

The University of the State of New York  
REGENTS HIGH SCHOOL EXAMINATION

# INTEGRATED ALGEBRA

Thursday, June 16, 2011—1:15 to 4:15 p.m., only

Student Name: Steve Watson

School Name: IHS @ PH

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 39 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

**Notice...**

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

The use of any communications device is strictly prohibited when taking this examination. If you use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

**DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.**

Part I

Answer all 30 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [60]

Difference of perfect squares:  $X^2 - Y^2 = (X+Y)(X-Y)$   
Use this space for computations.

1 The expression  $x^2 - 36y^2$  is equivalent to

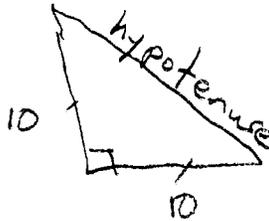
- (1)  $(x - 6y)(x - 6y)$        (2)  $(x + 6y)(x - 6y)$   
 (3)  $(x - 18y)(x - 18y)$       (4)  $(x + 18y)(x - 18y)$

$$x^2 - 36y^2$$

$$(x + 6y)(x - 6y)$$

2 The legs of an isosceles right triangle each measure 10 inches. What is the length of the hypotenuse of this triangle, to the nearest tenth of an inch?

- (1) 6.3       (3) 14.1  
 (2) 7.1      (4) 17.1



Pythagorean Theorem

$$a^2 + b^2 = c^2$$

$$(10)^2 + (10)^2 = c^2$$

$$100 + 100 = c^2$$

$$200 = c^2$$

$$\sqrt{200} = \sqrt{c^2}$$

$$\downarrow$$

$$14.14213562 = c$$

3 The expression  $\frac{12w^9y^3}{-3w^3y^3}$  is equivalent to

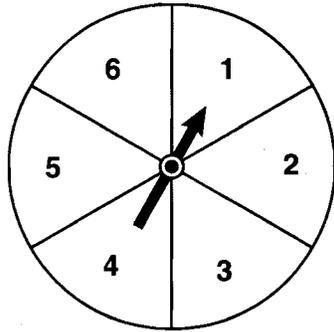
- (1)  $-4w^6$       (3)  $9w^6$   
 (2)  $-4w^3y$       (4)  $9w^3y$

12	$w^9$	$y^3$
-3	$w^3$	$y^3$
-4	$w^{(9-3)}$	$y^{(3-3)}$
-4	$w^6$	$y^0$
-4	$w^6$	

$-4w^6$  answer

Use this space for computations.

4 The spinner shown in the diagram below is divided into six equal sections.



Probability =  $\frac{\# \text{ desired outcomes}}{\# \text{ total outcomes}}$

Which outcome is *least* likely to occur on a single spin?

- (1) an odd number  $\frac{3}{6}$    
 (2) a prime number  $\frac{3}{6}$    
 (3) a perfect square  $\frac{2}{6}$    
 (4) a number divisible by 2  $\frac{3}{6}$
- {1, 3, 5}   
 {2, 3, 5}   
 {1 and 4}   
 {2, 4, 6}

5 What are the factors of the expression  $x^2 + x - 20$ ?

- (1)  $(x + 5)$  and  $(x + 4)$    
 (2)  $(x + 5)$  and  $(x - 4)$    
 (3)  $(x - 5)$  and  $(x + 4)$    
 (4)  $(x - 5)$  and  $(x - 4)$

Check  $(x + 5)(x - 4)$

$$x^2 + 5x - 4x - 20 = x^2 + x - 20 \checkmark$$

6 What is  $3\sqrt{250}$  expressed in simplest radical form?

- (1)  $5\sqrt{10}$    
 (2)  $8\sqrt{10}$    
 (3)  $15\sqrt{10}$    
 (4)  $75\sqrt{10}$

$$\begin{aligned}
 & 3\sqrt{250} \\
 & 3\sqrt{2} \sqrt{125} \\
 & 3\sqrt{2} \sqrt{5} \sqrt{25} \\
 & 3\sqrt{2} \sqrt{5} \sqrt{5} \sqrt{5} \\
 & 3\sqrt{2} \sqrt{5} \sqrt{5} \\
 & 15\sqrt{10}
 \end{aligned}$$

Check

$$\begin{aligned}
 3\sqrt{250} &= 47.4341649 \\
 15\sqrt{10} &= 47.4341649
 \end{aligned}$$

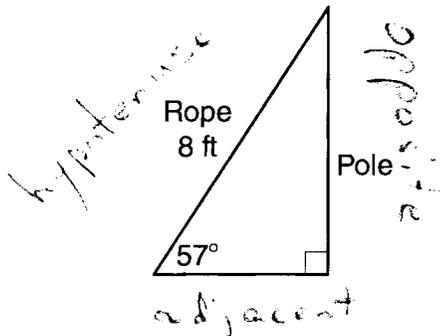
Use this space for computations.

7 A survey is being conducted to determine which school board candidate would best serve the Yonkers community. Which group, when randomly surveyed, would likely produce the most bias?

- (1) 15 employees of the Yonkers school district
- (2) 25 people driving past Yonkers High School
- (3) 75 people who enter a Yonkers grocery store
- (4) 100 people who visit the local Yonkers shopping mall

→ these people would be very biased because the school board candidate would be their boss.

8 An 8-foot rope is tied from the top of a pole to a stake in the ground, as shown in the diagram below.



Trig Formulas:  
 - See reference sheet -  
 SOH - CAH - TOA  
 $\sin = \frac{\text{opposite}}{\text{hypotenuse}}$   
 $\sin 57^\circ = \frac{\text{opposite}}{8}$

If the rope forms a  $57^\circ$  angle with the ground, what is the height of the pole, to the nearest tenth of a foot?

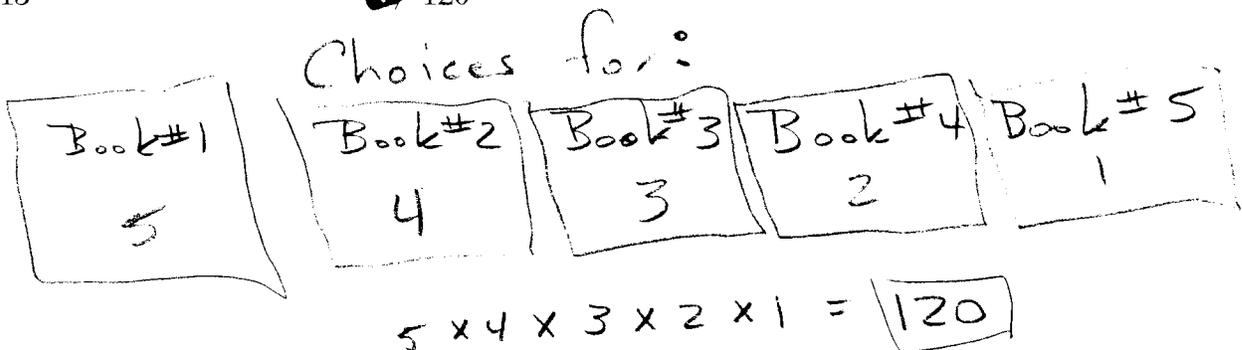
- (1) 4.4
- (2)  6.7
- (3) 9.5
- (4) 12.3

$8(\sin 57^\circ) = \text{opposite}$   
 $6.709364544 \dots$

Remember to set your calculator "mode" to "degrees"

9 How many different ways can five books be arranged on a shelf?

- (1) 5
- (2) 15
- (3) 25
- (4)  120



Answer

Use this space for computations.

10 What is the slope of the line passing through the points  $(-2,4)$  and  $(3,6)$ ?

(1)  $-\frac{5}{2}$

$\frac{2}{5}$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

See Reference Sheet for formula.

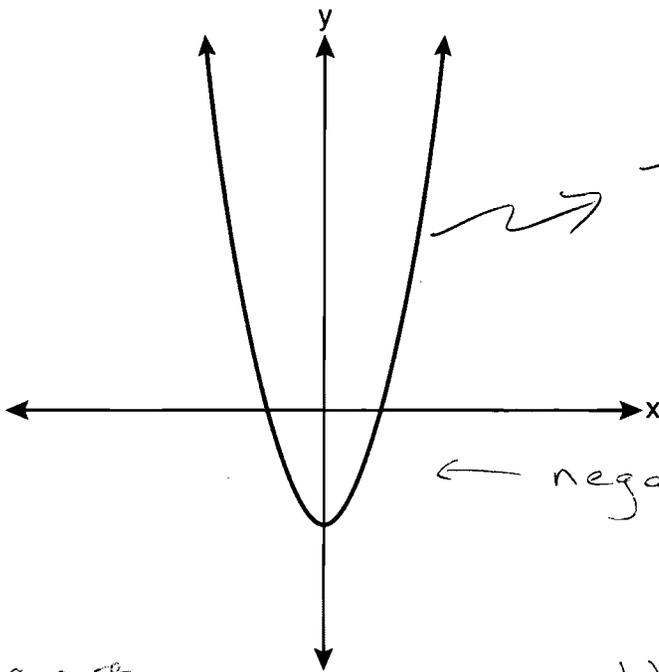
(2)  $-\frac{2}{5}$

(4)  $\frac{5}{2}$

(See Scrap Graph Paper for a second solution)

$$m = \frac{6 - 4}{3 - (-2)} = \boxed{\frac{2}{5}} \text{ answer}$$

11 Which type of function is represented by the graph shown below?



This is a parabola

negative x and negative y values

all values are positive

(1) absolute value

(2) exponential



(3) linear

quadratic

straight line  
parabola

12 Which equation represents a line parallel to the y-axis?

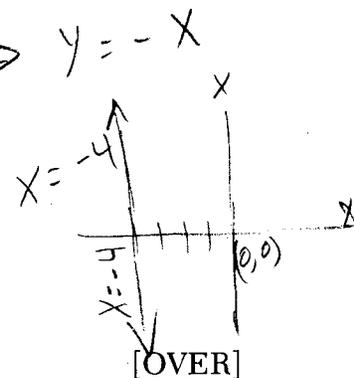
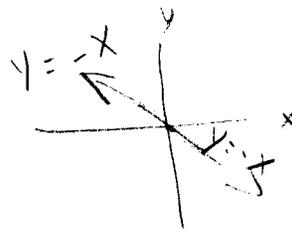
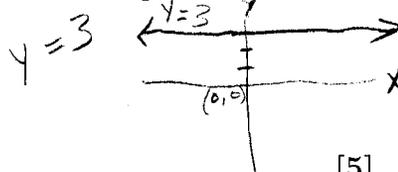
(1)  $y = x$

(2)  $y = 3$

(3)  $x = -y$

$x = -4$

$x = -y \Rightarrow y = -x$



Use this space for computations.

$$y = ax^2 + bx + c$$

$$y = x^2$$

$$y = -3x^2$$

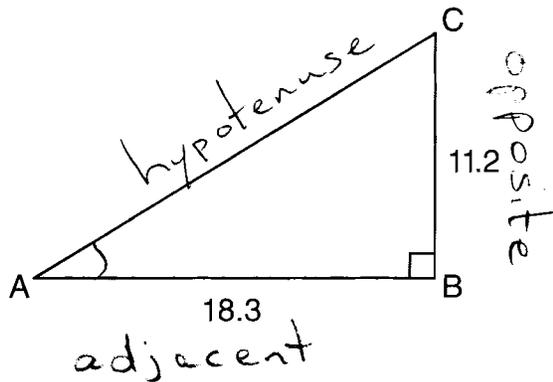
a is different

13 Melissa graphed the equation  $y = x^2$  and Dave graphed the equation  $y = -3x^2$  on the same coordinate grid. What is the relationship between the graphs that Melissa and Dave drew?

- (1) Dave's graph is wider and opens in the opposite direction from Melissa's graph.
- (2) Dave's graph is narrower and opens in the opposite direction from Melissa's graph.
- (3) Dave's graph is wider and is three units below Melissa's graph.
- (4) Dave's graph is narrower and is three units to the left of Melissa's graph.

The bigger the absolute value of a, the more narrow the parabola.  
Negative values of a open down. Positive open up.

14 In right triangle ABC shown below,  $AB = 18.3$  and  $BC = 11.2$ .



Trig Formulas: See Reference Sheet

SOH-CAH-TOA  
 $\tan = \frac{\text{opposite}}{\text{adjacent}}$

$$\tan \angle A = \frac{11.2}{18.3}$$

$$\arctan \frac{11.2}{18.3} = 31.46754337$$

$$\angle A = 31.5^\circ$$

What is the measure of  $\angle A$ , to the nearest tenth of a degree?

- (1) 31.5
- (2) 37.7
- (3) 52.3
- (4) 58.5

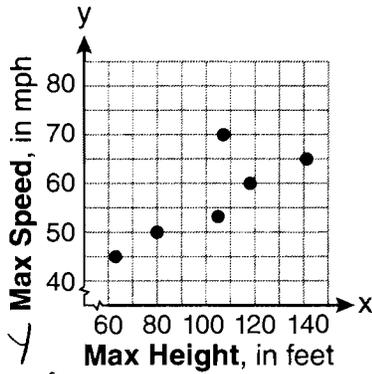
Remember: Set calculator to degree mode

Use this space for computations.

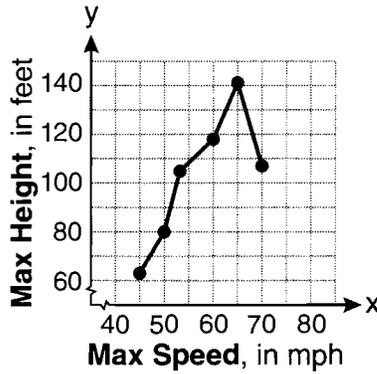
15 The maximum height and speed of various roller coasters in North America are shown in the table below.

<b>Maximum Speed,</b> in mph, (x)	45	50	54	60	65	70
<b>Maximum Height,</b> in feet, (y)	63	80	105	118	141	107

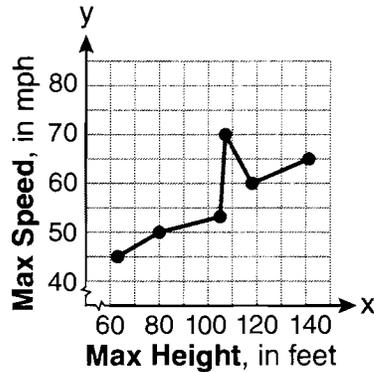
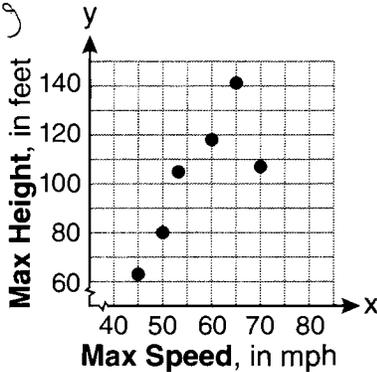
Which graph represents a correct scatter plot of the data?



*x and y variables are on wrong axes* (1)



(3) *This is a line graph - not a scatter plot*



(4) *This is a line graph and not a scatter plot, plus the variables are on the wrong axes.*

16 Which set of ordered pairs represents a function? *No*

(1)  $\{(0,4), (2,4), (2,5)\}$

(3)  $\{(4,1), (6,2), (6,3), (5,0)\}$

(2)  $\{(6,0), (5,0), (4,0)\}$

(4)  $\{(0,4), (1,4), (0,5), (1,5)\}$

*A function has one of y for each value of x.*

Use this space for computations.

17 A hiker walked 12.8 miles from 9:00 a.m. to noon. He walked an additional 17.2 miles from 1:00 p.m. to 6:00 p.m. What is his average rate for the entire walk, in miles per hour?

- 3.75  
 (2) 3.86

- (3) 4.27  
 (4) 7.71

$$\begin{array}{r} 12.8 \text{ miles} \quad 3 \text{ hours} \\ 17.2 \text{ miles} \quad 5 \text{ hours} \\ \hline 30 \text{ miles} \quad 8 \text{ hours} \\ \hline \frac{30 \text{ miles}}{8 \text{ hours}} = 3.75 \text{ miles/hour} \end{array}$$

18 Which ordered pair is a solution of the system of equations  $y = x + 3$  and  $y = x^2 - x$ ?

- (1) (6,9)  
 ● (3,6)

- (3) (3,-1)  
 (4) (2,5)

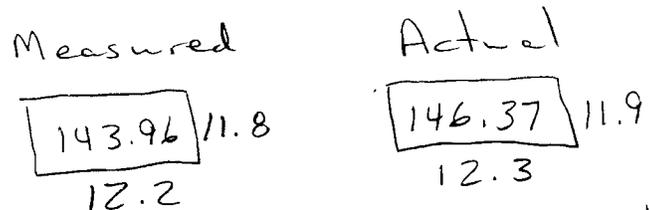
$$\begin{array}{l} y = x + 3 \quad y = x^2 - x \\ x + 3 = x^2 - x \\ 0 = x^2 - 2x - 3 \\ 0 = (x+1)(x-3) \\ \hline 0 = x + 1 \quad 0 = x - 3 \\ x = -1 \quad x = 3 \checkmark \\ y = x + 3 \\ y = 3 + 3 \\ y = 6 \\ (3, 6) \end{array}$$

19 Which verbal expression can be represented by  $2(x - 5)$ ?

- (1) 5 less than 2 times  $x$   $(2x - 5)$   
 (2) 2 multiplied by  $x$  less than 5  $2(5 - x)$   
 ● twice the difference of  $x$  and 5  
 (4) the product of 2 and  $x$ , decreased by 5  $2x - 5$

20 The dimensions of a rectangle are measured to be 12.2 inches by 11.8 inches. The actual dimensions are 12.3 inches by 11.9 inches. What is the relative error, to the nearest ten-thousandth, in calculating the area of the rectangle?

- (1) 0.0168  
 (2) 0.0167  
 ● 0.0165  
 (4) 0.0164



$$\begin{aligned} \text{Relative Error} &= \frac{\text{Measured} - \text{Actual}}{\text{Actual}} = \frac{143.96 - 146.37}{146.37} \\ &= \frac{-2.41}{146.37} = -0.016465172 \\ &\quad \boxed{0.0165} \text{ answer} \end{aligned}$$

21 An example of an algebraic expression is

an expression has no equal sign or inequality sign.

Use this space for computations.

~~(1)~~  $y = mx + b$

~~(3)~~  $2x + 3y \leq 18$

$3x + 4y - 7$

~~(4)~~  $(x + y)(x - y) = 25$

22 A study showed that a decrease in the cost of carrots led to an increase in the number of carrots sold. Which statement best describes this relationship?

- (1) positive correlation and a causal relationship
- (2) negative correlation and a causal relationship
- (3) positive correlation and not a causal relationship
- (4) negative correlation and not a causal relationship

23 Given:  $A = \{3, 6, 9, 12, 15\}$   
 $B = \{2, 4, 6, 8, 10, 12\}$

A union has all the elements in both sets

What is the union of sets A and B?

- (1)  $\{6\}$
- (2)  $\{6, 12\}$
- (3)  $\{2, 3, 4, 8, 9, 10, 15\}$
- (4)  $\{2, 3, 4, 6, 8, 9, 10, 12, 15\}$

$$\{2, 3, 4, 6, 8, 9, 10, 12, 15\}$$

24 The value of a car purchased for \$20,000 decreases at a rate of 12% per year. What will be the value of the car after 3 years?

- (1) \$12,800.00
- (2) \$13,629.44
- (3) \$17,600.00
- (4) \$28,098.56

Initial Value

$$(20,000) \times (100\% - 12\%)^3 = \text{value}$$

$$20,000 \times (.88)^3 = \text{value}$$

$$\$13,629.44 = \text{value}$$

# years

undefined occurs when denominator equals zero.

Use this space for computations.

25 For which set of values of  $x$  is the algebraic expression  $\frac{x^2 - 16}{x^2 - 4x - 12}$  undefined?

- (1)  $\{-6, 2\}$
- (2)  $\{-4, 3\}$
- (3)  $\{-4, 4\}$
- (4)  $\{-2, 6\}$

$$x^2 - 4x - 12 = 0$$

$$(x+2)(x-6) = 0$$


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$$x+2=0 \quad x-6=0$$

$$x=-2 \quad x=6$$

26 Michael is 25 years younger than his father. The sum of their ages is 53. What is Michael's age?

- (1) 14
- (2) 25
- (3) 28
- (4) 39

$$M + 25 = F \Rightarrow M - F = -25$$

$$M + F = 53 \Rightarrow M + F = 53$$


---


$$2M = 28$$

$$M = 14$$

27 What is the product of  $(6 \times 10^3)$ ,  $(4.6 \times 10^5)$ , and  $(2 \times 10^{-2})$  expressed in scientific notation?

- (1)  $55.2 \times 10^6$
- (2)  $5.52 \times 10^7$
- (3)  $55.2 \times 10^7$
- (4)  $5.52 \times 10^{10}$

$$6 \times 10^3$$

$$4.6 \times 10^5$$

$$2 \times 10^{-2}$$


---


$$(6 \times 4.6 \times 2) \times 10^{(3+5-2)}$$

$$55.2 \times 10^6$$

$$5.52 \times 10^7$$

28 Which notation describes  $\{1, 2, 3\}$ ?

- (1)  $\{x | 1 \leq x < 3, \text{ where } x \text{ is an integer}\}$  no 3
- (2)  $\{x | 0 < x \leq 3, \text{ where } x \text{ is an integer}\}$
- (3)  $\{x | 1 < x < 3, \text{ where } x \text{ is an integer}\}$  no 3
- (4)  $\{x | 0 \leq x \leq 3, \text{ where } x \text{ is an integer}\}$  includes 0

Use this space for computations.

29 What is  $\frac{7}{12x} - \frac{y}{6x^2}$  expressed in simplest form?

(1)  $\frac{7-y}{6x}$

(3)  $-\frac{7y}{12x^2}$

(2)  $\frac{7-y}{12x-6x^2}$

$\frac{7x-2y}{12x^2}$

$$\frac{7}{12x} - \frac{y}{6x^2}$$

$$\frac{7}{(6)(2)(x)} - \frac{y}{(6)(x)(x)}$$

$$\left(\frac{x}{x}\right) \frac{7}{(6)(2)(x)} - \left(\frac{2}{2}\right) \frac{y}{(6)(x)(x)}$$

$$\frac{7x}{12x^2} - \frac{2y}{12x^2}$$

$$\frac{7x-2y}{12x^2}$$

30 When  $5x + 4y$  is subtracted from  $5x - 4y$ , the difference is

(1) 0

(3)  $8y$

(2)  $10x$

$-8y$

$$\begin{array}{r} 5x - 4y \\ - (5x + 4y) \\ \hline 0x - 8y \\ \hline -8y \end{array} \Rightarrow \begin{array}{r} 5x - 4y \\ -5x - 4y \\ \hline 0x - 8y \\ \hline -8y \end{array}$$

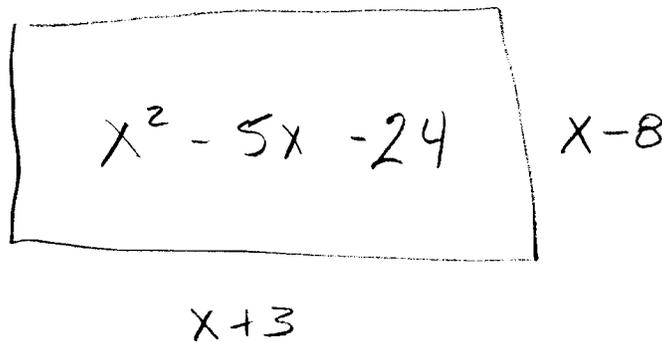
## Part II

Answer all 3 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [6]

- 31 The area of a rectangle is represented by  $x^2 - 5x - 24$ . If the width of the rectangle is represented by  $x - 8$ , express the length of the rectangle as a binomial.

factors  
sum to -5

factors multiply  
to -24



Check

$$(x+3)(x-8)$$

$$x^2 - 8x + 3x - 24$$

$$x^2 - 5x - 24 \quad \checkmark$$

32 A method for solving  $5(x - 2) - 2(x - 5) = 9$  is shown below. Identify the property used to obtain *each* of the two indicated steps.

$$5(x - 2) - 2(x - 5) = 9$$

(1)  $5x - 10 - 2x + 10 = 9$

(2)  $5x - 2x - 10 + 10 = 9$

$$3x + 0 = 9$$

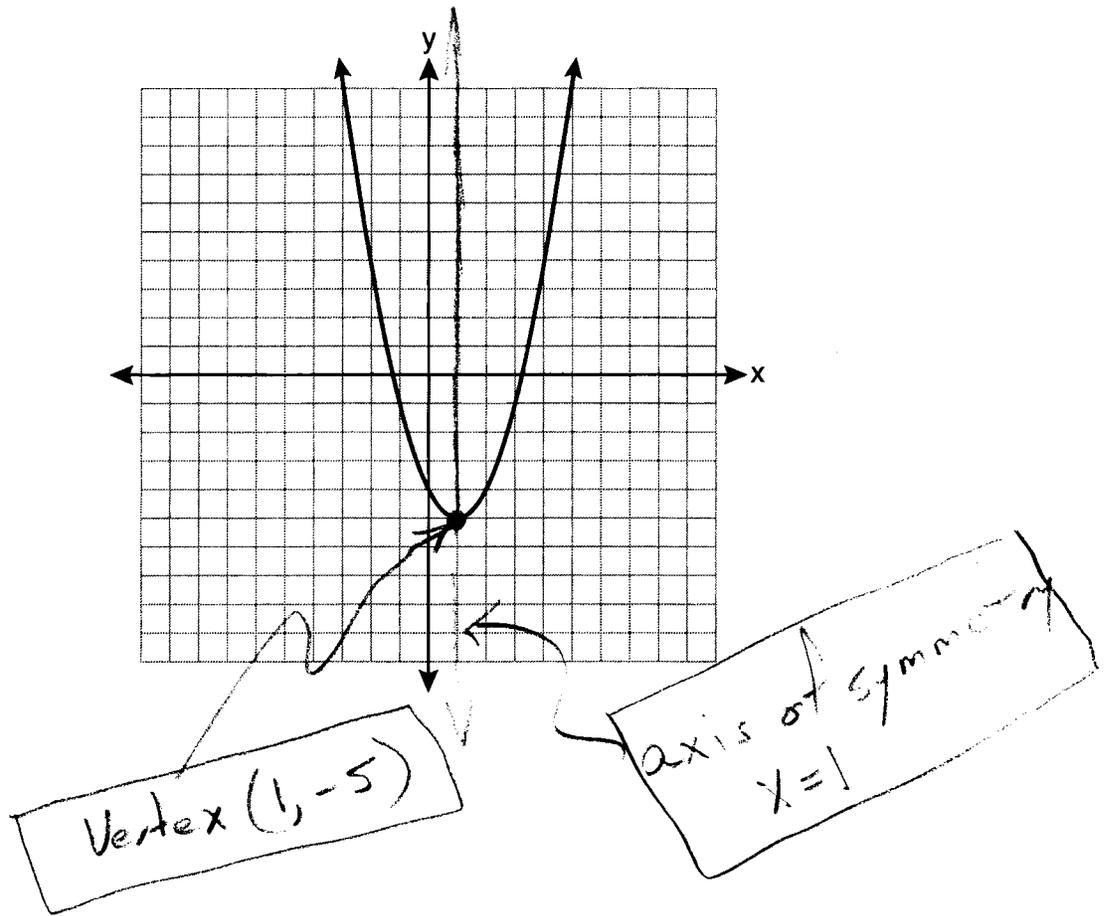
$$3x = 9$$

$$x = 3$$

(1) Distributive

(2) Commutative

33 State the equation of the axis of symmetry and the coordinates of the vertex of the parabola graphed below.



Part III

Answer all 3 questions in this part. Each correct answer will receive 3 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [9]

34 Given the following list of students' scores on a quiz: ~~5, 12, 7, 15, 20, 14, 7~~

5, 7, 7, 12, 14, 15, 20

w/ three more  $\rightarrow$  8, 10, 10, 15, 17, 18, 23

Determine the median of these scores.

$$\text{median} = 12$$

(the middle number in an ordered sequence)

Determine the mode of these scores.

$$\text{mode} = 7$$

(the most common frequent number in a sequence)

The teacher decides to adjust these scores by adding three points to each score. Explain the effect, if any, that this will have on the median and mode of these scores.

Both the median and the mode will increase by 3 points

$$\text{New median} = 15$$

$$\text{New mode} = 10$$

- 35 Chelsea has \$45 to spend at the fair. She spends \$20 on admission and \$15 on snacks. She wants to play a game that costs \$0.65 per game. Write an inequality to find the maximum number of times,  $x$ , Chelsea can play the game.

$$\frac{45 - (20 + 15)}{.65} \geq x$$

Using this inequality, determine the maximum number of times she can play the game.

$$\frac{45 - (20 + 15)}{.65} \geq x$$

$$\frac{45 - 35}{.65} \geq x$$

$$\frac{10}{.65} \geq x$$

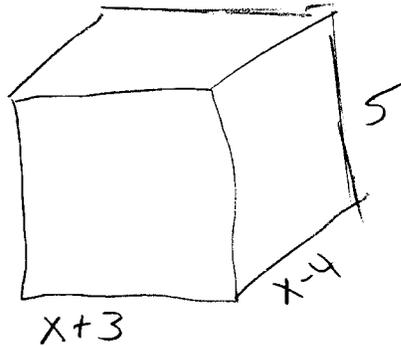
$$15.38461538 \geq x$$

15 is the maximum number of times she can play the game

36 A plastic storage box in the shape of a rectangular prism has a length of  $x + 3$ , a width of  $x - 4$ , and a height of 5.

Represent the surface area of the box as a trinomial in terms of  $x$ .

See reference sheet for surface area formulas



surface area equal 2 times each side

$$SA = \left[ \underset{\text{Side 1}}{2(x+3)(x-4)} \right] + \left[ \underset{\text{Side 2}}{2(5)(x+3)} \right] + \left[ \underset{\text{Side 3}}{2(5)(x-4)} \right]$$

$$= 2(x^2 - 4x + 3x - 12) + 10(x+3) + 10(x-4)$$

$$= 2x^2 - 8x + 6x - 24 + 10x + 30 + 10x - 40$$

$$= 2x^2 - 2x - 24 + 10x + 30 + 10x - 40$$

$$= \boxed{2x^2 + 18x - 34}$$

Part IV

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

37 Solve algebraically for  $x$ :  $\frac{3}{4} = \frac{-(x+11)}{4x} + \frac{1}{2x}$

$$\frac{3}{4} = \frac{-x-11}{4x} + \frac{1}{2x}$$

$$\frac{3}{4} = \frac{-x-11}{4x} + \frac{2}{4x}$$

$$\frac{3}{4} = \frac{-x-9}{4x}$$

$$12x = -4x - 36$$

$$16x = -36$$

$$x = \frac{-36}{16} = \boxed{\frac{-9}{4}}$$

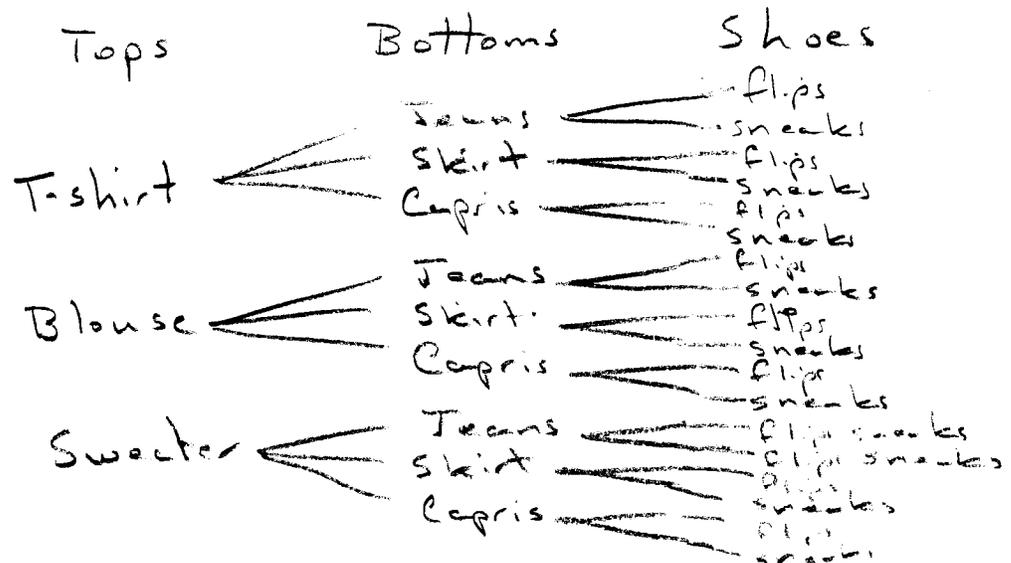
38 An outfit Jennifer wears to school consists of a top, a bottom, and shoes. Possible choices are listed below.

Tops: T-shirt, blouse, sweater 3

Bottoms: jeans, skirt, capris 3

Shoes: flip-flops, sneakers 2

List the sample space or draw a tree diagram to represent all possible outfits consisting of one type of top, one type of bottom, and one pair of shoes.



Determine how many different outfits contain jeans and flip-flops.

3

Determine how many different outfits do *not* include a sweater.

12

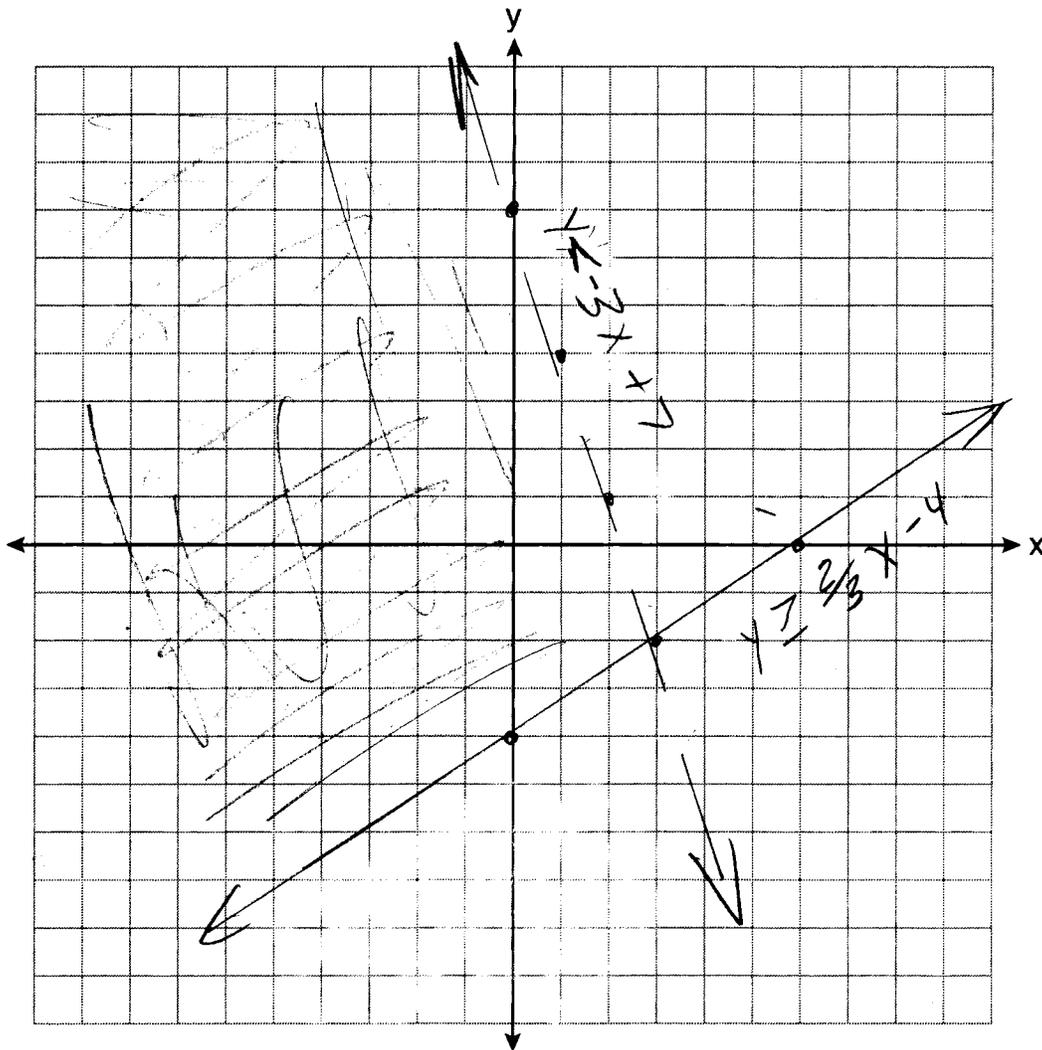
39 Solve the following system of inequalities graphically on the set of axes below.

$$3x + y < 7$$

$$y \geq \frac{2}{3}x - 4$$

$$y < -3x + 7$$

State the coordinates of a point in the solution set.



$(0,0)$

Check

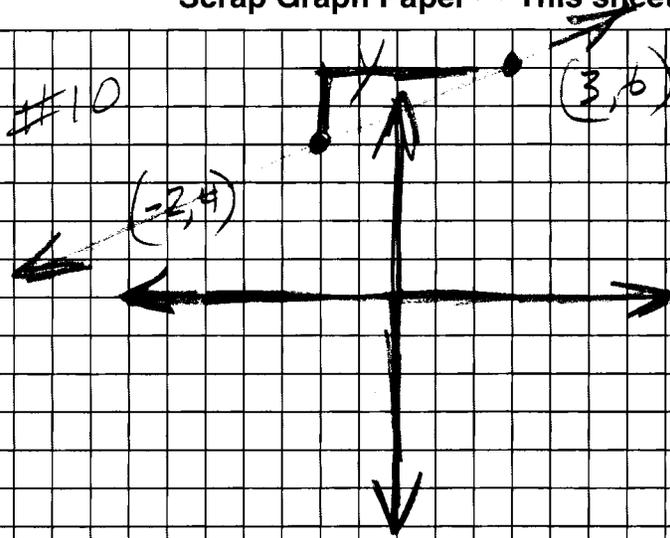
$$3x + y < 7$$

$$0 + 0 < 7 \checkmark$$

$$y \geq \frac{2}{3}x - 4$$

$$0 \geq 0 - 4$$

$$0 \geq -4 \checkmark$$



$$\frac{\text{rise}}{\text{run}} = \frac{2}{5}$$
$$\text{slope} = \frac{2}{5}$$