The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II (Common Core)

Thursday, August 18, 2016 — 12:30 to 3:30 p.m., only

Student Name: Mr. Sibol

School Name: JMAP

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 37 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice…

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 Which equation has $1 - i$ as a solution?

(1) $x^2 + 2x - 2 = 0$
(2) $x^2 + 2x + 2 = 0$
(3) $x^2 - 2x - 2 = 0$
(4) $x^2 - 2x + 2 = 0$

$x - 2x + 1 = -2 + 1$
$\sqrt{(x-1)^2} = \sqrt{-1}$
$x - 1 = \pm i$

If $1-i$ is a solution, $1+i$ is another
$(x-(1-i))(x-(1+i))=0$
$x^2 - x \cancel{-ix} - x \cancel{+ix} + (1-i^2) = 0$
$x^2 - 2x + 2 = 0$

2 Which statement(s) about statistical studies is true?

I. A survey of all English classes in a high school would be a good sample to determine the number of hours students throughout the school spend studying.

II. A survey of all ninth graders in a high school would be a good sample to determine the number of student parking spaces needed at that high school.

Ninth graders drive to school less often

III. A survey of all students in one lunch period in a high school would be a good sample to determine the number of hours adults spend on social media websites.

Students know little about adults.

IV. A survey of all Calculus students in a high school would be a good sample to determine the number of students throughout the school who don't like math.

Calculus students love math!

(1) I, only
(2) II, only
(3) I and III
(4) III and IV

Use this space for computations.

3 To the *nearest tenth*, the value of x that satisfies $2^x = -2x + 11$ is

(1) 2.5 (3) 5.8

(2) 2.6 (4) 5.9

The graph of $y = 2^x$
$y = -2x + 11$
intersects near
(2.56, 5.89)

4 The lifespan of a 60-watt lightbulb produced by a company is normally distributed with a mean of 1450 hours and a standard deviation of 8.5 hours. If a 60-watt lightbulb produced by this company is selected at random, what is the probability that its lifespan will be between 1440 and 1465 hours?

(1) 0.3803 (3) 0.8415

(2) 0.4612 (4) 0.9612

normCdf(1440, 1465, 1450, 8.5)

5 Which factorization is *incorrect*?

(1) $4k^2 - 49 = (2k + 7)(2k - 7)$

(2) $a^3 - 8b^3 = (a - 2b)(a^2 + 2ab + 4b^2)$

(3) $m^3 + 3m^2 - 4m + 12 = (m - 2)^2(m + 3)$

(4) $t^3 + 5t^2 + 6t + t^2 + 5t + 6 = (t + 1)(t + 2)(t + 3)$

$t(t+3)(t+2) + 1(t+3)(t+2)$
$(t+1)(t+3)(t+2)$

Use this space for computations.

6 Sally's high school is planning their spring musical. The revenue, R, generated can be determined by the function $R(t) = -33t^2 + 360t$, where t represents the price of a ticket. The production cost, C, of the musical is represented by the function $C(t) = 700 + 5t$. What is the highest ticket price, to *the nearest dollar*, they can charge in order to *not* lose money on the event?

(1) $t = 3$

(2) $t = 5$

(3) $t = 8$

(4) $t = 11$

$-33t^2 + 360t \geq 700 + 5t$

$-33t^2 + 355t - 700 \geq 0$

$t = \dfrac{-355 \pm \sqrt{355^2 - 4(-33)(-700)}}{2(-33)}$

$t \approx 2.60 \quad 8.16$

7 The set of data in the table below shows the results of a survey on the number of messages that people of different ages text on their cell phones each month.

Age Group	Text Messages per Month		
	0–10	11–50	Over 50
15–18	4	37	68
19–22	6	25	87
23–60	25	47	157

229

If a person from this survey is selected at random, what is the probability that the person texts over 50 messages per month given that the person is between the ages of 23 and 60?

(1) $\frac{157}{229}$

(2) $\frac{157}{312}$

(3) $\frac{157}{384}$

(4) $\frac{157}{456}$

8 A recursive formula for the sequence 18, 9, 4.5, ... is

(1) $g_1 = 18$
$g_n = \frac{1}{2} g_{n-1}$

(2) $g_n = 18\left(\frac{1}{2}\right)^{n-1}$

not recursive

(3) $g_1 = 18$
$g_n = 2g_{n-1}$

(4) $g_n = 18(2)^{n-1}$

Use this space for computations.

9 Kristin wants to increase her running endurance. According to experts, a gradual mileage increase of 10% per week can reduce the risk of injury. If Kristin runs 8 miles in week one, which expression can help her find the total number of miles she will have run over the course of her 6-week training program?

(1) $\sum_{n=1}^{6} 8(1.10)^{n-1}$

(2) $\sum_{n=1}^{6} 8(1.10)^{n}$

(3) $\dfrac{8 - 8(1.10)^6}{0.90}$

(4) $\dfrac{8 - 8(0.10)^n}{1.10}$

decreases

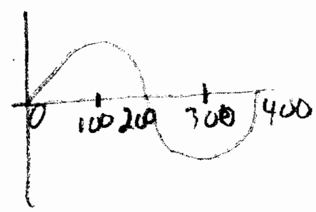
10 A sine function increasing through the origin can be used to model light waves. Violet light has a wavelength of 400 nanometers. Over which interval is the height of the wave *decreasing*, only?

(1) (0, 200)

(2) (100, 300)

(3) (200, 400)

(4) (300, 400)



11 The expression $\dfrac{x^3 + 2x^2 + x + 6}{x + 2}$ is equivalent to

(1) $x^2 + 3$

(2) $x^2 + 1 + \dfrac{4}{x + 2}$

(3) $2x^2 + x + 6$

(4) $2x^2 + 1 + \dfrac{4}{x + 2}$

$x^2 + 0x + 1 + \dfrac{4}{x+2}$

$x+2 \overline{) x^3 + 2x^2 + x + 6}$

$x^3 + 2x^2$

$0 + x$

$0 + 0x$

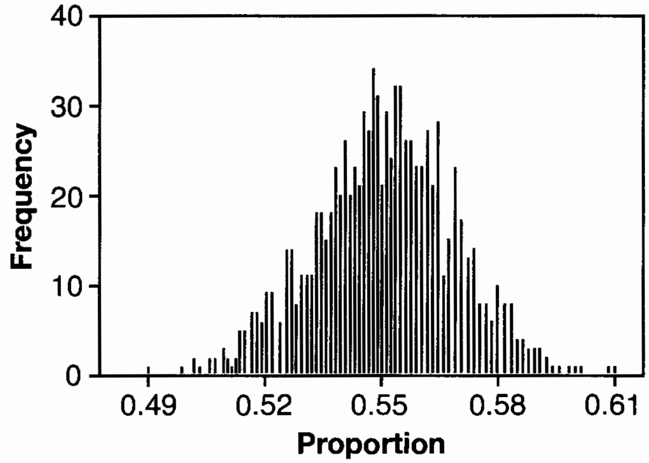
$x + 6$

$x + 2$

4

Use this space for computations.

12 A candidate for political office commissioned a poll. His staff received responses from 900 likely voters and 55% of them said they would vote for the candidate. The staff then conducted a simulation of 1000 more polls of 900 voters, assuming that 55% of voters would vote for their candidate. The output of the simulation is shown in the diagram below.



$ME = z \cdot \sqrt{\frac{p(1-p)}{n}}$

$= 1.96\sqrt{\frac{.55 \cdot .45}{900}} = .0325$

Given this output, and assuming a 95% confidence level, the margin of error for the poll is closest to

(1) 0.01
(2) 0.03
(3) 0.06
(4) 0.12

13 An equation to represent the value of a car after t months of ownership is $v = 32{,}000(0.81)^{\frac{t}{12}}$. Which statement is *not* correct?

(1) The car lost approximately 19% of its value each ~~month.~~ year
(2) The car maintained approximately 98% of its value each month.
(3) The value of the car when it was purchased was \$32,000.
(4) The value of the car 1 year after it was purchased was \$25,920.

Use this space for computations.

14 Which equation represents an odd function?

(1) $y = \sin x$ (3) $y = (x + 1)^3$

(2) $y = \cos x$ (4) $y = e^{5x}$

The graph of $y = \sin x$ is unchanged when rotated 180° about the origin

15 The completely factored form of $2d^4 + 6d^3 - 18d^2 - 54d$ is

(1) $2d(d^2 - 9)(d + 3)$ (3) $2d(d + 3)^2(d - 3)$

(2) $2d(d^2 + 9)(d + 3)$ (4) $2d(d - 3)^2(d + 3)$

$2d(d^3+3d^2-9d-27)$

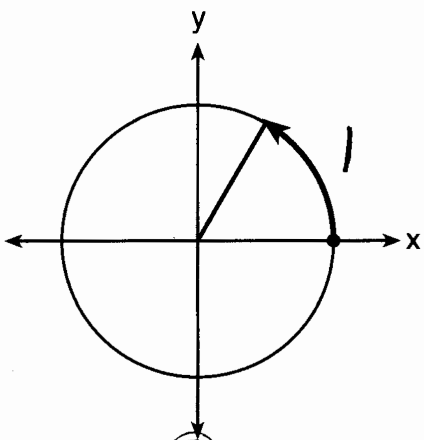
$2d(d^2(d+3)-9(d+3))$

$2d(d^2-9)(d+3)$

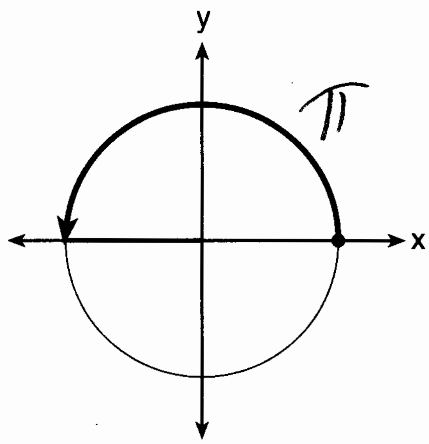
$2d(d-3)(d+3)(d+3)$

$2d(d+3)^2(d-3)$

16 Which diagram shows an angle rotation of 1 radian on the unit circle?



(1)



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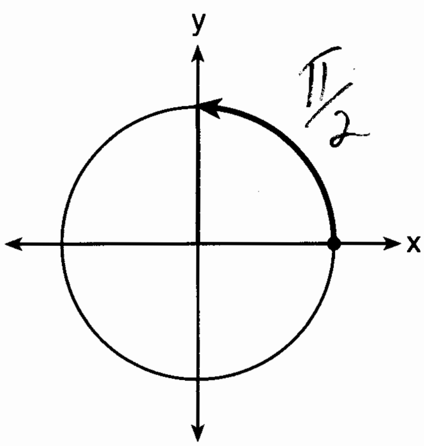
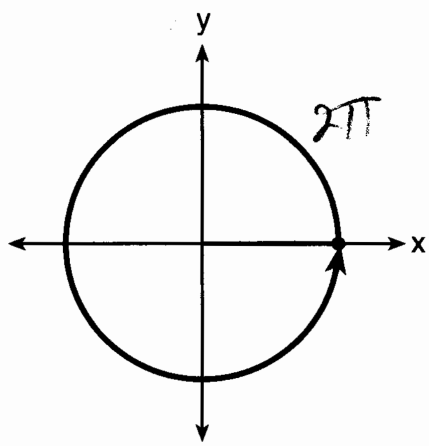


(2)



(4)

Use this space for computations.

17 The focal length, F, of a camera's lens is related to the distance of the object from the lens, J, and the distance to the image area in the camera, W, by the formula below.

$$\frac{1}{J} + \frac{1}{W} = \frac{1}{F}$$

When this equation is solved for J in terms of F and W, J equals

(1) $F - W$

(2) $\frac{FW}{F - W}$

(3) $\frac{FW}{W - F}$

(4) $\frac{1}{F} - \frac{1}{W}$

$\frac{J+W}{JW} = \frac{1}{F}$

$FJ + FW = JW$

$FJ - JW = -FW$

$\frac{J\cancel{(F-W)}}{\cancel{F-W}} = \frac{-FW}{F-W}$

$J = \frac{FW}{W-F}$

18 The sequence $a_1 = 6$, $a_n = 3a_{n-1}$ can also be written as

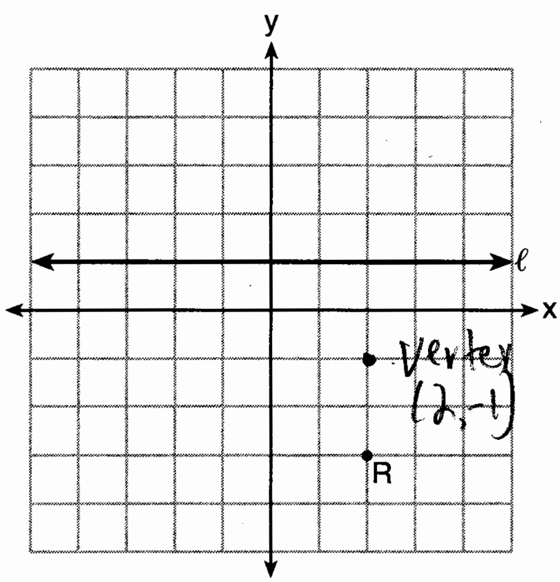
(1) $a_n = 6 \cdot 3^n$ $a_1 = 18$

(2) $a_n = 6 \cdot 3^{n+1}$ $a_1 = 54$

(3) $a_n = 2 \cdot 3^n$ $a_1 = 6$

(4) $a_n = 2 \cdot 3^{n+1}$ $a_1 = 18$

19 Which equation represents the set of points equidistant from line l and point R shown on the graph below?



(1) $y = -\frac{1}{8}(x + 2)^2 + 1$ $(-2,1)$

(2) $y = -\frac{1}{8}(x + 2)^2 - 1$ $(-2,-1)$

(3) $y = -\frac{1}{8}(x - 2)^2 + 1$ $(2,1)$

(4) $y = -\frac{1}{8}(x - 2)^2 - 1$ $(2,-1)$

Use this space for computations.

20 Mr. Farison gave his class the three mathematical rules shown below to either prove or disprove. Which rules can be proved for all real numbers?

I $\quad (m + p)^2 = m^2 + 2mp + p^2$ ✓

II $\quad (x + y)^3 = x^3 + 3xy + y^3$

III $\quad (a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$

(1) I, only

(2) I and II

(3) II and III

(4) I and III

II $(x^2+2xy+y^2)(x+y)$

$x^3+2x^2y+xy^2+x^2y+2xy^2+y^3$

$x^3+3x^2y+3xy^2+y^3$

III $(a^2+b^2)^2 = a^4+2a^2b^2+b^4$

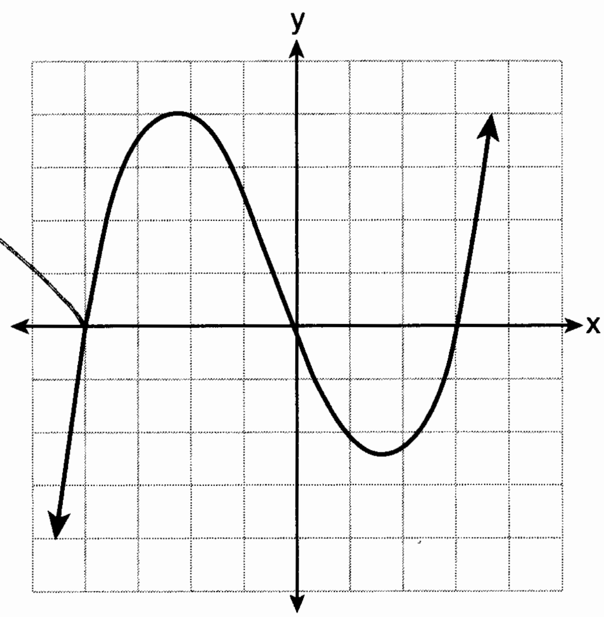
$(a^2-b^2)^2+(2ab)^2 = a^4-2a^2b^2+b^4+4a^2b^2$

$= a^4+2a^2b^2+b^4$

21 The graph of $p(x)$ is shown below.

$x=-4$

$x+4$



What is the remainder when $p(x)$ is divided by $x + 4$?

(1) $x - 4$

(2) -4

(3) 0

(4) 4

since $x+4$ is a factor of $p(x)$, there is no remainder

Use this space for computations.

22 A payday loan company makes loans between \$100 and \$1000 available to customers. Every 14 days, customers are charged 30% interest with compounding. In 2013, Remi took out a \$300 payday loan. Which expression can be used to calculate the amount she would owe, in dollars, after one year if she did not make payments?

(1) $300(.30)^{\frac{14}{365}}$

(2) $300(1.30)^{\frac{14}{365}}$

(3) $300(.30)^{\frac{365}{14}}$

(4) $300(1.30)^{\frac{365}{14}}$

23 Which value is *not* contained in the solution of the system shown below?

$-6c = -18$

$c = 3$

$$a + 5b - c = -20$$
$$4a - 5b + 4c = 19$$
$$-a - 5b - 5c = 2$$

$5a + 3c = -1$

$5a + 3(3) = -1$

$5a = -10$

$a = -2$

(1) -2

(2) 2

(3) $\cancel{3}$

(4) $\cancel{-3}$

$-(-2) - 5b - 5(3) = 2$

$2 - 5b - 15 = 2$

$b = -3$

24 In 2010, the population of New York State was approximately 19,378,000 with an annual growth rate of 1.5%. Assuming the growth rate is maintained for a large number of years, which equation can be used to predict the population of New York State t years after 2010?

(1) $P_t = 19{,}378{,}000(1.5)^t$ 1.015

(2) $P_0 = 19{,}378{,}000$
$P_t = 19{,}378{,}000 + 1.015P_{t-1}$

(3) $P_t = 19{,}378{,}000(1.015)^{t}$

(4) $P_0 = 19{,}378{,}000$
$P_t = 1.015P_{t-1}$

Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

25 The volume of air in a person's lungs, as the person breathes in and out, can be modeled by a sine graph. A scientist is studying the differences in this volume for people at rest compared to people told to take a deep breath. When examining the graphs, should the scientist focus on the amplitude, period, or midline? Explain your choice.

Amplitude, because the height of the graph shows the volume of air

26 Explain how $\left(3^{\frac{1}{5}}\right)^2$ can be written as the equivalent radical expression $\sqrt[5]{9}$.

Applying the commutative property, $\left(3^{\frac{1}{5}}\right)^2$ can be rewritteen as $\left(3^2\right)^{1/5}$ or $9^{1/5}$

A fractional exponent can be rewritten as a radical with the denominator as the index

$9^{1/5} = \sqrt[5]{9}$

27 Simplify $xi(i - 7i)^2$, where i is the imaginary unit.

$xi(-6i)^2$

$xi \cdot 36i^2$

$36xi^3$

$-36xi$

28 Using the identity $\sin^2 \theta + \cos^2 \theta = 1$, find the value of $\tan \theta$, to the *nearest hundredth*, if $\cos \theta$ is -0.7 and θ is in Quadrant II.

$\sin^2 \theta + (-0.7)^2 = 1$

$\sin^2 \theta + .49 = 1$

$\sin^2 \theta = .51$

$\sin \theta = \pm\sqrt{.51}$

Quadrant II $\rightarrow +\sqrt{.51}$

$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\sqrt{.51}}{-.7} \approx -1.02$

29 Elizabeth waited for 6 minutes at the drive thru at her favorite fast-food restaurant the last time she visited. She was upset about having to wait that long and notified the manager. The manager assured her that her experience was very unusual and that it would not happen again.

A study of customers commissioned by this restaurant found an approximately normal distribution of results. The mean wait time was 226 seconds and the standard deviation was 38 seconds. Given these data, and using a 95% level of confidence, was Elizabeth's wait time unusual? Justify your answer.

Using a 95% level of confidence, $\bar{x} \pm 2sd$ sets the usual wait time as 150-302 seconds.

360 seconds is unusual

30 The x-value of which function's x-intercept is larger, f or h? Justify your answer.

$$f(x) = \log(x - 4)$$

x	h(x)
−1	6
0	4
1	2
2	0
3	−2

$0 = \log_{10} x - 4$

$10^0 = x - 4$

$1 = x - 4$

$5 = x$

$f(x)$

31 The distance needed to stop a car after applying the brakes varies directly with the square of the car's speed. The table below shows stopping distances for various speeds.

Speed (mph)	10	20	30	40	50	60	70
Distance (ft)	6.25	25	56.25	100	156.25	225	306.25

Determine the average rate of change in braking distance, in ft/mph, between one car traveling at 50 mph and one traveling at 70 mph.

$$\frac{156.25-50.25}{70-50} = \frac{150}{20} = 7.5$$

Explain what this rate of change means as it relates to braking distance.

Between 50-70 mph, each additional mph in speed needs 7.5 more feet to stop

32 Given events A and B, such that $P(A) = 0.6$, $P(B) = 0.5$, and $P(A \cup B) = 0.8$, determine whether A and B are independent or dependent.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$.8 = .6 + .5 - P(A \cap B)$$

$$P(A \cap B) = .3$$

A & B are independent if

$$P(A \cap B) = P(A) \cdot P(B)$$

$$.3 = .6 \cdot .5$$

$$.3 = .3$$

independent

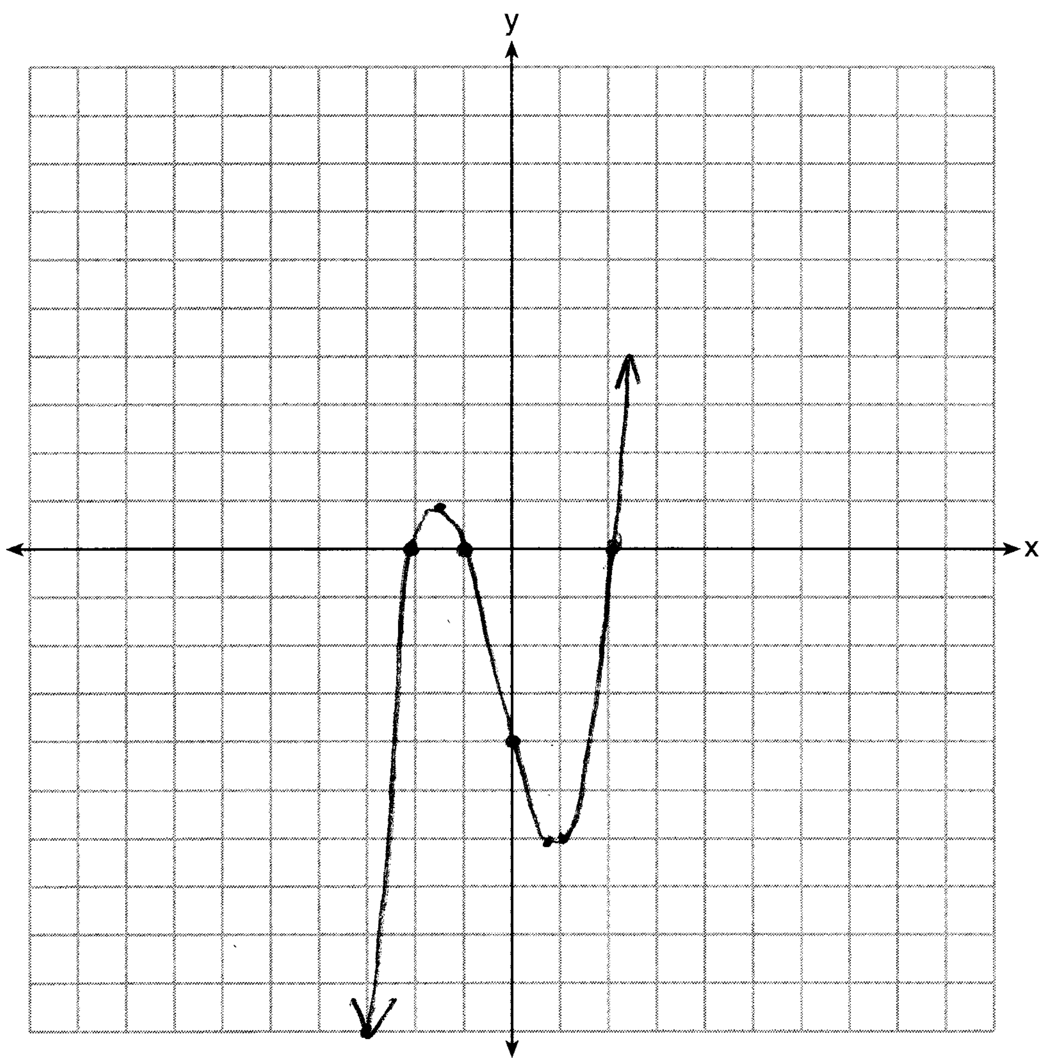
Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33 Find algebraically the zeros for $p(x) = x^3 + x^2 - 4x - 4$.

$$0 = x^2(x+1) - 4(x+1)$$
$$= (x^2-4)(x+1)$$
$$= (x+2)(x-2)(x+1)$$
$$-2 \quad 2 \quad -1$$

On the set of axes below, graph $y = p(x)$.



34 One of the medical uses of Iodine–131 (I–131), a radioactive isotope of iodine, is to enhance x-ray images. The half-life of I–131 is approximately 8.02 days. A patient is injected with 20 milligrams of I–131. Determine, to the *nearest day*, the amount of time needed before the amount of I–131 in the patient's body is approximately 7 milligrams.

$$7 = 20(.5)^{\frac{t}{8.02}}$$

$$\log .35 = \log .5^{\frac{t}{8.02}}$$

$$\log .35 = \frac{t \log .5}{8.02}$$

$$\frac{8.02 \log .35}{\log .5} = t$$

$$12 \approx t$$

35 Solve the equation $\sqrt{2x-7}+x=5$ algebraically, and justify the solution set.

$$(\sqrt{2x-7})^2=(5-x)^2$$
$$2x-7=25-10x+x^2$$
$$0=x^2-12x+32$$
$$0=(x-8)(x-4)$$
$$x=\cancel{8},4$$

$$\sqrt{2(8)-7}+8=5$$
$$\sqrt{9}=-3$$
extraneas

$$\sqrt{2(4)-7}+4=5$$
$$\sqrt{1}=1$$
✓

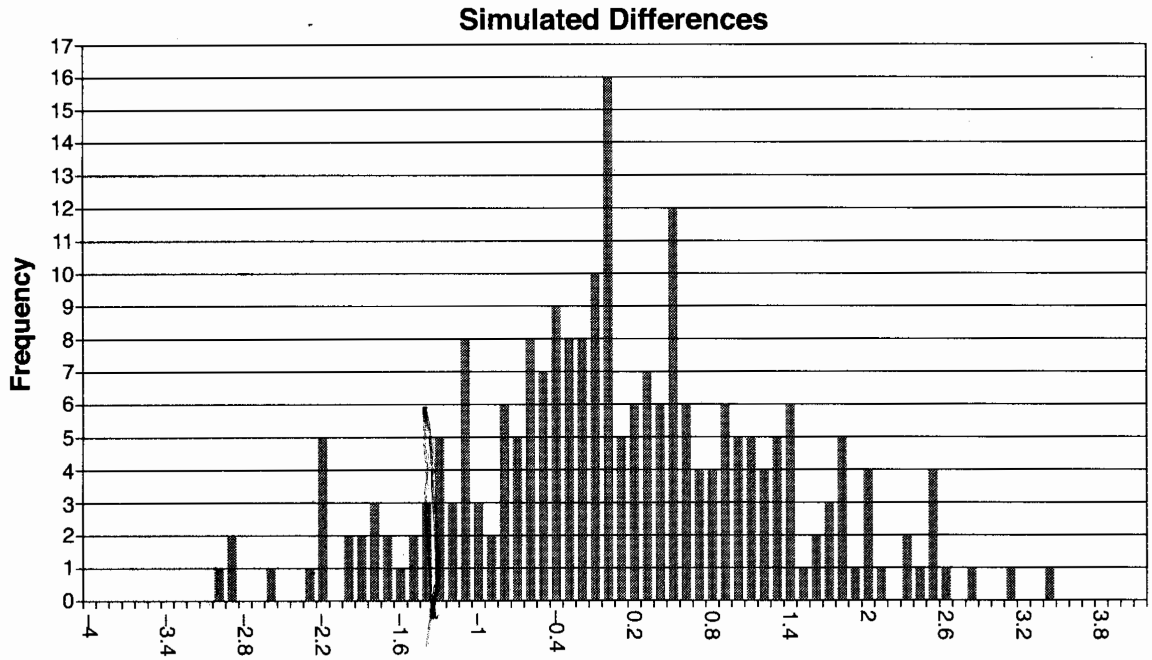
36 Ayva designed an experiment to determine the effect of a new energy drink on a group of 20 volunteer students. Ten students were randomly selected to form group 1 while the remaining 10 made up group 2. Each student in group 1 drank one energy drink, and each student in group 2 drank one cola drink. Ten minutes later, their times were recorded for reading the same paragraph of a novel. The results of the experiment are shown below.

Group 1 (seconds)	**Group 2** (seconds)
17.4	23.3
18.1	18.8
18.2	22.1
19.6	12.7
18.6	16.9
16.2	24.4
16.1	21.2
15.3	21.2
17.8	16.3
19.7	14.5
Mean = 17.7	Mean = 19.1

a) Ayva thinks drinking energy drinks makes students read faster. Using information from the experimental design or the results, explain why Ayva's hypothesis may be *incorrect*.

Some of the students who did not drink energy drinks read faster than those who did drink energy drinks

Using the given results, Ayva randomly mixes the 20 reading times, splits them into two groups of 10, and simulates the difference of the means 232 times.



17.7 - 19.1 = -1.4

b) Ayva has decided that the difference in mean reading times is *not* an unusual occurence. Support her decision using the results of the simulation. Explain your reasoning.

Differences of -1.4 & less occur $\frac{25}{232}$ or about 10% of the time, the difference is not unusual

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

37 Seth's parents gave him \$5000 to invest for his 16th birthday. He is considering two investment options. Option *A* will pay him 4.5% interest compounded annually. Option *B* will pay him 4.6% compounded quarterly.

Write a function of option *A* and option *B* that calculates the value of each account after *n* years.

$A = 5000(1.045)^n \qquad B = 5000\left(1 + \frac{.046}{4}\right)^{4n}$

Seth plans to use the money after he graduates from college in 6 years. Determine how much more money option *B* will earn than option *A* to the *nearest cent*.

$5000\left(1 + \frac{.046}{4}\right)^{4(6)} \approx 6578.87$

$5000(1.045)^6 \approx 6511.30$

67.57

Algebraically determine, to the *nearest tenth of a year*, how long it would take for option *B* to double Seth's initial investment.

$10000 = 5000(1.0115)^{4n}$

$\log 2 = 4n \log 1.0115$

$\frac{\log 2}{4 \log 1.0115} = n$

$15.2 \approx n$