

# GEOMETRY

The University of the State of New York  
REGENTS HIGH SCHOOL EXAMINATION

# GEOMETRY

Tuesday, August 13, 2013 — 8:30 to 11:30 a.m., only

Student Name: Mr. Sibol

School Name: JMAP

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 38 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...

A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

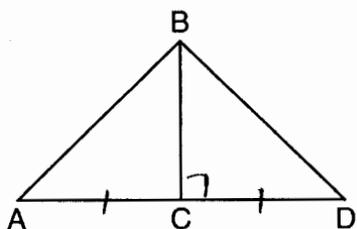
DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 28 questions in this part. Each correct answer will receive 2 credits. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [56]

Use this space for computations.

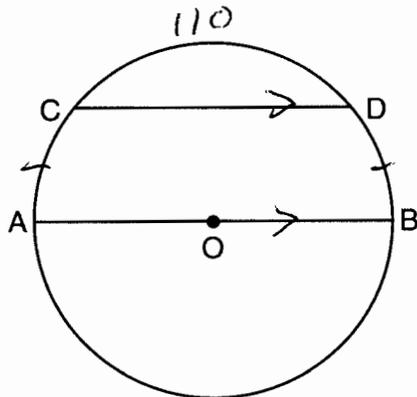
1 Given:  $\triangle ABD$ ,  $\overline{BC}$  is the perpendicular bisector of  $\overline{AD}$



Which statement can *not* always be proven?

- (1)  $\overline{AC} \cong \overline{DC}$
- (2)  $\overline{BC} \cong \overline{CD}$
- (3)  $\angle ACB \cong \angle DCB$
- (4)  $\triangle ABC \cong \triangle DBC$

2 In the diagram of circle  $O$  shown below, chord  $\overline{CD}$  is parallel to diameter  $\overline{AOB}$  and  $m\widehat{CD} = 110$ .



What is  $m\widehat{DB}$ ?

- (1) 35
- (2) 55
- (3) 70
- (4) 110

$$\frac{180 - 110}{2} = 35$$

3 Given the statement:

One is a prime number.

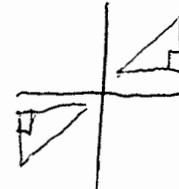
What is the negation and the truth value of the negation?

- (1) One is not a prime number; true
- (2) One is not a prime number; false
- (3) One is a composite number; true
- (4) One is a composite number; false

4 Triangle  $ABC$  has the coordinates  $A(1,2)$ ,  $B(5,2)$ , and  $C(5,5)$ .  
Triangle  $ABC$  is rotated  $180^\circ$  about the origin to form triangle  $A'B'C'$ .  
Triangle  $A'B'C'$  is

- (1) acute
- (2) isosceles
- (3) obtuse
- (4) right

*Distance is preserved after rotation*



5 What is an equation of the circle with center  $(-5,4)$  and a radius of 7?

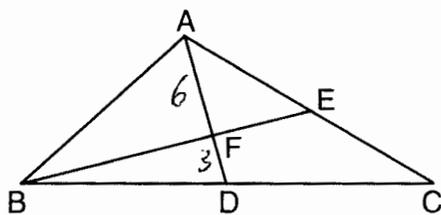
- (1)  $(x - 5)^2 + (y + 4)^2 = 14$
- (2)  $(x - 5)^2 + (y + 4)^2 = 49$
- (3)  $(x + 5)^2 + (y - 4)^2 = 14$
- (4)  $(x + 5)^2 + (y - 4)^2 = 49$

6 In  $\triangle ABC$ ,  $\angle A \cong \angle B$  and  $\angle C$  is an obtuse angle. Which statement is true?

- (1)  $\overline{AC} \cong \overline{AB}$  and  $\overline{BC}$  is the longest side.
- (2)  $\overline{AC} \cong \overline{BC}$  and  $\overline{AB}$  is the longest side.
- (3)  $\overline{AC} \cong \overline{AB}$  and  $\overline{BC}$  is the shortest side.
- (4)  $\overline{AC} \cong \overline{BC}$  and  $\overline{AB}$  is the shortest side.

Use this space for computations.

- 7 In the diagram of  $\triangle ABC$  below, medians  $\overline{AD}$  and  $\overline{BE}$  intersect at point  $F$ .



If  $AF = 6$ , what is the length of  $\overline{FD}$ ?

- (1) 6  
(2) 2  
(3) 3  
(4) 9

- 8 In circle  $O$ , diameter  $\overline{AB}$  intersects chord  $\overline{CD}$  at  $E$ . If  $CE = ED$ , then  $\angle CEA$  is which type of angle?

- (1) straight  
(2) obtuse  
(3) acute  
(4) right

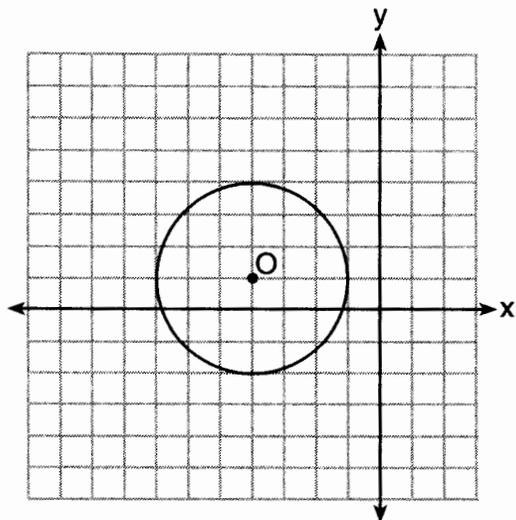
- 9 If  $\triangle ABC \cong \triangle JKL \cong \triangle RST$ , then  $\overline{BC}$  must be congruent to

- (1)  $\overline{JL}$   
(2)  $\overline{JK}$   
(3)  $\overline{ST}$   
(4)  $\overline{RS}$



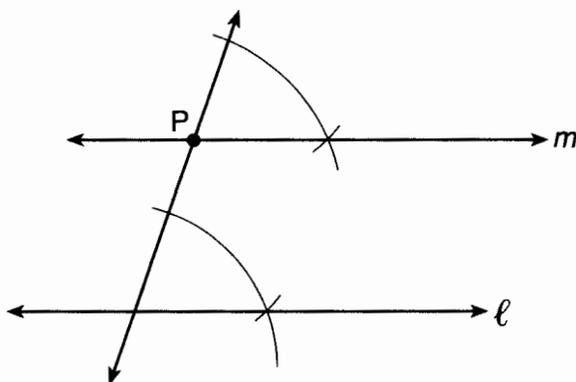
12 What is the equation of circle  $O$  shown in the diagram below?

Use this space for computations.



- (1)  $(x + 4)^2 + (y - 1)^2 = 3$
- (2)  $(x - 4)^2 + (y + 1)^2 = 3$
- (3)  $(x + 4)^2 + (y - 1)^2 = 9$
- (4)  $(x - 4)^2 + (y + 1)^2 = 9$

13 The diagram below shows the construction of line  $m$ , parallel to line  $\ell$ , through point  $P$ .



Which theorem was used to justify this construction?

- (1) If two lines are cut by a transversal and the alternate interior angles are congruent, the lines are parallel.
- (2) If two lines are cut by a transversal and the interior angles on the same side are supplementary, the lines are parallel.
- (3) If two lines are perpendicular to the same line, they are parallel.
- (4) If two lines are cut by a transversal and the corresponding angles are congruent, they are parallel.

Use this space for computations.

- 14 The lateral area of a right circular cone is equal to  $120\pi$  cm<sup>2</sup>. If the base of the cone has a diameter of 24 cm, what is the length of the slant height, in centimeters?  $r = 12$

- (1) 2.5  
(2) 5

- (3) 10  
(4) 15.7

$$\frac{120\pi}{12\pi} = \frac{\pi(12)(l)}{12\pi}$$

- 15 A student wrote the following equations:

$$3y + 6 = 2x$$
$$2y - 3x = 6$$

$$y = \frac{2}{3}x - 2$$
$$y = \frac{3}{2}x + 3$$

The lines represented by these equations are

- (1) parallel  
(2) the same line  
(3) perpendicular  
(4) intersecting, but *not* perpendicular

- 16 In a coordinate plane, the locus of points 5 units from the  $x$ -axis is the

- (1) lines  $x = 5$  and  $x = -5$   
(2) lines  $y = 5$  and  $y = -5$   
(3) line  $x = 5$ , only  
(4) line  $y = 5$ , only

- 17 The sides of a triangle are 8, 12, and 15. The longest side of a similar triangle is 18. What is the ratio of the perimeter of the smaller triangle to the perimeter of the larger triangle?

- (1) 2:3  
(2) 4:9  
(3) 5:6  
(4) 25:36

$$\frac{15}{18} = \frac{5}{6}$$

Use this space for computations.

18 Lines  $m$  and  $n$  are in plane  $\mathcal{A}$ . What is the converse of the statement "If lines  $m$  and  $n$  are parallel, then lines  $m$  and  $n$  do not intersect"?

- (1) If lines  $m$  and  $n$  are not parallel, then lines  $m$  and  $n$  intersect.
- (2) If lines  $m$  and  $n$  are not parallel, then lines  $m$  and  $n$  do not intersect.
- (3) If lines  $m$  and  $n$  intersect, then lines  $m$  and  $n$  are not parallel.
- (4) If lines  $m$  and  $n$  do not intersect, then lines  $m$  and  $n$  are parallel.

$$y = (x-4)^2 - 2 \quad y = -2x + 6$$

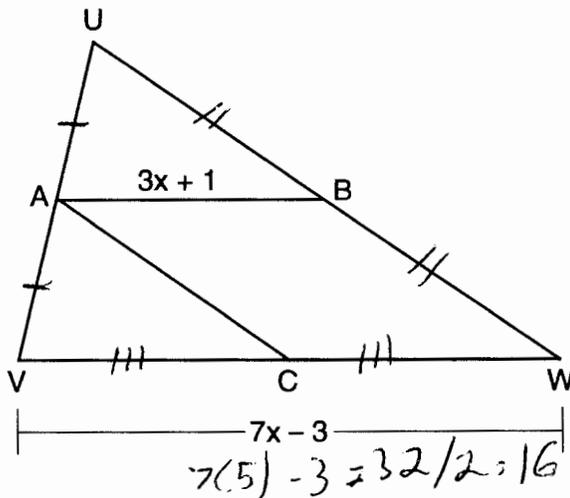
19 When the system of equations  $y + 2 = (x - 4)^2$  and  $2x + y - 6 = 0$  is solved graphically, the solution is

- (1)  $(-4, -2)$  and  $(-2, 2)$
- (2)  $(4, -2)$  and  $(2, 2)$
- (3)  $(-4, 2)$  and  $(-6, 6)$
- (4)  $(4, 2)$  and  $(6, 6)$

$$\begin{aligned} (x-4)^2 - 2 &= -2x + 6 \\ x^2 - 8x + 16 - 2 &= -2x + 6 \\ x^2 - 6x + 8 &= 0 \\ (x-4)(x-2) &= 0 \\ x &= 4, 2 \end{aligned}$$

$$\begin{aligned} y &= -2(4) + 6 = -2 \\ y &= -2(2) + 6 = 2 \end{aligned}$$

20 In the diagram of  $\triangle UVW$  below,  $A$  is the midpoint of  $\overline{UV}$ ,  $B$  is the midpoint of  $\overline{UW}$ ,  $C$  is the midpoint of  $\overline{VW}$ , and  $\overline{AB}$  and  $\overline{AC}$  are drawn.



$$\begin{aligned} 2(3x+1) &= 7x-3 \\ 6x+2 &= 7x-3 \\ 5 &= x \end{aligned}$$

If  $VW = 7x - 3$  and  $AB = 3x + 1$ , what is the length of  $\overline{VC}$ ?

- (1) 5
- (2) 13
- (3) 16
- (4) 32

Use this space for computations.

- 21 Two prisms have equal heights and equal volumes. The base of one is a pentagon and the base of the other is a square. If the area of the pentagonal base is 36 square inches, how many inches are in the length of each side of the square base?

- (1) 6  
(2) 9  
(3) 24  
(4) 36

If two prisms have equal heights and volumes, the areas of their bases are equal.

- 22 What is the difference between the sum of the measures of the interior angles of a regular pentagon and the sum of the measures of the exterior angles of a regular pentagon?

- (1) 36  
(2) 72  
(3) 108  
(4) 180

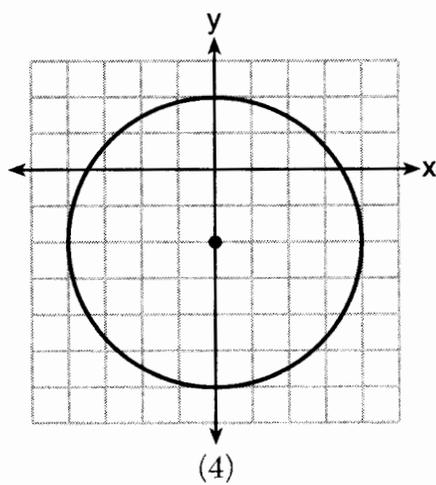
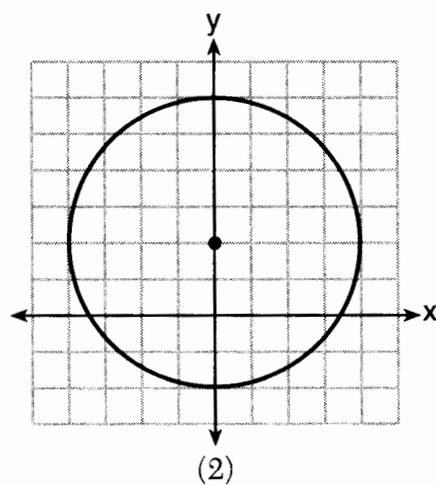
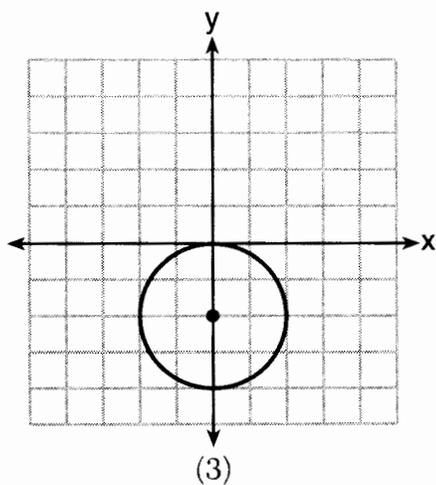
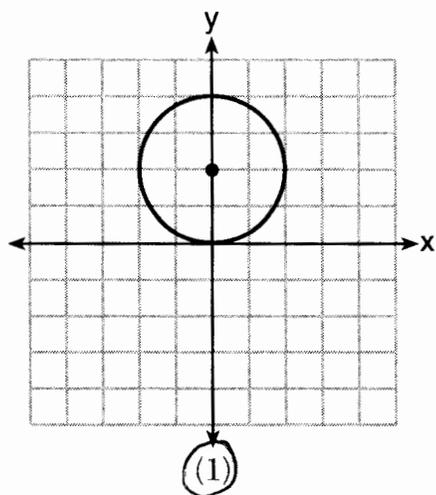
$$180(n-2) - n\left(180 - \frac{180(n-2)}{n}\right)$$
$$180n - 360 - 180n + 180n - 360$$
$$180n - 720$$
$$180(5) - 720 = 180$$

- 23 If line  $\ell$  is perpendicular to distinct planes  $\mathcal{P}$  and  $\mathcal{Q}$ , then planes  $\mathcal{P}$  and  $\mathcal{Q}$

- (1) are parallel  
(2) contain line  $\ell$   
(3) are perpendicular  
(4) intersect, but are *not* perpendicular

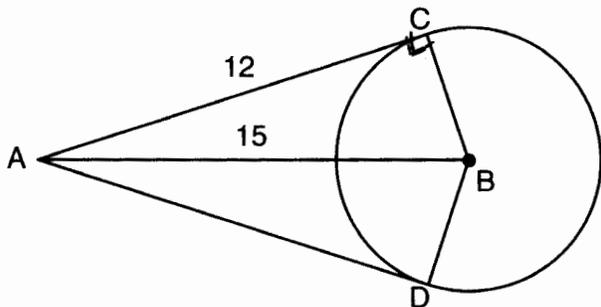
24 Which graph represents a circle whose equation is  $x^2 + (y - 2)^2 = 4$ ?

Use this space for computations.



Use this space for computations.

25 In the diagram below,  $\overline{AC}$  and  $\overline{AD}$  are tangent to circle  $B$  at points  $C$  and  $D$ , respectively, and  $\overline{BC}$ ,  $\overline{BD}$ , and  $\overline{BA}$  are drawn.



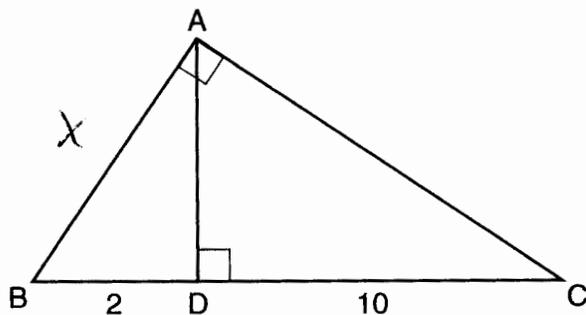
(Not drawn to scale)

If  $AC = 12$  and  $AB = 15$ , what is the length of  $\overline{BD}$ ?

- (1) 5.5                      (3) 12  
 (2) 9                         (4) 18

$$\sqrt{15^2 - 12^2} = 9$$

26 Triangle  $ABC$  shown below is a right triangle with altitude  $\overline{AD}$  drawn to the hypotenuse  $\overline{BC}$ .



If  $BD = 2$  and  $DC = 10$ , what is the length of  $\overline{AB}$ ?

- (1)  $2\sqrt{2}$                       (3)  $2\sqrt{6}$   
 (2)  $2\sqrt{5}$                       (4)  $2\sqrt{30}$

$$x^2 = 2(2+10)$$

$$x^2 = 24$$

$$x = \sqrt{24} = \sqrt{4 \cdot 6} = 2\sqrt{6}$$

Use this space for computations.

27 Triangle  $ABC$  has vertices  $A(0,0)$ ,  $B(6,8)$ , and  $C(8,4)$ . Which equation represents the perpendicular bisector of  $\overline{BC}$ ?

(1)  $y = 2x - 6$

(3)  $y = \frac{1}{2}x + \frac{5}{2}$

(2)  $y = -2x + 4$

(4)  $y = -\frac{1}{2}x + \frac{19}{2}$

midpoint of  $\overline{BC} = \left(\frac{6+8}{2}, \frac{8+4}{2}\right) = (7,6)$

slope of  $\overline{BC} = \frac{8-4}{6-8} = \frac{4}{-2} = -2$

$$m_{\perp} = \frac{1}{2}$$

$$6 = \frac{1}{2}(7) + b$$

$$\frac{12}{2} = \frac{7}{2} + b$$

$$\frac{5}{2} = b$$

28 Chords  $\overline{AB}$  and  $\overline{CD}$  intersect at point  $E$  in a circle with center at  $O$ . If  $AE = 8$ ,  $AB = 20$ , and  $DE = 16$ , what is the length of  $\overline{CE}$ ?

(1) 6

(3) 10

(2) 9

(4) 12

$$8 \cdot 12 = 16x$$

$$6 = x$$

Part II

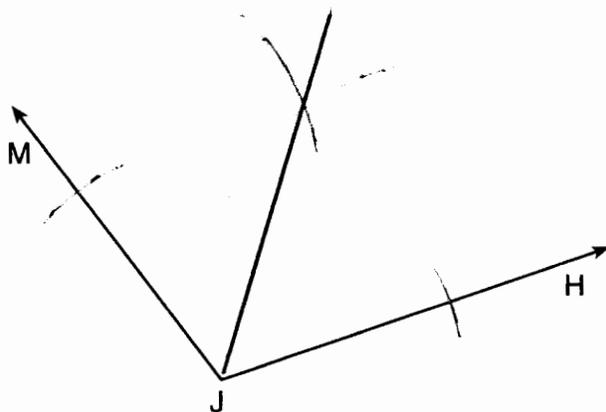
Answer all 6 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

29 Triangle  $ABC$  has vertices  $A(6,6)$ ,  $B(9,0)$ , and  $C(3,-3)$ . State and label the coordinates of  $\triangle A'B'C'$ , the image of  $\triangle ABC$  after a dilation of  $D_{\frac{1}{3}}$ .

$A'(2,2)$     $B'(3,0)$     $C'(1,-1)$

30 Using a compass and straightedge, construct the bisector of  $\angle MJH$ .

[Leave all construction marks.]



31 Find, in simplest radical form, the length of the line segment with endpoints whose coordinates are  $(-1, 4)$  and  $(3, -2)$ .

$$\sqrt{(-1-3)^2 + (4-(-2))^2}$$

$$\sqrt{16 + 36}$$

$$\sqrt{52}$$

$$\sqrt{4} \sqrt{13}$$

$$2\sqrt{13}$$

32 In  $\triangle ABC$ , the measure of angle  $A$  is fifteen less than twice the measure of angle  $B$ . The measure of angle  $C$  equals the sum of the measures of angle  $A$  and angle  $B$ . Determine the measure of angle  $B$ .

$$A = 2B - 15$$

$$C = A + B$$

$$A + B + C = 180$$

$$2B - 15 + B + 2B - 15 + B = 180$$

$$6B - 30 = 180$$

$$6B = 210$$

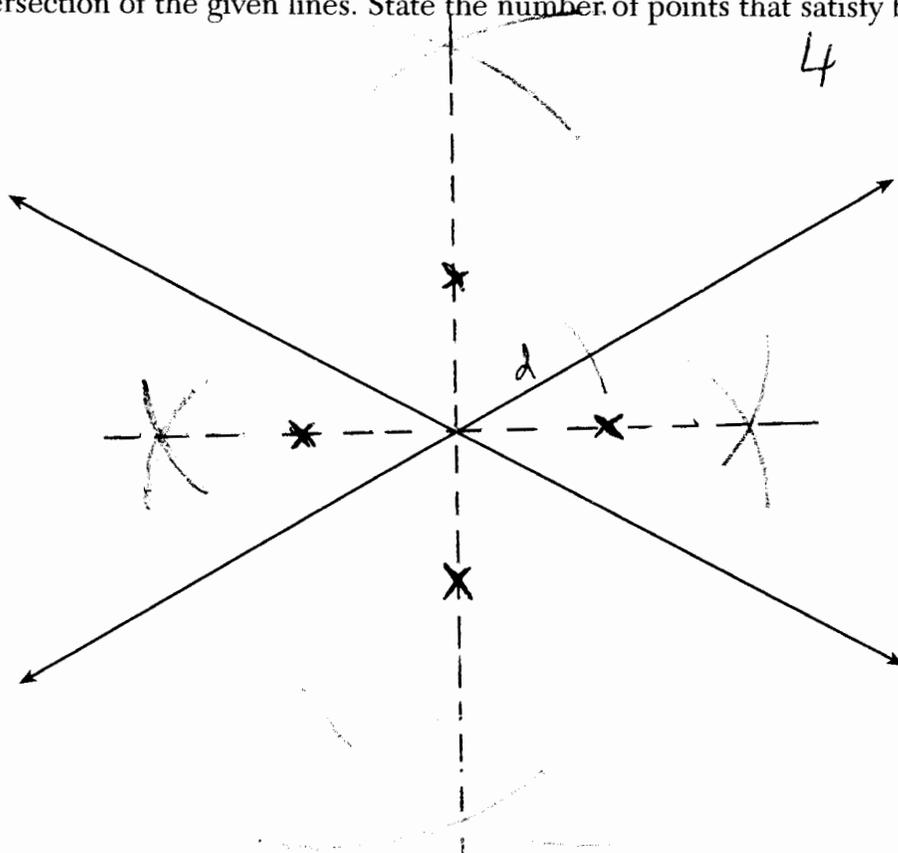
$$B = 35$$

**33** A circle has the equation  $(x - 3)^2 + (y + 4)^2 = 10$ . Find the coordinates of the center of the circle and the length of the circle's radius.

center  $(3, -4)$

radius  $\sqrt{10}$

34 Two intersecting lines are shown in the diagram below. Sketch the locus of points that are equidistant from the two lines. Sketch the locus of points that are a given distance,  $d$ , from the point of intersection of the given lines. State the number of points that satisfy both conditions.

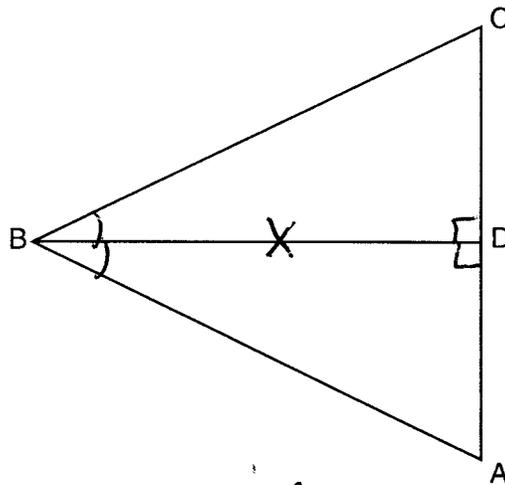


Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

35 Given:  $\triangle ABC$ ,  $\overline{BD}$  bisects  $\angle ABC$ ,  $\overline{BD} \perp \overline{AC}$

Prove:  $\overline{AB} \cong \overline{CB}$

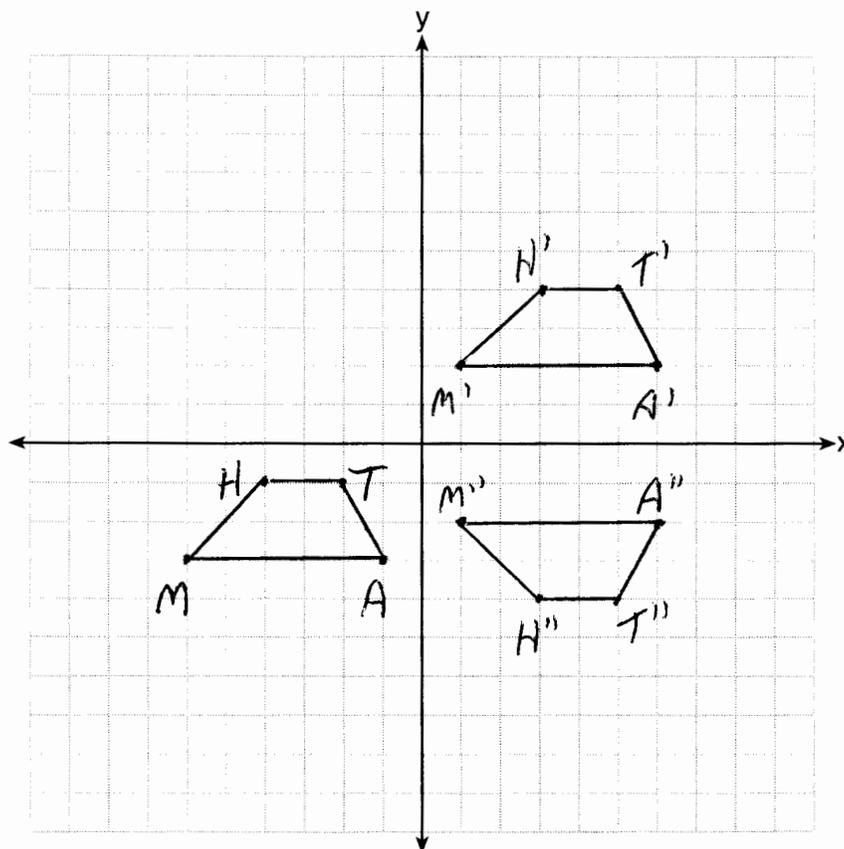


Statements	Reasons
① $\triangle ABC$ , $\overline{BD}$ bisects $\angle ABC$ , $\overline{BD} \perp \overline{AC}$	① Given
② $\angle CBD \cong \angle ABD$	② Definition of angle bisector
③ $\overline{BD} \cong \overline{BD}$	③ Reflexive Property
④ $\angle CDB$ & $\angle ADB$ are right angles	④ Definition of perpendicular
⑤ $\angle CDB \cong \angle ADB$	⑤ All right angles are congruent
⑥ $\triangle CDB \cong \triangle ADB$	⑥ ASA
⑦ $\overline{AB} \cong \overline{CB}$	⑦ CPCTC

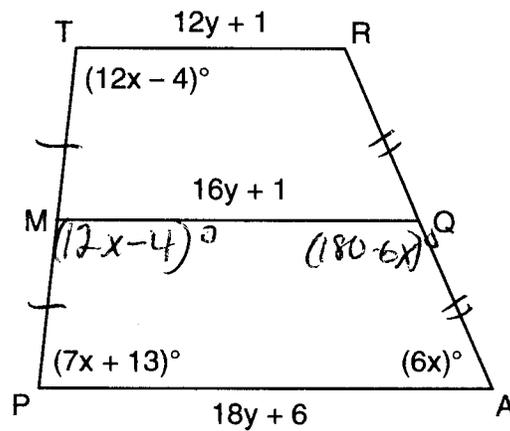
- 36 Quadrilateral  $MATH$  has coordinates  $M(-6, -3)$ ,  $A(-1, -3)$ ,  $T(-2, -1)$ , and  $H(-4, -1)$ . The image of quadrilateral  $MATH$  after the composition  $r_{x\text{-axis}} \circ T_{7,5}$  is quadrilateral  $M''A''T''H''$ . State and label the coordinates of  $M''A''T''H''$ . [The use of the set of axes below is optional.]

$$M'(1, 2) \quad A'(6, 2) \quad T'(5, 4) \quad H'(3, 4)$$

$$M''(1, -2) \quad A''(6, -2) \quad T''(5, -4) \quad H''(3, -4)$$



37 Trapezoid  $TRAP$ , with median  $\overline{MQ}$ , is shown in the diagram below. Solve algebraically for  $x$  and  $y$ .



$$12x - 4 + 180 - 6x + 6x + 7x + 13 = 360$$

$$19x + 189 = 360$$

$$\frac{19x}{19} = \frac{171}{19}$$

$$x = 9$$

$$16y + 1 = \frac{12y + 1 + 18y + 6}{2}$$

$$32y + 2 = 30y + 7$$

$$2y = \frac{5}{2}$$

$$y = \frac{5}{2}$$

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. A correct numerical answer with no work shown will receive only 1 credit. The answer should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

38 Quadrilateral  $ABCD$  with vertices  $A(-7,4)$ ,  $B(-3,6)$ ,  $C(3,0)$ , and  $D(1,-8)$  is graphed on the set of axes below. Quadrilateral  $MNPQ$  is formed by joining  $M$ ,  $N$ ,  $P$ , and  $Q$ , the midpoints of  $\overline{AB}$ ,  $\overline{BC}$ ,  $\overline{CD}$ , and  $\overline{AD}$ , respectively.

Prove that quadrilateral  $MNPQ$  is a parallelogram.

Prove that quadrilateral  $MNPQ$  is *not* a rhombus.

$$M\left(\frac{-7+3}{2}, \frac{4+6}{2}\right) = M(-2, 5)$$

$$N\left(\frac{-3+3}{2}, \frac{6+0}{2}\right) = N(0, 3)$$

$$P\left(\frac{3+1}{2}, \frac{0+(-8)}{2}\right) = P(2, -4)$$

$$Q\left(\frac{-7+1}{2}, \frac{4+(-8)}{2}\right) = Q(-3, -2)$$

$$m_{\overline{MN}} = \frac{5-3}{-2-0} = \frac{2}{-2}$$

$$m_{\overline{PQ}} = \frac{-4-(-2)}{2-(-3)} = \frac{-2}{5}$$

$$m_{\overline{NP}} = \frac{3-(-4)}{0-2} = \frac{7}{-2}$$

$$m_{\overline{QM}} = \frac{-2-5}{-3-(-2)} = \frac{-7}{-1} = 7$$

Both opposite sides have equal slopes & are parallel

$MNPQ$  is a parallelogram

$$\overline{MN} = \sqrt{(-2-0)^2 + (5-3)^2} = \sqrt{13}$$

$$\overline{NP} = \sqrt{(0-2)^2 + (3-(-4))^2} = \sqrt{33}$$

$\overline{MN} \neq \overline{NP}$ , so  $MNPQ$  is not a rhombus since all sides are not congruent.

