

INTEGRATED ALGEBRA

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

INTEGRATED ALGEBRA

Friday, June 18, 2010—1:15 to 4:15 p.m., only

Student Name: Steve Watson

School Name: JMAP.ORG

Print your name and the name of your school on the lines above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

This examination has four parts, with a total of 39 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

The use of any communications device is strictly prohibited when taking this examination. If you use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 30 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [60]

Use this space for computations.

1 Given:

$$\text{Set } U = \{S, \emptyset, P, H, I, A\}$$

$$\text{Set } B = \{A, I, \emptyset\}$$

If set B is a subset of set U , what is the complement of set B ?

- (1) $\{O, P, S\}$ (3) $\{A, H, P\}$
 (2) $\{I, P, S\}$ (4) $\{H, P, S\}$

2 How many different sandwiches consisting of one type of cheese, one condiment, and one bread choice can be prepared from five types of cheese, two condiments, and three bread choices?

- (1) 10 (3) 15
 (2) 13 (4) 30

Cheese Choices Condiment Choices Bread Choices

$$\boxed{5} \times \boxed{2} \times \boxed{3} = 30$$

3 The sum of $4x^3 + 6x^2 + 2x - 3$ and $3x^3 + 3x^2 - 5x - 5$ is

- (1) $7x^3 + 3x^2 - 3x - 8$ (3) $7x^3 + 9x^2 - 3x - 8$
 (2) $7x^3 + 3x^2 + 7x + 2$ (4) $7x^6 + 9x^4 - 3x^2 - 8$

add

$$\begin{array}{r} 4x^3 + 6x^2 + 2x - 3 \\ 3x^3 + 3x^2 - 5x - 5 \\ \hline 7x^3 + 9x^2 - 3x - 8 \end{array}$$

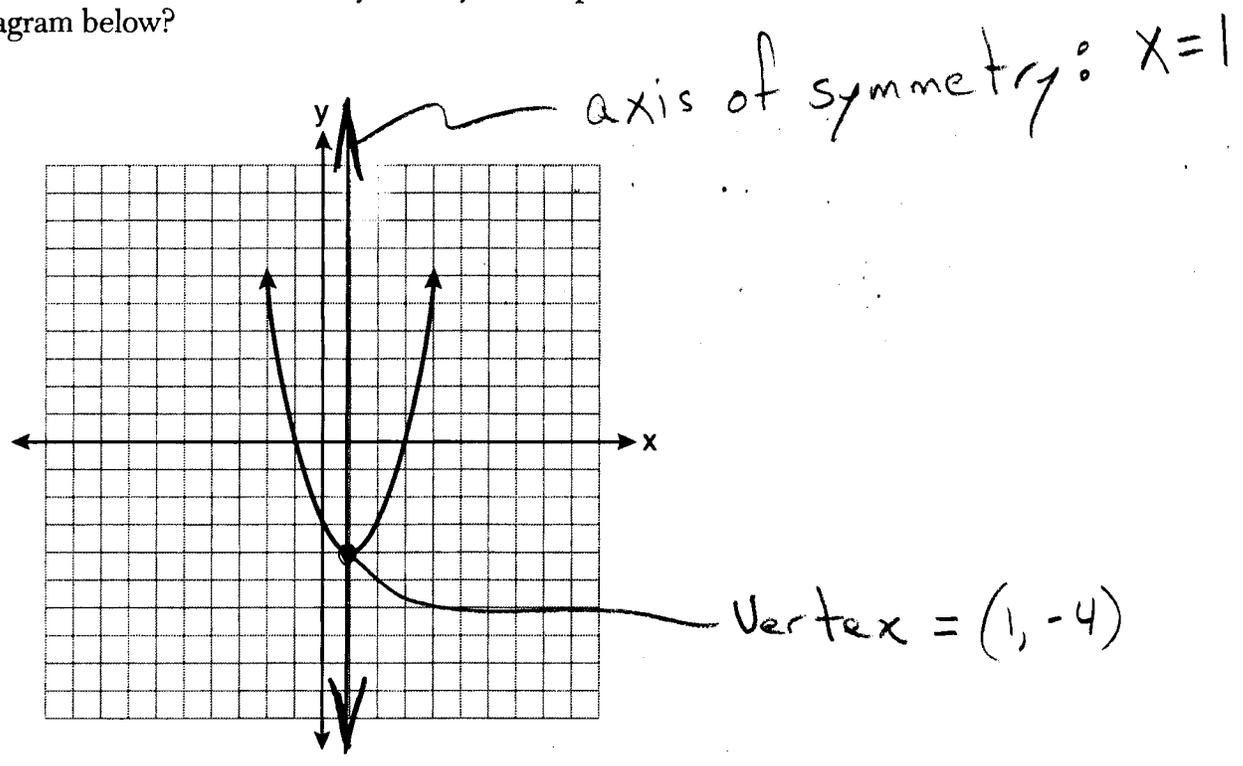
4 What is the slope of the line that passes through the points $(3,5)$ and $(-2,2)$? (x_1, y_1) Use this space for computations.

- (1) $\frac{1}{5}$ (3) $\frac{5}{3}$
 (2) $\frac{3}{5}$ (4) 5

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{2 - 5}{-2 - 3} = \frac{-3}{-5} = \frac{3}{5}$$

5 What are the vertex and axis of symmetry of the parabola shown in the diagram below?



- (1) vertex: $(1, -4)$; axis of symmetry: $x = 1$
 (2) vertex: $(1, -4)$; axis of symmetry: $x = -4$
 (3) vertex: $(-4, 1)$; axis of symmetry: $x = 1$
 (4) vertex: $(-4, 1)$; axis of symmetry: $x = -4$

Use this space for computations.

- 6 Three high school juniors, Reese, Matthew, and Chris, are running for student council president. A survey is taken a week before the election asking 40 students which candidate they will vote for in the election. The results are shown in the table below.

Candidate's Name	Number of Students Supporting Candidate
Reese	15
Matthew	13
Chris	12

Total 40

$$P(\text{event}) = \frac{\text{desired outcome}}{\text{total possible}}$$

$$P(\text{Reese}) = \frac{15}{40} = \boxed{\frac{3}{8}}$$

Based on the table, what is the probability that a student will vote for Reese?

(1) $\frac{1}{3}$

(3) $\frac{3}{8}$

(2) $\frac{3}{5}$

(4) $\frac{5}{8}$

- 7 Which linear equation represents a line containing the point $(1,3)$?

(1) $x + 2y = 5$

(3) $2x + y = 5$

(2) $x - 2y = 5$

(4) $2x - y = 5$

(1) $x + 2y \stackrel{?}{=} 5$
 $1 + 2(3) \stackrel{?}{=} 5$
 $7 \neq 5$

(2) $x - 2y \stackrel{?}{=} 5$
 $1 - 2(3) \stackrel{?}{=} 5$
 $-5 \neq 5$

(3) $2x + y = 5$
 $2(1) + 3 = 5$
 $5 = 5 \checkmark$

- 8 The expression $\sqrt{72} - 3\sqrt{2}$ written in simplest radical form is

(1) $5\sqrt{2}$

(3) $3\sqrt{2}$

(2) $3\sqrt{6}$

(4) $\sqrt{6}$

$$\begin{aligned} &\sqrt{72} - 3\sqrt{2} \\ &\sqrt{2} \sqrt{36} - 3\sqrt{2} \\ &\sqrt{2} 6 - 3\sqrt{2} \\ &6\sqrt{2} - 3\sqrt{2} \\ &3\sqrt{2} \end{aligned}$$

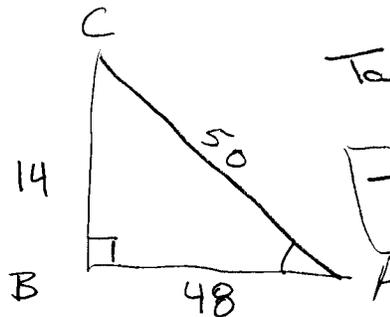
9 In $\triangle ABC$, the measure of $\angle B = 90^\circ$, $AC = 50$, $AB = 48$, and $BC = 14$. Which ratio represents the tangent of $\angle A$?

(1) $\frac{14}{50}$

(3) $\frac{48}{50}$

(2) $\frac{14}{48}$

(4) $\frac{48}{14}$



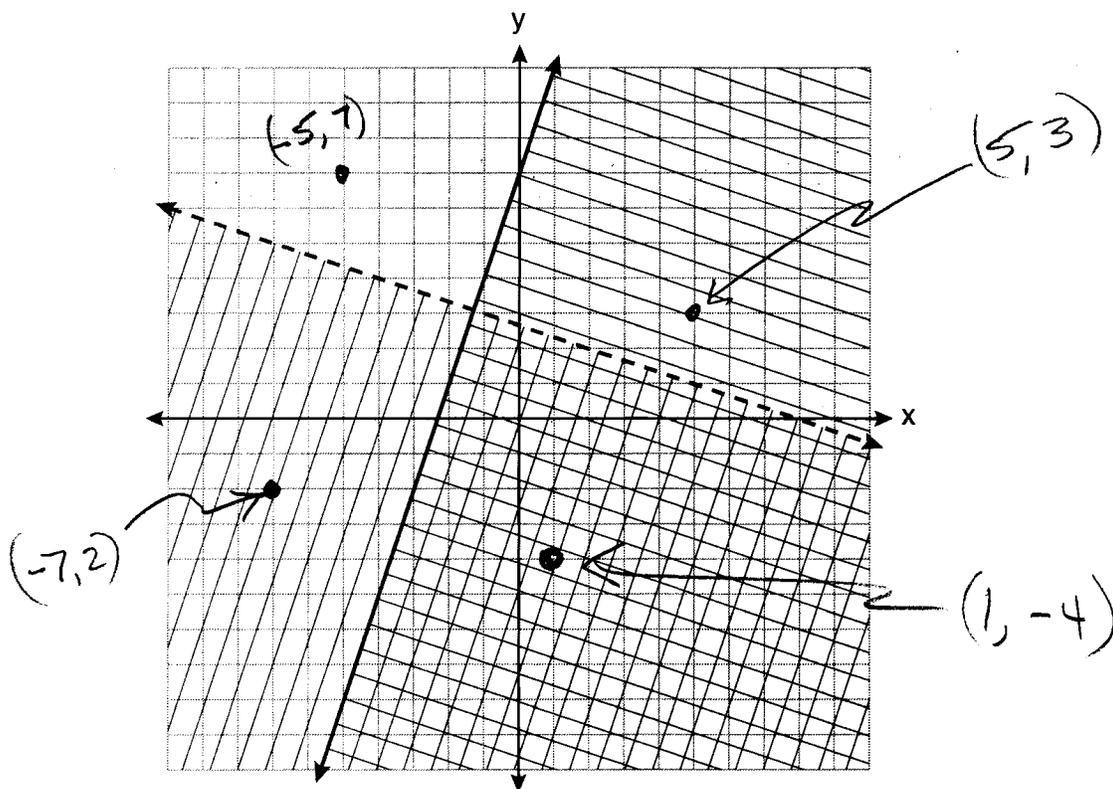
Use this space for computations.

SOH-CAH-TOA

Tangent = $\frac{\text{opposite}}{\text{adjacent}}$

$\tan A = \frac{14}{48}$

10 Which ordered pair is in the solution set of the system of linear inequalities graphed below?



(1) $(1, -4)$

(3) $(5, 3)$

(2) $(-5, 7)$

(4) $(-7, -2)$

Use this space for computations.

11 Which table does not show bivariate data?

(1)

Height (inches)	Weight (pounds)
39	50
48	70
60	90

2 variables
Height + Weight

(2)

Gallons	Miles Driven
15	300
20	400
25	500

2 variables
Gallons + Miles

(3)

Quiz Average	Frequency
70	12
80	15
90	6

1 variable
Quiz Average + How Many of Quiz Average

(4)

Speed (mph)	Distance (miles)
40	80
50	120
55	150

2 variables
Speed + Distance

Bivariate means two variables
bi and variate
2 - variables

12 What is the solution of the system of equations $c + 3d = 8$ and $c = 4d - 6$?

(1) $c = -14, d = -2$

(3) $c = 2, d = 2$

(2) $c = -2, d = 2$

(4) $c = 14, d = -2$

Step 1 $c + 3d = 8$

$c = 4d - 6 \Rightarrow c - 4d = -6$

Step 2

$c + 3d = 8$

$c - 4d = -6$

$7d = 14$

$d = 2$

Step 3

$c + 3d = 8$

$c + 3(2) = 8$

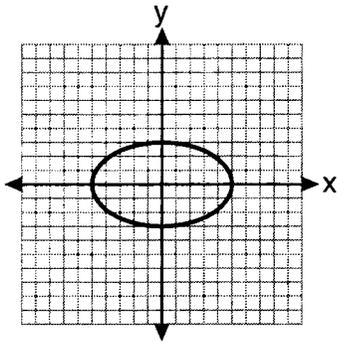
$c + 6 = 8$

$c = 2$

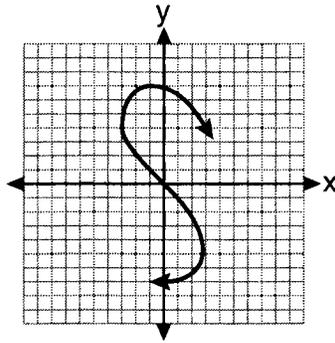
D(1)

Use this space for computations.

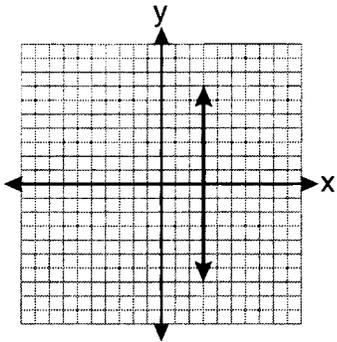
13 Which graph represents a function?



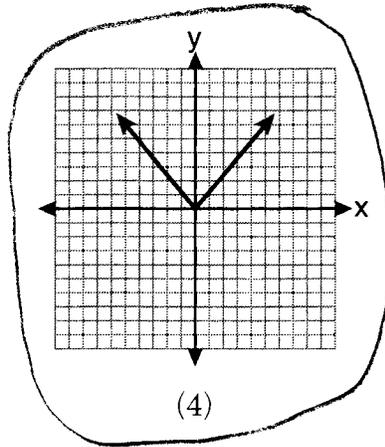
(1)



(3)



(2)



(4)

Use vertical line test

14 The algebraic expression $\frac{x-2}{x^2-9}$ is undefined when x is

(1) 0

(2) 2

(3) 3

(4) 9

Undefined occurs when the denominator equals zero.

$$x^2 - 9 = 0$$

$$x^2 = 9$$

$$x = \pm 3$$

Use this space for computations.

15 The graphs of the equations $y = 2x - 7$ and $y - kx = 7$ are parallel when k equals

- (1) -2
- (2) 2
- (3) -7
- (4) 7

↳ same slope, but different y-intercepts

$$y = 2x - 7$$

↳ slope = 2

$$y - kx = 7$$

$$y = kx + 7$$

↳ when $k = 2$, slopes are the same

16 Which verbal expression is represented by $\frac{1}{2}(n - 3)$?

- (1) one-half n decreased by 3 $\Rightarrow \frac{1}{2}n - 3$
- (2) one-half n subtracted from 3 $\Rightarrow 3 - \frac{1}{2}n$
- (3) the difference of one-half n and 3 $\Rightarrow \frac{1}{2}n - 3$
- (4) one-half the difference of n and 3

17 The freshman class held a canned food drive for 12 weeks. The results are summarized in the table below.

Canned Food Drive Results

Week	1	2	3	4	5	6	7	8	9	10	11	12
Number of Cans	20	35	32	45	58	46	28	23	31	79	65	62

Which number represents the second quartile of the number of cans of food collected?

- (1) 29.5
- (2) 30.5
- (3) 40
- (4) 60

Q_1 Q_2 Q_3
 \downarrow
 Median

Handwritten calculations for quartiles:

Q_1 : 20 23 28 | 31 32 35
 $\frac{28+31}{2} = 29.5$

Q_2 : 35 40 | 45 46
 $\frac{35+40}{2} = 40$

Q_3 : 58 | 62 65 79
 $\frac{58+62}{2} = 60$

18 Which expression represents $\frac{-14a^2c^8}{7a^3c^2}$ in simplest form?

(1) $-2ac^4$

(3) $\frac{-2c^4}{a}$

(2) $-2ac^6$

(4) $\frac{-2c^6}{a}$

Use this space for computations.

$$\begin{array}{r|l} -14 & a^2 c^8 \\ \hline 7 & a^3 c^2 \\ \hline -2 & a^{(2-3)} c^{(8-2)} \\ -2 & a^{-1} c^6 \Rightarrow \frac{-2c^6}{a} \end{array}$$

19 Which value of x is the solution of $\frac{x}{3} + \frac{x+1}{2} = x$?

(1) 1

(3) 3

(2) -1

(4) -3

M(6) $\frac{x}{3} + \frac{x+1}{2} = x$

$2x + 3x + 3 = 6x$

$5x + 3 = 6x$

$3 = x$

20 When 36 is subtracted from the square of a number, the result is five times the number. What is the positive solution?

(1) 9

(3) 3

(2) 6

(4) 4

$-36 + x^2 = 5x$
 $x^2 - 5x - 36 = 0$
 $(x-9)(x+4) = 0$

$x - 9 = 0$
 $x = 9$

$x + 4 = 0$
 $x = -4$

21 Which interval notation represents the set of all numbers greater than or equal to 5 and less than 12?

(1) $[5, 12)$

(3) $(5, 12)$

(2) $(5, 12]$

(4) $[5, 12]$

[includes the number]

(does not include the number)

$5 \leq x < 12$

$[5, 12)$

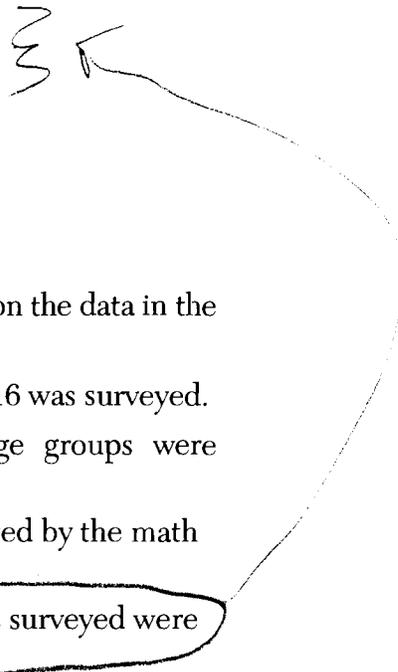
Use this space for computations.

- 22 Four hundred licensed drivers participated in the math club's survey on driving habits. The table below shows the number of drivers surveyed in each age group.

Ages of People in Survey on Driving Habits

Age Group	Number of Drivers
16-25	150
26-35	129
36-45	33
46-55	57
56-65	31

Total 400



Which statement best describes a conclusion based on the data in the table?

- (1) It may be biased because no one younger than 16 was surveyed.
- (2) It would be fair because many different age groups were surveyed.
- (3) It would be fair because the survey was conducted by the math club students.
- (4) It may be biased because the majority of drivers surveyed were in the younger age intervals.

- 23 A formula used for calculating velocity is $v = \frac{1}{2}at^2$. What is a expressed in terms of v and t ?

(1) $a = \frac{2v}{t}$

(2) $a = \frac{2v}{t^2}$

(3) $a = \frac{v}{t}$

(4) $a = \frac{v}{2t^2}$

M(2) $v = \frac{1}{2}at^2$
 $2v = at^2$

D(t²) $\frac{2v}{t^2} = a$

Use this space for computations.

24 What is the sum of $\frac{-x+7}{2x+4}$ and $\frac{2x+5}{2x+4}$?

(1) $\frac{x+12}{2x+4}$

(3) $\frac{x+12}{4x+8}$

(2) $\frac{3x+12}{2x+4}$

(4) $\frac{3x+12}{4x+8}$

common denominator, so add the numerators

$$\left(\frac{-x+7}{2x+4}\right) + \left(\frac{2x+5}{2x+4}\right) = \frac{-x+7+2x+5}{2x+4} = \frac{x+12}{2x+4}$$

25 Steve ran a distance of 150 meters in $1\frac{1}{2}$ minutes. What is his speed in meters per hour?

(1) 6

(3) 100

(2) 60

(4) 6,000

$$\frac{\text{Meters}}{\text{Minutes}} = \frac{150}{1.5} = \frac{x}{60}$$

Cross Multiply $60(150) = 1.5x$
 $9000 = 1.5x$
 $6000 = x$

26 How many different three-letter arrangements can be formed using the letters in the word ABSOLUTE if each letter is used only once?

(1) 56

(3) 168

(2) 112

(4) 336

ABSOLUTE has 8 letters

Choices for 1st Letter: 8
 Choices for 2nd Letter: 7
 Choices for 3rd Letter: 6

$$8 \times 7 \times 6 = 336$$

27 Factored completely, the expression $3x^2 - 3x - 18$ is equivalent to

(1) $3(x^2 - x - 6)$

(3) $(3x - 9)(x + 2)$

(2) $3(x - 3)(x + 2)$

(4) $(3x + 6)(x - 3)$

$$3x^2 - 3x - 18$$

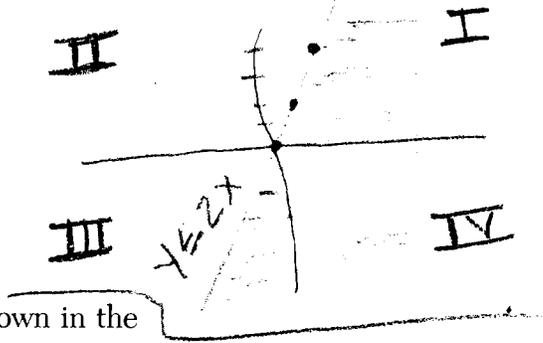
$$3(x^2 - x - 6)$$

$$3(x-3)(x+2)$$

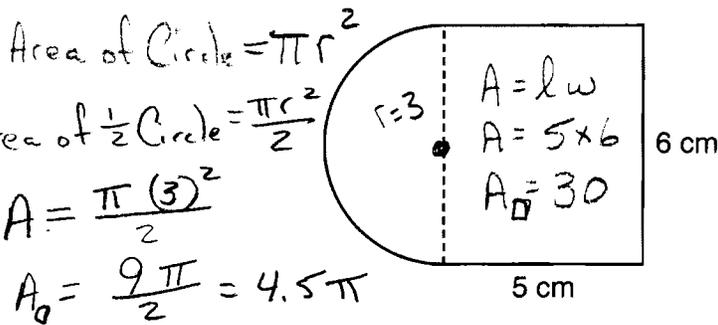
28 Which quadrant will be completely shaded in the graph of the inequality $y \leq 2x$?

- (1) Quadrant I
- (2) Quadrant II
- (3) Quadrant III
- (4) Quadrant IV

Use this space for computations.



29 A figure is made up of a rectangle and a semicircle as shown in the diagram below.



$$\begin{aligned} \text{Total Area} &= 4.5\pi + 30 \\ &= 44.13716694 \\ &= 44.1 \text{ cm}^2 \end{aligned}$$

What is the area of the figure, to the nearest tenth of a square centimeter?

- (1) 39.4
- (2) 44.1
- (3) 48.8
- (4) 58.3

30 The value, y , of a \$15,000 investment over x years is represented by the equation $y = 15000(1.2)^{\frac{x}{3}}$. What is the profit (interest) on a 6-year investment?

- (1) \$6,600
- (2) \$10,799
- (3) \$21,600
- (4) \$25,799

$$\begin{aligned} \text{Profit} &= \text{Revenues} \\ &\quad \text{minus} \\ &\quad \text{Expenses} \end{aligned}$$

21,600
↓
15,000

$$\begin{aligned} y &= 15000(1.2)^{\frac{x}{3}} \\ y &= 15000(1.2)^{\frac{6}{3}} \\ y &= 15000(1.2)^2 \\ y &= 15000(1.44) \\ y &= 21,600 \end{aligned}$$

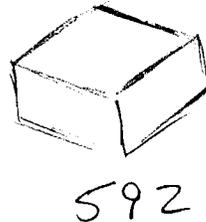
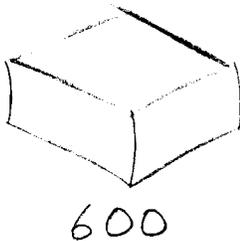
$$\begin{aligned} P &= 21,600 - 15,000 \\ P &= \boxed{6,600} \end{aligned}$$

This is what the investment is worth after 6 years.

Part II

Answer all 3 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [6]

- 31 Alexis calculates the surface area of a gift box as 600 square inches. The actual surface area of the gift box is 592 square inches. Find the relative error of Alexis' calculation expressed as a decimal to the nearest thousandth.



$$\frac{600 - 592}{592} = \frac{8}{592}$$

$$\frac{8}{592} = .0135135$$

$$\boxed{.014}$$

32 Perform the indicated operation: $-6(a - 7)$

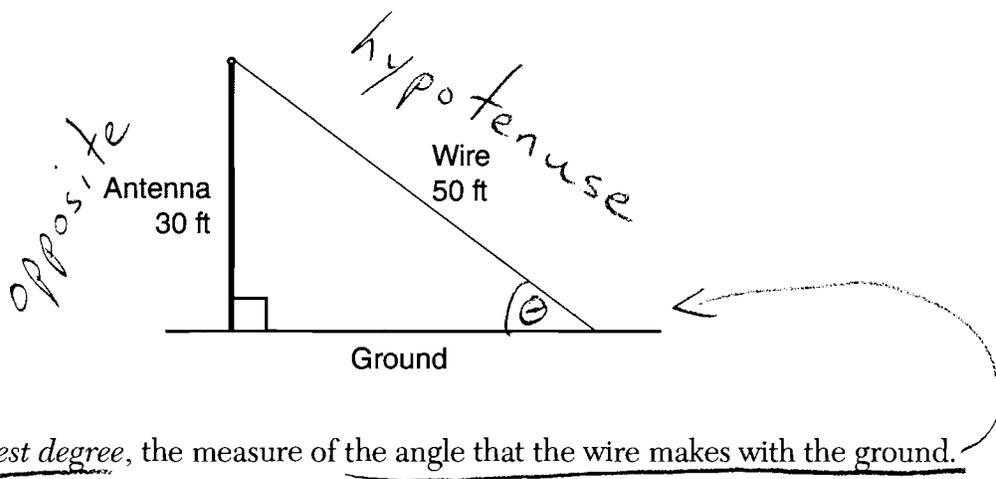
State the name of the property used.

$$-6(a - 7)$$

$$-6a + 42$$

Distributive Property

- 33 A communications company is building a 30-foot antenna to carry cell phone transmissions. As shown in the diagram below, a 50-foot wire from the top of the antenna to the ground is used to stabilize the antenna.



Find, to the nearest degree, the measure of the angle that the wire makes with the ground.

SOH - CAH - TOA

$$\sin = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{30}{50}$$

$$m\angle \theta = \arcsin \frac{30}{50}$$

$$m\angle \theta = \boxed{\text{second}} \boxed{\sin} \boxed{3} \boxed{0} \boxed{\div} \boxed{5} \boxed{0} \boxed{\text{Enter}}$$

$$m\angle \theta = 36.86989765$$

$$\boxed{m\angle \theta = 37^\circ}$$

Part III

Answer all 3 questions in this part. Each correct answer will receive 3 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [9]

34 Given: $A = \{18, 6, -3, -12\}$

Determine all elements of set A that are in the solution of the inequality $(\frac{2}{3}x + 3 < -2x - 7) \times 3$

$$2x + 9 < -6x - 21$$

$$\frac{8x}{8} < \frac{-30}{8}$$

$$x < -3\frac{3}{4}$$

-12 is less than $-3\frac{3}{4}$

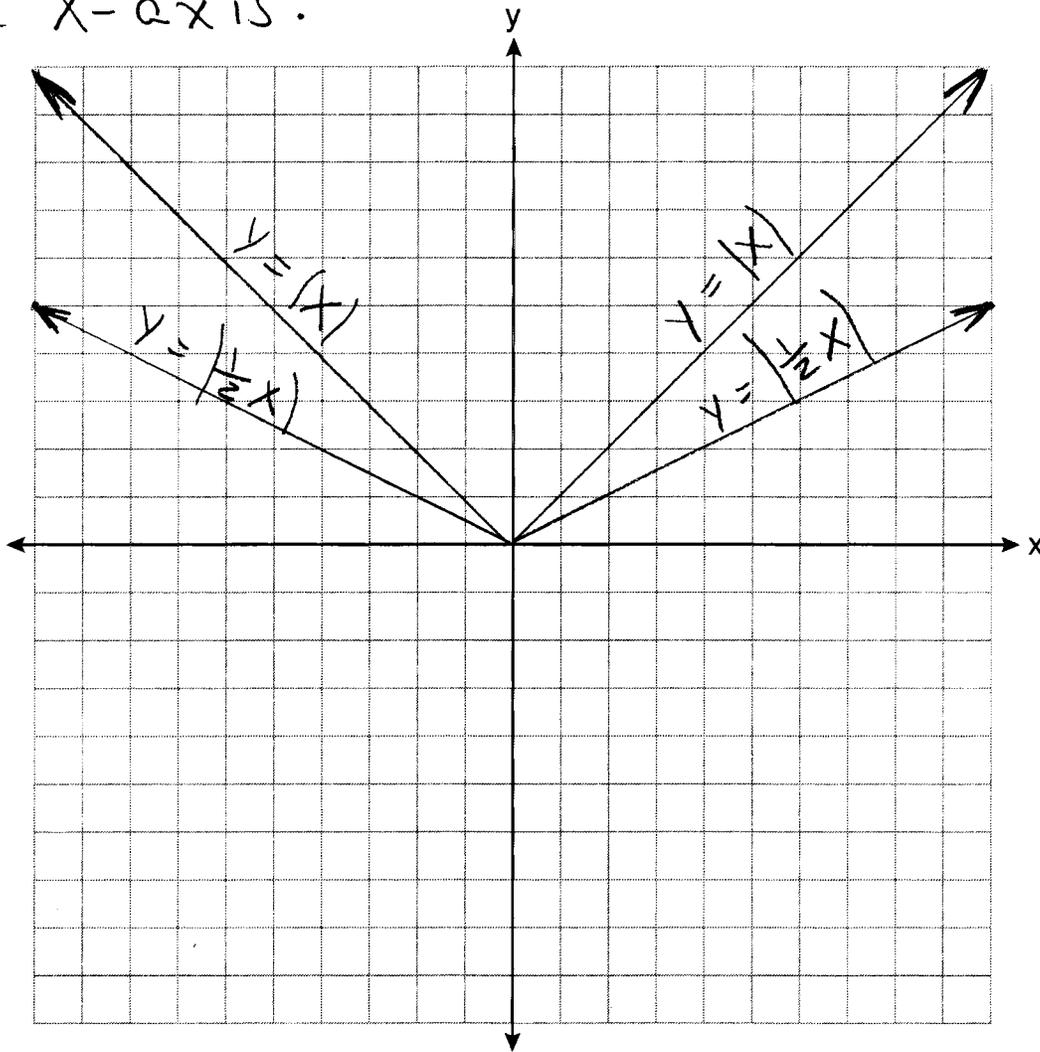
35 Graph and label the following equations on the set of axes below.

$$y = |x|$$

$$y = \left|\frac{1}{2}x\right|$$

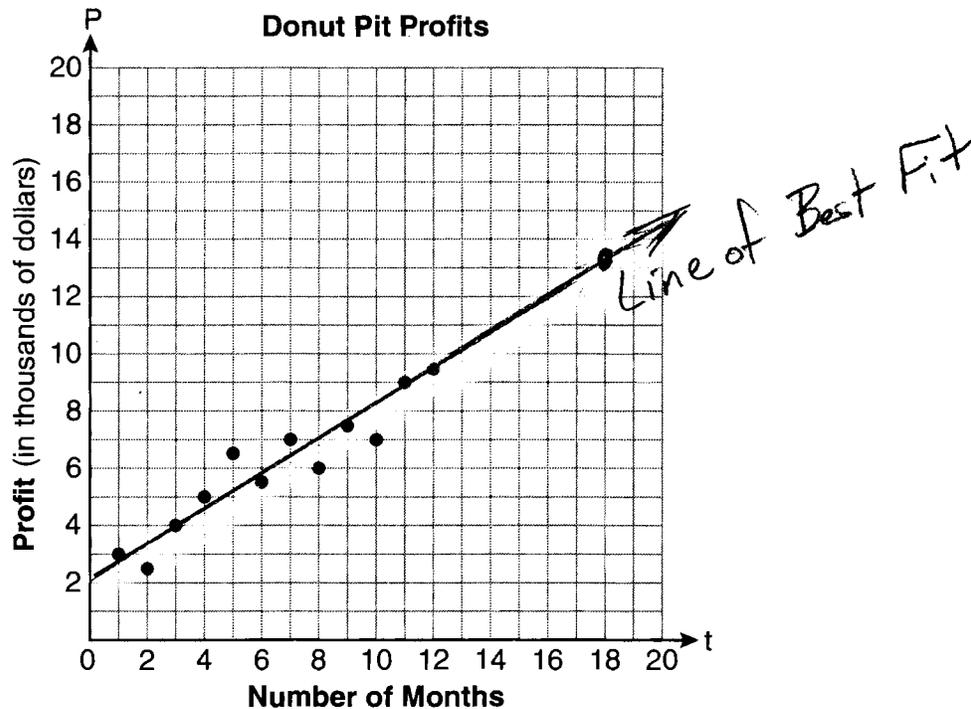
Explain how *decreasing* the coefficient of x affects the graph of the equation $y = |x|$.

As the coefficient of x approaches zero, the graph of $y = |x|$ approaches the x -axis.



36 Megan and Bryce opened a new store called the Donut Pit. Their goal is to reach a profit of \$20,000 in their 18th month of business. The table and scatter plot below represent the profit, P , in thousands of dollars, that they made during the first 12 months.

t (months)	1	2	3	4	5	6	7	8	9	10	11	12
P (profit, in thousands of dollars)	3.0	2.5	4.0	5.0	6.5	5.5	7.0	6.0	7.5	7.0	9.0	9.5



Draw a reasonable line of best fit.

Using the line of best fit, predict whether Megan and Bryce will reach their goal in the 18th month of their business.

Justify your answer.

No They will not meet their goal. The line of best fit suggests they will have less than \$14,000 profit after 18 months.

Part IV

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

37 Express in simplest form: $\frac{x^2 + 9x + 14}{x^2 - 49} \div \frac{3x + 6}{x^2 + x - 56}$

$$\frac{x^2 + 9x + 14}{x^2 - 49} \quad \div \quad \frac{3x + 6}{x^2 + x - 56}$$

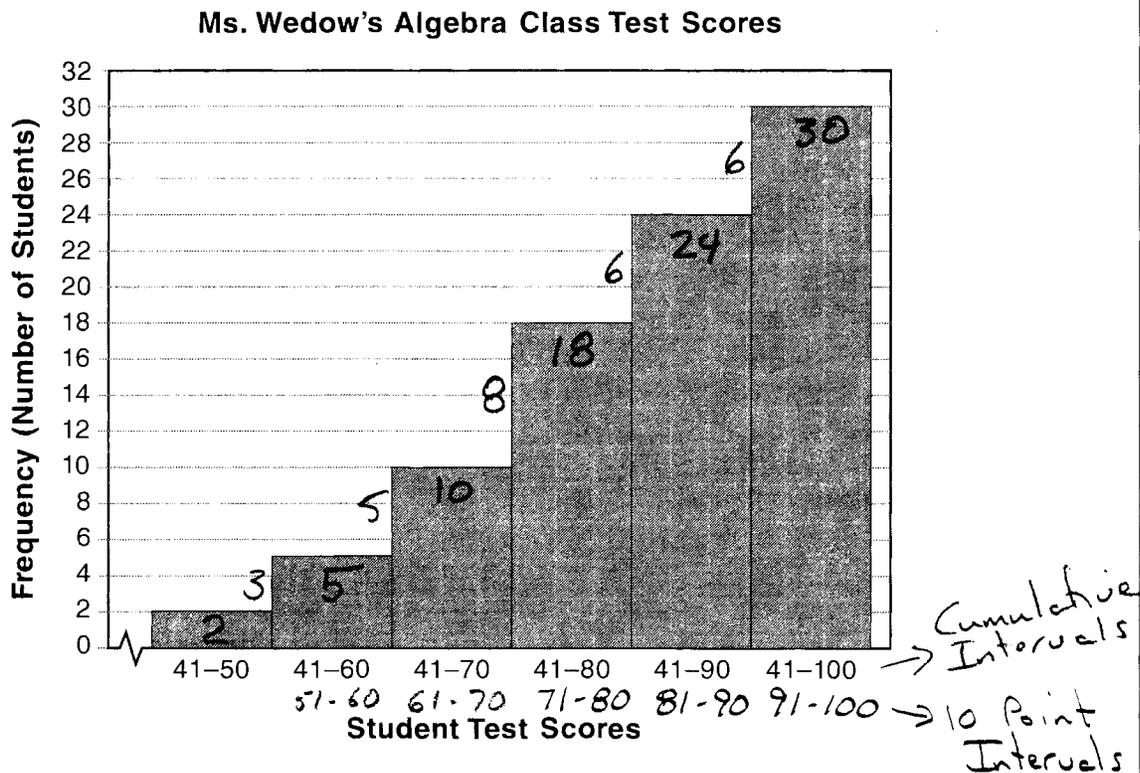
$$\frac{(x+7)(x+2)}{(x+7)(x-7)} \quad \div \quad \frac{3(x+2)}{(x+8)(x-7)}$$

keep change Flip

$$\frac{\cancel{(x+7)}(x+2)}{(x+7)(x-7)} \quad \times \quad \frac{(x+8)(x-7)}{3(x+2)}$$

$$\frac{x+8}{3}$$

- 38 The diagram below shows a cumulative frequency histogram of the students' test scores in Ms. Wedow's algebra class.



Determine the total number of students in the class.

30 Students

Determine how many students scored higher than 70.

$$8 + 6 + 6 = \boxed{20}$$

State which ten-point interval contains the median.

The 15th + 16th (middle) scores are in the 71-80 interval

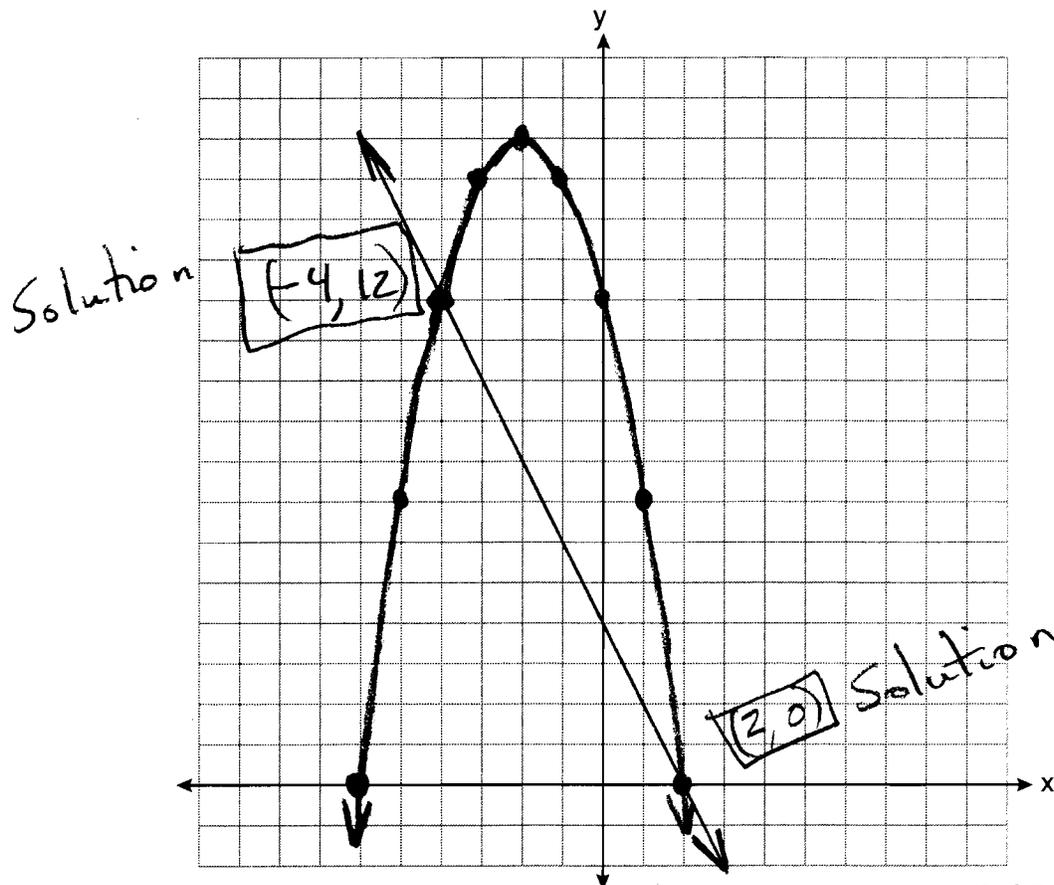
State which two ten-point intervals contain the same frequency.

The 81-90 and the 91-100 ten point intervals both have frequencies of six students.

39 On the set of axes below, solve the following system of equations graphically for all values of x and y .

$$y = -x^2 - 4x + 12$$

$$y = -2x + 4$$



From Graphing Calculator

$$Y_1 = -x^2 - 4x + 12$$

$$Y_2 = -2x + 4$$

x	Y_1	Y_2
-5	7	14
-4	12	12
-3	15	10
-2	16	8
-1	15	6
0	12	4
1	7	2
2	0	0