

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION**GEOMETRY**

Thursday, August 17, 2017 — 12:30 to 3:30 p.m., only

Student Name: Mr. SibalSchool Name: JMAP

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 36 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III, and IV** directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...

A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 A two-dimensional cross section is taken of a three-dimensional object. If this cross section is a triangle, what can *not* be the three-dimensional object?

- (1) cone
(2) cylinder
(3) pyramid
(4) rectangular prism

2 The image of $\triangle DEF$ is $\triangle D'E'F'$. Under which transformation will the triangles *not* be congruent?

- (1) a reflection through the origin
(2) a reflection over the line $y = x$
(3) a dilation with a scale factor of 1 centered at (2,3)
(4) a dilation with a scale factor of $\frac{3}{2}$ centered at the origin

Dilations do not preserve distance

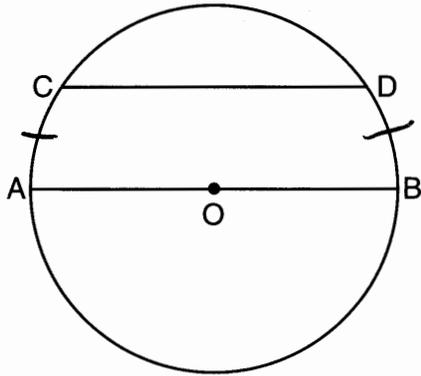
3 The vertices of square $RSTV$ have coordinates $R(-1,5)$, $S(-3,1)$, $T(-7,3)$, and $V(-5,7)$. What is the perimeter of $RSTV$?

- (1) $\sqrt{20}$
(2) $\sqrt{40}$
(3) $4\sqrt{20}$
(4) $4\sqrt{40}$

$$4\sqrt{(-1-(-3))^2 + (5-1)^2}$$
$$4\sqrt{20}$$

Use this space for computations.

- 4 In the diagram below of circle O , chord \overline{CD} is parallel to diameter \overline{AOB} and $m\widehat{CD} = 130$.

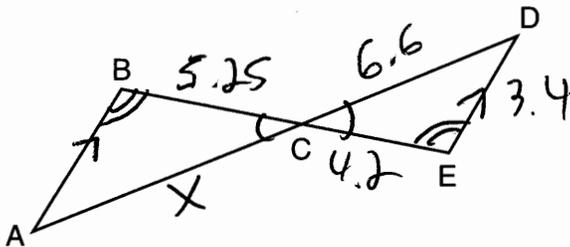


$$\frac{180 - 130}{2} = 25$$

What is $m\widehat{AC}$?

- (1) 25 (2) 50 (3) 65 (4) 115

- 5 In the diagram below, \overline{AD} intersects \overline{BE} at C , and $\overline{AB} \parallel \overline{DE}$.



$$\frac{6.6}{x} = \frac{4.2}{5.25}$$

$$4.2x = 34.65$$

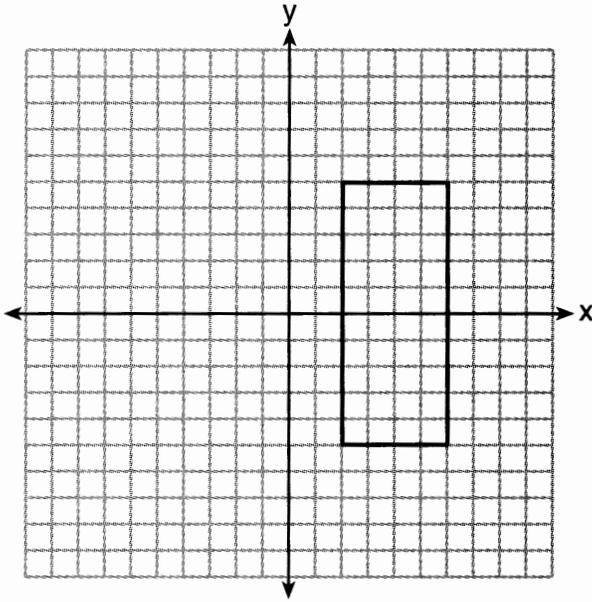
$$x = 8.25$$

If $CD = 6.6$ cm, $DE = 3.4$ cm, $CE = 4.2$ cm, and $BC = 5.25$ cm, what is the length of \overline{AC} , to the nearest hundredth of a centimeter?

- (1) 2.70 (2) 3.34 (3) 5.28 (4) 8.25

Use this space for computations.

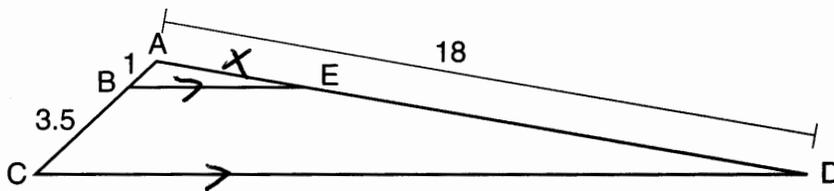
6 As shown in the graph below, the quadrilateral is a rectangle.



Which transformation would *not* map the rectangle onto itself?

- (1) a reflection over the x -axis
 - (2) a reflection over the line $x = 4$
 - (3) a rotation of 180° about the origin
 - (4) a rotation of 180° about the point $(4, 0)$
- } lines of symmetry
- point of symmetry

7 In the diagram below, triangle ACD has points B and E on sides \overline{AC} and \overline{AD} , respectively, such that $\overline{BE} \parallel \overline{CD}$, $AB = 1$, $BC = 3.5$, and $AD = 18$.



$$\frac{1}{3.5} = \frac{x}{18-x}$$

$$3.5x = 18 - x$$

$$4.5x = 18$$

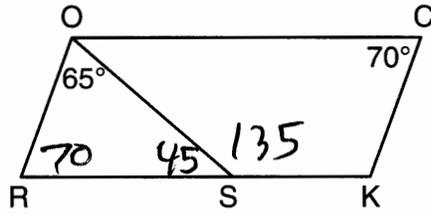
$$x = 4$$

What is the length of \overline{AE} , to the nearest tenth?

- (1) 14.0
- (2) 5.1
- (3) 3.3
- (4) 4.0

Use this space for computations.

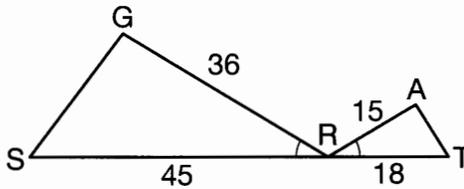
- 8 In the diagram below of parallelogram $ROCK$, $m\angle C$ is 70° and $m\angle ROS$ is 65° .



What is $m\angle KSO$?

- (1) 45° (3) 115°
 (2) 110° (4) 135°

- 9 In the diagram below, $\angle GRS \cong \angle ART$, $GR = 36$, $SR = 45$, $AR = 15$, and $RT = 18$.



$$\frac{36}{45} = \frac{15}{18}$$

$$\frac{4}{5} \neq \frac{5}{6}$$

Which triangle similarity statement is correct?

- (1) $\triangle GRS \sim \triangle ART$ by AA. (3) $\triangle GRS \sim \triangle ART$ by SSS.
 (2) $\triangle GRS \sim \triangle ART$ by SAS. (4) $\triangle GRS$ is not similar to $\triangle ART$.

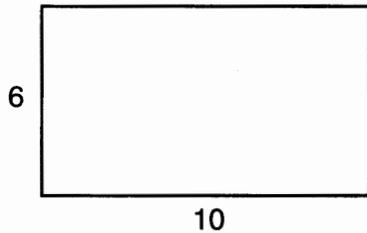
$$m = \frac{3}{4}$$

- 10 The line represented by the equation $4y = 3x + 7$ is transformed by a dilation centered at the origin. Which linear equation could represent its image?

- (1) $3x - 4y = 9$ $3/4$ (3) $4x - 3y = 9$ $4/3$
 (2) $3x + 4y = 9$ $-3/4$ (4) $4x + 3y = 9$ $-4/3$

Use this space for computations.

- 13 A rectangle whose length and width are 10 and 6, respectively, is shown below. The rectangle is continuously rotated around a straight line to form an object whose volume is 150π .



$$V = \pi r^2 h$$

$$150\pi = \pi r^2 h$$

$$(1) 6^2 \cdot 10 = 360$$

$$(2) 10^2 \cdot 6 = 600$$

$$(3) 5^2 \cdot 6 = 150$$

$$(4) 3^2 \cdot 10 = 90$$

Which line could the rectangle be rotated around?

- (1) a long side
(2) a short side

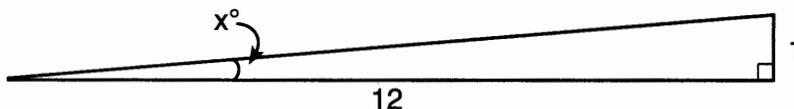
- (3) the vertical line of symmetry
(4) the horizontal line of symmetry

- 14 If $ABCD$ is a parallelogram, which statement would prove that $ABCD$ is a rhombus?

- (1) $\angle ABC \cong \angle CDA$
(2) $\overline{AC} \cong \overline{BD}$
could be

- (3) $\overline{AC} \perp \overline{BD}$
(4) $\overline{AB} \perp \overline{CD}$ *Not possible*

- 15 To build a handicapped-access ramp, the building code states that for every 1 inch of vertical rise in height, the ramp must extend out 12 inches horizontally, as shown in the diagram below.



$$\tan x = \frac{1}{12}$$

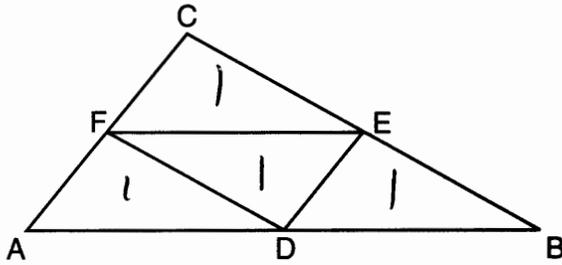
$$x \approx 4.76$$

What is the angle of inclination, x , of this ramp, to the nearest hundredth of a degree?

- (1) 4.76
(2) 4.78
(3) 85.22
(4) 85.24

Use this space for computations.

- 16 In the diagram below of $\triangle ABC$, D , E , and F are the midpoints of \overline{AB} , \overline{BC} , and \overline{CA} , respectively.



What is the ratio of the area of $\triangle CFE$ to the area of $\triangle CAB$?

- (1) 1:1 (3) 1:3
 (2) 1:2 (4) 1:4

- 17 The coordinates of the endpoints of \overline{AB} are $A(-8, -2)$ and $B(16, 6)$.

Point P is on \overline{AB} . What are the coordinates of point P , such that

$AP:PB$ is $3:5$?

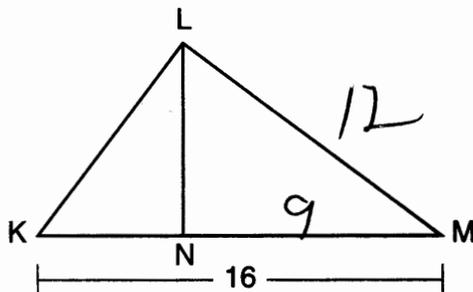
- (1) (1, 1) (3) (9.6, 3.6)
 (2) (7, 3) (4) (6.4, 2.8)

24

$$\frac{3}{8} \cdot 24 = 9 \quad x - 8 = 1$$

$$\frac{3}{8} \cdot 8 = 3 \quad y - 2 = 1$$

- 18 Kirstie is testing values that would make triangle KLM a right triangle when \overline{LN} is an altitude, and $KM = 16$, as shown below.



$$12^2 = 9 \cdot 16$$

$$144 = 144$$

Which lengths would make triangle KLM a right triangle?

- (1) $LM = 13$ and $KN = 6$ (3) $KL = 11$ and $KN = 7$
 (2) $LM = 12$ and $NM = 9$ (4) $LN = 8$ and $NM = 10$

Use this space for
computations.

23 In a circle with a diameter of 32, the area of a sector is $\frac{512\pi}{3}$. The measure of the angle of the sector, in radians, is

(1) $\frac{\pi}{3}$

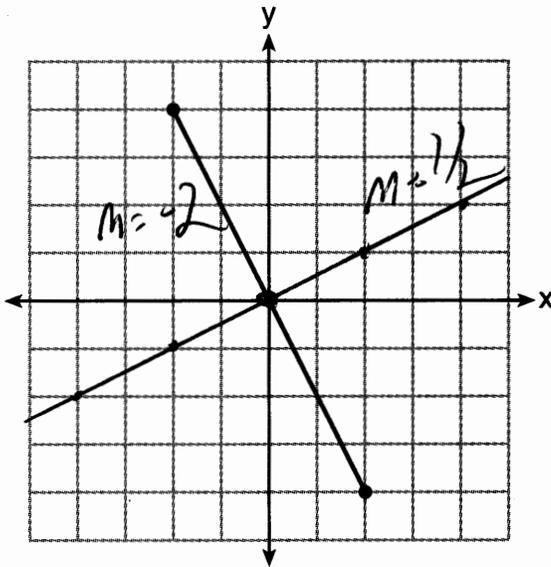
(3) $\frac{16\pi}{3}$

(2) $\frac{4\pi}{3}$

(4) $\frac{64\pi}{3}$

$$\frac{512\pi/3}{(\frac{32}{2})^2 \pi} \cdot 2\pi = \frac{4\pi}{3}$$

24 What is an equation of the perpendicular bisector of the line segment shown in the diagram below?



$$y = \frac{1}{2}x$$
$$2y - x = 0$$

(1) $y + 2x = 0$

(3) $2y + x = 0$

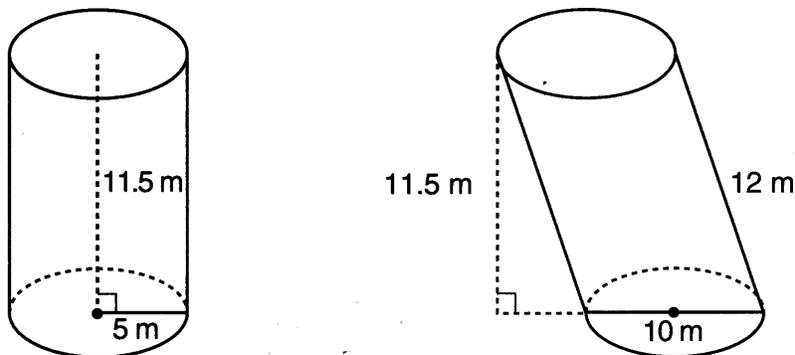
(2) $y - 2x = 0$

(4) $2y - x = 0$

Part II

Answer all 7 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [14]

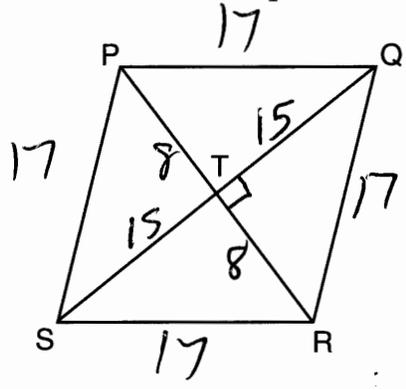
25 Sue believes that the two cylinders shown in the diagram below have equal volumes.



Is Sue correct? Explain why.

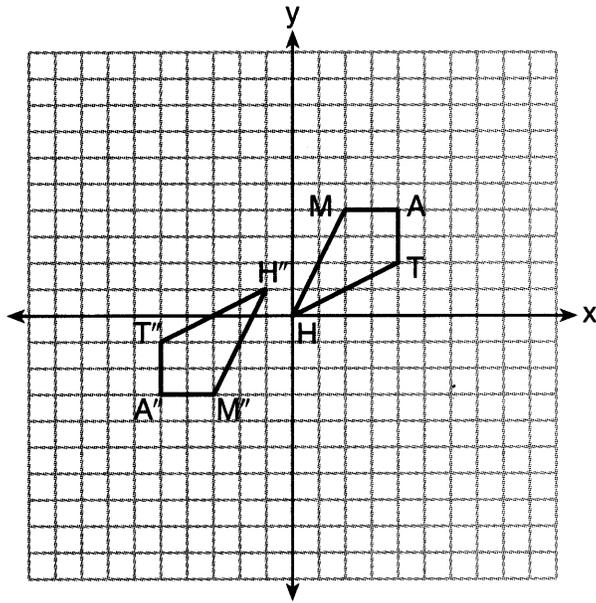
Yes. The bases of the cylinders have the same area & the cylinders have the same height.

26 In the diagram of rhombus $PQRS$ below, the diagonals \overline{PR} and \overline{QS} intersect at point T , $PR = 16$, and $QS = 30$. Determine and state the perimeter of $PQRS$.



$$17 \times 4 = 68$$

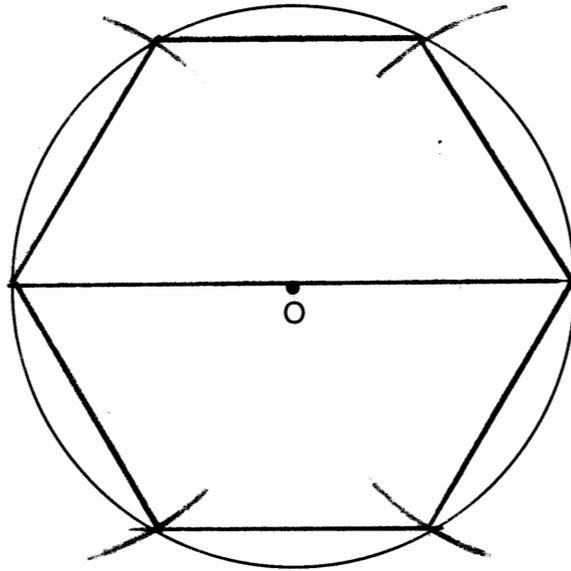
27 Quadrilateral $MATH$ and its image $M''A''T''H''$ are graphed on the set of axes below.



Describe a sequence of transformations that maps quadrilateral $MATH$ onto quadrilateral $M''A''T''H''$.

R_{180° about $(-\frac{1}{2}, \frac{1}{2})$

28 Using a compass and straightedge, construct a regular hexagon inscribed in circle O .
[Leave all construction marks.]



29 The coordinates of the endpoints of \overline{AB} are $A(2,3)$ and $B(5,-1)$. Determine the length of $\overline{A'B'}$, the image of \overline{AB} , after a dilation of $\frac{1}{2}$ centered at the origin.

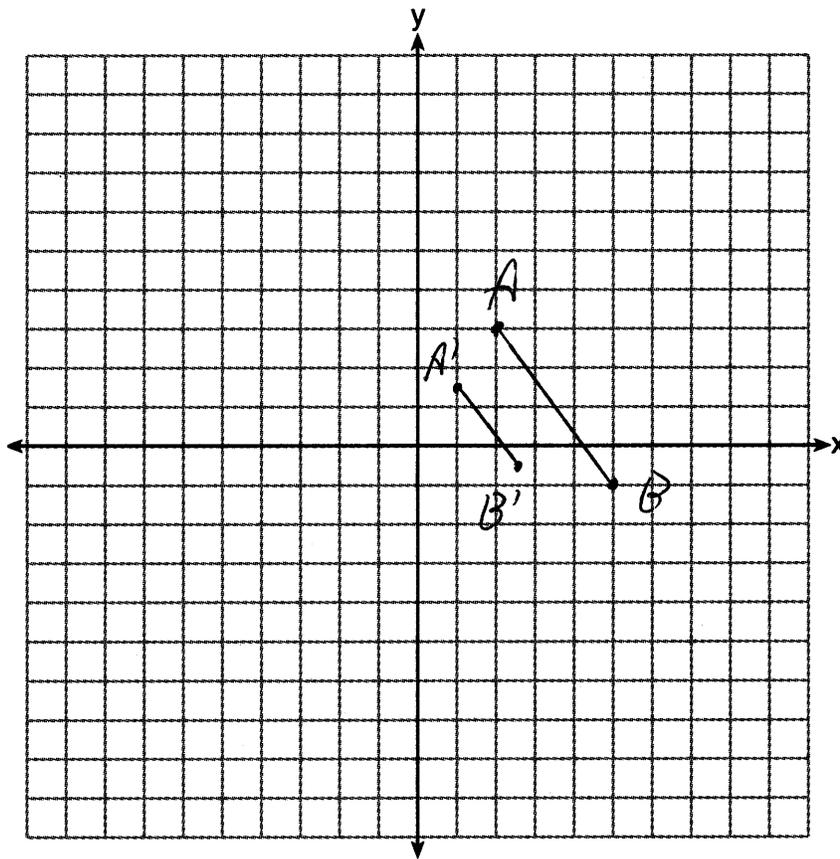
[The use of the set of axes below is optional.]

$$A'(1, 1.5)$$

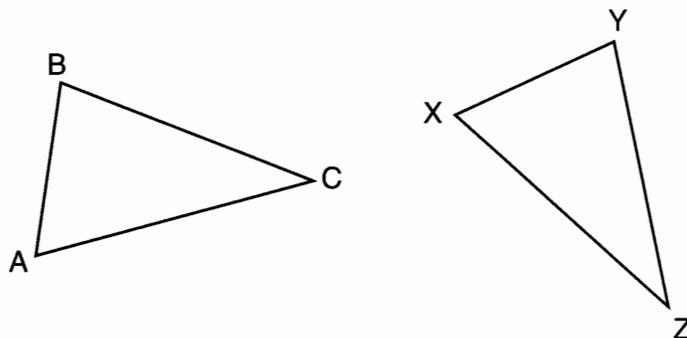
$$B'(2.5, -.5)$$

$$\sqrt{(2.5-1)^2 + (-.5-1.5)^2}$$

$$\sqrt{2.25 + 4} = \sqrt{6.25} = 2.5$$



30 In the diagram below of $\triangle ABC$ and $\triangle XYZ$, a sequence of rigid motions maps $\angle A$ onto $\angle X$, $\angle C$ onto $\angle Z$, and \overline{AC} onto \overline{XZ} .



Determine and state whether $\overline{BC} \cong \overline{YZ}$. Explain why.

Yes
 $\angle A \cong \angle X$, $\angle C \cong \angle Z$, $\overline{AC} \cong \overline{XZ}$ after a sequence of rigid motions which preserve distance & angle measure, so $\triangle ABC \cong \triangle XYZ$ by ASA. $\overline{BC} \cong \overline{YZ}$ by CPCTC.

31 Determine and state the coordinates of the center and the length of the radius of a circle whose equation is $x^2 + y^2 - 6x = 56 - 8y$.

$$x^2 - 6x + 9 + y^2 + 8y + 16 = 56 + 9 + 16$$

$$(x-3)^2 + (y+4)^2 = 81$$

$$(3, -4)$$

$$r = 9$$

Part III

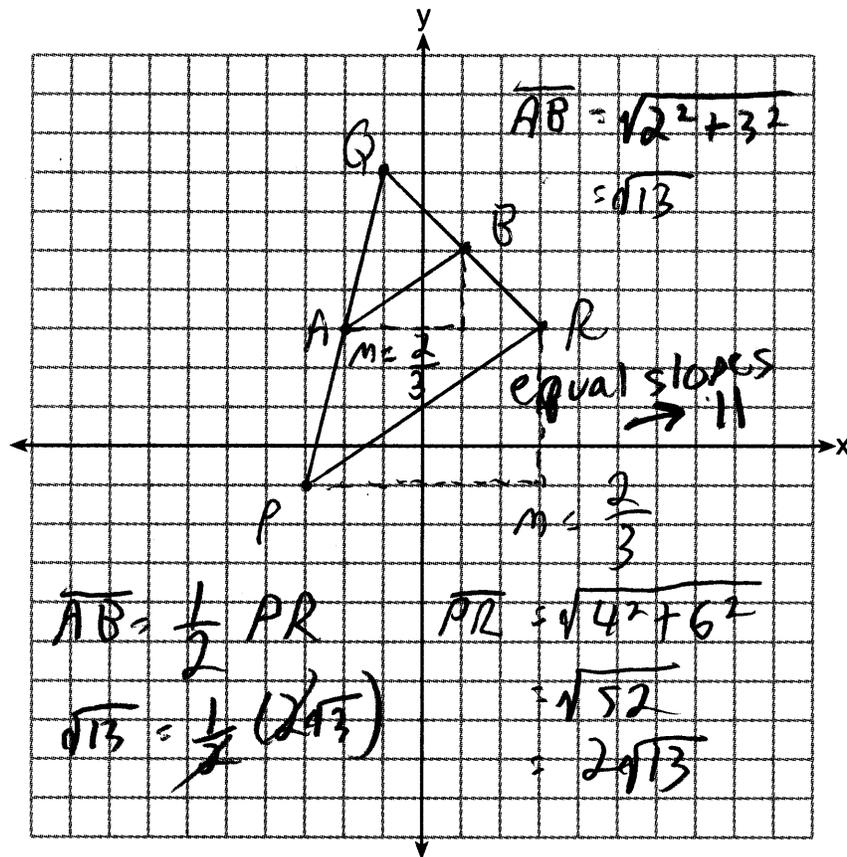
Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

32 Triangle PQR has vertices $P(-3,-1)$, $Q(-1,7)$, and $R(3,3)$, and points A and B are midpoints of \overline{PQ} and \overline{QR} , respectively. Use coordinate geometry to prove that \overline{AB} is parallel to \overline{PR} and is half the length of \overline{PR} .

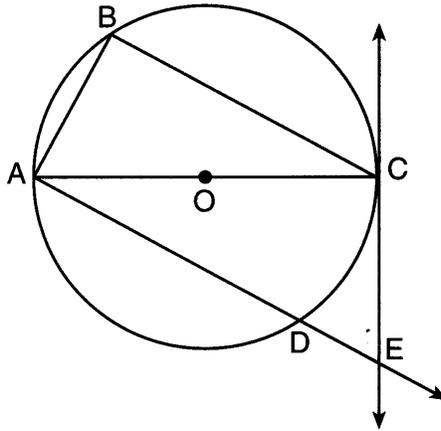
[The use of the set of axes below is optional.]

$$M_{\overline{PQ}} = \left(\frac{-3+(-1)}{2}, \frac{-1+7}{2} \right) = A(-2, 3)$$

$$M_{\overline{QR}} = \left(\frac{-1+3}{2}, \frac{7+3}{2} \right) = B(1, 5)$$



33 In the diagram below of circle O , tangent \overline{EC} is drawn to diameter \overline{AC} . Chord \overline{BC} is parallel to secant \overline{ADE} , and chord \overline{AB} is drawn.



Prove: $\frac{BC}{CA} = \frac{AB}{EC}$

STATEMENT

REASON

① Circle O , tangent \overline{EC} to diameter \overline{AC} , chord $\overline{BC} \parallel$ secant \overline{ADE} , chord \overline{AB}

① Given

② $\angle B$ is a right angle

② An angle inscribed in a semi-circle is a right \angle

③ $\overline{EC} \perp \overline{OC}$

③ A radius drawn to a point of tangency is \perp to the tangent

④ $\angle ECA$ is a right \angle

④ \perp lines form rt \angle s.

⑤ $\angle B \cong \angle ECA$

⑤ All right \angle s are \cong

⑥ $\angle BCA \cong \angle CAB$

⑥ The transversal of \parallel lines creates congruent alternate interior \angle s

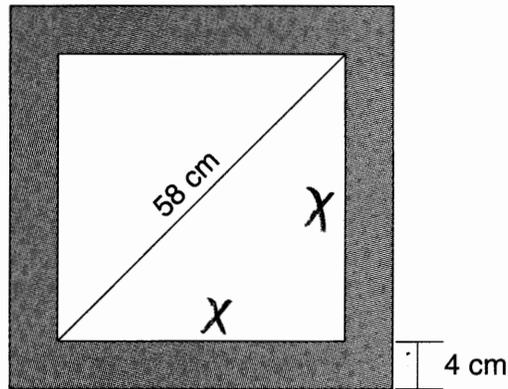
⑦ $\triangle ABC \sim \triangle ECA$

⑦ AA

⑧ $\frac{BC}{CA} = \frac{AB}{EC}$

⑧ Corresponding sides of similar \triangle s are in proportion

- 34 Keira has a square poster that she is framing and placing on her wall. The poster has a diagonal 58 cm long and fits exactly inside the frame. The width of the frame around the picture is 4 cm.



Determine and state the total area of the poster and frame to the *nearest tenth of a square centimeter*.

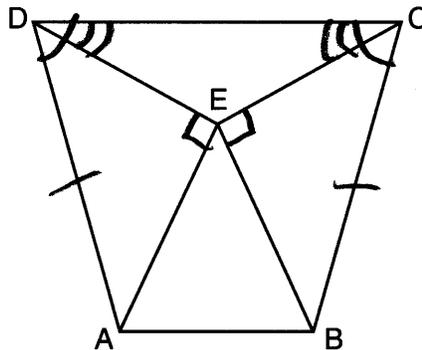
$$\begin{aligned}x^2 + x^2 &= 58^2 \\2x^2 &= 3364 \\x &= \sqrt{1682}\end{aligned}$$

$$A = (\sqrt{1682} + 8)^2 \approx 2402.2$$

Part IV

Answer the 2 questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

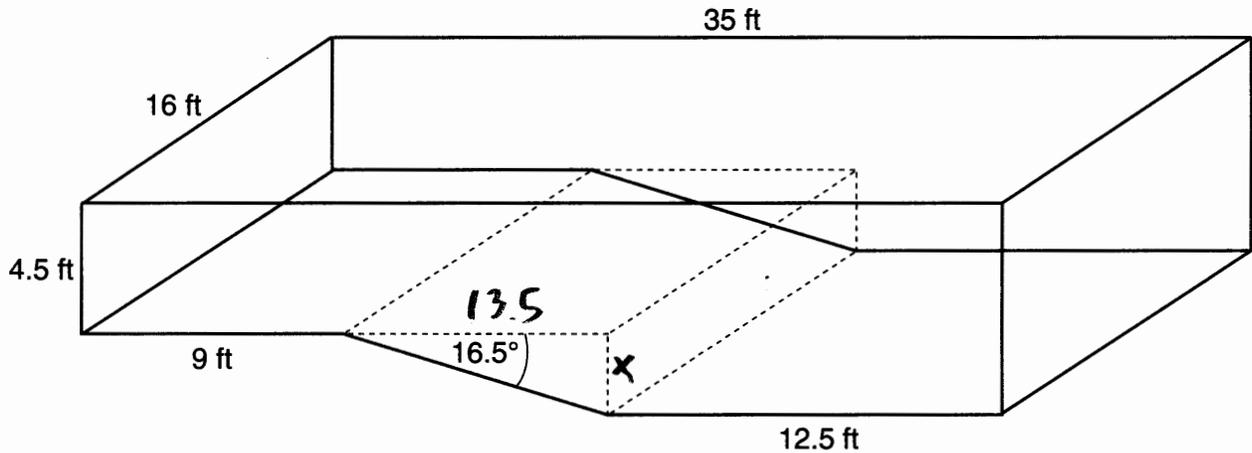
35 Isosceles trapezoid $ABCD$ has bases \overline{DC} and \overline{AB} with nonparallel legs \overline{AD} and \overline{BC} . Segments \overline{AE} , \overline{BE} , \overline{CE} , and \overline{DE} are drawn in trapezoid $ABCD$ such that $\angle CDE \cong \angle DCE$, $\overline{AE} \perp \overline{DE}$, and $\overline{BE} \perp \overline{CE}$.



Prove $\triangle ADE \cong \triangle BCE$ and prove $\triangle AEB$ is an isosceles triangle.

Statement	Reason
① $ABCD$ is an isosceles trapezoid, $\overline{AE} \perp \overline{DE}$, $\overline{BE} \perp \overline{CE}$, $\angle CDE \cong \angle DCE$	① Given
② $\overline{AD} \cong \overline{BC}$	② \cong legs of isosceles trapezoid
③ $\angle DBA$ & $\angle CBB$ are right \angle s	③ \perp lines form right \angle s
④ $\angle DCA \cong \angle CEB$	④ All right angles are \cong
⑤ $\angle CDA \cong \angle DCB$	⑤ Base angles of an isosceles trapezoid are congruent
⑥ $\angle CDA - \angle CDE \cong \angle DCB - \angle DCE$ $\angle EDA \cong \angle ECB$	⑥ subtraction postulate
⑦ $\triangle ADE \cong \triangle BCE$	⑦ AAS
⑧ $\overline{EA} \cong \overline{EB}$	⑧ CPCTC
⑨ $\triangle AEB$ is an isosceles \triangle	⑨ An isosceles \triangle has 2 \cong sides

36 A rectangular in-ground pool is modeled by the prism below. The inside of the pool is 16 feet wide and 35 feet long. The pool has a shallow end and a deep end, with a sloped floor connecting the two ends. Without water, the shallow end is 9 feet long and 4.5 feet deep, and the deep end of the pool is 12.5 feet long.



If the sloped floor has an angle of depression of 16.5 degrees, what is the depth of the pool at the deep end, to the *nearest tenth* of a foot?

$$\tan 16.5 = \frac{x}{13.5}$$

$$x \approx 4 + 4.5 = 8.5$$

Find the volume of the inside of the pool to the *nearest cubic foot*.

$$9 \times 16 \times 4.5 = 648 \quad 13.5 \times 16 \times 4.5 = 972 \quad 12.5 \times 16 \times 8.5 = 1700$$

$$\frac{1}{2} (13.5)(16)(4) = 432$$

$$648 + 972 + 432 + 1700 = 3752$$

Question 36 is continued on the next page.

Question 36 continued

A garden hose is used to fill the pool. Water comes out of the hose at a rate of 10.5 gallons per minute. How much time, to the *nearest hour*, will it take to fill the pool 6 inches from the top?
[1 ft³ = 7.48 gallons]

$$3752 - (35 \times 16 \times .5) = 3472$$
$$\begin{array}{r} \\ \times 7.48 \\ \hline 25,971 \\ \hline 10.5 \end{array} = \frac{2473.4}{60}$$
$$41$$