

**ALGEBRA II****Thursday, August 16, 2018 — 12:30 to 3:30 p.m., only****Student Name:**Mr. Sibol**School Name:**JMAP

**The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.**

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 37 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III, and IV** directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will not be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

**Notice...**

**A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.**

**DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.**

## Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

**Use this space for computations.**

- 1** The solution of  $87e^{0.3x} = 5918$ , to the *nearest thousandth*, is



- 2** A researcher randomly divides 50 bean plants into two groups. He puts one group by a window to receive natural light and the second group under artificial light. He records the growth of the plants weekly. Which data collection method is described in this situation?

- (1) observational study      (3) survey  
(2) controlled experiment      (4) systematic sample

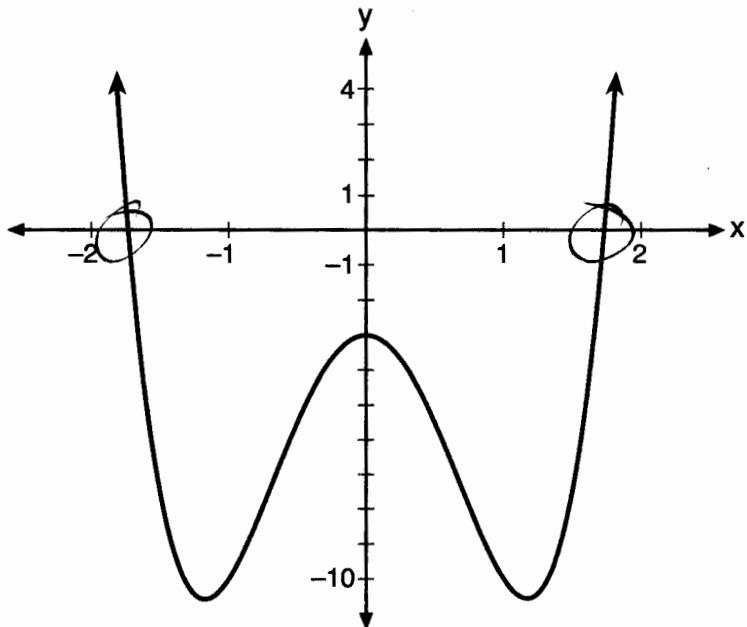
- 3** If  $f(x) = x^2 + 9$  and  $g(x) = x + 3$ , which operation would *not* result in a polynomial expression?

- (1)  $f(x) + g(x)$       (3)  $f(x) \bullet g(x)$   
(2)  $f(x) - g(x)$       (4)  $f(x) \div g(x)$

- (1)  $x^2 + x + 12$
- (2)  $x^2 - x + 6$
- (3)  $x^3 + 12x + 27$
- (4)  $\frac{x^2 + 9}{x + 3}$

**Use this space for computations.**

- 4 Consider the function  $p(x) = 3x^3 + x^2 - 5x$  and the graph of  $y = m(x)$  below.



Which statement is true?

- (1)  $p(x)$  has three real roots and  $m(x)$  has two real roots.  
(2)  $p(x)$  has one real root and  $m(x)$  has two real roots.  
(3)  $p(x)$  has two real roots and  $m(x)$  has three real roots.  
(4)  $p(x)$  has three real roots and  $m(x)$  has four real roots.

- 5 Which expression is equivalent to  $\frac{2x^4 + 8x^3 - 25x^2 - 6x + 14}{x + 6}$ ?

- (1)  $2x^3 + 4x^2 + x - 12 + \frac{86}{x+6}$

(2)  $2x^3 - 4x^2 - x + 14$

(3)  $2x^3 - 4x^2 - x + \frac{14}{x+6}$

(4)  $2x^3 - 4x^2 - x$

$$\begin{array}{r|rrrrr} -6 & 2 & 8 & -25 & -6 & 14 \\ & \underline{-12} & 24 & 6 & 0 \\ \hline & 2 & -4 & -1 & 0 & 14 \end{array}$$

6 Given  $f(x) = \frac{1}{2}x + 8$ , which equation represents the inverse,  $g(x)$ ?

- |   |                                 |
|---|---------------------------------|
| (1) $g(x) = 2x - 8$                                   | (3) $g(x) = -\frac{1}{2}x + 8$  |
| <input checked="" type="radio"/> (2) $g(x) = 2x - 16$ | (4) $g(x) = -\frac{1}{2}x - 16$ |

Use this space for computations.

$$x = \frac{1}{2}y + 8$$

$$2x = y + 16$$

7 The value(s) of  $x$  that satisfy  $\sqrt{x^2 - 4x - 5} = 2x - 10$  are

- |         |               |
|---------|---------------|
| (1) {5} | (3) {5, 7}    |
| (2) {7} | (4) {3, 5, 7} |

$$\begin{aligned} x^2 - 4x - 5 &= 4x^2 - 40x + 100 \\ 3x^2 - 36x + 105 &= 0 \\ x^2 - 12x + 35 &= 0 \\ (x-7)(x-5) &= 0 \end{aligned}$$

$$y = 2x - 16$$

8 Stephanie found that the number of white-winged crossbills in an area can be represented by the formula  $C = 550(1.08)^t$ , where  $t$  represents the number of years since 2010. Which equation correctly represents the number of white-winged crossbills in terms of the monthly rate of population growth?

- |   |                                       |
|---|---------------------------------------|
| (1) $C = 550(1.00643)^t$                                      | (3) $C = 550(1.00643)^{\frac{t}{12}}$ |
| <input checked="" type="radio"/> (2) $C = 550(1.00643)^{12t}$ | (4) $C = 550(1.00643)^{t+12}$         |

$$(1.00643)^{12} \approx 1.08$$

9 The roots of the equation  $3x^2 + 2x = -7$  are

- |                        |   |
|------------------------|---|
| (1) $-2, -\frac{1}{3}$ | (3) $-\frac{1}{3} \pm \frac{2i\sqrt{5}}{3}$ |
| (2) $-\frac{7}{3}, 1$  | (4) $-\frac{1}{3} \pm \frac{\sqrt{11}}{3}$  |

$$3x^2 + 2x + 7 = 0$$

$$\frac{-2 \pm \sqrt{2^2 - 4(3)(7)}}{2(3)}$$

$$\frac{-2 \pm \sqrt{-80}}{6} = \frac{-2 \pm 4i\sqrt{5}}{6}$$

**Use this space for computations.**

- 10 The average depreciation rate of a new boat is approximately 8% per year. If a new boat is purchased at a price of \$75,000, which model is a recursive formula representing the value of the boat  $n$  years after it was purchased? *92% dep.*

(1)  $a_n = 75,000(0.08)^n$

(2)  $a_0 = 75,000$

$a_n = (0.92)^n$

*missing initial  
value*

(3)  $a_n = 75,000(1.08)^n$  *growth*

(4)  $a_0 = 75,000$

$a_n = 0.92(a_{n-1})$

- 11 Given  $\cos \theta = \frac{7}{25}$ , where  $\theta$  is an angle in standard position terminating in quadrant IV, and  $\sin^2 \theta + \cos^2 \theta = 1$ , what is the value of  $\tan \theta$ ?

(1)  $-\frac{24}{25}$

(3)  $\frac{24}{25}$

(2)  $-\frac{24}{7}$

(4)  $\frac{24}{7}$

$$\left(\frac{7}{25}, -\frac{24}{25}\right)$$

- 12 For  $x > 0$ , which expression is equivalent to  $\frac{\sqrt[3]{x^2} \cdot \sqrt{x^5}}{\sqrt[6]{x}}$ ?

(1)  $x$

(3)  $x^3$

(2)  $x^{\frac{3}{2}}$

(4)  $x^{10}$

$$\frac{x^{\frac{2}{3}} \cdot x^{\frac{5}{2}}}{x^{\frac{1}{6}}}$$

$$\frac{x^{\frac{4}{6}} \cdot x^{\frac{15}{6}}}{x^{\frac{1}{6}}}$$

$$x^{\frac{18}{6}}$$

$$x^3$$

- 13 Jake wants to buy a car and hopes to save at least \$5000 for a down payment. The table below summarizes the amount of money he plans to save each week.

Use this space for computations.

Week	1	2	3	4	5
Money Saved, in Dollars	2	5	12.5	31.25	...

Based on this plan, which expression should he use to determine how much he has saved in  $n$  weeks?

(1)  $\frac{2 - 2(2.5^n)}{1 - 2.5}$

(3)  $\frac{1 - 2.5^n}{1 - 2.5}$

$$S_n = \frac{a_1 - a_1 r^n}{1 - r}$$

(2)  $\frac{2 - 2(2.5^{n-1})}{1 - 2.5}$

(4)  $\frac{1 - 2.5^{n-1}}{1 - 2.5}$

- 14 Which expression is equivalent to  $x^6y^4(x^4 - 16) - 9(x^4 - 16)$ ?

(1)  $x^{10}y^4 - 16x^6y^4 - 9x^4 - 144$

(2)  $(x^6y^4 - 9)(x + 2)^3(x - 2)$

(3)  $(x^3y^2 + 3)(x^3y^2 - 3)(x + 2)^2(x - 2)^2$

(4)  $(x^3y^2 + 3)(x^3y^2 - 3)(x^2 + 4)(x^2 - 4)$

$$(x^6y^4 - 9)(x^4 - 16)$$

$$(x^3y^2 + 3)(x^3y^2 - 3)(x^2 + 4)(x^2 - 4)$$

- 15 If  $A = -3 + 5i$ ,  $B = 4 - 2i$ , and  $C = 1 + 6i$ , where  $i$  is the imaginary unit, then  $A - BC$  equals

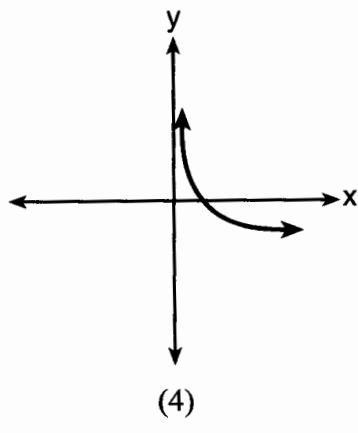
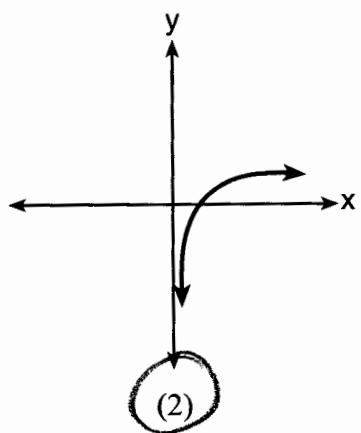
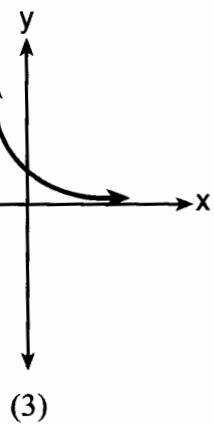
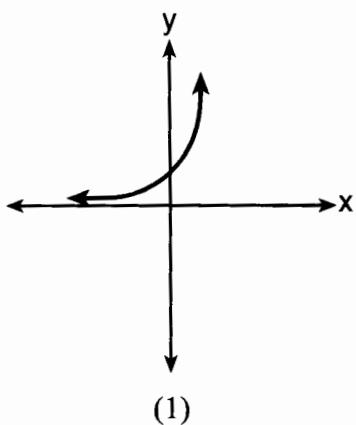
(1)  $5 - 17i$

(2)  $5 + 27i$

$$\begin{aligned} (3) \quad & -19 - 17i & -3 + 5i - (4 + 24i - 2i - 12i^2) \\ & (4) \quad -19 + 27i & -3 + 5i - (16 + 22i) \\ & & -19 - 17i \end{aligned}$$

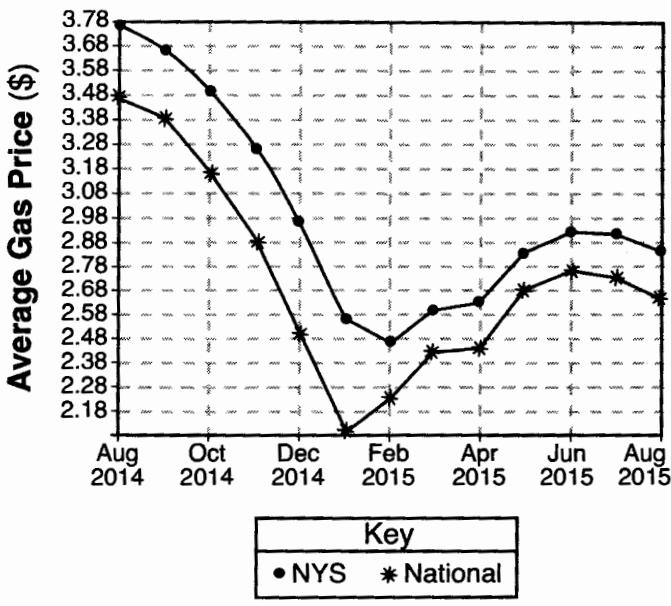
**Use this space for computations.**

- 16 Which sketch best represents the graph of  $x = 3^y$ ?



- 17 The graph below represents national and New York State average gas prices.

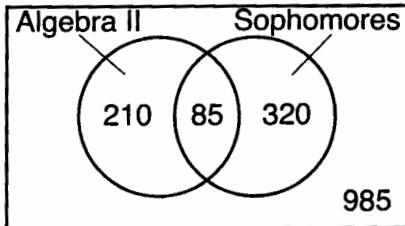
Use this space for computations.



If New York State's gas prices are modeled by  $G(x)$  and  $C > 0$ , which expression best approximates the national average  $x$  months from August 2014?

- (1)  $G(x + C)$       (3)  $G(x - C)$   
 (2)  $G(x) + C$       (4)  $G(x) - C$

- 18 Data for the students enrolled in a local high school are shown in the Venn diagram below.



If a student from the high school is selected at random, what is the probability that the student is a sophomore given that the student is enrolled in Algebra II?

- (1)  $\frac{85}{210}$       (3)  $\frac{85}{405}$   
 (2)  $\frac{85}{295}$       (4)  $\frac{85}{1600}$

Use this space for computations.

- 19 If  $p(x) = 2\ln(x) - 1$  and  $m(x) = \ln(x + 6)$ , then what is the solution for  $p(x) = m(x)$ ?

- (1) 1.65  
(2) 3.14

- (3) 5.62  
(4) no solution

$$2\ln x - 1 = \ln(x+6)$$

$$x \approx 5.62$$

$$-4(-1) - 3 = 4 - 3 > 1$$

- 20 Which function's graph has a period of 8 and reaches a maximum height of 1 if at least one full period is graphed?

- (1)  $y = -4\cos\left(\frac{\pi}{4}x\right) - 3$   
(2)  $y = -4\cos\left(\frac{\pi}{4}x\right) + 5$

- (3)  $y = -4\cos(8x) - 3$   
(4)  $y = -4\cos(8x) + 5$

$$P = \frac{2\pi}{b}$$

$$8 = \frac{2\pi}{b}$$

$$b = \frac{2\pi}{8} = \frac{\pi}{4}$$

- 21 Given  $c(m) = m^3 - 2m^2 + 4m - 8$ , the solution of  $c(m) = 0$  is

- (1)  $\pm 2$   
(2) 2, only

- (3)  $2i, 2$   
(4)  $\pm 2i, 2$

$$m^2(m-2) + 4(m-2) = 0$$

$$(m^2+4)(m-2) = 0$$

- 22 The height above ground for a person riding a Ferris wheel after  $t$  seconds is modeled by  $h(t) = 150\sin\left(\frac{\pi}{45}t + 67.5\right) + 160$  feet.

How many seconds does it take to go from the bottom of the wheel to the top of the wheel?

- (1) 10  
(2) 45

- (3) 90  
(4) 150

$$P = \frac{2\pi}{b}$$

$$P = \frac{45 \cdot 2\pi}{\pi}$$

$$P = 90$$

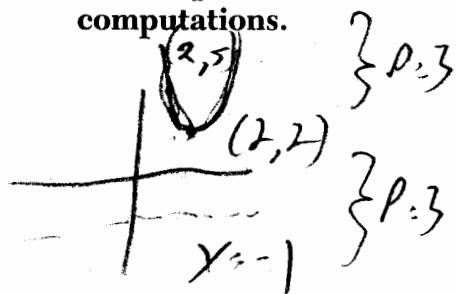
$$\frac{90}{2} = 45$$

*V(2,2)*

- 23 The parabola described by the equation  $y = \frac{1}{12}(x - 2)^2 + 2$  has the directrix at  $y = -1$ . The focus of the parabola is

- (1) (2, -1)      (3) (2, 3)  
(2) (2, 2)      (4) (2, 5)

Use this space for computations.



- 24 A fast-food restaurant analyzes data to better serve its customers. After its analysis, it discovers that the events  $D$ , that a customer uses the drive-thru, and  $F$ , that a customer orders French fries, are independent. The following data are given in a report:

$$P(F) = 0.8$$
$$P(F \cap D) = 0.456$$

Given this information,  $P(F|D)$  is

- (1) 0.344      (3) 0.57  
(2) 0.3648      (4) 0.8
-

**Part II**

**Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil.** [16]

- 25** Over the set of integers, factor the expression  $x^4 - 4x^2 - 12$ .

$$(x^2 - 6)(x^2 + 2)$$

**26** Express the fraction  $\frac{2x^{\frac{3}{2}}}{(16x^4)^{\frac{1}{4}}}$  in simplest radical form.

$$\frac{2x^{\frac{3}{2}}}{2x^{\frac{1}{2}}} = x^{\frac{1}{2}} = \sqrt{x}$$

- 27** The world population was 2560 million people in 1950 and 3040 million in 1960 and can be modeled by the function  $p(t) = 2560e^{0.017185t}$ , where  $t$  is time in years after 1950 and  $p(t)$  is the population in millions. Determine the average rate of change of  $p(t)$  in *millions of people per year*, from  $4 \leq t \leq 8$ . Round your answer to the *nearest hundredth*.

$$\frac{p(8) - p(4)}{8 - 4} \approx 48.78$$

- 28** The scores of a recent test taken by 1200 students had an approximately normal distribution with a mean of 225 and a standard deviation of 18. Determine the number of students who scored between 200 and 245.

$$1200 \cdot .784 \approx 941$$

29 Algebraically solve for  $x$ :

$$\frac{-3}{x+3} + \frac{1}{2} = \frac{x}{6} - \frac{1}{2}$$

$$-6(x+3) \left( \frac{-3}{x+3} - \frac{x}{6} + 1 = 0 \right)$$

$$18 + x(x+3) - 6(x+3) = 0$$

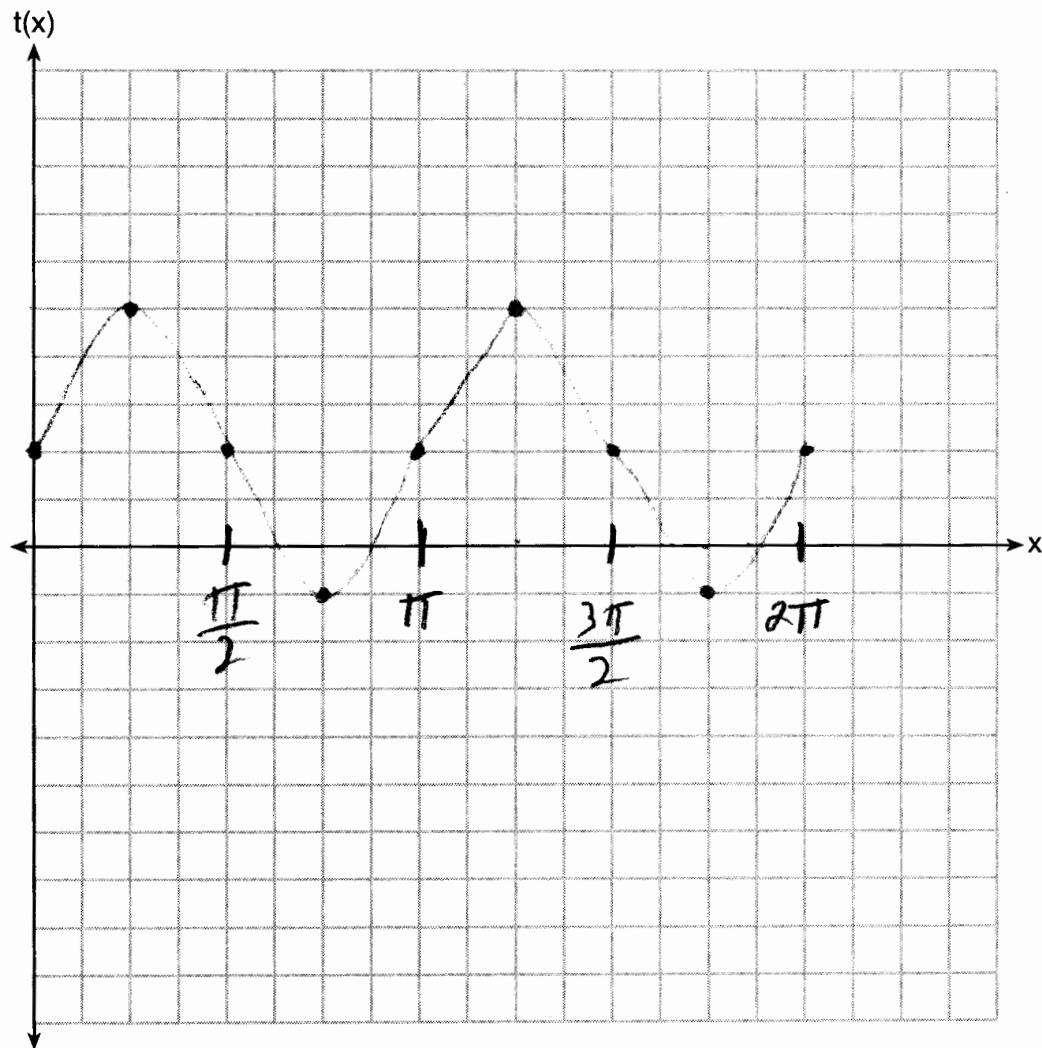
$$18 + x^2 + 3x - 6x - 18 = 0$$

$$x^2 - 3x = 0$$

$$x(x-3) = 0$$

$$x = 0, 3$$

**30** Graph  $t(x) = 3\sin(2x) + 2$  over the domain  $[0, 2\pi]$  on the set of axes below.



**31** Solve the following system of equations algebraically.

$$x^2 + y^2 = 400$$

$$y = x - 28$$

$$x^2 + (x-28)^2 = 400$$

$$x^2 + x^2 - 56x + 784 = 400$$

$$2x^2 - 56x + 384 = 0$$

$$x^2 - 28x + 192 = 0$$

$$(x-16)(x-12) = 0$$

$$x = 12, 16$$

$$y = 12 - 28$$

$$= -16$$

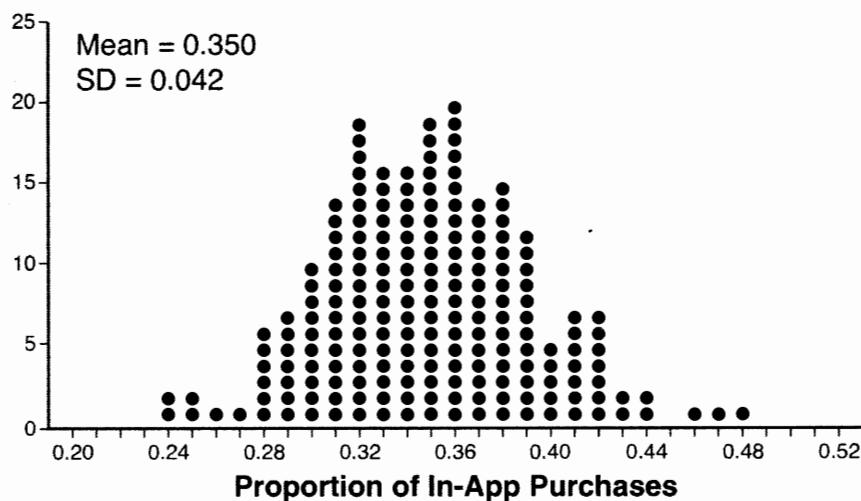
$$(12, -16)$$

$$y = 16 - 28$$

$$= -12$$

$$(16, -12)$$

- 32** Some smart-phone applications contain “in-app” purchases, which allow users to purchase special content within the application. A random sample of 140 users found that 35 percent made in-app purchases. A simulation was conducted with 200 samples of 140 users assuming 35 percent of the samples make in-app purchases. The approximately normal results are shown below.



Considering the middle 95% of the data, determine the margin of error, to the *nearest hundredth*, for the simulated results. In the given context, explain what this value represents.

$$2(0.042) = 0.084 \approx 0.08$$

The percent of users making in-app purchases will be 27-43%

### Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

- 33 Solve the following system of equations algebraically for all values of  $x$ ,  $y$ , and  $z$ .

$$\begin{array}{l} 2(2x + 3y - 4z = -1) \\ 4(x - 2y + 5z = 3) \\ \hline -4x + y + z = 16 \end{array}$$

$$\begin{array}{r} 2x + 6y - 8z = -2 \\ -4x + y + z = 16 \\ \hline 7y - 7z = 14 \\ y - z = 2 \\ y = 2 + z \end{array}$$

$$\begin{array}{r} 4x - 8y + 20z = 12 \\ -4x + y + z = 16 \\ \hline -7y + 21z = 28 \\ y - 3z = -4 \\ y = 3z - 4 \end{array}$$

$$\begin{array}{r} y = 3z - 4 \\ 2 + z = 3z - 4 \\ 6 = 2z \\ z = 3 \\ -4x + 5 + 3 = 16 \\ -4x = 8 \\ x = -2 \end{array}$$

- 34 Evaluate  $j(-1)$  given  $j(x) = 2x^4 - x^3 - 35x^2 + 16x + 48$ . Explain what your answer tells you about  $x + 1$  as a factor.

$x+1$  is a factor  
of  $j(x)$ .

$$\begin{array}{r} -1 | 2 \ -1 \ -35 \ 16 \ 48 \\ \quad \quad \quad \underline{-2 \ \ \ 3 \ 32 \ -48} \\ \quad \quad \quad 2 \ -3 \ -32 \ 48 \ 0 \\ \end{array}$$

$f(-1) = 0$

Algebraically find the remaining zeros of  $j(x)$ .

$$\begin{aligned} 2x^3 - 3x^2 - 32x + 48 &= 0 \\ x^2(2x-3) - 16(2x-3) &= 0 \\ (x^2-16)(2x-3) &= 0 \\ (x+4)(x-4)(2x-3) &= 0 \\ -4 &\quad 4 \quad \frac{3}{2} \end{aligned}$$

- 35** Determine, to the *nearest tenth of a year*, how long it would take an investment to double at a  $3\frac{3}{4}\%$  interest rate, compounded continuously.

$$2 = e^{.0375t}$$
$$t \approx 18.5$$

solve  $(2 = e^{.0375t}, t)$

- 36 To determine if the type of music played while taking a quiz has a relationship to results, 16 students were randomly assigned to either a room softly playing classical music or a room softly playing rap music. The results on the quiz were as follows:

Classical: 74, 83, 77, 77, 84, 82, 90, 89

Rap: 77, 80, 78, 74, 69, 72, 78, 69

$$\overline{X} = \frac{82 + 74 + 62 + 5}{4} =$$

$$7.375$$

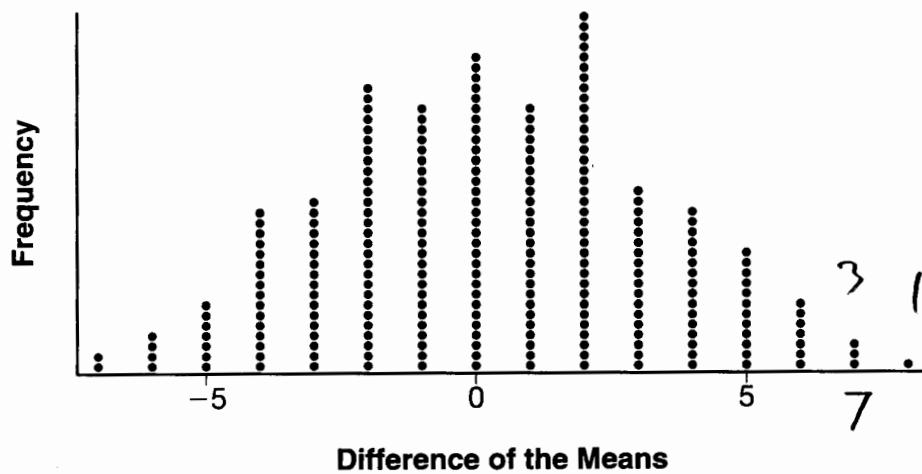
John correctly rounded the difference of the means of his experimental groups as 7. How did John obtain this value and what does it represent in the given context? Justify your answer.

John found the means of the scores of the two rooms, & subtracted the means.

The mean score for the classical room was 7 higher than the rap room.

To determine if there is any significance in this value, John rerandomized the 16 scores into two groups of 8, calculated the difference of the means, and simulated this process 250 times as shown below.

**Classical vs. Rap**



Does the simulation support the theory that there may be a significant difference in quiz scores? Explain.

Yes. There is less than a 5% chance this difference occurring due to random chance. It is likely the difference was due to the music.

**Part IV**

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

- 37 A major car company analyzes its revenue,  $R(x)$ , and costs  $C(x)$ , in millions of dollars over a fifteen-year period. The company represents its revenue and costs as a function of time, in years,  $x$ , using the given functions.

$$R(x) = 550x^3 - 12,000x^2 + 83,000x + 7000$$

$$C(x) = 880x^3 - 21,000x^2 + 150,000x - 160,000$$

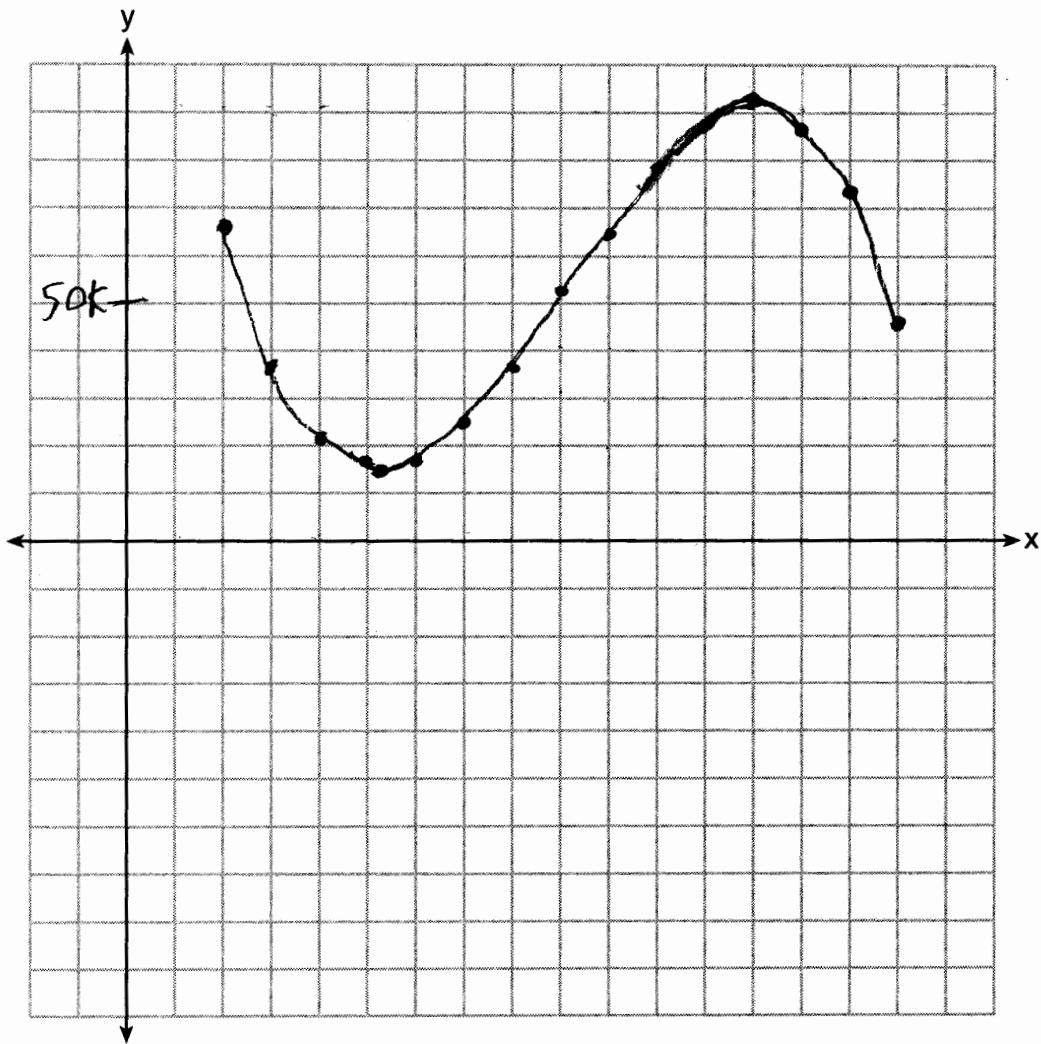
The company's profits can be represented as the difference between its revenue and costs. Write the profit function,  $P(x)$ , as a polynomial in standard form.

$$\begin{aligned}P(x) &= R(x) - C(x) \\&= -330x^3 + 9000x^2 - 67000x + 167000\end{aligned}$$

**Question 37 is continued on the next page.**

**Question 37 continued**

Graph  $y = P(x)$  on the set of axes below over the domain  $2 \leq x \leq 16$ .



Over the given domain, state when the company was the least profitable and the most profitable, to the *nearest year*. Explain how you determined your answer.

least profitable at year 5 because there is a minimum in  $P(x)$

most profitable at year 13 because there is a maximum in  $P(x)$