

INTEGRATED ALGEBRA

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

INTEGRATED ALGEBRA

SPECIAL ADMINISTRATION

Thursday, February 25, 2016 — 9:15 a.m. to 12:15 p.m., only

Student Name: Steve Watson

School Name: www.jmap.org

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 39 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 30 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [60]

Use this space for computations.

1 If h represents a number, which equation is a correct translation of "Sixty more than 9 times a number is 375"?

- (1) $9h = 375$ (3) $9h - 60 = 375$
 (2) $9h + 60 = 375$ (4) $60h + 9 = 375$

$$\begin{array}{rcl} 9h & + 60 & = 375 \\ \text{9 times a number} & \text{sixty more} & \text{is} \end{array}$$

2 Which expression is equivalent to $9x^2 - 16$?

- (1) $(3x + 4)(3x - 4)$ (3) $(3x + 8)(3x - 8)$
 (2) $(3x - 4)(3x - 4)$ (4) $(3x - 8)(3x - 8)$

Difference of Perfect Squares
 $a^2 - b^2 = (a + b)(a - b)$
 $9x^2 - 16 = (3x + 4)(3x - 4)$

3 Which expression represents $(3x^2y^4)(4xy^2)$ in simplest form?

- (1) $12x^2y^8$ (3) $12x^3y^8$
 (2) $12x^2y^6$ (4) $12x^3y^6$

3	x^2	y^4
4	x	y^2
12	x^{2+1}	y^{4+2}
12	x^3	y^6

4 An online music club has a one-time registration fee of \$13.95 and charges \$0.49 to buy each song. If Emma has \$50.00 to join the club and buy songs, what is the maximum number of songs she can buy?

- (1) 73 (3) 130
 (2) 74 (4) 131

one time fee cost per song

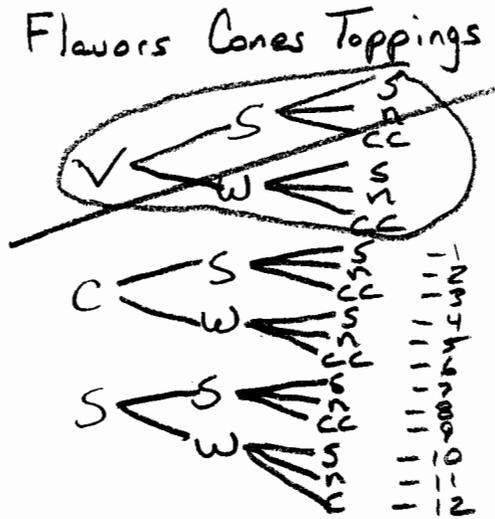
$$\begin{aligned} \text{Total} &\rightarrow y \leq 13.95 + 0.49x \\ 50 &\leq 13.95 + 0.49x \\ 50 - 13.95 &\leq 0.49x \\ 36.05 &\leq 0.49x \\ \frac{36.05}{0.49} &\leq x \\ 73.57 &\leq x \\ \boxed{73} &\leq x \end{aligned}$$

5 The local ice cream stand offers three flavors of soft-serve ice cream: vanilla, chocolate, and strawberry; two types of cone: sugar and wafer; and three toppings: sprinkles, nuts, and cookie crumbs. If Dawn does not order vanilla ice cream, how many different choices can she make that have one flavor of ice cream, one type of cone, and one topping?

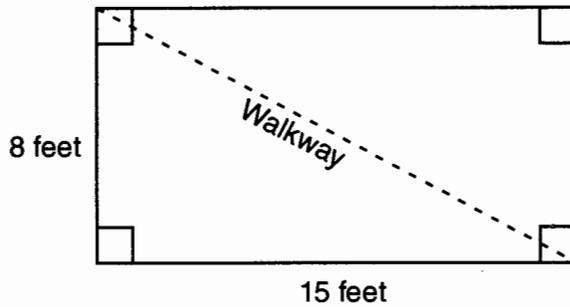
- (1) 7
- (2) 8

- (3) 12
- (4) 18

Use this space for computations.



6 Nancy's rectangular garden is represented in the diagram below.



If a diagonal walkway crosses her garden, what is its length, in feet?

- (1) 17
- (2) 22

- (3) $\sqrt{161}$
- (4) $\sqrt{529}$

$$a^2 + b^2 = c^2$$

$$8^2 + 15^2 = c^2$$

$$64 + 225 = c^2$$

$$289 = c^2$$

$$\sqrt{289} = \sqrt{c^2}$$

$17 = c$

Check

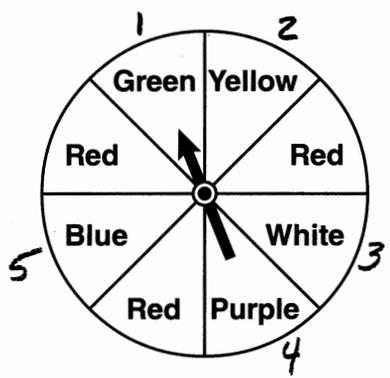
$$8^2 + 15^2 = 17^2$$

$$64 + 225 = 289$$

$$289 = 289 \quad \checkmark$$

Use this space for computations.

7 The spinner below is divided into eight equal regions and is spun once. What is the probability of not getting red?



Five out of eight regions are not red.

$$P(\text{not red}) = \frac{5 \text{ (# times not red happens)}}{8 \text{ (possible outcomes)}}$$

- (1) $\frac{3}{5}$
- (2) $\frac{3}{8}$
- (3) $\frac{5}{8}$
- (4) $\frac{7}{8}$

8 Which relationship can best be described as causal?

"A" causes "B"

- (1) height and intelligence *height does not cause intelligence.*
- (2) shoe size and running speed *shoe size does not cause running speed.*
- (3) number of correct answers on a test and test score *# correct answers do cause test score*
- (4) number of students in a class and number of students with brown hair *# students does not cause hair color.*

9 Solve for x: $\frac{3}{5}(x+2) = x-4$

- (1) 8
- (2) 13
- (3) 15
- (4) 23

$$\frac{3}{5}(x+2) = x-4$$

$$5 \left[\frac{3}{5}(x+2) \right] = 5(x-4)$$

$$3(x+2) = 5x - 20$$

$$3x + 6 = 5x - 20$$

$$26 = 2x$$

$$13 = x$$

Check

$$\frac{3}{5}(13+2) = 13-4$$

$$\frac{3}{5}(15) = 9$$

$$3(3) = 9$$

$$9 = 9 \checkmark$$

Use this space for computations.

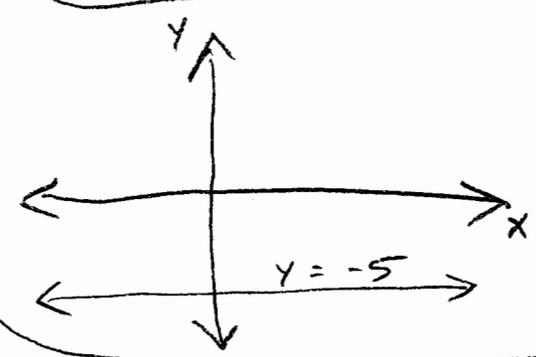
10 Erica is conducting a survey about the proposed increase in the sports budget in the Hometown School District. Which survey method would likely contain the most bias?

- (1) Erica asks every third person entering the Hometown Grocery Store.
- (2) Erica asks every third person leaving the Hometown Shopping Mall this weekend.
- (3) Erica asks every fifth student entering Hometown High School on Monday morning.
- (4) Erica asks every fifth person leaving Saturday's Hometown High School football game.

People who go to high school are probably biased in favor of sports.

11 Which equation represents a line parallel to the x-axis?

- (1) $y = -5$
- (2) $y = -5x$
- (3) $x = 3$
- (4) $x = 3y$



12 Given:

$A = \{\text{All even integers from 2 to 20, inclusive}\}$
 $B = \{10, 12, 14, 16, 18\}$

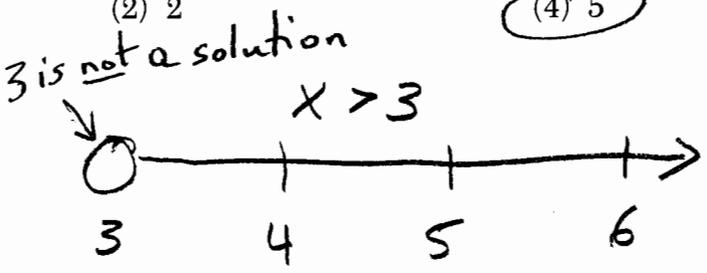
What is the complement of set B within the universe of set A?

- (1) $\{4, 6, 8\}$
- (2) $\{2, 4, 6, 8\}$
- (3) $\{4, 6, 8, 20\}$
- (4) $\{2, 4, 6, 8, 20\}$

A	2	4	6	8	10	12	14	16	18	20
B					10	12	14	16	18	
Complement	2	4	6	8						20

13 Which value of x is in the solution set of the inequality $-2(x - 5) < 4$?

- (1) 0
- (2) 2
- (3) 3
- (4) 5

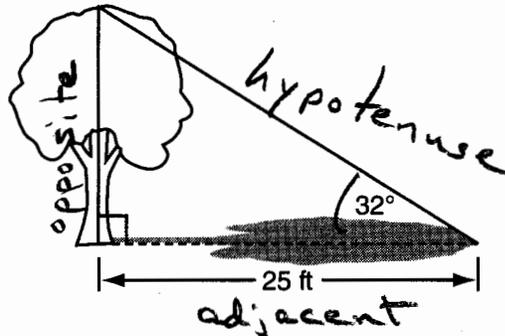


$$\begin{aligned}
 -2(x-5) &< 4 \\
 -2x + 10 &< 4 \\
 -2x &< -6 \\
 \frac{-2x}{-2} &> \frac{-6}{-2} \\
 x &> 3
 \end{aligned}$$

Change direction of inequality!

14 A tree casts a 25-foot shadow on a sunny day, as shown in the diagram below.

Use this space for computations.



If the angle of elevation from the tip of the shadow to the top of the tree is 32° , what is the height of the tree to the nearest tenth of a foot?

- (1) 13.2 (3) 21.2
 (2) 15.6 (4) 40.0

SOH-CAH-TOA

$$\tan = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 32^\circ = \frac{\text{opp}}{\text{adj}} = \frac{\text{opp}}{25}$$

$$\tan 32^\circ = \frac{\text{opp}}{25}$$

$$25 \tan 32^\circ = \text{opp}$$

$$15.6217338 = \text{opp}$$

15 What is the slope of the line that passes through the points $(-5, 4)$ and $(15, -4)$?

- (1) $-\frac{2}{5}$ (3) $-\frac{5}{2}$
 (2) 0 (4) undefined

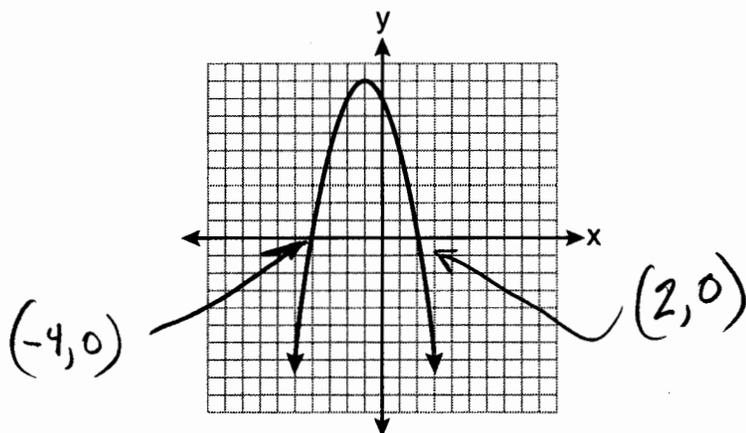
$$\begin{matrix} (x_1, y_1) & (x_2, y_2) \\ (-5, 4) & (15, -4) \end{matrix}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-4 - 4}{15 - (-5)} = \frac{-8}{20} = -\frac{2}{5}$$

16 The equation $y = -x^2 - 2x + 8$ is graphed on the set of axes below.

Use this space for computations.



The roots are the X values of the points where the graph crosses the X-axis.

Based on this graph, what are the roots of the equation $-x^2 - 2x + 8 = 0$?

(1) 8 and 0

(3) 9 and -1

(2) 2 and -4

(4) 4 and -2

17 What is the sum of $\frac{3}{2x}$ and $\frac{4}{3x}$ expressed in simplest form?

(1) $\frac{12}{6x^2}$

(3) $\frac{7}{5x}$

(2) $\frac{17}{6x}$

(4) $\frac{17}{12x}$

$$\begin{aligned} & \frac{3}{2x} + \frac{4}{3x} \\ & \left(\frac{3}{3}\right) \frac{3}{2x} + \frac{4}{3x} \left(\frac{2}{2}\right) \\ & \frac{9}{6x} + \frac{8}{6x} \\ & \boxed{\frac{17}{6x}} \end{aligned}$$

when the denominator = zero

Use this space for computations.

18 Which value of x makes the expression $\frac{x^2 - 9}{x^2 + 7x + 10}$ undefined?

- (1) -5
- (2) 2
- (3) 3
- (4) -3

$$x^2 + 7x + 10 = 0$$

$$(x+5)(x+2) = 0$$

$$x+5 = 0$$

$$x = -5$$

$$x+2 = 0$$

$$x = -2$$

19 Which relation is not a function?

- (1) $\{(1,5), (2,6), (3,6), (4,7)\}$
- (2) $\{(4,7), (2,1), (-3,6), (3,4)\}$
- (3) $\{(-1,6), (1,3), (2,5), (1,7)\}$
- (4) $\{(-1,2), (0,5), (5,0), (2,-1)\}$

A function has 1 and only 1 value of y for every value of x .

There are two values of y when $x = 1$

20 What is the value of the y -coordinate of the solution to the system of equations $x - 2y = 1$ and $x + 4y = 7$?

- (1) 1
- (2) -1
- (3) 3
- (4) 4

$$\begin{array}{r}
 \text{Eq. 1} \quad x - 2y = 1 \\
 \text{Eq. 2} \quad -(x + 4y = 7) \\
 \hline
 -6y = -6 \\
 \frac{-6y}{-6} = \frac{-6}{-6} \\
 \boxed{y = 1}
 \end{array}$$

21 The solution to the equation $x^2 - 6x = 0$ is

- (1) 0, only
- (2) 6, only
- (3) 0 and 6
- (4) $\pm\sqrt{6}$

$$x^2 - 6x = 0$$

$$x(x-6) = 0$$

$$x = 0$$

$$x-6 = 0$$

$$x = 6$$

Use this space for computations.

22 When $5\sqrt{20}$ is written in simplest radical form, the result is $k\sqrt{5}$.
What is the value of k ?

- (1) 20 (3) 7
(2) 10 (4) 4

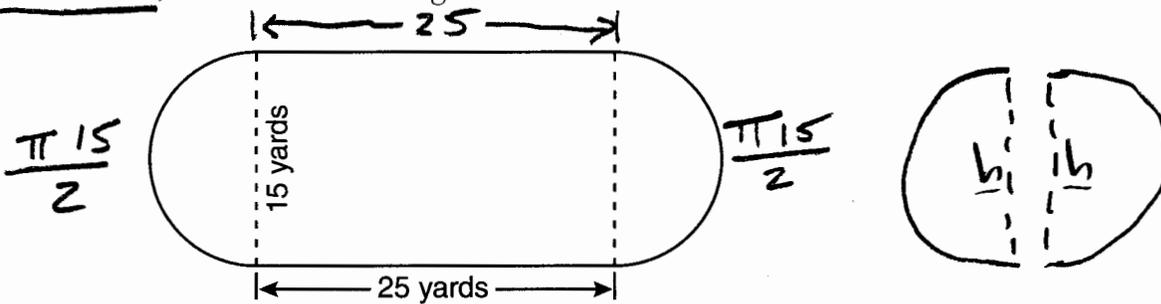
$$\begin{aligned}
 &5\sqrt{20} \\
 &5\sqrt{4 \times 5} \\
 &5\sqrt{4} \sqrt{5} \\
 &5(2)\sqrt{5} \\
 &10\sqrt{5} \\
 &k\sqrt{5} \quad \boxed{k=10}
 \end{aligned}$$

23 What is the value of the expression $|-5x + 12|$ when $x = 5$?

- (1) -37 (3) 13
(2) -13 (4) 37

$$\begin{aligned}
 &|-5x + 12| \\
 &|-5(5) + 12| \\
 &|-25 + 12| \\
 &|-13| \rightarrow \boxed{13}
 \end{aligned}$$

24 A playground in a local community consists of a rectangle and two semicircles, as shown in the diagram below.



Which expression represents the amount of fencing, in yards, that would be needed to completely enclose the playground?

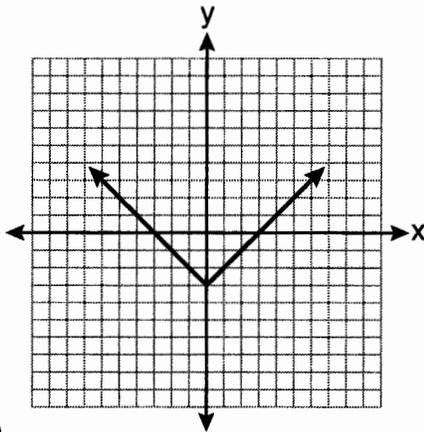
- (1) $15\pi + 50$ (3) $30\pi + 50$
(2) $15\pi + 80$ (4) $30\pi + 80$

The perimeter of a circle is πd
The perimeter of a semi-circle is $\frac{\pi d}{2}$

$$\begin{aligned}
 &25 + 25 + \frac{15\pi}{2} + \frac{15\pi}{2} \\
 &25 + 25 + \frac{15\pi + 15\pi}{2} \\
 &25 + 25 + \frac{30\pi}{2} \\
 &25 + 25 + 15\pi \\
 &\boxed{50 + 15\pi}
 \end{aligned}$$

25 Which equation is represented by the graph below?

Use this space for computations.



Check in graphing calculator.

~~(1) parabola~~
~~(2) parabola~~

- (3) $y = |x| - 3$
- (4) $y = |x - 3|$

26 Carrie bought new carpet for her living room. She calculated the area of the living room to be 174.2 square feet. The actual area was 149.6 square feet. What is the relative error of the area to the nearest ten-thousandth?

- (1) 0.1412
- (2) 0.1644
- (3) 1.8588
- (4) 2.1644

$$\frac{|\text{actual} - \text{measured}|}{\text{actual}}$$

$$\frac{|149.6 - 174.2|}{149.6}$$

27 What is an equation of the line that passes through the point (3, -1) and has a slope of 2?

- (1) $y = 2x + 5$
- (2) $y = 2x - 1$
- (3) $y = 2x - 4$
- (4) $y = 2x - 7$

↑ x ↑ y

$$|-0.164438|$$

0.1644

$$y = mx + b$$

$$y = 2x + b \Rightarrow y = 2x - 7$$

$$-1 = 2(3) + b$$

$$-1 = 6 + b$$

$$-7 = b$$

28 The ages of three brothers are consecutive even integers. ^{3x} Three times the age of the youngest brother exceeds the oldest brother's age by 48 years. What is the age of the *youngest* brother?

- (1) 14 (3) 22
 (2) 18 (4) 26

Use this space for computations.

x youngest
 x+2 middle
 x+4 oldest

$$3x = (x+4) + 48$$

$$3x = x + 52$$

$$2x = 52$$

$$x = 26$$

29 Cassandra bought an antique dresser for \$500. If the value of her dresser increases 6% annually, what will be the value of Cassandra's dresser at the end of 3 years to the *nearest dollar*?

- (1) \$415 (3) \$596
 (2) \$590 (4) \$770

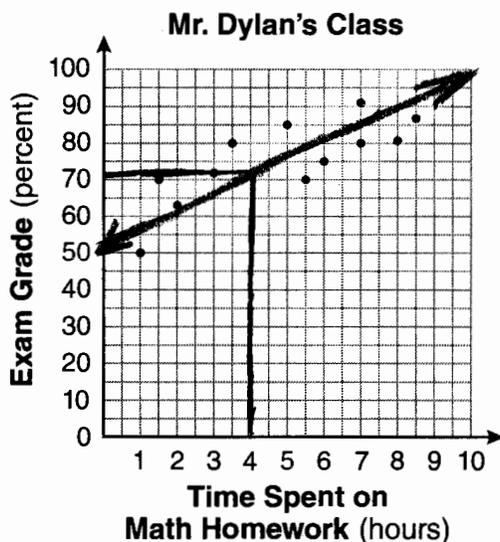
$$A = P(1+r)^t$$

$$A = 500(1+0.06)^3$$

$$A = 500(1.06)^3$$

$$A = 595.508$$

30 The number of hours spent on math homework each week and the final exam grades for twelve students in Mr. Dylan's algebra class are plotted below.



Based on a line of best fit, which exam grade is the best prediction for a student who spends about 4 hours on math homework each week?

- (1) 62 (3) 82
 (2) 72 (4) 92

Part II

Answer all 3 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

31 Chad complained to his friend that he had five equations to solve for homework. Are all of the homework problems equations? Justify your answer.

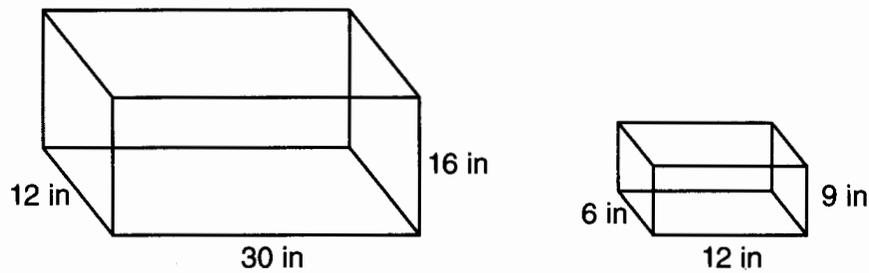
Math Homework

1. $3x^2 \cdot 2x^4$
2. $5 - 2x = 3x$
3. $3(2x + 7)$
4. $7x^2 + 2x - 3x^2 - 9$
5. $\frac{2}{3} = \frac{x+2}{6}$

Name Chad

No. All of the problems are not equations. An equation must have an equal sign. Three of the problems do not contain equal signs.

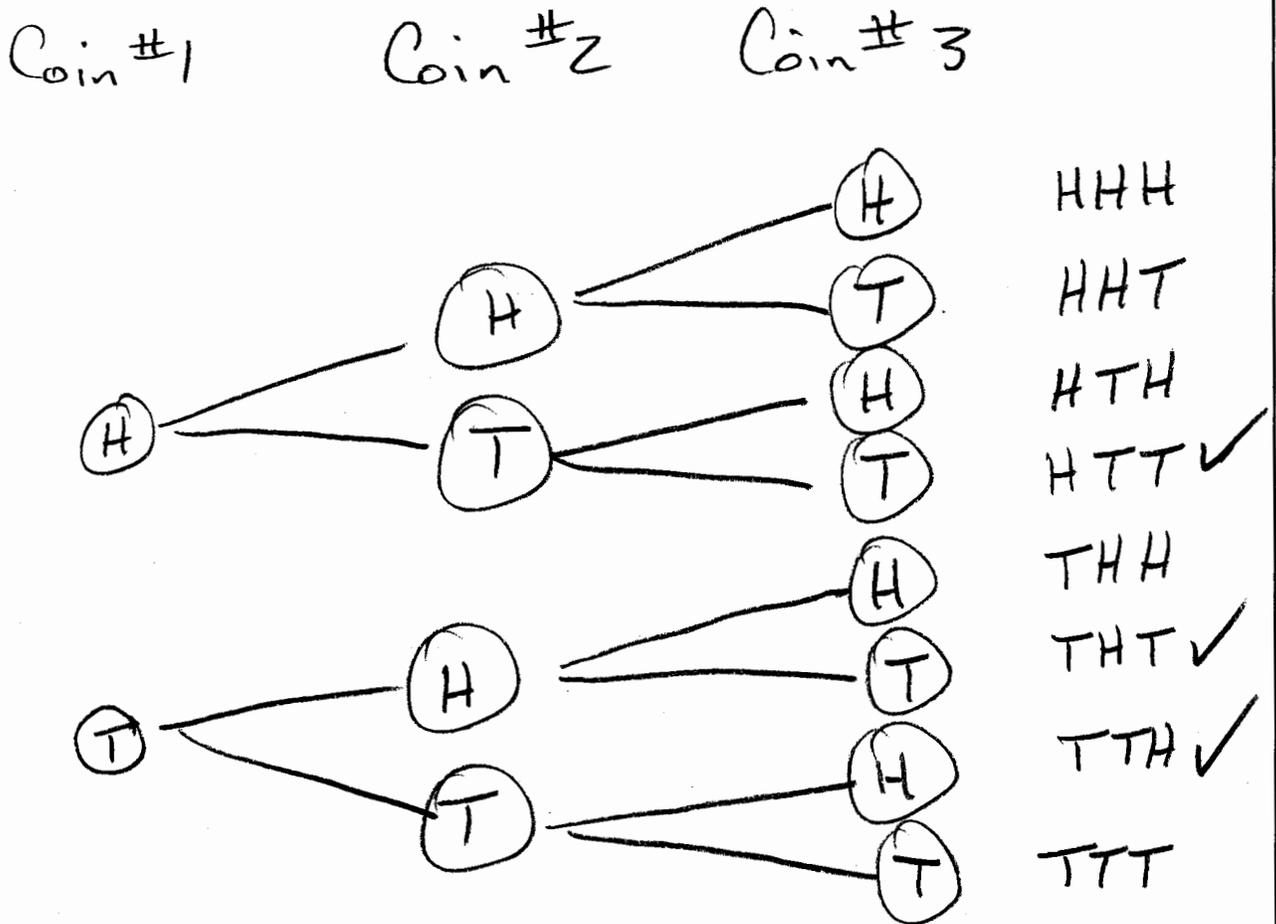
32 The diagram below represents Joe's two fish tanks.



Joe's larger tank is completely filled with water. He takes water from it to completely fill the small tank. Determine how many cubic inches of water will remain in the larger tank.

$$\text{Volume} = (\text{length})(\text{width})(\text{height})$$
$$(30)(12)(16) - (12)(6)(9) = 5,112$$

33 Clayton has three fair coins. Find the probability that he gets two tails and one head when he flips the three coins.



$$P(\text{event}) = \frac{\# \text{ times event happens}}{\# \text{ possible outcomes}}$$

$$P(2 \text{ tails and } 1 \text{ head}) = \boxed{\frac{3}{8}}$$

Part III

Answer all 3 questions in this part. Each correct answer will receive 3 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [9]

34 Find algebraically the equation of the axis of symmetry and the coordinates of the vertex of the parabola whose equation is $y = -2x^2 - 8x + 3$.

$$0 = ax^2 + bx + c$$

$$a = -2$$

$$b = -8$$

$$c = 3$$

$$\text{axis of symmetry} = \frac{-b}{2a}$$

$$x = \frac{-(-8)}{2(-2)}$$

$$x = \frac{8}{-4} = -2$$

axis of symmetry
 $x = -2$

$$y = -2x^2 - 8x + 3$$

$$y = -2(-2)^2 - 8(-2) + 3$$

$$y = -2(4) + 16 + 3$$

$$y = -8 + 19$$

$$y = 11$$

Vertex
 $(-2, 11)$
x y

Check in Graphing Calculator

x	y
-5	-7
-4	3
-3	9
-2	11
-1	9
0	3
1	-7

← This is the "mirror"

35 At the end of week one, a stock had increased in value from \$5.75 a share to \$7.50 a share. Find the percent of increase at the end of week one to the nearest tenth of a percent.

At the end of week two, the same stock had decreased in value from \$7.50 to \$5.75. Is the percent of decrease at the end of week two the same as the percent of increase at the end of week one? Justify your answer.

$$5.75(1+r) = 7.50$$

$$5.75 + 5.75r = 7.50$$

$$5.75r = 7.50 - 5.75$$

$$r = \frac{7.50 - 5.75}{5.75}$$

$$r = .3043478261$$

% increase at
end of week one

→ $r = 30.4\%$

$$7.50(1-r) = 5.75$$

$$7.50 - 7.50r = 5.75$$

$$-7.50r = 5.75 - 7.50$$

$$r = \frac{5.75 - 7.50}{-7.50}$$

$$r = .233$$

$$r = 23.3\%$$

No The stock increased 30.4% the first week and decreased 23.3% the second week.

36 The chart below compares two runners.

Runner	Distance, in miles	Time, in hours
Greg	11	2
Dave	16	3

Based on the information in this chart, state which runner has the faster rate. Justify your answer.

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$\text{Greg} = \frac{11}{2} = 5\frac{1}{2} \text{ mph}$$

$$\text{Dave} = \frac{16}{3} = 5\frac{1}{3} \text{ mph}$$

$$5\frac{1}{2} > 5\frac{1}{3}$$

Greg has the faster rate

Part IV

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

37 Express in simplest form: $\left(\frac{2x^2 - 8x - 42}{6x^2}\right) \div \left(\frac{x^2 - 9}{x^2 - 3x}\right)$

To divide fractions, multiply by the inverse of the denominator.

$$\left(\frac{2x^2 - 8x - 42}{6x^2}\right) \left(\frac{x^2 - 3x}{x^2 - 9}\right)$$

difference of perfect squares

$$\frac{\cancel{2}(x^2 - 4x - 21)\cancel{(x)}\cancel{(x-3)}}{\cancel{2}\cancel{3}\cancel{(x)}\cancel{(x)}(x+3)\cancel{(x-3)}}$$

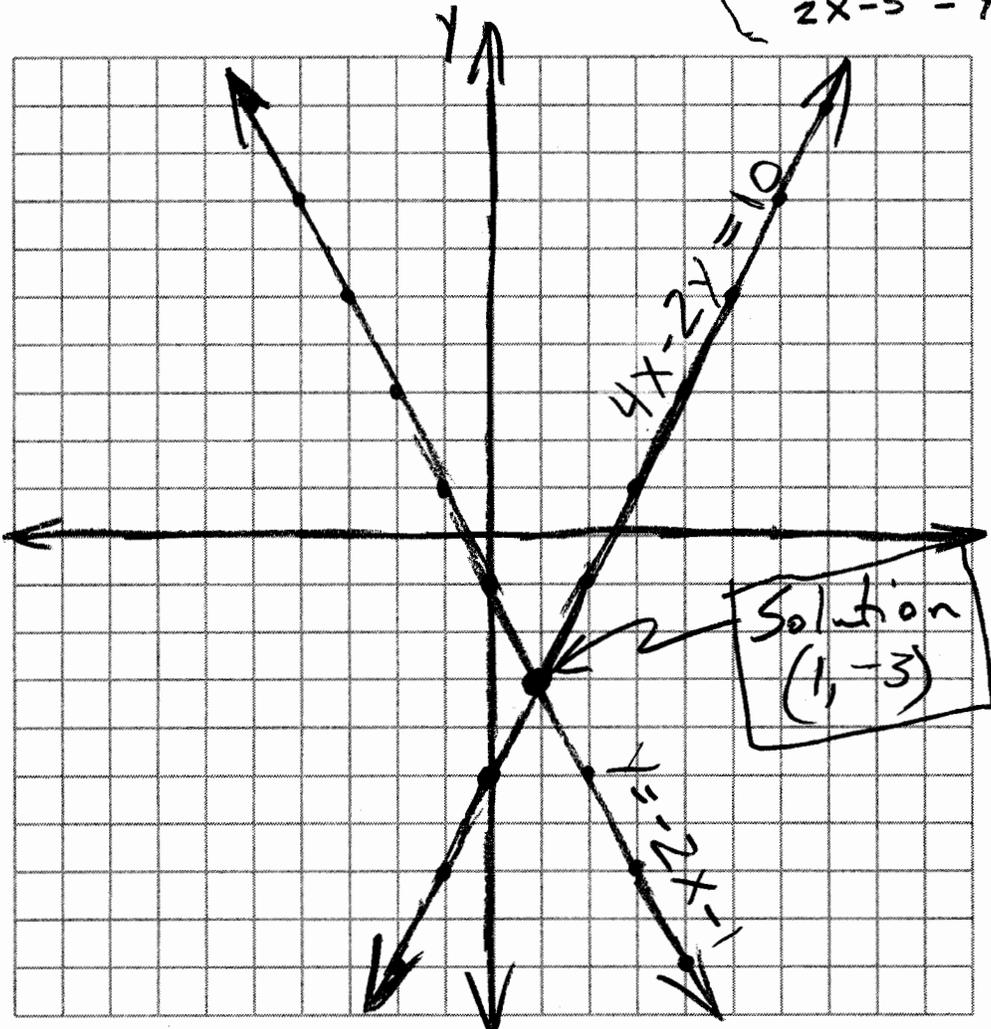
$$\frac{(x-7)\cancel{(x+3)}}{\cancel{3}\cancel{(x)}\cancel{(x+3)}}$$

$$\boxed{\frac{x-7}{3x}}$$

38 On the grid below, solve the system of equations graphically for x and y .

$$4x - 2y = 10$$
$$y = -2x - 1$$

$$\left\{ \begin{array}{l} \frac{4x - 2y}{2} = \frac{10}{2} \\ 2x - y = 5 \\ 2x - 5 = y \end{array} \right.$$



$y = mx + b$ — slope
— y -intercept

$$y = 2x - 5$$
$$y = -2x - 1$$

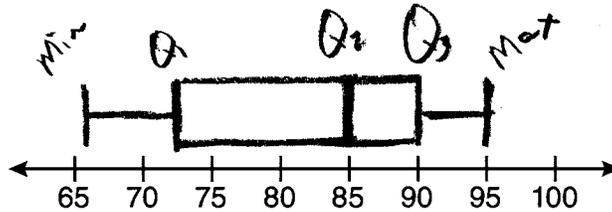
} Use slope intercept form to graph the equations

39 The test scores from Mrs. Gray's math class are shown below.

72, 73, 66, 71, 82, 85, 95, 85, 86, 89, 91, 92

66 71 72 | 73 82 85 | 85 86 89 | 91 92 95

Construct a box-and-whisker plot to display these data.



$$\text{Min} = 66$$

$$Q1 = 72.5$$

$$Q2 = 85$$

$$Q3 = 90$$

$$\text{Max} = 95$$