4x-10 \\
 & x^4+2x^3 \\
\hline
 & 0+4x-10 \\
 & 4x+8 \\
\hline
 & -18
\end{array}
$$

$x^3+4-\frac{18}{x+2}$

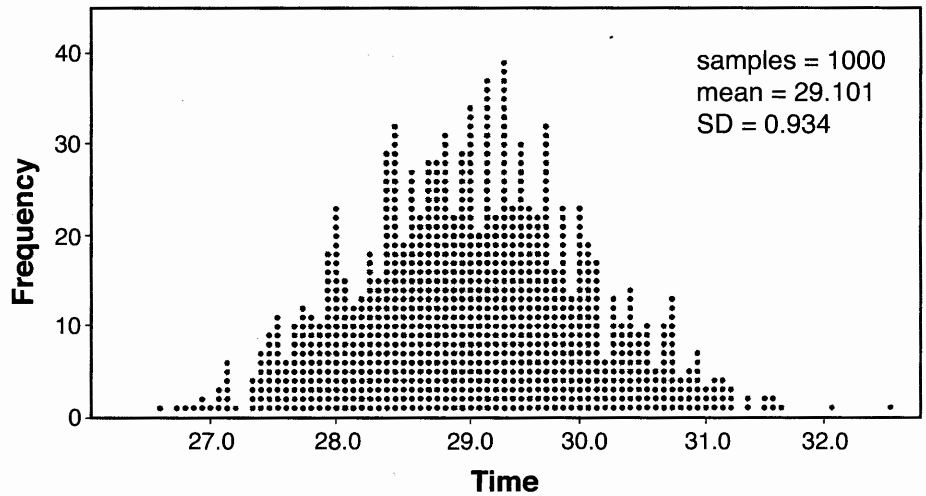
Is $b(x)$ a factor of $a(x)$? Explain.

No, becausethe remainder is not zero

35 A radio station claims to its advertisers that the mean number of minutes commuters listen to the station is 30. The station conducted a survey of 500 of their listeners who commute. The sample statistics are shown below.

$\bar{x}$	29.11
s_x	20.718

A simulation was run 1000 times based upon the results of the survey. The results of the simulation appear below.



Based on the simulation results, is the claim that commuters listen to the station on average 30 minutes plausible? Explain your response including an interval containing the middle 95% of the data, rounded to the *nearest hundredth*.

$29.101 \pm 2(.934)$

$27.23 - 30.97$

Yes, 30 falls withing the interval,

36 Solve the given equation algebraically for all values of x.

$$3\sqrt{x} - 2x = -5$$

$$3\sqrt{x} = 2x - 5$$

$$9x = 4x^2 - 20x + 25$$

$$0 = 4x^2 - 29x + 25$$

$$0 = (4x - 25)(x - 1)$$

$$x = \frac{25}{4} \quad x = \cancel{1}$$

extraneous

Part IV

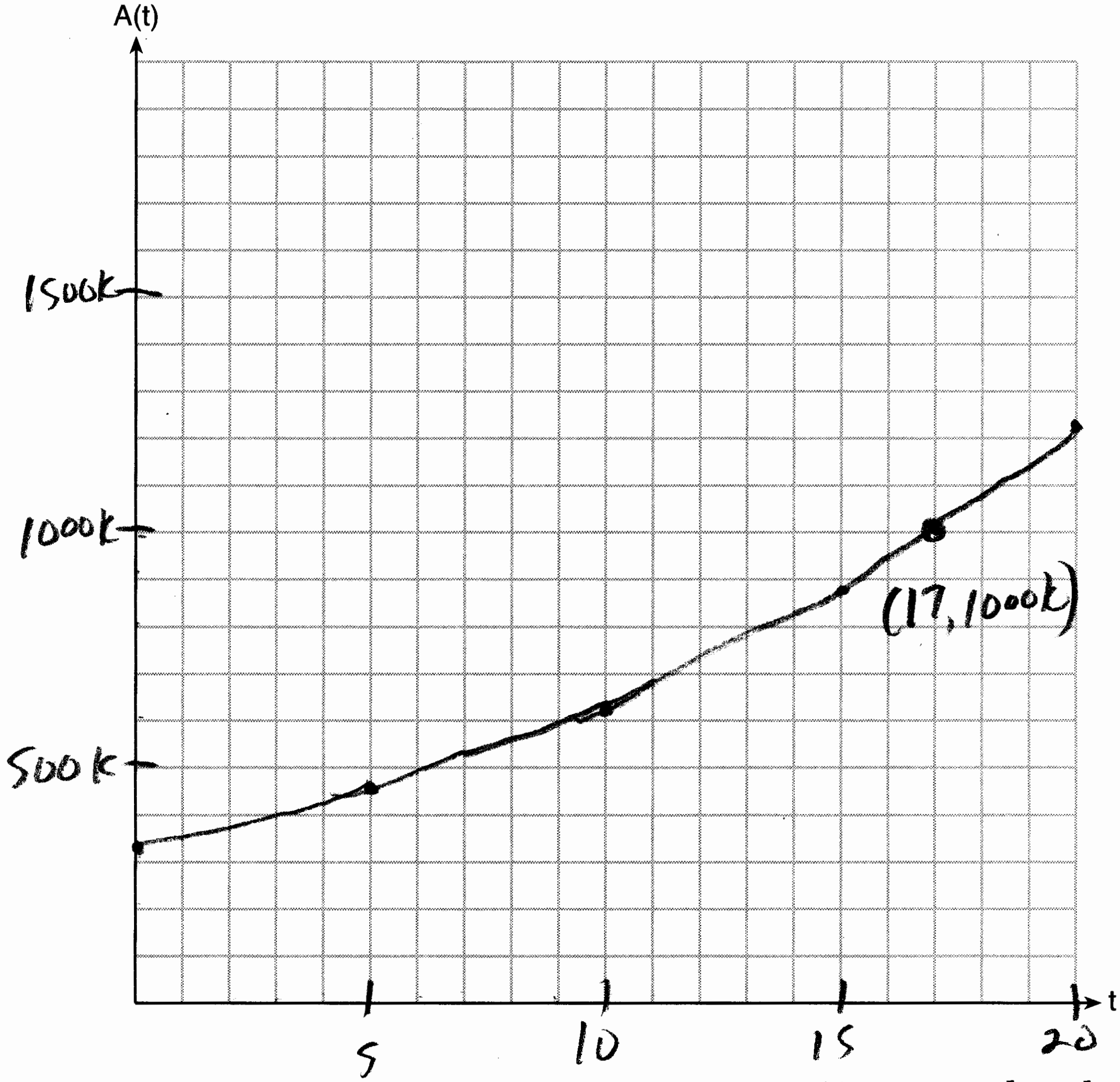
Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

37 Tony is evaluating his retirement savings. He currently has $318,000 in his account, which earns an interest rate of 7% compounded annually. He wants to determine how much he will have in the account in the future, even if he makes no additional contributions to the account.

Write a function, $A(t)$, to represent the amount of money that will be in his account in t years.

$A(t) = 318000(1.07)^t$

Graph $A(t)$ where $0 \leq t \leq 20$ on the set of axes below.



Question 37 is continued on the next page.

Question 37 continued

Tony's goal is to save $1,000,000. Determine algebraically, to the *nearest year*, how many years it will take for him to achieve his goal.

$$318000(1.07)^t = 1000000$$

$$\frac{t \log 1.07^{\cancel{t}}}{\log 1.07} = \frac{\log \frac{1000}{318}}{\log 1.07}$$

$$t \approx 17$$

Explain how your graph of $A(t)$ confirms your answer.

The graph of A(t) nearly intersects the point (17, 1000k)