Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

1 After a counterclockwise rotation about point $X$, scalene triangle $ABC$ maps onto $\triangle RST$, as shown in the diagram below.

Which statement must be true?

1. $\angle A \equiv \angle R$
2. $\angle A \equiv \angle S$
3. $\overline{CB} \equiv \overline{TR}$
4. $\overline{CA} \equiv \overline{TS}$

2 In the diagram below, $\overline{AB} \parallel \overline{DEF}$, $\overline{AE}$ and $\overline{BD}$ intersect at $C$, $m\angle B = 43^\circ$, and $m\angle CEF = 152^\circ$.

Which statement is true?

1. $m\angle D = 28^\circ$
2. $m\angle A = 43^\circ$
3. $m\angle ACD = 71^\circ$
4. $m\angle BCE = 109^\circ$
3 In the diagram below, line $m$ is parallel to line $n$. Figure 2 is the image of Figure 1 after a reflection over line $m$. Figure 3 is the image of Figure 2 after a reflection over line $n$.

Which single transformation would carry Figure 1 onto Figure 3?

(1) a dilation    (3) a reflection
(2) a rotation    (4) a translation

4 In the diagram below, $\overline{AF}$ and $\overline{DB}$ intersect at $C$, and $\overline{AD}$ and $\overline{FBE}$ are drawn such that $m\angle D = 65^\circ$, $m\angle CBE = 115^\circ$, $DC = 7.2$, $AC = 9.6$, and $FC = 21.6$.

What is the length of $\overline{CB}$?

(1) 3.2    (3) 16.2
(2) 4.8    (4) 19.2
5 Given square $RSTV$, where $RS = 9$ cm. If square $RSTV$ is dilated by a scale factor of 3 about a given center, what is the perimeter, in centimeters, of the image of $RSTV$ after the dilation?

(1) 12  (3) 36
(2) 27  (4) 108

6 In right triangle $ABC$, hypotenuse $\overline{AB}$ has a length of 26 cm, and side $\overline{BC}$ has a length of 17.6 cm. What is the measure of angle $B$, to the nearest degree?

(1) 48°  (3) 43°
(2) 47°  (4) 34°

7 The greenhouse pictured below can be modeled as a rectangular prism with a half-cylinder on top. The rectangular prism is 20 feet wide, 12 feet high, and 45 feet long. The half-cylinder has a diameter of 20 feet.

To the nearest cubic foot, what is the volume of the greenhouse?

(1) 17,869  (3) 39,074
(2) 24,937  (4) 67,349
8 In a right triangle, the acute angles have the relationship \( \sin (2x + 4) = \cos (46) \).

What is the value of \( x \)?
(1) 20  
(2) 21  
(3) 24  
(4) 25

9 In the diagram below, \( \overline{AB} \parallel \overline{DFC}, \overline{EDA} \parallel \overline{CBG} \), and \( \overline{EFB} \) and \( \overline{AG} \) are drawn.

Which statement is always true?
(1) \( \triangle DEF \cong \triangle CBF \)  
(2) \( \triangle BAG \cong \triangle BAE \)  
(3) \( \triangle BAG \sim \triangle AEB \)  
(4) \( \triangle DEF \sim \triangle AEB \)

10 The base of a pyramid is a rectangle with a width of 4.6 cm and a length of 9 cm. What is the height, in centimeters, of the pyramid if its volume is 82.8 cm\(^3\)?
(1) 6  
(2) 2  
(3) 9  
(4) 18

Use this space for computations.
11 In the diagram below of right triangle $AED$, $BC \parallel DE$.

Which statement is always true?

\[ \begin{align*}
(1) \quad & \frac{AC}{BC} = \frac{DE}{AE} \\
(2) \quad & \frac{AB}{AD} = \frac{BC}{DE} \\
(3) \quad & \frac{AC}{CE} = \frac{BC}{DE} \\
(4) \quad & \frac{DE}{BC} = \frac{DB}{AB}
\end{align*} \]

12 What is an equation of the line that passes through the point (6,8) and is perpendicular to a line with equation $y = \frac{3}{2}x + 5$?

\[ \begin{align*}
(1) \quad & y - 8 = \frac{3}{2}(x - 6) \\
(2) \quad & y - 8 = -\frac{2}{3}(x - 6) \\
(3) \quad & y + 8 = \frac{3}{2}(x + 6) \\
(4) \quad & y + 8 = -\frac{2}{3}(x + 6)
\end{align*} \]
13. The diagram below shows parallelogram $ABCD$ with diagonals $AC$ and $BD$ intersecting at $E$.

What additional information is sufficient to prove that parallelogram $ABCD$ is also a rhombus?

(1) $BD$ bisects $AC$.  
(2) $AB$ is parallel to $CD$.  
(3) $AC$ is congruent to $BD$.  
(4) $AC$ is perpendicular to $BD$.

14. Directed line segment $DE$ has endpoints $D(-4,-2)$ and $E(1,8)$. Point $F$ divides $DE$ such that $DF:FE$ is 2:3. What are the coordinates of $F$?

(1) $(-3,0)$  
(2) $(-2,2)$  
(3) $(-1,4)$  
(4) $(2,4)$
15 Triangle $\triangle DAN$ is graphed on the set of axes below. The vertices of $\triangle DAN$ have coordinates $D(-6,-1)$, $A(6,3)$, and $N(-3,10)$.

What is the area of $\triangle DAN$?

(1) 60  
(2) 120  
(3) $20\sqrt{13}$  
(4) $40\sqrt{13}$
16 Triangle $ABC$, with vertices at $A(0,0)$, $B(3,5)$, and $C(0,5)$, is graphed on the set of axes shown below.

Which figure is formed when $\triangle ABC$ is rotated continuously about $\overline{BC}$?

(1)  

(2)  

(3)  

(4)
17 In the diagram below of circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at $E$.

If $m\overarc{AC} = 72^\circ$ and $m\angle AEC = 58^\circ$, how many degrees are in $m\overarc{DB}$?

(1) $108^\circ$  
(2) $65^\circ$  
(3) $44^\circ$  
(4) $14^\circ$

18 In triangle $SRK$ below, medians $\overline{SC}$, $\overline{KE}$, and $\overline{RL}$ intersect at $M$.

Which statement must always be true?

(1) $3(MC) = SC$  
(2) $MC = \frac{1}{3}(SM)$  
(3) $RM = 2MC$  
(4) $SM = KM$
19 The regular polygon below is rotated about its center.

Which angle of rotation will carry the figure onto itself?

(1) 60°  (2) 108°
(3) 216°  (4) 540°

20 What is an equation of circle $O$ shown in the graph below?

(1) $x^2 + 10x + y^2 + 4y = -13$
(2) $x^2 - 10x + y^2 - 4y = -13$
(3) $x^2 + 10x + y^2 + 4y = -25$
(4) $x^2 - 10x + y^2 - 4y = -25$
21 In the diagram below of $\triangle PQR$, $\overline{ST}$ is drawn parallel to $\overline{PR}$, $PS = 2$, $SQ = 5$, and $TR = 5$.

What is the length of $\overline{QR}$?

(1) 7  
(2) 2  
(3) $12\frac{1}{2}$  
(4) $17\frac{1}{2}$

22 The diagram below shows circle $O$ with radii $\overline{OA}$ and $\overline{OB}$. The measure of angle $AOB$ is $120^\circ$, and the length of a radius is 6 inches.

Which expression represents the length of arc $AB$, in inches?

(1) $\frac{120}{360}(6\pi)$  
(2) $120(6)$  
(3) $\frac{1}{3}(36\pi)$  
(4) $\frac{1}{3}(12\pi)$
23 Line segment $CD$ is the altitude drawn to hypotenuse $EF$ in right triangle $ECF$. If $EC = 10$ and $EF = 24$, then, to the nearest tenth, $ED$ is

(1) 4.2  (3) 15.5
(2) 5.4  (4) 21.8

24 Line $MN$ is dilated by a scale factor of 2 centered at the point $(0,6)$. If $\overline{MN}$ is represented by $y = -3x + 6$, which equation can represent $\overline{M'N'}$, the image of $\overline{MN}$?

(1) $y = -3x + 12$  (3) $y = -6x + 12$
(2) $y = -3x + 6$  (4) $y = -6x + 6$
Part II

Answer all 7 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [14]

25 Triangle $A'B'C'$ is the image of triangle $ABC$ after a translation of 2 units to the right and 3 units up. Is triangle $ABC$ congruent to triangle $A'B'C'$? Explain why.
Triangle $ABC$ and point $D(1,2)$ are graphed on the set of axes below.

Graph and label $\triangle A'B'C'$, the image of $\triangle ABC$, after a dilation of scale factor 2 centered at point $D$. 
27 Quadrilaterals *BIKE* and *GOLF* are graphed on the set of axes below.

Describe a sequence of transformations that maps quadrilateral *BIKE* onto quadrilateral *GOLF*.
28 In the diagram below, secants $\overline{RST}$ and $\overline{RQP}$, drawn from point $R$, intersect circle $O$ at $S$, $T$, $Q$, and $P$.

If $RS = 6$, $ST = 4$, and $RP = 15$, what is the length of $\overline{RQ}$?
Using a compass and straightedge, construct the median to side $\overline{AC}$ in $\triangle ABC$ below.

[Leave all construction marks.]
30 Skye says that the two triangles below are congruent. Margaret says that the two triangles are similar.

Are Skye and Margaret both correct? Explain why.
Randy’s basketball is in the shape of a sphere with a maximum circumference of 29.5 inches. Determine and state the volume of the basketball, to the nearest cubic inch.
32 Triangle $ABC$ has vertices with coordinates $A(-1,-1)$, $B(4,0)$, and $C(0,4)$. Prove that $\triangle ABC$ is an isosceles triangle but not an equilateral triangle. [The use of the set of axes below is optional.]
The map of a campground is shown below. Campsite $C$, first aid station $F$, and supply station $S$ lie along a straight path. The path from the supply station to the tower, $T$, is perpendicular to the path from the supply station to the campsite. The length of path $FS$ is 400 feet. The angle formed by path $TF$ and path $FS$ is $72^\circ$. The angle formed by path $TC$ and path $CS$ is $55^\circ$.

Determine and state, to the nearest foot, the distance from the campsite to the tower.
Shae has recently begun kickboxing and purchased training equipment as modeled in the diagram below. The total weight of the bag, pole, and unfilled base is 270 pounds. The cylindrical base is 18 inches tall with a diameter of 20 inches. The dry sand used to fill the base weighs 95.46 lbs per cubic foot.

To the nearest pound, determine and state the total weight of the training equipment if the base is filled to 85% of its capacity.
Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for the question to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

35 Given: Parallelogram $ABCD$, $BF \perp AFD$, and $DE \perp BEC$

Prove: $BEDF$ is a rectangle

Work space for question 35 is continued on the next page.
Question 35 continued
Scrap Graph Paper — This sheet will *not* be scored.
Scrap Graph Paper — This sheet will not be scored.
High School Math Reference Sheet

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td>( A = \frac{1}{2}bh )</td>
</tr>
<tr>
<td>Parallelogram</td>
<td>( A = bh )</td>
</tr>
<tr>
<td>Circle</td>
<td>( A = \pi r^2 )</td>
</tr>
<tr>
<td>Circle</td>
<td>( C = \pi d \text{ or } C = 2\pi r )</td>
</tr>
<tr>
<td>General Prisms</td>
<td>( V = Bh )</td>
</tr>
<tr>
<td>Cylinder</td>
<td>( V = \pi r^2h )</td>
</tr>
<tr>
<td>Sphere</td>
<td>( V = \frac{4}{3}\pi r^3 )</td>
</tr>
<tr>
<td>Cone</td>
<td>( V = \frac{1}{3}\pi r^2h )</td>
</tr>
<tr>
<td>Pyramid</td>
<td>( V = \frac{1}{3}Bh )</td>
</tr>
<tr>
<td>Pythagorean Theorem</td>
<td>( a^2 + b^2 = c^2 )</td>
</tr>
<tr>
<td>Quadratic Formula</td>
<td>( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} )</td>
</tr>
<tr>
<td>Arithmetic Sequence</td>
<td>( a_n = a_1 + (n - 1)d )</td>
</tr>
<tr>
<td>Geometric Sequence</td>
<td>( a_n = a_1 r^{n-1} )</td>
</tr>
<tr>
<td>Geometric Series</td>
<td>( S_n = \frac{a_1 - a_1 r^n}{1 - r} \text{ where } r \neq 1 )</td>
</tr>
<tr>
<td>Radians</td>
<td>1 radian = ( \frac{180}{\pi} ) degrees</td>
</tr>
<tr>
<td>Degrees</td>
<td>1 degree = ( \frac{\pi}{180} ) radians</td>
</tr>
<tr>
<td>Exponential Growth/Decay</td>
<td>( A = A_0e^{kt} + B_0 )</td>
</tr>
</tbody>
</table>
The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY

Tuesday, June 19, 2018 — 9:15 a.m. to 12:15 p.m., only

**Updated June 20, 2018**

SCORING KEY AND RATING GUIDE

Mechanics of Rating

The following procedures are to be followed for scoring student answer papers for the Regents Examination in Geometry. More detailed information about scoring is provided in the publication *Information Booklet for Scoring the Regents Examination in Geometry*.

Do not attempt to correct the student’s work by making insertions or changes of any kind. In scoring the open-ended questions, use check marks to indicate student errors. Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Each student’s answer paper is to be scored by a minimum of three mathematics teachers. No one teacher is to score more than approximately one-third of the open-ended questions on a student’s paper. Teachers may not score their own students’ answer papers. On the student’s separate answer sheet, for each question, record the number of credits earned and the teacher’s assigned rater/scorer letter.

Schools are not permitted to rescore any of the open-ended questions on this exam after each question has been rated once, regardless of the final exam score. Schools are required to ensure that the raw scores have been added correctly and that the resulting scale score has been determined accurately.

Raters should record the student’s scores for all questions and the total raw score on the student’s separate answer sheet. Then the student’s total raw score should be converted to a scale score by using the conversion chart that will be posted on the Department’s web site at: [http://www.p12.nysed.gov/assessment/](http://www.p12.nysed.gov/assessment/) on Tuesday, June 19, 2018. Because scale scores corresponding to raw scores in the conversion chart may change from one administration to another, it is crucial that, for each administration, the conversion chart provided for that administration be used to determine the student’s final score. The student’s scale score should be entered in the box provided on the student’s separate answer sheet. The scale score is the student’s final examination score.
If the student’s responses for the multiple-choice questions are being hand scored prior to being scanned, the scorer must be careful not to make any marks on the answer sheet except to record the scores in the designated score boxes. Marks elsewhere on the answer sheet will interfere with the accuracy of the scanning.

Part I

Allow a total of 48 credits, 2 credits for each of the following. Allow credit if the student has written the correct answer instead of the numeral 1, 2, 3, or 4.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>(1).....</td>
<td>(2).....</td>
<td>(3).....</td>
<td>(4).....</td>
</tr>
<tr>
<td>(9).....</td>
<td>(10).....</td>
<td>(11).....</td>
<td>(12).....</td>
</tr>
<tr>
<td>(17).....</td>
<td>(18).....</td>
<td>(19).....</td>
<td>(20).....</td>
</tr>
<tr>
<td>(24).....</td>
<td>(23).....</td>
<td>(22).....</td>
<td>(21).....</td>
</tr>
<tr>
<td>(16).....</td>
<td>(15).....</td>
<td>(14).....</td>
<td>(13).....</td>
</tr>
<tr>
<td>(8).....</td>
<td>(7).....</td>
<td>(6).....</td>
<td>(5).....</td>
</tr>
</tbody>
</table>

Updated information regarding the rating of this examination may be posted on the New York State Education Department's web site during the rating period. Check this web site at: http://www.p12.nysed.gov/assessment/ and select the link “Scoring Information” for any recently posted information regarding this examination. This site should be checked before the rating process for this examination begins and several times throughout the Regents Examination period.

The Department is providing supplemental scoring guidance, the “Model Response Set,” for the Regents Examination in Geometry. This guidance is intended to be part of the scorer training. Schools should use the Model Response Set along with the rubrics in the Scoring Key and Rating Guide to help guide scoring of student work. While not reflective of all scenarios, the Model Response Set illustrates how less common student responses to constructed-response questions may be scored. The Model Response Set will be available on the Department’s web site at: http://www.nysedregents.org/geometryre/.
General Rules for Applying Mathematics Rubrics

I. General Principles for Rating

The rubrics for the constructed-response questions on the Regents Examination in Geometry are designed to provide a systematic, consistent method for awarding credit. The rubrics are not to be considered all-inclusive; it is impossible to anticipate all the different methods that students might use to solve a given problem. Each response must be rated carefully using the teacher’s professional judgment and knowledge of mathematics; all calculations must be checked. The specific rubrics for each question must be applied consistently to all responses. In cases that are not specifically addressed in the rubrics, raters must follow the general rating guidelines in the publication Information Booklet for Scoring the Regents Examination in Geometry, use their own professional judgment, confer with other mathematics teachers, and/or contact the State Education Department for guidance. During each Regents Examination administration period, rating questions may be referred directly to the Education Department. The contact numbers are sent to all schools before each administration period.

II. Full-Credit Responses

A full-credit response provides a complete and correct answer to all parts of the question. Sufficient work is shown to enable the rater to determine how the student arrived at the correct answer. When the rubric for the full-credit response includes one or more examples of an acceptable method for solving the question (usually introduced by the phrase “such as”), it does not mean that there are no additional acceptable methods of arriving at the correct answer. Unless otherwise specified, mathematically correct alternative solutions should be awarded credit. The only exceptions are those questions that specify the type of solution that must be used; e.g., an algebraic solution or a graphic solution. A correct solution using a method other than the one specified is awarded half the credit of a correct solution using the specified method.

III. Appropriate Work

Full-Credit Responses: The directions in the examination booklet for all the constructed-response questions state: “Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.” The student has the responsibility of providing the correct answer and showing how that answer was obtained. The student must “construct” the response; the teacher should not have to search through a group of seemingly random calculations scribbled on the student paper to ascertain what method the student may have used.

Responses With Errors: Rubrics that state “Appropriate work is shown, but…” are intended to be used with solutions that show an essentially complete response to the question but contain certain types of errors, whether computational, rounding, graphing, or conceptual. If the response is incomplete; i.e., an equation is written but not solved or an equation is solved but not all of the parts of the question are answered, appropriate work has not been shown. Other rubrics address incomplete responses.

IV. Multiple Errors

Computational Errors, Graphing Errors, and Rounding Errors: Each of these types of errors results in a 1-credit deduction. Any combination of two of these types of errors results in a 2-credit deduction. No more than 2 credits should be deducted for such mechanical errors in a 4-credit question and no more than 3 credits should be deducted in a 6-credit question. The teacher must carefully review the student’s work to determine what errors were made and what type of errors they were.

Conceptual Errors: A conceptual error involves a more serious lack of knowledge or procedure. Examples of conceptual errors include using the incorrect formula for the area of a figure, choosing the incorrect trigonometric function, or multiplying the exponents instead of adding them when multiplying terms with exponents.

If a response shows repeated occurrences of the same conceptual error, the student should not be penalized twice. If the same conceptual error is repeated in responses to other questions, credit should be deducted in each response.

For 4- and 6-credit questions, if a response shows one conceptual error and one computational, graphing, or rounding error, the teacher must award credit that takes into account both errors. Refer to the rubric for specific scoring guidelines.
Part II

For each question, use the specific criteria to award a maximum of 2 credits. Unless otherwise specified, mathematically correct alternative solutions should be awarded appropriate credit.

(25) [2] Yes, and a correct explanation is written.

[1] An appropriate explanation is written, but one conceptual error is made.

or

[1] An incomplete or partially correct explanation is written.

[0] Yes, but no explanation is written.

or

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

(26) [2] Triangle \( A'B'C' \) is graphed and labeled correctly.

[1] Appropriate work is shown, but one graphing error is made.

or

[1] One conceptual error is made, but appropriate vertices are graphed and labeled.

or

[1] The image of \( \triangle ABC \) is graphed correctly, but it is not labeled or is labeled incorrectly.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
A correct sequence of transformations is described.

An appropriate sequence is described, but one graphing error is made.  

or

An appropriate sequence is described, but one conceptual error is made.  

or

A reflection and translation are identified, but no specific description is written.

A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

4, and correct work is shown.

Appropriate work is shown, but one computational error is made.

or

Appropriate work is shown, but one conceptual error is made.

or

4, but no work is shown.

A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
(29)  [2] A correct construction is drawn showing all appropriate arcs, and the median to $AC$ is drawn.

[1] Appropriate work is drawn showing all appropriate arcs, but the median to $AB$ or $BC$ is drawn.

or

[1] A correct construction is drawn showing all appropriate arcs, but the median to $AC$ is not drawn.

[0] A drawing that is not an appropriate construction is shown.

or

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

(30)  [2] Yes, and a correct explanation is written.

[1] An appropriate explanation is written, but one conceptual error is made.

or

[1] An incomplete or partially correct explanation is written.

[0] Yes, but the explanation is missing or incorrect.

or

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

(31)  [2] 433 or 434, and correct work is shown.

[1] Appropriate work is shown, but one computational or rounding error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

or

[1] 433 or 434, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
Part III

For each question, use the specific criteria to award a maximum of 4 credits. Unless otherwise specified, mathematically correct alternative solutions should be awarded appropriate credit.

(32)  [4] Correct work is shown to prove $\triangle ABC$ is an isosceles triangle and not an equilateral triangle, and correct concluding statements are made.

[3] Appropriate work is shown, but one computational or graphing error is made. Appropriate concluding statements are made.

or

[3] Appropriate work is shown to prove $\triangle ABC$ is an isosceles triangle, and a correct concluding statement is made, but no further correct work is shown.

[2] Appropriate work is shown, but two or more computational or graphing errors are made. Appropriate concluding statements are made.

or

[2] Appropriate work is shown, but one conceptual error is made. Appropriate concluding statements are made.

or

[2] Appropriate work is shown to find the lengths of all three sides, but no further correct work is shown.

[1] Appropriate work is shown to find the lengths of the two sides, but no further correct work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
[4] 1503, and correct work is shown.

[3] Appropriate work is shown, but one computational error is made.

[2] Appropriate work is shown, but two or more computational or rounding errors are made.

or

[2] Appropriate work is shown, but one conceptual error is made.

or

[2] Appropriate work is shown to find $TS$, but no further correct work is shown.

[1] Appropriate work is shown, but one conceptual error and one computational or rounding error are made.

or

[1] A correct trigonometric equation is written to find $TS$, but no further correct work is shown.

or

[1] 1503, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
(34) [4] 536, and correct work is shown.

[3] Appropriate work is shown, but one computational or rounding error is made.

or

[3] Appropriate work is shown to find the weight of the base, but no further correct work is shown.

[2] Appropriate work is shown, but two or more computational or rounding errors are made.

or

[2] Appropriate work is shown, but one conceptual error is made.

or

[2] Correct work is shown to find the volume of the base in cubic feet and/or 85% of the volume in cubic inches, but no further correct work is shown.

[1] Correct work is shown to find the volume of the base in cubic inches, but no further correct work is shown.

or

[1] Appropriate work is shown, but one conceptual error and one computational or rounding error are made.

or

[1] 536, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
Part IV

For this question, use the specific criteria to award a maximum of 6 credits. Unless otherwise specified, mathematically correct alternative solutions should be awarded appropriate credit.

(35) [6] A complete and correct proof that includes a concluding statement is written.

[5] A proof is written that demonstrates a thorough understanding of the method of proof and contains no conceptual errors, but one statement and/or reason is missing or incorrect.

[4] A proof is written that demonstrates a good understanding of the method of proof and contains no conceptual errors, but two statements and/or reasons are missing or incorrect.

or

[4] A proof is written that demonstrates a good understanding of the method of proof, but one conceptual error is made.

[3] A proof is written that demonstrates a method of proof, but three statements and/or reasons are missing or incorrect.

or

[3] \( \triangle ABF \cong \triangle CDE \) is proven, but no further correct work is shown.

or

[3] A proof is written that demonstrates a method of proof, but one conceptual error is made, and one statement and/or reason is missing or incorrect.

[2] Some correct relevant statements about the proof are made, but four or more statements and/or reasons are missing or incorrect.

or

[2] A proof is written with some understanding of the method of proof, but two conceptual errors are made.

[1] Only one correct relevant statement and reason are written.

[0] The “given” and/or the “prove” statements are rewritten in the style of a formal proof, but no further correct relevant statements are written.

or

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
<table>
<thead>
<tr>
<th>Question</th>
<th>Type</th>
<th>Credits</th>
<th>Cluster</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-CO.B</td>
</tr>
<tr>
<td>2</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-CO.C</td>
</tr>
<tr>
<td>3</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-CO.A</td>
</tr>
<tr>
<td>4</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-SRT.B</td>
</tr>
<tr>
<td>5</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-SRT.A</td>
</tr>
<tr>
<td>6</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-SRT.C</td>
</tr>
<tr>
<td>7</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-MG.A</td>
</tr>
<tr>
<td>8</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-SRT.C</td>
</tr>
<tr>
<td>9</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-SRT.B</td>
</tr>
<tr>
<td>10</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-GMD.A</td>
</tr>
<tr>
<td>11</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-SRT.B</td>
</tr>
<tr>
<td>12</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-GPE.B</td>
</tr>
<tr>
<td>13</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-CO.C</td>
</tr>
<tr>
<td>14</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-GPE.B</td>
</tr>
<tr>
<td>15</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-GPE.B</td>
</tr>
<tr>
<td>16</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-GMD.B</td>
</tr>
<tr>
<td>17</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-C.A</td>
</tr>
<tr>
<td>18</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-CO.C</td>
</tr>
<tr>
<td>19</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-CO.A</td>
</tr>
<tr>
<td>20</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-GPE.A</td>
</tr>
<tr>
<td>21</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-SRT.B</td>
</tr>
<tr>
<td>22</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-C.B</td>
</tr>
<tr>
<td>23</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-SRT.B</td>
</tr>
<tr>
<td>24</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-SRT.A</td>
</tr>
<tr>
<td>25</td>
<td>Constructed Response</td>
<td>2</td>
<td>G-CO.B</td>
</tr>
<tr>
<td>26</td>
<td>Constructed Response</td>
<td>2</td>
<td>G-SRT.A</td>
</tr>
<tr>
<td>27</td>
<td>Constructed Response</td>
<td>2</td>
<td>G-CO.A</td>
</tr>
<tr>
<td>28</td>
<td>Constructed Response</td>
<td>2</td>
<td>G-C.A</td>
</tr>
<tr>
<td>29</td>
<td>Constructed Response</td>
<td>2</td>
<td>G-CO.D</td>
</tr>
<tr>
<td>30</td>
<td>Constructed Response</td>
<td>2</td>
<td>G-SRT.B</td>
</tr>
<tr>
<td>31</td>
<td>Constructed Response</td>
<td>2</td>
<td>G-MG.A</td>
</tr>
<tr>
<td>32</td>
<td>Constructed Response</td>
<td>4</td>
<td>G-GPE.B</td>
</tr>
<tr>
<td>33</td>
<td>Constructed Response</td>
<td>4</td>
<td>G-SRT.C</td>
</tr>
<tr>
<td>34</td>
<td>Constructed Response</td>
<td>4</td>
<td>G-MG.A</td>
</tr>
<tr>
<td>35</td>
<td>Constructed Response</td>
<td>6</td>
<td>G-CO.C</td>
</tr>
</tbody>
</table>
Online Submission of Teacher Evaluations of the Test to the Department

Suggestions and feedback from teachers provide an important contribution to the test development process. The Department provides an online evaluation form for State assessments. It contains spaces for teachers to respond to several specific questions and to make suggestions. Instructions for completing the evaluation form are as follows:


2. Select the test title.

3. Complete the required demographic fields.

4. Complete each evaluation question and provide comments in the space provided.

5. Click the SUBMIT button at the bottom of the page to submit the completed form.
25 Triangle $A'B'C'$ is the image of triangle $ABC$ after a translation of 2 units to the right and 3 units up. Is triangle $ABC$ congruent to triangle $A'B'C'$? Explain why.

$\Delta ABC$ must be congruent to $\Delta A'B'C'$ because a translation is a basic rigid motion which preserves angle measure and side length. Therefore the 2 $\Delta$'s have all corresponding parts congruent.

Score 2: The student gave a complete and correct response.
25 Triangle $A'B'C'$ is the image of triangle $ABC$ after a translation of 2 units to the right and 3 units up. Is triangle $ABC$ congruent to triangle $A'B'C'$? Explain why.

Yes, the $\triangle$s are $\cong$ because a translation is a rigid motion so it preserves side lengths, and $\angle$ measures. Because corr. sides have the same lengths, the $\triangle$s are $\cong$ by SSS.

Score 2: The student gave a complete and correct response.
Question 25

25 Triangle $A'B'C'$ is the image of triangle $ABC$ after a translation of 2 units to the right and 3 units up. Is triangle $ABC$ congruent to triangle $A'B'C'$? Explain why.

Yes, because a translation keeps the triangles the same size.

Score 1: The student wrote an incomplete explanation.
Triangle $A'B'C'$ is the image of triangle $ABC$ after a translation of 2 units to the right and 3 units up. Is triangle $ABC$ congruent to triangle $A'B'C'$? Explain why.

Yes, because it was translated, not dilated. Dilations change sizes of shapes causing them to not be congruent.

**Score 1:** The student wrote a partially correct explanation.
Triangle $A'B'C'$ is the image of triangle $ABC$ after a translation of 2 units to the right and 3 units up. Is triangle $ABC$ congruent to triangle $A'B'C'$? Explain why.

Score 0: The student did not show enough correct relevant work to receive any credit.
Graph and label $\triangle A'B'C'$, the image of $\triangle ABC$, after a dilation of scale factor 2 centered at point $D$. 

**Score 2:** The student gave a complete and correct response.
26 Triangle $ABC$ and point $D(1,2)$ are graphed on the set of axes below.

Graph and label $\triangle A'B'C'$, the image of $\triangle ABC$, after a dilation of scale factor 2 centered at point $D$.

**Score 2:** The student gave a complete and correct response.
26 Triangle $ABC$ and point $D(1,2)$ are graphed on the set of axes below.

Graph and label $\triangle A'B'C'$, the image of $\triangle ABC$, after a dilation of scale factor 2 centered at point $D$.

**Score 2:** The student gave a complete and correct response. The student drew a new set of axes whose origin is at point $D$. Then the student dilated and graphed $\triangle ABC$ by a scale factor of 2 centered at the origin, point $D$, with respect to the new axes. The result is a graph of $\triangle A'B'C'$, the image of $\triangle ABC$ after a dilation of 2 centered at point $D$ $(1,2)$, with respect to the original set of axes.
26 Triangle $ABC$ and point $D(1,2)$ are graphed on the set of axes below.

Graph and label $\triangle A'B'C'$, the image of $\triangle ABC$, after a dilation of scale factor 2 centered at point $D$.

Score 1: The student used the origin as the center of dilation.
Triangle $ABC$ and point $D(1,2)$ are graphed on the set of axes below. Graph and label $\triangle A'B'C'$, the image of $\triangle ABC$, after a dilation of scale factor 2 centered at point $D$.

Score 1: The student made one graphing error when graphing point $B$. 
26 Triangle $ABC$ and point $D(1,2)$ are graphed on the set of axes below.

Graph and label $\triangle A'B'C'$, the image of $\triangle ABC$, after a dilation of scale factor 2 centered at point $D$.

Score 1: The student stated the vertices of triangle $A'B'C'$, but did not draw the triangle.
26 Triangle $ABC$ and point $D(1,2)$ are graphed on the set of axes below.

Graph and label $\triangle A'B'C'$, the image of $\triangle ABC$, after a dilation of scale factor 2 centered at point $D$.

Score 0: The student gave a completely incorrect response.
27 Quadrilaterals BIKE and GOLF are graphed on the set of axes below.

Describe a sequence of transformations that maps quadrilateral BIKE onto quadrilateral GOLF.

Score 2: The student gave a complete and correct response.
Describe a sequence of transformations that maps quadrilateral BIKE onto quadrilateral GOLF.

Reflection across the y-axis, followed by a translation 5 units up.

Score 2: The student gave a complete and correct response.
27 Quadrilaterals BIKE and GOLF are graphed on the set of axes below.

Describe a sequence of transformations that maps quadrilateral BIKE onto quadrilateral GOLF.

- Rotate Quad BIKE, 180° around the origin. Image is B’I’K’E’
- Reflect B’I’K’E’ over the line y = 2.5
- Then Quad BIKE will be onto Quad GOLF

**Score 2:** The student gave a complete and correct response.
27 Quadrilaterals BIKE and GOLF are graphed on the set of axes below.

Describe a sequence of transformations that maps quadrilateral BIKE onto quadrilateral GOLF.

Reflection over \( x = -1 \)

Followed by

Translation up 5, right 2

Score 2: The student gave a complete and correct response.
27 Quadrilaterals BIKE and GOLF are graphed on the set of axes below.

Describe a sequence of transformations that maps quadrilateral BIKE onto quadrilateral GOLF.

Translation of \((x+8, y+5)\) and a reflection

Score 1: The student gave an incomplete response. The student did not describe the reflection.
Question 27

Quadrilaterals $BIKE$ and $GOLF$ are graphed on the set of axes below.

Describe a sequence of transformations that maps quadrilateral $BIKE$ onto quadrilateral $GOLF$.

$A$ translation of $\langle 8, 5 \rangle$.

**Score 1:** The student correctly described a translation that carries quadrilateral $BIKE$ onto quadrilateral $LOGF$, not accounting for the orientation of the quadrilateral.
27 Quadrilaterals BIKE and GOLF are graphed on the set of axes below.

Describe a sequence of transformations that maps quadrilateral BIKE onto quadrilateral GOLF.

Score 0: The student gave a completely incorrect response.
28 In the diagram below, secants $\overline{RST}$ and $\overline{RQP}$, drawn from point $R$, intersect circle $O$ at $S$, $T$, $Q$, and $P$.

If $RS = 6$, $ST = 4$, and $RP = 15$, what is the length of $\overline{RQ}$?

\[
15x = 600
\]
\[
\frac{15}{15} = \frac{600}{15}
\]
\[
x = 4
\]

Score 2: The student gave a complete and correct response.
28 In the diagram below, secants $\overline{RST}$ and $\overline{RQP}$, drawn from point $R$, intersect circle $O$ at $S$, $T$, $Q$, and $P$.

If $RS = 6$, $ST = 4$, and $RP = 15$, what is the length of $\overline{RQ}$?

Score 2: The student gave a complete and correct response.
In the diagram below, secants $\overline{RST}$ and $\overline{RQP}$, drawn from point $R$, intersect circle $O$ at $S$, $T$, $Q$, and $P$.

If $RS = 6$, $ST = 4$, and $RP = 15$, what is the length of $\overline{RQ}$?

\[
\begin{align*}
4.6 &= 15x \\
2y &= \frac{15x}{4} \\
\frac{15}{4} &= x \\
x &= 3.75
\end{align*}
\]

**Score 1:** The student wrote an incorrect equation, but solved it correctly for the length of $\overline{RQ}$.
28 In the diagram below, secants $\overline{RST}$ and $\overline{RQP}$, drawn from point $R$, intersect circle $O$ at $S$, $T$, $Q$, and $P$.

If $RS = 6$, $ST = 4$, and $RP = 15$, what is the length of $\overline{RQ}$?

$$60 = x(x+15)$$

$$60 = x^2 + 15x$$

$$x^2 + 15x - 60 = 0$$

$$\frac{x^2 + 15x}{15x} = 0$$

$$\sqrt{x^2 + 15x} = 4$$

$$\sqrt{x} = 2$$

**Score 0:** The student gave a completely incorrect response.
Question 29

29 Using a compass and straightedge, construct the median to side $\overline{AC}$ in $\triangle ABC$ below. [Leave all construction marks.]

Score 2: The student gave a complete and correct response.
29 Using a compass and straightedge, construct the median to side $\overline{AC}$ in $\triangle ABC$ below. [Leave all construction marks.]

Score 2: The student gave a complete and correct response.
29 Using a compass and straightedge, construct the median to side $\overline{AC}$ in $\triangle ABC$ below. [Leave all construction marks.]

Score 1: The student correctly constructed the perpendicular bisector of side $\overline{AC}$, but did not draw the median to side $\overline{AC}$. 
29 Using a compass and straightedge, construct the median to side $\overline{AC}$ in $\triangle ABC$ below. [Leave all construction marks.]

Score 1: The student had an appropriate construction of a median, but constructed it to the wrong side.
Question 29

29 Using a compass and straightedge, construct the median to side $\overline{AC}$ in $\triangle ABC$ below. [Leave all construction marks.]

Score 0: The student gave a completely incorrect response.
Skye says that the two triangles below are congruent. Margaret says that the two triangles are similar.

Are Skye and Margaret both correct? Explain why.

Yes, they are both correct.

Using Pythagorean theorem, both triangles are 5-12-13 triples.

So, \( \triangle \)s are \( \cong \) by SSS.

All \( \cong \) \( \triangle \)s are also similar.

Score 2: The student gave a complete and correct response.
Skye says that the two triangles below are congruent. Margaret says that the two triangles are similar.

Are Skye and Margaret both correct? Explain why.

They are both correct due to all sides in the triangle, along with their right angles being congruent and similar. After doing Pythagorean theorem you find how all the sides are the same.

Score 1: The student wrote an incomplete explanation. The student did not explain why the triangles were also similar.
30 Skye says that the two triangles below are congruent. Margaret says that the two triangles are similar.

Are Skye and Margaret both correct? Explain why.

\[ 5^2 + 12^2 = x^2 \\
25 + 144 = x^2 \\
\sqrt{169} = x \\
x = 13 \]

\[ 5^2 + y^2 = y^2 \\
25 + 144 = 169 \\
\sqrt{y^2} = \sqrt{144} \\
y = 12 \]

No, only Skye is correct. The triangles have the same measurements. Margaret is wrong because one triangle is not bigger or smaller than the other one.

---

**Score 1:** The student wrote a partially correct explanation. The student incorrectly concluded that similar triangles must be different sizes.
30 Skye says that the two triangles below are congruent. Margaret says that the two triangles are similar.

Are Skye and Margaret both correct? Explain why.

They are both correct because the triangles are right triangles. All right triangles are similar and congruent.

Score 0: The student wrote a completely incorrect explanation as to why the triangles were congruent and similar.
31 Randy’s basketball is in the shape of a sphere with a maximum circumference of 29.5 inches. Determine and state the volume of the basketball, to the nearest cubic inch.

\[
C = 2\pi r
\]
\[
\frac{29.5\text{ in}}{2\pi} = 2\pi r
\]
\[
4.69507082\text{ in} = r
\]

\[
V = \frac{4}{3}\pi r^3
\]
\[
V = \frac{4}{3}\pi (4.69507082\text{ in})^3
\]
\[
V = 423.525903\text{ in}^3
\]

**Score 2:** The student gave a complete and correct response.
Randy’s basketball is in the shape of a sphere with a maximum circumference of 29.5 inches. Determine and state the volume of the basketball, to the nearest cubic inch.

\[
\begin{align*}
\text{C} &= \pi d \\
\frac{29.5}{\pi} &= \frac{\pi d}{\pi} \\
9.390141642 &= d \\
4.7 &\approx \text{radius} \\
V &= \frac{4}{3} \pi r^3 \\
V &\approx 43.5 \text{ in}^3
\end{align*}
\]

Score 1: The student rounded the radius, leading to an incorrect volume of the sphere.
Randy’s basketball is in the shape of a sphere with a maximum circumference of 29.5 inches. Determine and state the volume of the basketball, to the nearest cubic inch.

\[
V = \frac{4}{3} \pi r^3
\]

\[
\frac{29.5}{\pi} = R = 9.4
\]

\[
V = 3479 \text{ in}^3
\]

**Score 1:** The student made an error in finding the length of the radius to find the volume of the sphere.
31 Randy's basketball is in the shape of a sphere with a maximum circumference of 29.5 inches. Determine and state the volume of the basketball, to the nearest cubic inch.

\[ V = \frac{1}{3} \pi r^2 \]
\[ V = \frac{1}{3} \pi (14.75)^2 \]
\[ V \approx 227.8309172 \]

Score 0: The student gave a completely incorrect response.
Question 32

32 Triangle $ABC$ has vertices with coordinates $A(-1,-1)$, $B(4,0)$, and $C(0,4)$. Prove that $\triangle ABC$ is an isosceles triangle but not an equilateral triangle. [The use of the set of axes below is optional.]

Distance $AB = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

\begin{align*}
\text{Distance } AB & = \sqrt{(4-(-1))^2 + (0-(-1))^2} \\
& = \sqrt{5^2 + 1^2} \\
& = \sqrt{26}
\end{align*}

Distance $BC = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

\begin{align*}
\text{Distance } BC & = \sqrt{(0-4)^2 + (4-0)^2} \\
& = \sqrt{(0-4)^2 + (4-0)^2} \\
& = \sqrt{20} \\
& = \sqrt{4 \times 5}
\end{align*}

Distance $CA = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

\begin{align*}
\text{Distance } CA & = \sqrt{(0-(-1))^2 + (4-(-1))^2} \\
& = \sqrt{1 + 25} \\
& = \sqrt{26}
\end{align*}

\[ \therefore \] $\triangle ABC$ is isosceles but not equilateral.

Score 4: The student gave a complete and correct response.
Question 32

32 Triangle $ABC$ has vertices with coordinates $A(-1,-1)$, $B(4,0)$, and $C(0,4)$. Prove that $\triangle ABC$ is an isosceles triangle but not an equilateral triangle. [The use of the set of axes below is optional.]

1. $\overline{AB}$ and $\overline{AC}$ are sides on equal length because of the Pythagorean theorem.
2. Two sides of triangle $ABC$ are congruent, therefore $\triangle ABC$ is an isosceles triangle.
3. Side $\overline{BC}$ is not congruent to the other sides.
4. 3 equal sides are necessary for an equilateral triangle, which is not present in the figure below.
5. Therefore $\triangle ABC$ is isosceles but not equilateral.

Score 4: The student gave a complete and correct response.
Question 32

Triangle $ABC$ has vertices with coordinates $A(-1, -1)$, $B(4,0)$, and $C(0,4)$. Prove that $\triangle ABC$ is an isosceles triangle but not an equilateral triangle. [The use of the set of axes below is optional.]

Two sides are congruent, and the third one is not, creating an isosceles triangle.

Score 3: The student proved $\triangle ABC$ is an isosceles triangle, but did not write a concluding statement that $\triangle ABC$ is not an equilateral triangle.
32 Triangle $ABC$ has vertices with coordinates $A(-1,-1)$, $B(4,0)$, and $C(0,4)$. Prove that $\triangle ABC$ is an isosceles triangle but not an equilateral triangle. [The use of the set of axes below is optional.]

\[
\begin{align*}
\text{distance formula: } & \frac{d}{1} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \\
AC &= \sqrt{(-1)^2 + (3)^2} = \sqrt{10} \\
AB &= \sqrt{(3)^2 + (-1)^2} = \sqrt{10} \\
BC &= \sqrt{4^2 + 4^2} = \sqrt{32} \\
\end{align*}
\]

\[\begin{align*}
AC &= AB \neq BC \\
&\therefore \triangle ABC \text{ is not equilateral, because all 3 sides are not equal.} \\
\end{align*}\]

Score 3: The student made a computational error in finding the lengths of $AB$ and $AC$ by stating that $-1 - 4 = 3$. 
Triangle $ABC$ has vertices with coordinates $A(-1,-1)$, $B(4,0)$, and $C(0,4)$. Prove that $\triangle ABC$ is an isosceles triangle but not an equilateral triangle. [The use of the set of axes below is optional.]

\[
\begin{align*}
AB & : \sqrt{(-1-4)^2 + (-1-0)^2} = \sqrt{26} \\
BC & : \sqrt{(4-0)^2 + (0-4)^2} = \sqrt{32} \\
AC & : \sqrt{(-1-0)^2 + (-1-4)^2} = \sqrt{26}
\end{align*}
\]

$\triangle ABC$ is isosceles but not equilateral.

Score 2: The student wrote an incomplete concluding statement by not stating why the lengths of the sides of $\triangle ABC$ led to the triangle being isosceles but not equilateral.
32 Triangle $ABC$ has vertices with coordinates $A(-1, -1), B(4, 0)$, and $C(0, 4)$. Prove that $\triangle ABC$ is an isosceles triangle but not an equilateral triangle. [The use of the set of axes below is optional.]

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \overline{AC} \]
\[ = \sqrt{(0 + 1) + (4 + 1)^2} \]
\[ = \sqrt{26} \]

\[ d = \sqrt{(y + 1)^2 + (0 + 1)^2} \quad \overline{AB} \]
\[ = \sqrt{26} \]

**Score 1:** The student correctly found the lengths of $\overline{AB}$ and $\overline{AC}$, but no further correct work was shown.
32 Triangle $ABC$ has vertices with coordinates $A(-1, -1)$, $B(4,0)$, and $C(0,4)$. Prove that $\triangle ABC$ is an isosceles triangle but not an equilateral triangle. [The use of the set of axes below is optional.]

\[ AB = \sqrt{(-1-4)^2 + (-1-0)^2} = \sqrt{26} \]

\[ BC = \sqrt{(4-0)^2 + (0-4)^2} = \sqrt{32} \]

$\triangle ABC$ is not equilateral

Score 1: The student found the lengths of two noncongruent sides, but the concluding statement was incomplete.
32 Triangle $ABC$ has vertices with coordinates $A(-1, -1)$, $B(4,0)$, and $C(0,4)$. Prove that $\triangle ABC$ is an isosceles triangle but not an equilateral triangle. [The use of the set of axes below is optional.]

**Score 0:** The student did not show enough correct relevant work to receive any credit.
The map of a campground is shown below. Campsite C, first aid station F, and supply station S lie along a straight path. The path from the supply station to the tower, T, is perpendicular to the path from the supply station to the campsite. The length of path FS is 400 feet. The angle formed by path TF and path FS is 72°. The angle formed by path TC and path CS is 55°.

Determine and state, to the nearest foot, the distance from the campsite to the tower.

\[
\tan 72 = \frac{y}{400} \\
400 \tan 72 = y \\
1231.0731 \approx y
\]

\[
\sin 55 = \frac{y}{x} \\
\sin 55 = \frac{400 \tan 72}{x} \\
x = \frac{400 \tan 72}{\sin 55} \approx 1503 \text{ ft}
\]

Score 4: The student gave a complete and correct response.
The map of a campground is shown below. Campsite $C$, first aid station $F$, and supply station $S$ lie along a straight path. The path from the supply station to the tower, $T$, is perpendicular to the path from the supply station to the campsite. The length of path $FS$ is 400 feet. The angle formed by path $TF$ and path $FS$ is $72^\circ$. The angle formed by path $TC$ and path $CS$ is $55^\circ$.

Determine and state, to the nearest foot, the distance from the campsite to the tower.

\[ x = \sqrt{1502.863} \]

\[ y = 1231.073 \]

Score 4: The student gave a complete and correct response.
The map of a campground is shown below. Campsite $C$, first aid station $F$, and supply station $S$ lie along a straight path. The path from the supply station to the tower, $T$, is perpendicular to the path from the supply station to the campsite. The length of path $FS$ is 400 feet. The angle formed by path $TF$ and path $FS$ is $72^\circ$. The angle formed by path $TC$ and path $CS$ is $55^\circ$.

Determine and state, to the nearest foot, the distance from the campsite to the tower.

Score 3: The student made one computational error in determining the length of $CT$ by incorrectly dividing: $1231 + 0.8192 = 1230$. 

\[ \text{Score 3: The student made one computational error in determining the length of } CT \text{ by incorrectly dividing: } 1231 + 0.8192 = 1230. \]
33 The map of a campground is shown below. Campsite $C$, first aid station $F$, and supply station $S$ lie along a straight path. The path from the supply station to the tower, $T$, is perpendicular to the path from the supply station to the campsite. The length of path $FS$ is 400 feet. The angle formed by path $TF$ and path $FS$ is $72^\circ$. The angle formed by path $TC$ and path $CS$ is $55^\circ$.

\[
\tan 72 = \frac{y}{400} \quad \text{or} \quad x = 1485 \text{ ft}
\]

\[
\sin 56 = \frac{1231.073415}{x} \quad \text{or} \quad x = 1485 \text{ ft}
\]

Determine and state, to the nearest foot, the distance from the campsite to the tower.

**Score 3:** The student made a transcription error by writing $\sin 56$ instead of $\sin 55$. 
Question 33

33 The map of a campground is shown below. Campsite C, first aid station F, and supply station S lie along a straight path. The path from the supply station to the tower, T, is perpendicular to the path from the supply station to the campsite. The length of path FS is 400 feet. The angle formed by path TF and path FS is 72°. The angle formed by path TC and path CS is 55°.

Determine and state, to the nearest foot, the distance from the campsite to the tower.

\[
\tan 72^\circ = \frac{x}{400} \quad x = \tan(72^\circ) \cdot 400 = 1231.07\text{ feet} \\
1231^2 + 400^2 = x^2 \\
1557581 = x^2 \\
x = 1244.36 \\
x = 1244\text{ ft}
\]

Determine and state, to the nearest foot, the distance from the campsite to the tower.

Score 2: The student made one conceptual error by using a proportion in non-similar triangles to find CF.
33 The map of a campground is shown below. Campsite $C$, first aid station $F$, and supply station $S$ lie along a straight path. The path from the supply station to the tower, $T$, is perpendicular to the path from the supply station to the campsite. The length of path $FS$ is 400 feet. The angle formed by path $TF$ and path $FS$ is $72^\circ$. The angle formed by path $TC$ and path $CS$ is $55^\circ$.

\[
\cos(72^\circ) = \frac{400}{x} \\
x = \frac{400}{\cos(72^\circ)} \\
x \approx 1294.4 \\
\sin(55^\circ) = \frac{1294.4}{x} \\
x = \frac{1294.4}{\sin(55^\circ)} \\
x \approx 1580 \text{ ft}
\]

**Score 2:** The student made one conceptual error by incorrectly using the sine function in non-right triangle $CFT$. 
The map of a campground is shown below. Campsite $C$, first aid station $F$, and supply station $S$ lie along a straight path. The path from the supply station to the tower, $T$, is perpendicular to the path from the supply station to the campsite. The length of path $FS$ is 400 feet. The angle formed by path $TF$ and path $FS$ is $72^\circ$. The angle formed by path $TC$ and path $CS$ is $55^\circ$.

Determine and state, to the nearest foot, the distance from the campsite to the tower.

\[
\tan 72^\circ = \frac{x}{400} \quad x = \frac{400}{\tan 72^\circ} = 129.9678785 \quad 130
\]

**Score 1:** The student wrote one correct trigonometric equation to find the length of $TS$, but no further correct work was shown.
The map of a campground is shown below. Campsite \( C \), first aid station \( F \), and supply station \( S \) lie along a straight path. The path from the supply station to the tower, \( T \), is perpendicular to the path from the supply station to the campsite. The length of path \( FS \) is 400 feet. The angle formed by path \( TF \) and path \( FS \) is \( 72^\circ \). The angle formed by path \( TC \) and path \( CS \) is \( 55^\circ \).

Determine and state, to the nearest foot, the distance from the campsite to the tower.

\[
72^2 + 55^2 = c^2
\]

\[
5184 + 3025 = c^2
\]

\[
c = \sqrt{8209} 
\]

\[
88.8 \text{ feet}
\]

**Score 0:** The student gave a completely incorrect response.
34 Shae has recently begun kickboxing and purchased training equipment as modeled in the diagram below. The total weight of the bag, pole, and unfilled base is 270 pounds. The cylindrical base is 18 inches tall with a diameter of 20 inches. The dry sand used to fill the base weighs 95.46 lbs per cubic foot.

\[
V = \pi r^2 h
\]
\[
V = \pi \cdot 10^2 \cdot 18
\]
\[
V = 5654.866776 \text{ ft}^3
\]
\[
V \times .85 = 4806.63676 \text{ ft}^3
\]

To the nearest pound, determine and state the total weight of the training equipment if the base is filled to 85% of its capacity.

Score 4: The student gave a complete and correct response.
34 Shae has recently begun kickboxing and purchased training equipment as modeled in the diagram below. The total weight of the bag, pole, and unfilled base is 270 pounds. The cylindrical base is 18 inches tall with a diameter of 20 inches. The dry sand used to fill the base weighs 95.46 lbs per cubic foot.

To the nearest pound, determine and state the total weight of the training equipment if the base is filled to 85% of its capacity.

Score 4: The student gave a complete and correct response.
34 Shae has recently begun kickboxing and purchased training equipment as modeled in the diagram below. The total weight of the bag, pole, and unfilled base is 270 pounds. The cylindrical base is 18 inches tall with a diameter of 20 inches. The dry sand used to fill the base weighs 95.46 lbs per cubic foot.

To the nearest pound, determine and state the total weight of the training equipment if the base is filled to 85% of its capacity.

The total weight of the freestanding training bag if the base is filled to 85% of its capacity is 459.112 pounds.

Score 3: The student did not convert cubic inches to cubic feet.
34 Shae has recently begun kickboxing and purchased training equipment as modeled in the diagram below. The total weight of the bag, pole, and unfilled base is 270 pounds. The cylindrical base is 18 inches tall with a diameter of 20 inches. The dry sand used to fill the base weighs 95.46 lbs per cubic foot.

To the nearest pound, determine and state the total weight of the training equipment if the base is filled to 85% of its capacity.

\[
\begin{align*}
V &= \pi r^2 h \\
V &= \pi \cdot (10^2) (18) \\
V &= 1800\pi \\
V &= 5654.86676 \\
\text{change to ft}^3 \\
\frac{5654.86676}{12^3} &= \frac{5654.86676}{1728} \\
&= 3.27249467 \\
&= 3.272492347 \\
\text{cubic feet}
\end{align*}
\]

Score 2: The student found the volume of the base in cubic feet, but no further correct work was shown.
34 Shae has recently begun kickboxing and purchased training equipment as modeled in the diagram below. The total weight of the bag, pole, and unfilled base is 270 pounds. The cylindrical base is 18 inches tall with a diameter of 20 inches. The dry sand used to fill the base weighs 95.46 lbs per cubic foot.

![Diagram of training equipment]

To the nearest pound, determine and state the total weight of the training equipment if the base is filled to 85% of its capacity.

\[ V = \pi r^2 h \]
\[ V = \pi (20)^2 (18) \]
\[ V = 22619.4671 \]
\[ \frac{22619.4671}{12} = 1.89995309167 \]

**Score 1:** The student made an error in finding the volume in cubic feet by using the diameter of the base in the volume formula.
Shae has recently begun kickboxing and purchased training equipment as modeled in the diagram below. The total weight of the bag, pole, and unfilled base is 270 pounds. The cylindrical base is 18 inches tall with a diameter of 20 inches. The dry sand used to fill the base weighs 95.46 lbs per cubic foot.

To the nearest pound, determine and state the total weight of the training equipment if the base is filled to 85% of its capacity.

\[
V = \pi r^2 h \\
= \pi \times 10^2 \times 18 \\
= 1800\pi \\
= 5659.87...
\]

\[
\frac{270 + 80 = 350}{1800\pi} = \frac{95.46}{100}
\]

\[
\frac{954.60}{85} = 85x \\
11.2 = x
\]

**Score 1:** The student found the volume of the base in cubic inches, but no further correct work was shown.
Shae has recently begun kickboxing and purchased training equipment as modeled in the diagram below. The total weight of the bag, pole, and unfilled base is 270 pounds. The cylindrical base is 18 inches tall with a diameter of 20 inches. The dry sand used to fill the base weighs 95.46 lbs per cubic foot.

To the nearest pound, determine and state the total weight of the training equipment if the base is filled to 85% of its capacity.

\[
\begin{align*}
565.38 \times 0.85 &= 452.304 \\
565.38 \div 270 &= 2.094 \\
95.46 \times 2.1 &= 200.466 \\
\frac{200.466}{6} &= 33.4105 \\
200.466 \times (0.85) &= 170.34
\end{align*}
\]

\[
\begin{align*}
\pi r^2 &= \pi \times 10^2 \\
\pi &= 31.4189 \\
31.4189 \times 18 &= 565.38
\end{align*}
\]

The total weight is 170 lbs.

Score 0: The student gave a completely incorrect response.
35 Given: Parallelogram $ABCD$, $BF \perp AFD$, and $DE \perp BEC$

Prove: $BEDF$ is a rectangle

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Parallelogram $ABCD$, $BF \perp AD$ and $DE \perp BEC$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\angle BFA \cong \angle BFD \cong \angle DEC \cong \angle DEB$</td>
<td>2. $\perp$ lines form right angles</td>
</tr>
<tr>
<td>3. $\cong \angle BFA \cong \angle BFD \cong \angle DEC \cong \angle DEB$</td>
<td>3. All right $\angle s \cong$</td>
</tr>
<tr>
<td>4. $\angle A \cong \angle C$</td>
<td>4. In a parallelogram, opp. $\angle s \cong$</td>
</tr>
<tr>
<td>5. $\overline{AB} \cong \overline{CD}$</td>
<td>5. In a parallelogram, opp. sides $\cong$</td>
</tr>
<tr>
<td>6. $\overline{AB} \cong \overline{CD}$</td>
<td>6. $\triangle AFB \cong \triangle CDE$ AAS $\cong$ AAS</td>
</tr>
<tr>
<td>7. $\overline{BF} \cong \overline{DE}$</td>
<td>7. CPCTC</td>
</tr>
<tr>
<td>8. $\overline{BF} \parallel \overline{AD}$</td>
<td>8. In a parallelogram, opp. sides $\parallel$</td>
</tr>
<tr>
<td>9. $\overline{BF} \parallel \overline{ED}$</td>
<td>9. $\perp$ lines $\parallel \perp$ lines are $\parallel$</td>
</tr>
<tr>
<td>10. $\overline{BEDF}$ is a Parallelogram</td>
<td>10. one pair of opp. sides $\cong$ and $\parallel$ Parallelogram</td>
</tr>
<tr>
<td>11. $\overline{BEDF}$ is a rectangle</td>
<td>11. a parallelogram with a right $\angle$ is a rectangle</td>
</tr>
</tbody>
</table>

Score 6: The student gave a complete and correct response.
35 Given: Parallelogram $ABCD$, $BF \perp AFD$, and $DE \perp BEC$

Prove: $BEDF$ is a rectangle

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) $ABCD$, $BF \perp AFD$</td>
<td>1) Given</td>
</tr>
<tr>
<td>$DE \perp BEC$</td>
<td>2) opp. sides of a $\square$ are $\parallel$</td>
</tr>
<tr>
<td>2) $BC \parallel AD$</td>
<td>3) parts of $\parallel$ lines are $\parallel$</td>
</tr>
<tr>
<td>3) $BE \parallel FD$</td>
<td>4) 2 lines $\perp$ to $\parallel$ lines are $\parallel$</td>
</tr>
<tr>
<td>4) $BF \parallel DE$</td>
<td>5) A $\square$ w/ both $\parallel$ sides $\perp$ is a $\square$</td>
</tr>
<tr>
<td>$\square$ $BEDF$</td>
<td>6) $\perp$ lines form $\square$ $\perp$ $\angle$s</td>
</tr>
<tr>
<td>6) $\angle$ $DEB$ is a $\angle$ $\angle$ $BEDF$</td>
<td>7) A $\square$ with one $\angle$ $\perp$ $\angle$ is a rectangle</td>
</tr>
<tr>
<td>7) is a rectangle</td>
<td></td>
</tr>
</tbody>
</table>

Score 6: The student gave a complete and correct response.
Given: Parallelogram $ABCD$, $BF \perp AFD$, and $DE \perp BEC$

Prove: $BEDF$ is a rectangle

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) $ABCD$ is a parallelogram, $BF \perp AD$, $DE \perp BC$</td>
<td>1) given</td>
</tr>
<tr>
<td>2) $\angle A \cong \angle C$</td>
<td>2) opposite $\angle$s of a parallelogram are $\cong$</td>
</tr>
<tr>
<td>3) $\left{ \begin{array}{l} AB \cong BC \ AD \cong DC \end{array} \right.$</td>
<td>3) opposite sides of a parallelogram are $\cong$</td>
</tr>
<tr>
<td>4) $\angle BFD &lt; \angle DEB, \angle &lt; BFA, \angle DEC$ right $\angle$s</td>
<td>4) $\perp$ lines form right angles</td>
</tr>
<tr>
<td>4a) $\angle BFD \cong \angle DEB$</td>
<td>4a) all right $\angle$s are $\cong$</td>
</tr>
<tr>
<td>$\angle BFA \cong \angle DEC$</td>
<td>5) $\triangle AFB \cong \triangle DEC$</td>
</tr>
<tr>
<td>5) $\triangle AFB \cong \triangle DEC$</td>
<td>6) CPCTC</td>
</tr>
<tr>
<td>6) $BF \cong DE$</td>
<td>7) $\angle$s subt from $s$ are $=$</td>
</tr>
<tr>
<td>7) $BE \cong FD$</td>
<td>8) Because both pairs of opposite sides are $\cong$</td>
</tr>
<tr>
<td>8) $BEDF$ is a parallelogram</td>
<td>9) Four $=$ sides and four $=$ angles $(90^\circ)$</td>
</tr>
<tr>
<td>9) $BEDF$ is a rectangle</td>
<td>So it is a quadrilateral from $8,6,4$</td>
</tr>
</tbody>
</table>

Score 5: The student had an incorrect reason in step 9.
Question 35

35 Given: Parallelogram $ABCD$, $BF \perp AFD$, and $DE \perp BEC$

Prove: $BEDF$ is a rectangle

<table>
<thead>
<tr>
<th>S</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Parallelogram $ABCD$, $BF \perp AFD$, $DE \perp BEC$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $AB \cong CD$</td>
<td>2. Opposite sides of a parallelogram are $\cong$.</td>
</tr>
<tr>
<td>3. $\angle A \cong \angle C$</td>
<td>3. Opposite $\angle$s of a parallelogram are $\cong$.</td>
</tr>
<tr>
<td>4. $\angle BFA \cong \angle BFD$, $\angle DEC$ are all right $\angle$s.</td>
<td>4. Perpendicular lines meet to form right angles.</td>
</tr>
<tr>
<td>5. $\triangle BFA \cong \triangle DEC$</td>
<td>5. All right $\angle$s are $\cong$.</td>
</tr>
<tr>
<td>6. $\triangle BFA \cong \triangle DEC$</td>
<td>6. AAS</td>
</tr>
<tr>
<td>7. $BF \cong ED$</td>
<td>7. CPCTC</td>
</tr>
<tr>
<td>8. $BF \parallel ED$</td>
<td>8. If 2 segments are $\perp$ to // lines, then they are // to each other.</td>
</tr>
<tr>
<td>9. $BEDF$ is a parallelogram</td>
<td>9. A quad w/ one set of opposite sides $\cong$ and // is a parallelogram.</td>
</tr>
</tbody>
</table>

Score 4: The student did not state that $AD \parallel BC$ in order to prove $BF \parallel ED$. The student did not prove $BEDF$ is a rectangle.
35 Given: Parallelogram $ABCD$, $BF \perp AFD$, and $DE \perp BEC$

Prove: $BEDF$ is a rectangle

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 $ABCD \parallel BF \parallel AD \parallel DE \parallel CE$</td>
<td>1 Given</td>
</tr>
<tr>
<td>2 $\angle 1, \angle 2, \angle 3, \angle 4$ are $90^\circ$</td>
<td>2 Definition of Perpendicular</td>
</tr>
<tr>
<td>3 $\angle 3 \cong \angle 4$</td>
<td>3 All $\angle$ equal</td>
</tr>
<tr>
<td>4 $\angle A \cong \angle C$</td>
<td>4 Opposite $\angle$s $\equiv$</td>
</tr>
<tr>
<td>5 $AB \cong CD$</td>
<td>5 Opposite sides $\equiv$</td>
</tr>
<tr>
<td>6 $\triangle BAF \cong \triangle DCE$</td>
<td>6 AAS</td>
</tr>
<tr>
<td>7 $BF \cong ED$</td>
<td>7 CPCTC</td>
</tr>
<tr>
<td>8 $BEDF$ is a Rectangle</td>
<td>8 A Rectangle has $\angle$ &amp; $\cong$ opposite sides</td>
</tr>
</tbody>
</table>

Score 4: The student made one conceptual error by concluding a rectangle is a quadrilateral with one pair of opposite sides congruent and two right angles (step 8).
Question 35

Given: Parallelogram $ABCD$, $BF \perp AFD$, and $DE \perp BEC$

Prove: $BEDF$ is a rectangle

Score 3: The student proved $\triangle AFB \cong \triangle CED$. 
Given: Parallelogram $ABCD$, $BF \perp AFD$, and $DE \perp BEC$

Prove: $BEDF$ is a rectangle

Score 2: The student made two correct relevant statements and reasons about parallelogram $ABCD$ (steps 2 and 3).
35 Given: Parallelogram $ABCD$, $BF \perp AFD$, and $DE \perp BEC$

Prove: $BEDF$ is a rectangle

Score 1: The student made a correct relevant statement and reason in step 4.
35 Given: Parallelogram $ABCD$, $BF \perp AFD$, and $DE \perp BEC$

Prove: $BEDF$ is a rectangle

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $BF \perp AD$ &amp; $DE \perp BC$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\angle BFDE \cong \angle DEB$</td>
<td>2. Both right angles</td>
</tr>
<tr>
<td>3. $BE \cong FD$</td>
<td>3. Parallel lines</td>
</tr>
<tr>
<td>4. $BF \cong ED$</td>
<td>4. Parallel lines</td>
</tr>
<tr>
<td>5. $\angle B \cong \angle D$</td>
<td>5. Both right angles</td>
</tr>
<tr>
<td>6. $BEDF$ is a rectangle</td>
<td>6. All right angles but two pairs of matching sides</td>
</tr>
</tbody>
</table>

Score 0: The student did not show enough correct relevant work to receive any credit.
Schools are not permitted to rescore any of the open-ended questions on this exam after each question has been rated once, regardless of the final exam score. Schools are required to ensure that the raw scores have been added correctly and that the resulting scale score has been determined accurately.

Because scale scores corresponding to raw scores in the conversion chart change from one administration to another, it is crucial that for each administration the conversion chart provided for that administration be used to determine the student’s final score. The chart above is usable only for this administration of the Regents Examination in Geometry.

<table>
<thead>
<tr>
<th>Raw Score</th>
<th>Scale Score</th>
<th>Performance Level</th>
<th>Raw Score</th>
<th>Scale Score</th>
<th>Performance Level</th>
<th>Raw Score</th>
<th>Scale Score</th>
<th>Performance Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>100</td>
<td>5</td>
<td>53</td>
<td>80</td>
<td>4</td>
<td>26</td>
<td>61</td>
<td>2</td>
</tr>
<tr>
<td>79</td>
<td>99</td>
<td>5</td>
<td>52</td>
<td>80</td>
<td>4</td>
<td>25</td>
<td>60</td>
<td>2</td>
</tr>
<tr>
<td>78</td>
<td>98</td>
<td>5</td>
<td>51</td>
<td>79</td>
<td>3</td>
<td>24</td>
<td>59</td>
<td>2</td>
</tr>
<tr>
<td>77</td>
<td>97</td>
<td>5</td>
<td>50</td>
<td>79</td>
<td>3</td>
<td>23</td>
<td>58</td>
<td>2</td>
</tr>
<tr>
<td>76</td>
<td>96</td>
<td>5</td>
<td>49</td>
<td>78</td>
<td>3</td>
<td>22</td>
<td>56</td>
<td>2</td>
</tr>
<tr>
<td>75</td>
<td>95</td>
<td>5</td>
<td>48</td>
<td>78</td>
<td>3</td>
<td>21</td>
<td>55</td>
<td>2</td>
</tr>
<tr>
<td>74</td>
<td>94</td>
<td>5</td>
<td>47</td>
<td>77</td>
<td>3</td>
<td>20</td>
<td>53</td>
<td>1</td>
</tr>
<tr>
<td>73</td>
<td>94</td>
<td>5</td>
<td>46</td>
<td>77</td>
<td>3</td>
<td>19</td>
<td>51</td>
<td>1</td>
</tr>
<tr>
<td>72</td>
<td>93</td>
<td>5</td>
<td>45</td>
<td>76</td>
<td>3</td>
<td>18</td>
<td>50</td>
<td>1</td>
</tr>
<tr>
<td>71</td>
<td>92</td>
<td>5</td>
<td>44</td>
<td>76</td>
<td>3</td>
<td>17</td>
<td>48</td>
<td>1</td>
</tr>
<tr>
<td>70</td>
<td>91</td>
<td>5</td>
<td>43</td>
<td>75</td>
<td>3</td>
<td>16</td>
<td>46</td>
<td>1</td>
</tr>
<tr>
<td>69</td>
<td>90</td>
<td>5</td>
<td>42</td>
<td>74</td>
<td>3</td>
<td>15</td>
<td>44</td>
<td>1</td>
</tr>
<tr>
<td>68</td>
<td>90</td>
<td>5</td>
<td>41</td>
<td>74</td>
<td>3</td>
<td>14</td>
<td>42</td>
<td>1</td>
</tr>
<tr>
<td>67</td>
<td>89</td>
<td>5</td>
<td>40</td>
<td>73</td>
<td>3</td>
<td>13</td>
<td>40</td>
<td>1</td>
</tr>
<tr>
<td>66</td>
<td>88</td>
<td>5</td>
<td>39</td>
<td>73</td>
<td>3</td>
<td>12</td>
<td>38</td>
<td>1</td>
</tr>
<tr>
<td>65</td>
<td>88</td>
<td>5</td>
<td>38</td>
<td>72</td>
<td>3</td>
<td>11</td>
<td>35</td>
<td>1</td>
</tr>
<tr>
<td>64</td>
<td>87</td>
<td>5</td>
<td>37</td>
<td>71</td>
<td>3</td>
<td>10</td>
<td>33</td>
<td>1</td>
</tr>
<tr>
<td>63</td>
<td>86</td>
<td>5</td>
<td>36</td>
<td>71</td>
<td>3</td>
<td>9</td>
<td>30</td>
<td>1</td>
</tr>
<tr>
<td>62</td>
<td>86</td>
<td>5</td>
<td>35</td>
<td>70</td>
<td>3</td>
<td>8</td>
<td>28</td>
<td>1</td>
</tr>
<tr>
<td>61</td>
<td>85</td>
<td>5</td>
<td>34</td>
<td>69</td>
<td>3</td>
<td>7</td>
<td>25</td>
<td>1</td>
</tr>
<tr>
<td>60</td>
<td>84</td>
<td>4</td>
<td>33</td>
<td>68</td>
<td>3</td>
<td>6</td>
<td>22</td>
<td>1</td>
</tr>
<tr>
<td>59</td>
<td>84</td>
<td>4</td>
<td>32</td>
<td>67</td>
<td>3</td>
<td>5</td>
<td>19</td>
<td>1</td>
</tr>
<tr>
<td>58</td>
<td>83</td>
<td>4</td>
<td>31</td>
<td>67</td>
<td>3</td>
<td>4</td>
<td>15</td>
<td>1</td>
</tr>
<tr>
<td>57</td>
<td>82</td>
<td>4</td>
<td>30</td>
<td>66</td>
<td>3</td>
<td>3</td>
<td>12</td>
<td>1</td>
</tr>
<tr>
<td>56</td>
<td>82</td>
<td>4</td>
<td>29</td>
<td>65</td>
<td>3</td>
<td>2</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>55</td>
<td>81</td>
<td>4</td>
<td>28</td>
<td>64</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>54</td>
<td>81</td>
<td>4</td>
<td>27</td>
<td>63</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>