## 0118geo

1 In the diagram below, a sequence of rigid motions maps $A B C D$ onto JKLM.


If $\mathrm{m} \angle A=82^{\circ}, \mathrm{m} \angle B=104^{\circ}$, and $\mathrm{m} \angle L=121^{\circ}$, the measure of $\angle M$ is

1) $53^{\circ}$
2) $82^{\circ}$
3) $104^{\circ}$
4) $121^{\circ}$

2 Parallelogram HAND is drawn below with diagonals $\overline{H N}$ and $\overline{A D}$ intersecting at $S$.


Which statement is always true?

1) $A N=\frac{1}{2} A D$
2) $A S=\frac{1}{2} A D$
3) $\angle A H S \cong \angle A N S$
4) $\angle H D S \cong \angle N D S$

3 The graph below shows two congruent triangles, $A B C$ and $A^{\prime} B^{\prime} C^{\prime}$.


Which rigid motion would map $\triangle A B C$ onto $\triangle A^{\prime} B^{\prime} C^{\prime \prime}$ ?

1) a rotation of 90 degrees counterclockwise about the origin
2) a translation of three units to the left and three units up
3) a rotation of 180 degrees about the origin
4) a reflection over the line $y=x$

4 A man was parasailing above a lake at an angle of elevation of $32^{\circ}$ from a boat, as modeled in the diagram below.


If 129.5 meters of cable connected the boat to the parasail, approximately how many meters above the lake was the man?

1) 68.6
2) 80.9
3) 109.8
4) 244.4

5 A right hexagonal prism is shown below. A two-dimensional cross section that is perpendicular to the base is taken from the prism.


Which figure describes the two-dimensional cross section?

1) triangle
2) rectangle
3) pentagon
4) hexagon

6 In the diagram below, $\overline{A C}$ has endpoints with coordinates $A(-5,2)$ and $C(4,-10)$.


If $B$ is a point on $\overline{A C}$ and $A B: B C=1: 2$, what are the coordinates of $B$ ?

1) $(-2,-2)$
2) $\left(-\frac{1}{2},-4\right)$
3) $\left(0,-\frac{14}{3}\right)$
4) $(1,-6)$

7 An ice cream waffle cone can be modeled by a right circular cone with a base diameter of 6.6 centimeters and a volume of $54.45 \pi$ cubic centimeters. What is the number of centimeters in the height of the waffle cone?

1) $3 \frac{3}{4}$
2) 5
3) 15
4) $24 \frac{3}{4}$

8 The vertices of $\triangle P Q R$ have coordinates $P(2,3)$, $Q(3,8)$, and $R(7,3)$. Under which transformation of $\triangle P Q R$ are distance and angle measure preserved?

1) $(x, y) \rightarrow(2 x, 3 y)$
2) $(x, y) \rightarrow(x+2,3 y)$
3) $(x, y) \rightarrow(2 x, y+3)$
4) $(x, y) \rightarrow(x+2, y+3)$

9 In $\triangle A B C$ shown below, side $\overline{A C}$ is extended to point $D$ with $\mathrm{m} \angle D A B=(180-3 x)^{\circ}$,
$\mathrm{m} \angle B=(6 x-40)^{\circ}$, and $\mathrm{m} \angle C=(x+20)^{\circ}$.


What is $\mathrm{m} \angle B A C$ ?

1) $20^{\circ}$
2) $40^{\circ}$
3) $60^{\circ}$
4) $80^{\circ}$

10 Circle $O$ is centered at the origin. In the diagram below, a quarter of circle $O$ is graphed.


Which three-dimensional figure is generated when the quarter circle is continuously rotated about the $y$-axis?

1) cone
2) sphere
3) cylinder
4) hemisphere

11 Rectangle $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$ is the image of rectangle $A B C D$ after a dilation centered at point $A$ by a scale factor of $\frac{2}{3}$. Which statement is correct?

1) Rectangle $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$ has a perimeter that is $\frac{2}{3}$ the perimeter of rectangle $A B C D$.
2) Rectangle $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$ has a perimeter that is $\frac{3}{2}$ the perimeter of rectangle $A B C D$.
3) Rectangle $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$ has an area that is $\frac{2}{3}$ the area of rectangle $A B C D$.
4) Rectangle $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$ has an area that is $\frac{3}{2}$ the area of rectangle $A B C D$.

12 The equation of a circle is $x^{2}+y^{2}-6 x+2 y=6$. What are the coordinates of the center and the length of the radius of the circle?

1) center $(-3,1)$ and radius 4
2) center $(3,-1)$ and radius 4
3) center $(-3,1)$ and radius 16
4) center $(3,-1)$ and radius 16

13 In the diagram of $\triangle A B C$ below, $\overline{D E}$ is parallel to $\overline{A B}, C D=15, A D=9$, and $A B=40$.


The length of $\overline{D E}$ is

1) 15
2) 24
3) 25
4) 30

14 The line whose equation is $3 x-5 y=4$ is dilated by a scale factor of $\frac{5}{3}$ centered at the origin. Which statement is correct?

1) The image of the line has the same slope as the pre-image but a different $y$-intercept.
2) The image of the line has the same $y$-intercept as the pre-image but a different slope.
3) The image of the line has the same slope and the same $y$-intercept as the pre-image.
4) The image of the line has a different slope and a different $y$-intercept from the pre-image.

15 Which transformation would not carry a square onto itself?

1) a reflection over one of its diagonals
2) a $90^{\circ}$ rotation clockwise about its center
3) a $180^{\circ}$ rotation about one of its vertices
4) a reflection over the perpendicular bisector of one side

16 In circle $M$ below, diameter $\overline{A C}$, chords $\overline{A B}$ and $\overline{B C}$, and radius $\overline{M B}$ are drawn.


Which statement is not true?

1) $\triangle A B C$ is a right triangle.
2) $\triangle A B M$ is isosceles.
3) $\mathrm{m} \overparen{B C}=\mathrm{m} \angle B M C$
4) $\mathrm{m} \overparen{A B}=\frac{1}{2} \mathrm{~m} \angle A C B$

17 In the diagram below, $\overline{X S}$ and $\overline{Y R}$ intersect at $Z$. Segments $X Y$ and $R S$ are drawn perpendicular to $\overline{Y R}$ to form triangles $X Y Z$ and $S R Z$.


Which statement is always true?

1) $(X Y)(S R)=(X Z)(R Z)$
2) $\triangle X Y Z \cong \triangle S R Z$
3) $\overline{X S} \cong \overline{Y R}$
4) $\frac{X Y}{S R}=\frac{Y Z}{R Z}$

18 As shown in the diagram below, $\overleftrightarrow{A B C} \| \overleftrightarrow{E F G}$ and $\overline{B F} \cong \overline{E F}$.


If $\mathrm{m} \angle C B F=42.5^{\circ}$, then $\mathrm{m} \angle E B F$ is

1) $42.5^{\circ}$
2) $68.75^{\circ}$
3) $95^{\circ}$
4) $137.5^{\circ}$

19 A parallelogram must be a rhombus if its diagonals

1) are congruent
2) bisect each other
3) do not bisect its angles
4) are perpendicular to each other

20 What is an equation of a line which passes through $(6,9)$ and is perpendicular to the line whose equation is $4 x-6 y=15$ ?

1) $y-9=-\frac{3}{2}(x-6)$
2) $y-9=\frac{2}{3}(x-6)$
3) $y+9=-\frac{3}{2}(x+6)$
4) $y+9=\frac{2}{3}(x+6)$

21 Quadrilateral $A B C D$ is inscribed in circle $O$, as shown below.


If $\mathrm{m} \angle A=80^{\circ}, \mathrm{m} \angle B=75^{\circ}, \mathrm{m} \angle C=(y+30)^{\circ}$, and $\mathrm{m} \angle D=(x-10)^{\circ}$, which statement is true?

1) $x=85$ and $y=50$
2) $x=90$ and $y=45$
3) $x=110$ and $y=75$
4) $x=115$ and $y=70$

22 A regular pyramid has a square base. The perimeter of the base is 36 inches and the height of the pyramid is 15 inches. What is the volume of the pyramid in cubic inches?

1) 180
2) 405
3) 540
4) 1215

23 In the diagram below of $\triangle A B C, \angle A B C$ is a right angle, $A C=12, A D=8$, and altitude $\overline{B D}$ is drawn.


What is the length of $\overline{B C}$ ?

1) $4 \sqrt{2}$
2) $4 \sqrt{3}$
3) $4 \sqrt{5}$
4) $4 \sqrt{6}$

24 In the diagram below, two concentric circles with center $O$, and radii $\overline{O C}, \overline{O D}, \overline{O G E}$, and $\overline{O D F}$ are drawn.


If $O C=4$ and $O E=6$, which relationship between the length of arc $E F$ and the length of arc $C D$ is always true?

1) The length of arc $E F$ is 2 units longer than the length of arc $C D$.
2) The length of arc $E F$ is 4 units longer than the length of arc $C D$.
3) The length of arc $E F$ is 1.5 times the length of $\operatorname{arc} C D$.
4) The length of arc $E F$ is 2.0 times the length of $\operatorname{arc} C D$.

25 Given: Parallelogram $A B C D$ with diagonal $\overline{A C}$ drawn


Prove: $\triangle A B C \cong \triangle C D A$

26 The diagram below shows circle $O$ with diameter $\overline{A B}$. Using a compass and straightedge, construct a square that is inscribed in circle $O$. [Leave all construction marks.]


27 Given: Right triangle $A B C$ with right angle at $C$. If $\sin A$ increases, does $\cos B$ increase or decrease?
Explain why.

28 In the diagram below, the circle has a radius of 25 inches. The area of the unshaded sector is $500 \pi$ in $^{2}$.


Determine and state the degree measure of angle $Q$, the central angle of the shaded sector.

29 A machinist creates a solid steel part for a wind turbine engine. The part has a volume of 1015 cubic centimeters. Steel can be purchased for $\$ 0.29$ per kilogram, and has a density of 7.95 $\mathrm{g} / \mathrm{cm}^{3}$. If the machinist makes 500 of these parts, what is the cost of the steel, to the nearest dollar?

30 In the graph below, $\triangle A B C$ has coordinates $A(-9,2), B(-6,-6)$, and $C(-3,-2)$, and $\triangle R S T$ has coordinates $R(-2,9), S(5,6)$, and $T(2,3)$.


Is $\triangle A B C$ congruent to $\triangle R S T$ ? Use the properties of rigid motions to explain your reasoning.

31 Bob places an 18 -foot ladder 6 feet from the base of his house and leans it up against the side of his house. Find, to the nearest degree, the measure of the angle the bottom of the ladder makes with the ground.

32 Triangle $A B C$ and triangle $A D E$ are graphed on the set of axes below.


Describe a transformation that maps triangle $A B C$ onto triangle $A D E$. Explain why this transformation makes triangle $A D E$ similar to triangle $A B C$.

33 A storage tank is in the shape of a cylinder with a hemisphere on the top. The highest point on the inside of the storage tank is 13 meters above the floor of the storage tank, and the diameter inside the cylinder is 8 meters. Determine and state, to the nearest cubic meter, the total volume inside the storage tank.


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34 As shown in the diagram below, an island $(I)$ is due north of a marina $(M)$. A boat house $(H)$ is 4.5 miles due west of the marina. From the boat house, the island is located at an angle of $54^{\circ}$ from the marina.


Determine and state, to the nearest tenth of a mile, the distance from the boat house $(H)$ to the island (I). Determine and state, to the nearest tenth of a mile, the distance from the island $(I)$ to the marina (M).

35 In the coordinate plane, the vertices of triangle $P A T$ are $P(-1,-6), A(-4,5)$, and $T(5,-2)$. Prove that $\triangle P A T$ is an isosceles triangle. [The use of the set of axes below is optional.] State the coordinates of $R$ so that quadrilateral $P A R T$ is a parallelogram.
Prove that quadrilateral $P A R T$ is a parallelogram.


## 0118geo

Answer Section
1 ANS: 1
$360-(82+104+121)=53$
PTS: 2 REF: 011801geo NAT: G.CO.B. 6 TOP: Properties of Transformations
KEY: basic
2 ANS: 2
PTS: 2
REF: 011802geo NAT: G.CO.C. 11
TOP: Parallelograms
3 ANS: 4 PTS: 2
TOP: Identifying Transformations
REF: 011803geo NAT: G.CO.A. 2
KEY: graphics
4 ANS: 1
$\sin 32=\frac{O}{129.5}$
$O \approx 68.6$
PTS: 2 REF: 011804geo NAT: G.SRT.C. 8 TOP: Using Trigonometry to Find a Side 5 ANS: 2 PTS: 2 REF: 011805geo NAT: G.GMD.B. 4

TOP: Cross-Sections of Three-Dimensional Objects
6 ANS: 1
$x=-5+\frac{1}{3}(4--5)=-5+3=-2 \quad y=2+\frac{1}{3}(-10-2)=2-4=-2$
PTS: 2 REF: 011806geo NAT: G.GPE.B. 6 TOP: Directed Line Segments
7 ANS: 3

$$
V=\frac{1}{3} \pi r^{2} h
$$

$54.45 \pi=\frac{1}{3} \pi(3.3)^{2} h$
$h=15$
PTS: 2 REF: 011807geo NAT: G.GMD.A. 3 TOP: Volume
KEY: cones
8 ANS: 4
PTS: 2
REF: 011808geo
NAT: G.CO.A. 2
TOP: Analytical Representations of Transformations
KEY: basic
9 ANS: 3

$$
\begin{aligned}
6 x-40+x+20 & =180-3 x \mathrm{~m} \angle B A C=180-(80+40)=60 \\
10 x & =200 \\
x & =20
\end{aligned}
$$

PTS: 2 REF: 011809geo NAT: G.CO.C. 10 TOP: Exterior Angle Theorem
10 ANS: 4 PTS: 2 REF: 011810geo NAT: G.GMD.B. 4
TOP: Rotations of Two-Dimensional Objects

11 ANS: 1
PTS: 2
REF: 011811geo NAT: G.SRT.A. 2
TOP: Dilations
12 ANS: 2
$x^{2}+y^{2}-6 x+2 y=6$
$x^{2}-6 x+9+y^{2}+2 y+1=6+9+1$
$(x-3)^{2}+(y+1)^{2}=16$

PTS: 2 REF: 011812geo NAT: G.GPE.A. 1 TOP: Equations of Circles
KEY: completing the square
13 ANS: 3
$\frac{24}{40}=\frac{15}{x}$
$24 x=600$

$$
x=25
$$

PTS: 2 REF: 011813geo NAT: G.SRT.B. 5 TOP: Side Splitter Theorem
14 ANS: 1
PTS: 2
REF: 011814geo NAT: G.SRT.A. 1
TOP: Line Dilations
15 ANS: 3 PTS: 2
TOP: Mapping a Polygon onto Itself
16 ANS: 4 PTS: 2
TOP: Chords, Secants and Tangents
17 ANS: 4 PTS: 2 TOP: Similarity KEY: basic
18 ANS: 2


PTS: 2 REF: 011818geo NAT: G.CO.C. 9 TOP: Lines and Angles
19 ANS: 4 PTS: 2 REF: 011819geo NAT: G.CO.C. 11
TOP: Special Quadrilaterals

20 ANS: 1
$m=\frac{-4}{-6}=\frac{2}{3}$
$m_{\perp}=-\frac{3}{2}$
PTS: 2 REF: 011820geo NAT: G.GPE.B. 5 TOP: Parallel and Perpendicular Lines
KEY: write equation of perpendicular line
21 ANS: 4
Opposite angles of an inscribed quadrilateral are supplementary.
PTS: 2 REF: 011821geo NAT: G.C.A. 3 TOP: Inscribed Quadrilaterals
22 ANS: 2
$V=\frac{1}{3}\left(\frac{36}{4}\right)^{2} \cdot 15=405$
PTS: 2 REF: 011822geo NAT: G.GMD.A. 3 TOP: Volume
KEY: pyramids
23 ANS: 2
$x^{2}=12(12-8)$
$x^{2}=48$
$x=4 \sqrt{3}$
PTS: 2 REF: 011823geo NAT: G.SRT.B. 5 TOP: Similarity
KEY: leg
24 ANS: 3
$\frac{s_{L}}{s_{S}}=\frac{6 \theta}{4 \theta}=1.5$
PTS: 2 REF: 011824geo NAT: G.C.B. 5 TOP: Arc Length
KEY: arc length
25 ANS:
Parallelogram $A B C D$ with diagonal $\overline{A C}$ drawn (given). $\overline{A C} \cong \overline{A C}$ (reflexive property). $\overline{A D} \cong \overline{C B}$ and $\overline{B A} \cong \overline{D C}$ (opposite sides of a parallelogram are congruent). $\triangle A B C \cong \triangle C D A$ (SSS).

PTS: 2
REF: 011825geo NAT: G.SRT.B. 5 TOP: Quadrilateral Proofs

26 ANS:


PTS: 2 REF: 011826geo NAT: G.CO.D. 13 TOP: Constructions
27 ANS:
$\cos B$ increases because $\angle A$ and $\angle B$ are complementary and $\sin A=\cos B$.
PTS: 2 REF: 011827geo NAT: G.SRT.C. 7 TOP: Cofunctions
28 ANS:
$\frac{Q}{360}(\pi)\left(25^{2}\right)=(\pi)\left(25^{2}\right)-500 \pi$
$Q=\frac{125 \pi(360)}{625 \pi}$
$Q=72$
PTS: 2 REF: 011828geo NAT: G.C.B. 5 TOP: Sectors
29 ANS:
$500 \times 1015 \mathrm{cc} \times \frac{\$ 0.29}{\mathrm{~kg}} \times \frac{7.95 \mathrm{~g}}{\mathrm{cc}} \times \frac{1 \mathrm{~kg}}{1000 \mathrm{~g}}=\$ 1170$
PTS: 2 REF: 011829geo NAT: G.MG.A. 2 TOP: Density
30 ANS:
No. Since $\overline{B C}=5$ and $\overline{S T}=\sqrt{18}$ are not congruent, the two triangles are not congruent. Since rigid motions preserve distance, there is no rigid motion that maps $\triangle A B C$ onto $\triangle R S T$.

PTS: 2 REF: 011830geo NAT: G.CO.B. 7 TOP: Triangle Congruency
31 ANS:
$\cos W=\frac{6}{18}$

$$
W \approx 71
$$

PTS: 2
REF: 011831geo NAT: G.SRT.C. 8 TOP: Using Trigonometry to Find an Angle

32 ANS:
A dilation of 3 centered at $A$. A dilation preserves angle measure, so the triangles are similar.
PTS: 4 REF: 011832geo NAT: G.SRT.A. 2 TOP: Dilations
33 ANS:
$V=(\pi)\left(4^{2}\right)(9)+\left(\frac{1}{2}\right)\left(\frac{4}{3}\right)(\pi)\left(4^{3}\right) \approx 586$
PTS: 4 REF: 011833geo NAT: G.GMD.A. 3 TOP: Volume
KEY: compositions
34 ANS:
$\cos 54=\frac{4.5}{m} \tan 54=\frac{h}{4.5}$

$$
m \approx 7.7 \quad h \approx 6.2
$$

PTS: 4 REF: 011834geo NAT: G.SRT.C. 8 TOP: Using Trigonometry to Find a Side
35 ANS:
$\triangle P A T$ is an isosceles triangle because sides $\overline{A P}$ and $\overline{A T}$ are congruent $\left(\sqrt{3^{2}+11^{2}}=\sqrt{7^{2}+9^{2}}=\sqrt{130}\right)$.
$R(2,9)$. Quadrilateral PART is a parallelogram because the opposite sides are parallel since they have equal slopes

$$
\left(m_{\overline{A R}}=\frac{4}{6}=\frac{2}{3} ; m_{\overline{P T}}=\frac{4}{6}=\frac{2}{3} ; m_{\overline{P A}}=-\frac{11}{3} ; m_{\overline{R T}}=-\frac{11}{3}\right)
$$



PTS: 6
REF: 011835 geo NAT: G.GPE.B. 4 TOP: Quadrilaterals in the Coordinate Plane KEY: grids

