1 In the diagram below, lines \( \ell, m, n, \) and \( p \) intersect line \( r. \)

Which statement is true?
1) \( \ell \parallel n \)
2) \( \ell \parallel p \)
3) \( m \parallel p \)
4) \( m \parallel n \)

2 Which transformation would not always produce an image that would be congruent to the original figure?
1) translation
2) dilation
3) rotation
4) reflection

3 If an equilateral triangle is continuously rotated around one of its medians, which 3-dimensional object is generated?
1) cone
2) pyramid
3) prism
4) sphere

4 In the diagram below, \( \angle BDC = 100^\circ, \)
\( \angle A = 50^\circ, \) and \( \angle DBC = 30^\circ. \)

Which statement is true?
1) \( \triangle ABD \) is obtuse.
2) \( \triangle ABC \) is isosceles.
3) \( \angle ABD = 80^\circ \)
4) \( \triangle ABD \) is scalene.

5 Which point shown in the graph below is the image of point \( P \) after a counterclockwise rotation of 90° about the origin?

1) \( A \)
2) \( B \)
3) \( C \)
4) \( D \)
6. In \( \triangle ABC \), where \( \angle C \) is a right angle, \( \cos A = \frac{21}{\sqrt{5}} \). What is \( \sin B \)?

1) \( \frac{\sqrt{21}}{5} \)
2) \( \frac{\sqrt{21}}{2} \)
3) \( \frac{2}{5} \)
4) \( \frac{5}{\sqrt{21}} \)

7. Quadrilateral \( ABCD \) with diagonals \( \overline{AC} \) and \( \overline{BD} \) is shown in the diagram below.

[Diagram of a quadrilateral]

Which information is not enough to prove \( ABCD \) is a parallelogram?

1) \( AB \cong CD \) and \( AB \parallel DC \)
2) \( AB \cong CD \) and \( BC \cong DA \)
3) \( AB \cong CD \) and \( BC \parallel AD \)
4) \( AB \parallel DC \) and \( BC \parallel AD \)

8. An equilateral triangle has sides of length 20. To the nearest tenth, what is the height of the equilateral triangle?

1) 10.0
2) 11.5
3) 17.3
4) 23.1

9. Given: \( \triangle AEC, \triangle DEF, \) and \( \overline{FE} \perp \overline{CE} \)

[Diagram of a triangle]

What is a correct sequence of similarity transformations that shows \( \triangle AEC \sim \triangle DEF \)?

1) a rotation of 180 degrees about point \( E \) followed by a horizontal translation
2) a counterclockwise rotation of 90 degrees about point \( E \) followed by a horizontal translation
3) a rotation of 180 degrees about point \( E \) followed by a dilation with a scale factor of 2 centered at point \( E \)
4) a counterclockwise rotation of 90 degrees about point \( E \) followed by a dilation with a scale factor of 2 centered at point \( E \)

10. In the diagram of right triangle \( ABC, \overline{CD} \) intersects hypotenuse \( \overline{AB} \) at \( D \).

[Diagram of a right triangle]

If \( AD = 4 \) and \( DB = 6 \), which length of \( \overline{AC} \) makes \( \overline{CD} \perp \overline{AB} \)?

1) \( 2\sqrt{6} \)
2) \( 2\sqrt{10} \)
3) \( 2\sqrt{15} \)
4) \( 4\sqrt{2} \)
11 Segment $CD$ is the perpendicular bisector of $AB$ at $E$. Which pair of segments does not have to be congruent?
1) $AD, BD$
2) $AC, BC$
3) $AE, BE$
4) $DE, CE$

12 In triangle $CHR$, $O$ is on $HR$, and $D$ is on $CR$ so that $\angle H \cong \angle RDO$.

![Diagram of triangle CHR with points H, O, R, and D]

If $RD = 4$, $RO = 6$, and $OH = 4$, what is the length of $CD$?
1) $2 \frac{2}{3}$
2) $6 \frac{2}{3}$
3) 11
4) 15

13 The cross section of a regular pyramid contains the altitude of the pyramid. The shape of this cross section is a
1) circle
2) square
3) triangle
4) rectangle

14 The diagonals of rhombus $TEAM$ intersect at $P(2,1)$. If the equation of the line that contains diagonal $TA$ is $y = -x + 3$, what is the equation of a line that contains diagonal $EM$?
1) $y = x - 1$
2) $y = x - 3$
3) $y = -x - 1$
4) $y = -x - 3$

15 The coordinates of vertices $A$ and $B$ of $\triangle ABC$ are $A(3,4)$ and $B(3,12)$. If the area of $\triangle ABC$ is 24 square units, what could be the coordinates of point $C$?
1) $(3,6)$
2) $(8,-3)$
3) $(-3,8)$
4) $(6,3)$

16 What are the coordinates of the center and the length of the radius of the circle represented by the equation $x^2 + y^2 - 4x + 8y + 11 = 0$?
1) center $(2,-4)$ and radius 3
2) center $(-2,4)$ and radius 3
3) center $(2,-4)$ and radius 9
4) center $(-2,4)$ and radius 9

17 The density of the American white oak tree is 752 kilograms per cubic meter. If the trunk of an American white oak tree has a circumference of 4.5 meters and the height of the trunk is 8 meters, what is the approximate number of kilograms of the trunk?
1) 13
2) 9694
3) 13,536
4) 30,456
18. Point $P$ is on the directed line segment from point $X(-6, -2)$ to point $Y(6, 7)$ and divides the segment in the ratio 1:5. What are the coordinates of point $P$?

1) $(4, 5\frac{1}{2})$
2) $\left(-\frac{1}{2}, -4\right)$
3) $\left(-4\frac{1}{2}, 0\right)$
4) $\left(-4, -\frac{1}{2}\right)$

19. In circle $O$, diameter $AB$, chord $BC$, and radius $OC$ are drawn, and the measure of arc $BC$ is $108^\circ$.

Some students wrote these formulas to find the area of sector $COB$:

Amy: $\frac{3}{10} \cdot \pi \cdot (BC)^2$
Beth: $\frac{108}{360} \cdot \pi \cdot (OC)^2$
Carl: $\frac{3}{10} \cdot \pi \cdot \left(\frac{1}{2} AB\right)^2$
Dex: $\frac{108}{360} \cdot \pi \cdot \left(\frac{1}{2} AB\right)^2$

Which students wrote correct formulas?
1) Amy and Dex
2) Beth and Carl
3) Carl and Amy
4) Dex and Beth

20. Tennis balls are sold in cylindrical cans with the balls stacked one on top of the other. A tennis ball has a diameter of $6.7$ cm. To the nearest cubic centimeter, what is the minimum volume of the can that holds a stack of 4 tennis balls?

1) 236
2) 282
3) 564
4) 945

21. Line segment $A'B'$, whose endpoints are $(4, -2)$ and $(16, 14)$, is the image of $AB$ after a dilation of $\frac{1}{2}$ centered at the origin. What is the length of $AB$?

1) 5
2) 10
3) 20
4) 40

22. Given: $\triangle ABE$ and $\triangle CBD$ shown in the diagram below with $DB \cong BE$

Which statement is needed to prove $\triangle ABE \cong \triangle CBD$ using only SAS $\cong$ SAS?

1) $\angle CDB \cong \angle AEB$
2) $\angle AFD \cong \angle EFC$
3) $AD \cong CE$
4) $AE \cong CD$
23 In the diagram below, $\overline{BC}$ is the diameter of circle $A$.

Point $D$, which is unique from points $B$ and $C$, is plotted on circle $A$. Which statement must always be true?
1) $\triangle BCD$ is a right triangle.
2) $\triangle BCD$ is an isosceles triangle.
3) $\triangle BAD$ and $\triangle CBD$ are similar triangles.
4) $\triangle BAD$ and $\triangle CAD$ are congruent triangles.

24 In the diagram below, $ABCD$ is a parallelogram, $\overline{AB}$ is extended through $B$ to $E$, and $\overline{CE}$ is drawn.

If $\overline{CE} \cong \overline{BE}$ and $m\angle D = 112^\circ$, what is $m\angle E$?
1) $44^\circ$
2) $56^\circ$
3) $68^\circ$
4) $112^\circ$

25 Lines $AE$ and $BD$ are tangent to circles $O$ and $P$ at $A$, $E$, $B$, and $D$, as shown in the diagram below. If $AC:CE = 5:3$, and $BD = 56$, determine and state the length of $CD$.

26 In the diagram below, $\triangle ABC$ has coordinates $A(1,1)$, $B(4,1)$, and $C(4,5)$. Graph and label $\triangle A'B'C'$, the image of $\triangle ABC$ after the translation five units to the right and two units up followed by the reflection over the line $y = 0$.

27 A regular hexagon is rotated in a counterclockwise direction about its center. Determine and state the minimum number of degrees in the rotation such that the hexagon will coincide with itself.
28 In the diagram of $\triangle ABC$ shown below, use a compass and straightedge to construct the median to $AB$. [Leave all construction marks.]

![Diagram of triangle ABC with median drawn]

29 Triangle $MNP$ is the image of triangle $JKL$ after a $120^\circ$ counterclockwise rotation about point $Q$. If the measure of angle $L$ is $47^\circ$ and the measure of angle $N$ is $57^\circ$, determine the measure of angle $M$. Explain how you arrived at your answer.

![Diagram of triangle JKL with rotation]

30 A circle has a center at $(1, -2)$ and radius of 4. Does the point $(3.4, 1.2)$ lie on the circle? Justify your answer.

31 In the diagram below, a window of a house is 15 feet above the ground. A ladder is placed against the house with its base at an angle of $75^\circ$ with the ground. Determine and state the length of the ladder to the nearest tenth of a foot.

![Diagram of ladder against a house with angle and height indicated]

32 Using a compass and straightedge, construct and label $\triangle A'B'C'$, the image of $\triangle ABC$ after a dilation with a scale factor of 2 and centered at $B$. [Leave all construction marks.] Describe the relationship between the lengths of $AC$ and $A'C'$.

![Diagram of dilation of triangle ABC]

31 In the diagram below, a window of a house is 15 feet above the ground. A ladder is placed against the house with its base at an angle of $75^\circ$ with the ground. Determine and state the length of the ladder to the nearest tenth of a foot.

![Diagram of ladder against a house with angle and height indicated]

32 Using a compass and straightedge, construct and label $\triangle A'B'C'$, the image of $\triangle ABC$ after a dilation with a scale factor of 2 and centered at $B$. [Leave all construction marks.] Describe the relationship between the lengths of $AC$ and $A'C'$.

![Diagram of dilation of triangle ABC]
33 The grid below shows \( \triangle ABC \) and \( \triangle DEF \).

Let \( \triangle A'B'C' \) be the image of \( \triangle ABC \) after a rotation about point \( A \). Determine and state the location of \( B' \) if the location of point \( C' \) is \((8,-3)\). Explain your answer. Is \( \triangle DEF \) congruent to \( \triangle A'B'C' \)? Explain your answer.

34 As modeled below, a movie is projected onto a large outdoor screen. The bottom of the 60-foot-tall screen is 12 feet off the ground. The projector sits on the ground at a horizontal distance of 75 feet from the screen.

Determine and state, to the nearest tenth of a degree, the measure of \( \theta \), the projection angle.

35 Given: Circle \( O \), chords \( \overline{AB} \) and \( \overline{CD} \) intersect at \( E \)

Theorem: If two chords intersect in a circle, the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord. Prove this theorem by proving \( AE \cdot EB = CE \cdot ED \).

36 A snow cone consists of a paper cone completely filled with shaved ice and topped with a hemisphere of shaved ice, as shown in the diagram below. The inside diameter of both the cone and the hemisphere is 8.3 centimeters. The height of the cone is 10.2 centimeters.

The desired density of the shaved ice is 0.697 g/cm\(^3\), and the cost, per kilogram, of ice is $3.83. Determine and state the cost of the ice needed to make 50 snow cones.
0816geo

Answer Section

1 ANS: 2
   PTS: 2
   REF: 081601geo
   NAT: G.CO.C.9
   TOP: Lines and Angles

2 ANS: 2
   PTS: 2
   REF: 081602geo
   NAT: G.CO.A.2
   TOP: Identifying Transformations
       KEY: basic

3 ANS: 1
   PTS: 2
   REF: 081603geo
   NAT: G.GMD.B.4
   TOP: Rotations of Two-Dimensional Objects

4 ANS: 2
   PTS: 2
   REF: 081604geo
   NAT: G.CO.C.10
   TOP: Interior and Exterior Angles of Triangles

5 ANS: 1
   PTS: 2
   REF: 081605geo
   NAT: G.CO.A.5
   TOP: Rotations
       KEY: grids

6 ANS: 1
   PTS: 2
   REF: 081606geo
   NAT: G.SRT.C.7
   TOP: Cofunctions

7 ANS: 3
   (3) Could be a trapezoid.
   PTS: 2
   REF: 081607geo
   NAT: G.CO.C.11
   TOP: Parallelograms

8 ANS: 3
   \[ \sqrt{20^2 - 10^2} \approx 17.3 \]
   PTS: 2
   REF: 081608geo
   NAT: G.SRT.C.8
   TOP: Pythagorean Theorem
   KEY: without graphics

9 ANS: 4
   PTS: 2
   REF: 081609geo
   NAT: G.SRT.A.2
   TOP: Compositions of Transformations
       KEY: grids

10 ANS: 2
    \[ x^2 = 4 \cdot 10 \]
    \[ x = \sqrt{40} \]
    \[ x = 2\sqrt{10} \]
    PTS: 2
    REF: 081610geo
    NAT: G.SRT.B.5
    TOP: Similarity
    KEY: leg

11 ANS: 4
   PTS: 2
   REF: 081611geo
   NAT: G.CO.C.9
   TOP: Lines and Angles
12 ANS: 3
\[
x = \frac{6}{4} \quad \text{CD} = 15 - 4 = 11
\]
\[
x = 15
\]

PTS: 2  REF: 081612geo  NAT: G.SRT.B.5  TOP: Similarity
KEY: basic

13 ANS: 3  PTS: 2  REF: 081613geo  NAT: G.GMD.B.4
TOP: Cross-Sections of Three-Dimensional Objects

14 ANS: 1
\[
m_{TA} = -1 \quad y = mx + b
\]
\[
m_{EM} = 1 \quad 1 = 1(2) + b
\]
\[
-1 = b
\]

PTS: 2  REF: 081614geo  NAT: G.GPE.B.4  TOP: Quadrilaterals in the Coordinate Plane
KEY: general

15 ANS: 3
\[
A = \frac{1}{2} ab \quad 3 - 6 = -3 = x
\]
\[
24 = \frac{1}{2} a(8) \quad \frac{4+12}{2} = 8 = y
\]
\[
a = 6
\]

PTS: 2  REF: 081615geo  NAT: G.GPE.B.7  TOP: Polygons in the Coordinate Plane

16 ANS: 1
\[
x^2 - 4x + 4 + y^2 + 8y + 16 = -11 + 4 + 16
\]
\[
(x - 2)^2 + (y + 4)^2 = 9
\]

PTS: 2  REF: 081616geo  NAT: G.GPE.A.1  TOP: Equations of Circles
KEY: completing the square

17 ANS: 2
\[
C = \pi d \quad V = \pi \left( \frac{2.25}{\pi} \right)^2 \cdot 8 \approx 12.8916 \quad W = 12.8916 \cdot 752 \approx 9694
\]
\[
4.5 = \pi d
\]
\[
\frac{4.5}{\pi} = d
\]
\[
\frac{2.25}{\pi} = r
\]

PTS: 2  REF: 081617geo  NAT: G.MG.A.2  TOP: Density
18 ANS: 4

\[ x = -6 + \frac{1}{6} (6 - 6) = -6 + 2 = -4 \quad y = -2 + \frac{1}{6} (7 - 2) = -2 + \frac{9}{6} = -\frac{1}{2} \]

PTS: 2  REF: 081618geo  NAT: G.GPE.B.6  TOP: Directed Line Segments

19 ANS: 2  PTS: 2  REF: 081619geo  NAT: G.C.B.5

TOP: Sectors

20 ANS: 4

\[ V = \pi \left( \frac{6.7}{2} \right)^2 \cdot (4 \cdot 6.7) \approx 945 \]

PTS: 2  REF: 081620geo  NAT: G.GMD.A.3  TOP: Volume

KEY: cylinders

21 ANS: 4

\[ \sqrt{(32 - 8)^2 + (28 - 4)^2} = \sqrt{576 + 1024} = \sqrt{1600} = 40 \]

PTS: 2  REF: 081621geo  NAT: G.SRT.A.1  TOP: Line Dilations

22 ANS: 3  PTS: 2  REF: 081622geo  NAT: G.SRT.B.5

TOP: Triangle Proofs  KEY: statements

23 ANS: 1

The other statements are true only if \( AD \perp BC \).

PTS: 2  REF: 081623geo  NAT: G.C.A.2  TOP: Chords, Secants and Tangents

KEY: inscribed

24 ANS: 1

\[ 180 - (68 \cdot 2) \]

PTS: 2  REF: 081624geo  NAT: G.CO.C.11  TOP: Interior and Exterior Angles of Polygons

25 ANS:

\[ \frac{3}{8} \cdot 56 = 21 \]

PTS: 2  REF: 081625geo  NAT: G.C.A.2  TOP: Chords, Secants and Tangents

KEY: common tangents
26 ANS: 

\[
\begin{array}{c}
\text{\includegraphics[width=0.5\textwidth]{diagram.png}}
\end{array}
\]

PTS: 2  REF: 081626geo  NAT: G.CO.A.5  TOP: Compositions of Transformations

KEY: grids

27 ANS: 

\[
\frac{360}{6} = 60
\]

PTS: 2  REF: 081627geo  NAT: G.CO.A.3  TOP: Mapping a Polygon onto Itself

28 ANS: 

\[
\begin{array}{c}
\text{\includegraphics[width=0.5\textwidth]{diagram.png}}
\end{array}
\]

PTS: 2  REF: 081628geo  NAT: G.CO.D.12  TOP: Constructions

KEY: line bisector

29 ANS: 

\[M = 180 - (47 + 57) = 76\] Rotations do not change angle measurements.

PTS: 2  REF: 081629geo  NAT: G.CO.B.6  TOP: Properties of Transformations

30 ANS: 

\[Yes.\quad (x - 1)^2 + (y + 2)^2 = 4^2\]

\[=(3.4 - 1)^2 + (1.2 + 2)^2 = 16\]

\[5.76 + 10.24 = 16\]

\[16 = 16\]

PTS: 2  REF: 081630geo  NAT: G.GPE.B.4  TOP: Circles in the Coordinate Plane
31 ANS:
\[ \sin 75 = \frac{15}{x} \]
\[ x = \frac{15}{\sin 75} \]
\[ x \approx 15.5 \]

PTS: 2 REF: 081631geo NAT: G.SRT.C.8 TOP: Using Trigonometry to Find a Side
KEY: graphics

32 ANS:

The length of \( \overline{A'C'} \) is twice \( \overline{AC} \).

PTS: 4 REF: 081632geo NAT: G.CO.D.12 TOP: Constructions
KEY: congruent and similar figures

33 ANS:

\( ABC \) – point of reflection \( \rightarrow (-y, x) \) + point of reflection \( \Delta DEF \cong \Delta A'B'C' \) because \( \Delta DEF \) is a reflection of

\( A(2, -3) - (2, -3) = (0, 0) \rightarrow (0, 0) + (2, -3) = A'(2, -3) \)
\( B(6, -8) - (2, -3) = (4, -5) \rightarrow (5, 4) + (2, -3) = B'(7, 1) \)
\( C(2, -9) - (2, -3) = (0, -6) \rightarrow (6, 0) + (2, -3) = C'(8, -3) \)
\( \Delta A'B'C' \) and reflections preserve distance.

PTS: 4 REF: 081633geo NAT: G.CO.A.5 TOP: Rotations
KEY: grids

34 ANS:

\[ \tan x = \frac{12}{75} \quad \tan y = \frac{72}{75} \quad 43.83 - 9.09 \approx 34.7 \]
\[ x \approx 9.09 \quad y \approx 43.83 \]

PTS: 4 REF: 081634geo NAT: G.SRT.C.8 TOP: Using Trigonometry to Find an Angle
35 ANS:
Circle \( O \), chords \( \overline{AB} \) and \( \overline{CD} \) intersect at \( E \) (Given); Chords \( \overline{CB} \) and \( \overline{AD} \) are drawn (auxiliary lines drawn); \( \angle CEB \cong \angle AED \) (vertical angles); \( \angle C \cong \angle A \) (Inscribed angles that intercept the same arc are congruent);
\( \triangle BCE \sim \triangle DAE \) (AA); \( \frac{AE}{CE} = \frac{ED}{EB} \) (Corresponding sides of similar triangles are proportional);
\( AE \cdot EB = CE \cdot ED \) (The product of the means equals the product of the extremes).

PTS: 6 REF: 081635geo NAT: G.SRT.B.5 TOP: Circle Proofs

36 ANS:
\[
V = \frac{1}{3} \pi \left( \frac{8.3}{2} \right)^2 (10.2) + \frac{1}{2} \cdot \frac{4}{3} \pi \left( \frac{8.3}{2} \right)^3 \approx 183.961 + 149.693 \approx 333.65 \text{ cm}^3 \quad 333.65 \times 50 = 16682.7 \text{ cm}^3
\]
\[
16682.7 \times 0.697 = 11627.8 \text{ g} \quad 11.6278 \times 3.83 = 44.53
\]

PTS: 6 REF: 081636geo NAT: G.MG.A.2 TOP: Density