

MATHEMATICS A

Thursday, January 26, 2006 — 1:15 to 4:15 p.m., only

Print Your Name:

Imaginary Student

Print Your School's Name:

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Print your name and the name of your school in the boxes above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. Any work done on this sheet of scrap graph paper will *not* be scored. All work should be written in pen, except graphs and drawings, which should be done in pencil.

This examination has four parts, with a total of 39 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice . . .

A minimum of a scientific calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

The use of any communications device is strictly prohibited when taking this examination. If you use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [60]

1 What is the value of x in the equation $5(2x - 7) = 15x - 10$?

- (1) 1
- (2) 0.6

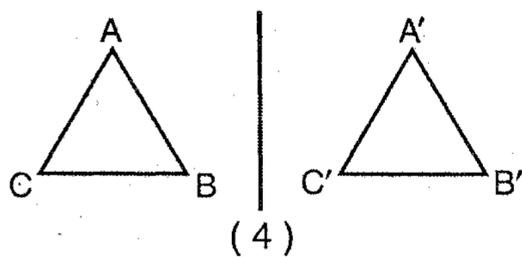
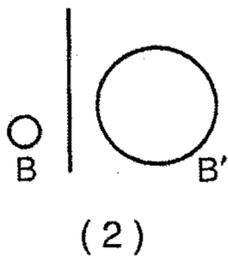
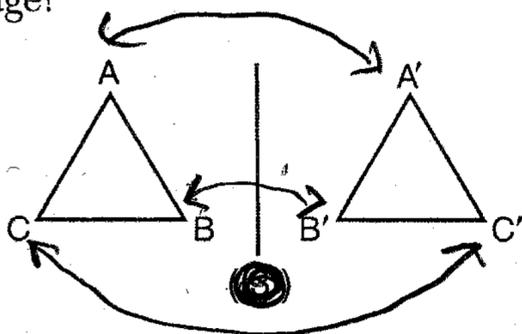
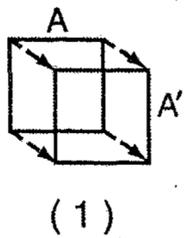
-5
 -9
 D.P. $5(2x-7) = 15x - 10$
 $10x - 35 = 15x - 10$
 $5(10x) - 10x \quad -10x$
 $-35 = 5x - 10$

Use this space for computations.

$-35 = 5x - 10$
 $+10 \quad +10$

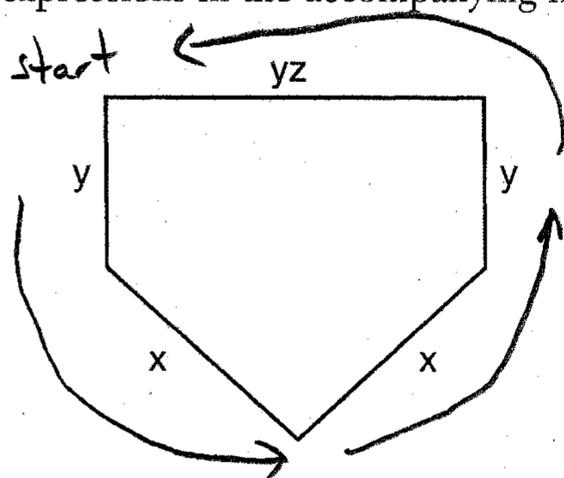
 $-25 = 5x$
 $\frac{-25}{5} = \frac{5x}{5}$
 $-5 = x$

2 Ms. Brewer's art class is drawing reflected images. She wants her students to draw images reflected in a line. Which diagram represents a correctly drawn image?



a reflection in a line is like a mirror image.

3 The lengths of the sides of home plate in a baseball field are represented by the expressions in the accompanying figure.



perimeter means the length around the outside

Which expression represents the perimeter of the figure?

- (1) $5xyz$
- (2) $x^2 + y^3z$
- (3) $2x + 3yz$
- $2x + 2y + yz$

$y + x + x + y + yz = \text{perimeter}$
 $x + x + y + y + yz = \text{perimeter}$
 $2x + 2y + yz = \text{perimeter}$

4 Which expression represents "5 less than the product of 7 and x"?

- (1) $7(x - 5)$
 $7x - 5$

- (3) $7 + x - 5$
 (4) $5 - 7x$

Use this space for computations.

5 less than product of 7 and x
 (-5) $(+7x)$

$+7x - 5$

5 What is the y-intercept of the graph of the line whose equation is

$y = -\frac{2}{5}x + 4$?

- (1) $-\frac{5}{2}$

- (3) 0

- (2) $-\frac{2}{5}$

- 4

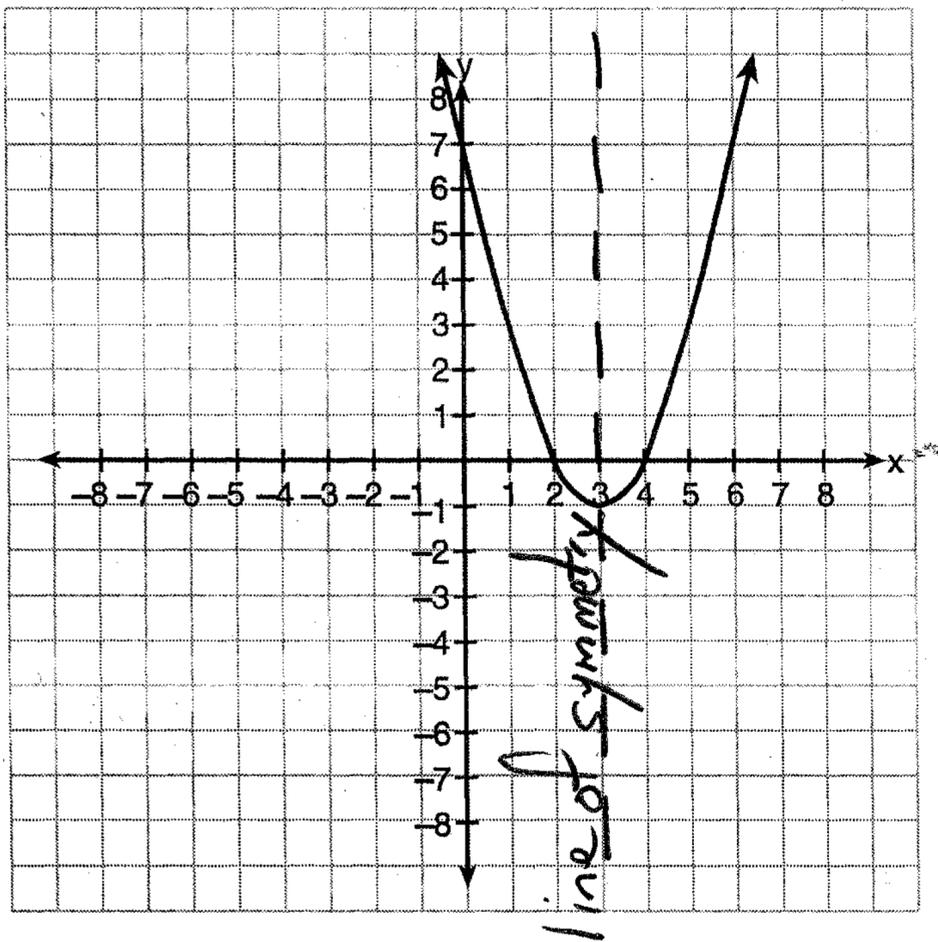
y intercept occurs when $x = 0$

$y = -\frac{2}{5}\left(\frac{0}{x}\right) + 4$

$y = 0 + 4$

$y = 4$

6 Which is an equation of the line of symmetry for the parabola in the accompanying diagram?



- (1) $x = 2$
 (2) $x = 4$

- $x = 3$
 (4) $y = 3$

The line of symmetry of a parabola goes down the middle of the parabola and divides the parabola into two equal left and right halves.

7 For which value of x will the fraction $\frac{3}{2x+4}$ be undefined?

Use this space for computations.

- (1) -2
- (2) 2

- (3) 0
- (4) -4

"undefined" happens when the denominator equals zero.

$$2x + 4 = 0$$

$$2x = -4$$

$$x = -2$$

8 The equation $A = \frac{1}{2}(12)(3 + 7)$ is used to find the area of a trapezoid. Which calculation would *not* result in the correct area?

(1) $\frac{12(3+7)}{2} = 60$

(3) $0.5(12)(10) = 60$

(2) $6(3 + 7) = 60$

(4) $\frac{12}{2} \times \frac{10}{2} = 30$

9 The size of a certain type of molecule is 0.00009078 inch. If this number is expressed as 9.078×10^n , what is the value of n ?

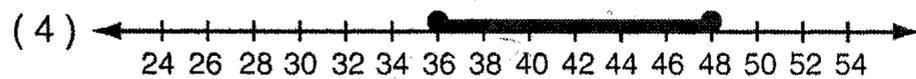
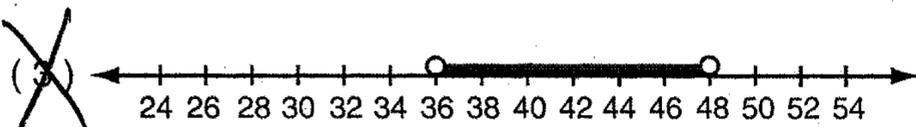
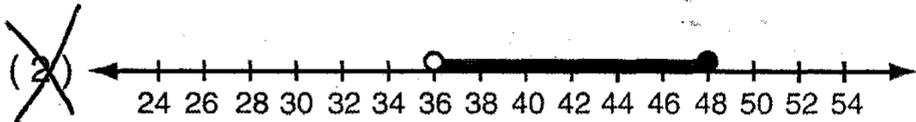
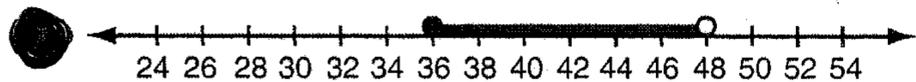
- (1) -5
- (2) 5

- (3) -8
- (4) 8

0.00009078
 -1 -2 -3 -4 -5

Move to the left is positive.
 Move to the right is negative.

10 In order to be admitted for a certain ride at an amusement park, a child must be greater than or equal to 36 inches tall and less than 48 inches tall. Which graph represents these conditions?



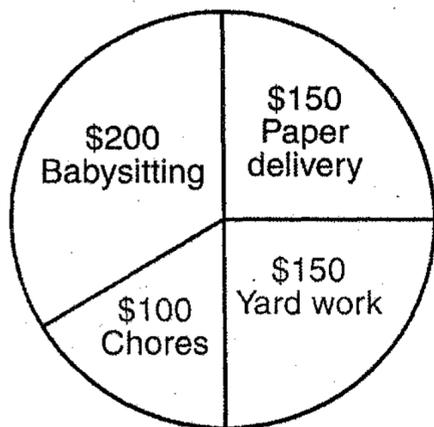
Color In \geq $<$
 Color in \leq $>$

The equal sign with the inequality reminds us to color in the dot.

≥ 36 is colored in
 < 48 is an open dot

11 The accompanying circle graph shows how Shannon earned \$600 during her summer vacation.

Use this space for computations.



Part
Whole

$$\frac{100}{600} = \frac{x}{360}$$

$$600x = 36,000$$

$$x = \frac{36,000}{600}$$

$$x = 60$$

What is the measure of the central angle of the section labeled "Chores"?

- (1) 30°
 (2) 60°

- (3) 90°
 (4) 120°

12 Robin has 8 blouses, 6 skirts, and 5 scarves. Which expression can be used to calculate the number of different outfits she can choose, if an outfit consists of a blouse, a skirt, and a scarf?

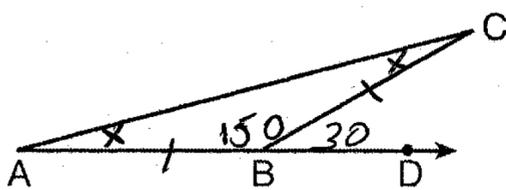
- (1) $8 + 6 + 5$
 (2) $8 \cdot 6 \cdot 5$

- (3) $8!6!5!$
 (4) ${}_{19}C_3$

Blouse Skirt Scarf
 Choices Choices Choices

$$\boxed{8} \times \boxed{6} \times \boxed{5}$$

13 In the accompanying diagram of $\triangle ABC$, \overline{AB} is extended through D , $m\angle CBD = 30$, and $\overline{AB} \cong \overline{BC}$.



What is the measure of $\angle A$?

- (1) 15°
 (2) 30°

- (3) 75°
 (4) 150°

$$2x + 150 = 180$$

$$-150 \quad -150$$

$$2x = 30$$

$$x = 15$$

14 The image of point $(-2,3)$ under translation T is $(3,-1)$. What is the image of point $(4,2)$ under the same translation?

- (1) $(-1,6)$ (3) $(5,4)$
 (2) $(0,7)$ (4) $(9,-2)$

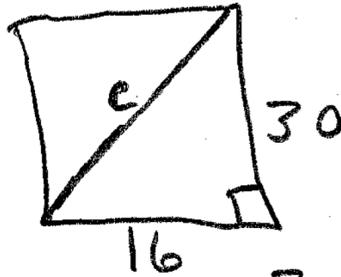
Use this space for computations.

$$\begin{aligned} -2 + X_T &= 3 \\ X_T &= 5 \\ 4 + 5 &= 9 \end{aligned}$$

$$\begin{aligned} 3 + Y_T &= -1 \\ Y_T &= -4 \\ 2 + (-4) &= -2 \end{aligned}$$

15 A builder is building a rectangular deck with dimensions of 16 feet by 30 feet. To ensure that the sides form 90° angles, what should each diagonal measure?

- (1) 16 ft (3) 34 ft
 (2) 30 ft (4) 46 ft



$$\begin{aligned} 16^2 + 30^2 &= c^2 \\ 256 + 900 &= c^2 \\ 1156 &= c^2 \\ \sqrt{1156} &= c \\ &= c \end{aligned}$$

16 Which statement is the inverse of "If the waves are small, I do not go surfing"?

- (1) If the waves are not small, I do not go surfing.
 (2) If I do not go surfing, the waves are small.
 (3) If I go surfing, the waves are not small.
 (4) If the waves are not small, I go surfing.

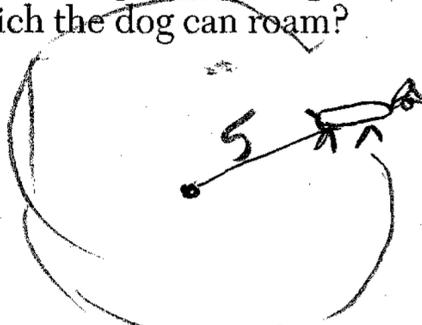
rhymes with "n" which stands for "not."

If not 1, then not 2

If waves are not small, then, I do ~~not not~~ go surfing (these cancel)

17 A dog is tied with a rope to a stake in the ground. The length of the rope is 5 yards. What is the area, in square yards, in which the dog can roam?

- (1) 25π (3) 25
 (2) 10π (4) 20



$$\begin{aligned} A &= \pi r^2 \\ A &= \pi (5)^2 \\ A &= \pi 25 \\ A &= 25\pi \end{aligned}$$

18 Melissa's test scores are 75, 83, and 75. Which statement is true about this set of data?

- (1) mean < mode (3) mode = median
 (2) mode < median (4) mean = median

mode
 75, 75, 83
 ↑
 median

mean =

$$\bar{x} = \frac{75 + 75 + 83}{3} = 77\frac{2}{3} = \text{mean}$$

19 When $3a^2 - 7a + 6$ is subtracted from $4a^2 - 3a + 4$, the result is

- (1) $a^2 + 4a - 2$ (3) $-a^2 - 4a + 2$
 (2) $a^2 - 10a - 2$ (4) $7a^2 - 10a + 10$

$$\begin{array}{r} 4a^2 - 3a + 4 \\ - 3a^2 + 7a - 6 \\ \hline a^2 + 4a - 2 \end{array}$$

Use this space for computations.

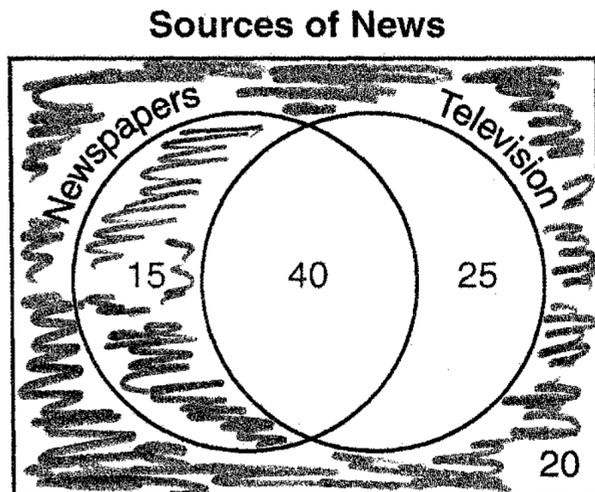
20 In the equation $A = p + prt$, t is equivalent to

- (1) $\frac{A - pr}{p}$ (3) $\frac{A}{pr} - p$ Given
 (2) $\frac{A - p}{pr}$ (4) $\frac{A}{p} - pr$ S(A)

D(p)

$$\begin{array}{l} A = p + prt \\ -p \\ \hline A - p = prt \\ \frac{A - p}{pr} = \frac{prt}{pr} \\ \frac{A - p}{pr} = t \end{array}$$

21 The accompanying Venn diagram shows the results of a survey asking 100 people if they get news by reading newspapers or by watching television.



$$p(\text{not television}) = \frac{\text{not television}}{\text{total}}$$

television = $40 + 25 = 65$
 not television = $15 + 20 = 35$
 Total = $15 + 40 + 25 + 20 = 100$

What is the probability that a person selected at random from this survey does not claim television as a source of getting the news?

- (1) $\frac{15}{100}$ (3) $\frac{55}{100}$
 (2) $\frac{35}{100}$ (4) $\frac{75}{100}$

$$p(\text{not television}) = \frac{35}{100}$$

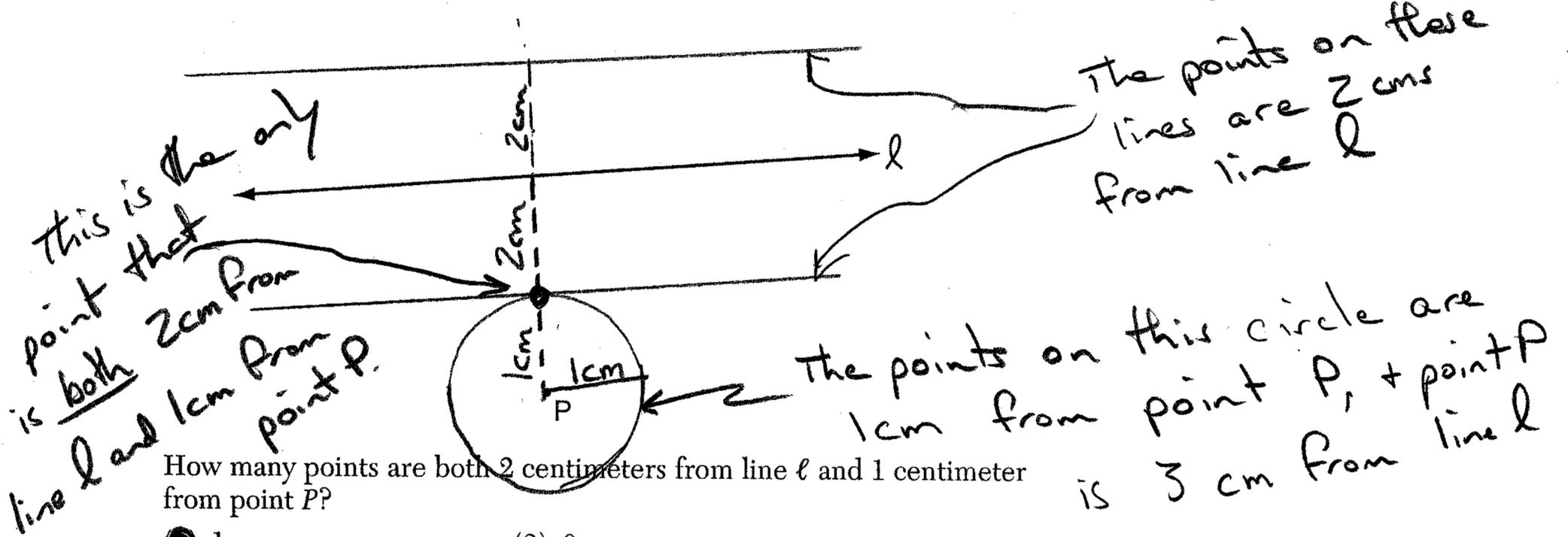
22 The expression $\frac{6\sqrt{20}}{3\sqrt{5}}$ is equivalent to

- (1) $3\sqrt{15}$ (3) 8
 (2) $2\sqrt{15}$ (4) 4

$$\frac{6\sqrt{20}}{3\sqrt{5}} = \frac{(2)(\cancel{3})(\sqrt{4})(\sqrt{5})}{(\cancel{3})(\sqrt{5})} = \frac{(2)(2)}{1} = 4$$

23 In the accompanying diagram, point P lies 3 centimeters from line ℓ .

Use this space for computations.

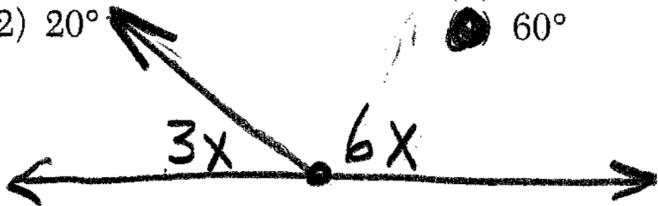


How many points are both 2 centimeters from line ℓ and 1 centimeter from point P ?

- (1) 1
 (2) 2
 (3) 0
 (4) 4

24 The ratio of two supplementary angles is 3:6. What is the measure of the *smaller* angle?

- (1) 10°
 (2) 20°
 (3) 30°
 (4) 60°



means they add up to 180°

$$\begin{aligned}
 3x + 6x &= 180^\circ \\
 9x &= 180^\circ \\
 x &= 20^\circ
 \end{aligned}$$

The smaller angle is $3x$

$$3(20) = 60^\circ$$

25 Which point is on the circle whose equation is $x^2 + y^2 = 289$?

- (1) $(-12, 12)$
 (2) $(7, -10)$
 (3) $(-1, -16)$
 (4) $(8, -15)$

$$(-12)^2 + (12)^2 \stackrel{?}{=} 289 \quad \text{No (298)}$$

$$(-1)^2 + (-16)^2 \stackrel{?}{=} 289 \quad \text{No (257)}$$

$$(7)^2 + (-10)^2 \stackrel{?}{=} 289 \quad \text{No (149)}$$

$$(8)^2 + (-15)^2 \stackrel{?}{=} 289 \rightarrow 64 + 225 = 289$$

26 The Edison Lightbulb Company tests 5% of their daily production of lightbulbs. If 500 bulbs were tested on Tuesday, what was the total number of bulbs produced that day?

- (1) 25
 (2) 1,000
 (3) 10,000
 (4) 100,000

$$5\% = \frac{5}{100}$$

Tested
Total

$$\frac{5}{100} = \frac{500}{x}$$

$$5x = 100(500)$$

$$5x = 50,000$$

$$x = \frac{50,000}{5}$$

$$x = 10,000$$

must have "if and only if" wording.

27 Which statement is expressed as a biconditional?

Use this space for computations.

- (1) Two angles are congruent if they have the same measure.
- (2) If two angles are both right angles, then they are congruent.
- (3) Two angles are congruent if and only if they have the same measure.
- (4) If two angles are congruent, then they are both right angles.

28 A committee of five members is to be randomly selected from a group of nine freshmen and seven sophomores. Which expression represents the number of different committees of three freshmen and two sophomores that can be chosen?

This is probability of multiple events. The first event is a subcommittee of

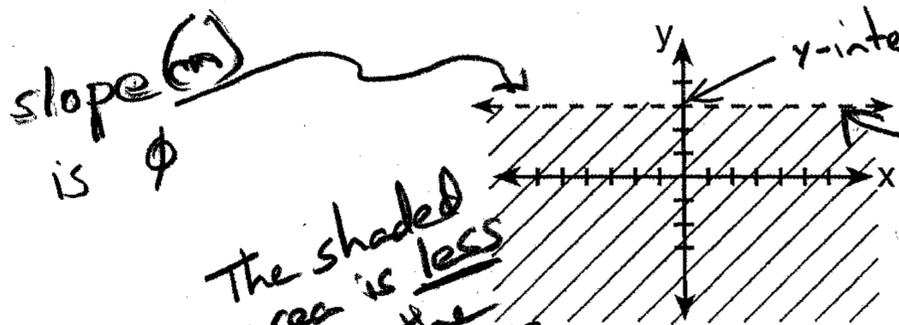
- (1) ${}^9C_3 + {}^7C_2$
- (2) ${}^9C_3 \cdot {}^7C_2$
- (3) ${}^{16}C_3 \cdot {}^{16}C_2$
- (4) ${}^9P_3 \cdot {}^7P_2$

3 freshmen chosen from 9 choices,

which is 9C_3 . The second event is a

subcommittee of 2 sophomores chosen from 7 sophomores, or 7C_2 . We multiply to find the probability of multiple events. Order is not important in choosing committees, so we use C and not P.

29 Which inequality is represented by the accompanying graph?



- (1) $y < 3$
- (2) $y > 3$
- ~~(3) $y \leq 3$~~
- ~~(4) $y \geq 3$~~

The equation of this line is

$$y = mx + b$$

$$m = 0 \quad b = 3$$

$$y = 0x + 3$$

$$y = 3$$

But the line is dotted, which means $y \neq 3$, so we eliminate options (3) + (4).

30 Which equation illustrates the multiplicative inverse property?

- (1) $1 \cdot x = x$
- (2) $x \cdot \frac{1}{x} = 1$
- (3) $1 \cdot 0 = 0$
- (4) $-1 \cdot x = -x$

The inverse property always results in the identity element.

This is the multiplicative identity property

Part II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [10]

31 Simplify: $\frac{x^2 + 6x + 5}{x^2 - 25} \Rightarrow \frac{x^2 + 6x + 5}{(x+5)(x-5)} \Rightarrow \frac{\cancel{(x+5)}(x+1)}{\cancel{(x+5)}(x-5)}$

$$\frac{x+1}{x-5}$$

32 Write an irrational number and explain why it is irrational.

π

This number never repeats, never ends, and cannot be expressed as the ratio of two integers.

33 In a circle whose center is (2,3), one endpoint of a diameter is (-1,5). Find the coordinates of the other endpoint of that diameter. [The use of the accompanying grid is optional.]

Solution #1

$$MP = \left[\left(\frac{x_1 + x_2}{2} \right), \left(\frac{y_1 + y_2}{2} \right) \right]$$

$$(2,3) = \left[\left(\frac{-1 + x_2}{2} \right), \left(\frac{5 + y_2}{2} \right) \right]$$

$$2 = \frac{-1 + x_2}{2}$$

$$3 = \frac{5 + y_2}{2}$$

$$4 = -1 + x_2$$

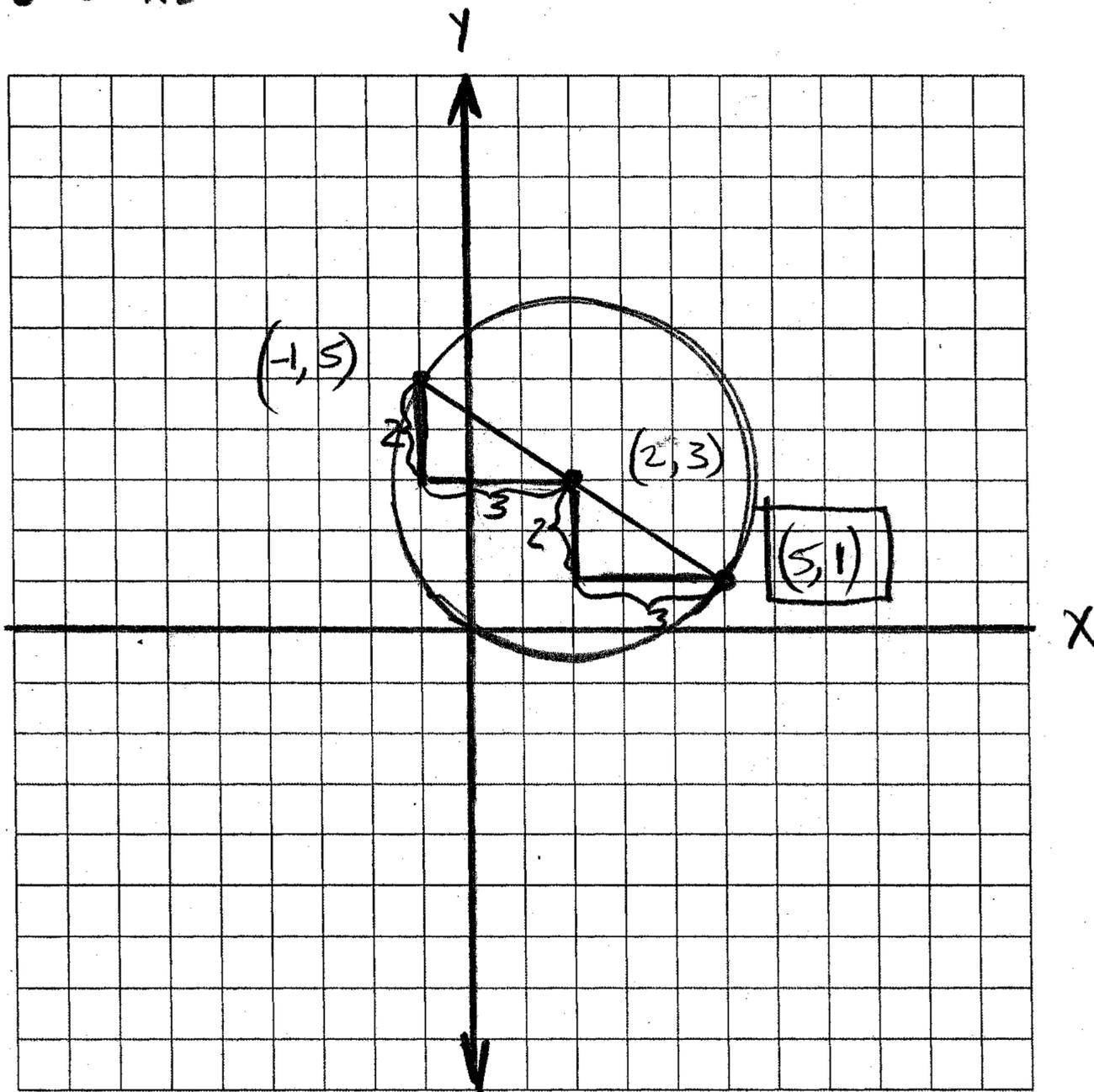
$$6 = 5 + y_2$$

$$5 = x_2$$

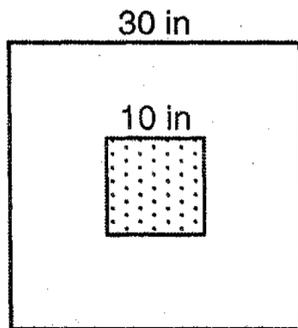
$$1 = y_2$$

$$(x_2, y_2) = (5, 1)$$

Solution #2



- 34 The accompanying diagram shows a square dartboard. The side of the dartboard measures 30 inches. The square shaded region at the center has a side that measures 10 inches. If darts thrown at the board are equally likely to land anywhere on the board, what is the theoretical probability that a dart does not land in the shaded region?



$$\begin{aligned} \text{Total area} &= (30\text{ in})(30\text{ in}) = 900\text{ in}^2 \\ \text{Shaded area} &= (10\text{ in})(10\text{ in}) = 100\text{ in}^2 \\ \text{Not Shaded area} &= 900\text{ in}^2 - 100\text{ in}^2 = 800\text{ in}^2 \end{aligned}$$

$$\text{Probability (not land in shaded region)} = \frac{\text{not shaded area}}{\text{total area}}$$

$$P = \frac{800\text{ in}^2}{900\text{ in}^2}$$

$$P = \boxed{\frac{8}{9}}$$

35 A candy store sells 8-pound bags of mixed hazelnuts and cashews. If c pounds of cashews are in a bag, the price p of the bag can be found using the formula $p = 2.59c + 1.72(8 - c)$. If one bag is priced at \$18.11, how many pounds of cashews does it contain?

Given	$p = 2.59c + 1.72(8 - c)$
Given	$p = 18.11$
Sub.	$18.11 = 2.59c + 1.72(8 - c)$
Dist. Prop.	$18.11 = 2.59c + 1.72(8) + 1.72(-c)$
Combine Like Terms	$18.11 = 2.59c + 13.76 - 1.72c$
	$18.11 = 0.87c + 13.76$
S (13.76)	$-13.76 = -13.76$
	$4.35 = 0.87c$
D (.87)	$\frac{4.35}{.87} = \frac{0.87c}{.87}$
Answer	$5 = c$ There are 5 pounds of cashews
Check	$18.11 = 2.59(5) + 1.72(8 - 5)$
	$18.11 = 12.95 + 1.72(3)$
	$18.11 = 12.95 + 5.16$
✓	$18.11 = 18.11$

$$\begin{array}{r} 2.59 \\ -1.72 \\ \hline .87 \end{array}$$

Part III

Answer all questions in this part. Each correct answer will receive 3 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [6]

36 Solve for x: $\frac{1}{16}x + \frac{1}{4} = \frac{1}{2}$

Given	$\frac{1}{16} \left(\frac{x}{1} \right) + \frac{x}{4} = \frac{1}{2}$
$S\left(\frac{1}{4}\right)$	$\frac{x}{4} - \frac{x}{4} = \frac{1}{4}$
Cross Multiply	$\frac{x}{16} = \frac{1}{4}$
	$x(4) = 16(1)$
	$4x = 16$
D_4	$\frac{4x}{4} = \frac{16}{4}$

$x = 4$

Check

$\frac{1}{16}(4) + \frac{1}{4} = \frac{1}{2}$

$\frac{1}{16}\left(\frac{4}{1}\right) + \frac{1}{4} = \frac{1}{2}$

$\frac{4}{16} + \frac{1}{4} = \frac{1}{2}$

$\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

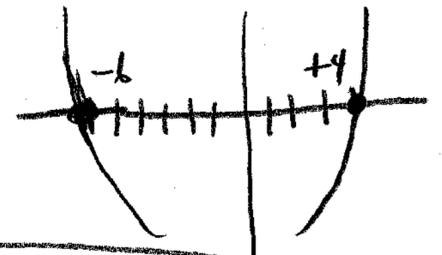
$\frac{1}{2} = \frac{1}{2} \checkmark$

37 Solve for x: $x^2 + 2x - 24 = 0$

Solution #1: Put in graphing calculator $y = x^2 + 2x - 24$

Look at graph

Solutions are $(-6, 4)$



Solution #2

$$x^2 + 2x - 24 = 0$$

$$(x+6)(x-4) = 0$$

$$x+6 = 0 \qquad x-4 = 0$$

$x = -6 \qquad x = 4$

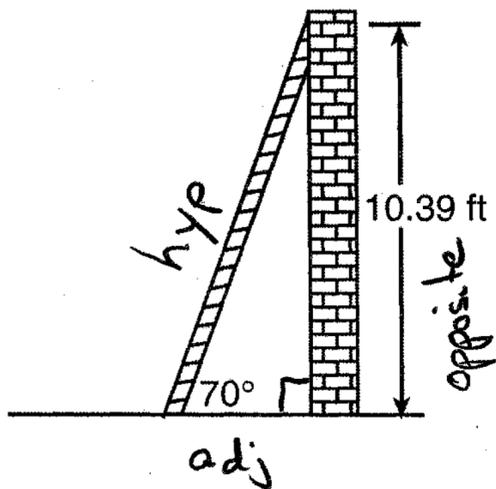
Part IV

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [8]

38 As shown in the accompanying diagram, a ladder is leaning against a vertical wall, making an angle of 70° with the ground and reaching a height of 10.39 feet on the wall.

Find, to the nearest foot, the length of the ladder. 11 ft.

Find, to the nearest foot, the distance from the base of the ladder to the wall. 4 ft.



We have:
 1) a right Δ
 2) an angle (70°)
 3) an opposite length (10.39 ft)

We want to know hyp.

$$\sin 70^\circ = \frac{10.39}{\text{hyp}}$$

We also want to know adj.

$$\tan 70^\circ = \frac{10.39}{\text{adj}}$$

SOH-CAH-TOA

$$\sin 70^\circ(\text{hyp}) = 10.39$$

$$\text{hyp} = \frac{10.39}{\sin 70^\circ}$$

$$\text{hyp} = 11.05680706$$

$$\boxed{\text{hyp} = 11}$$

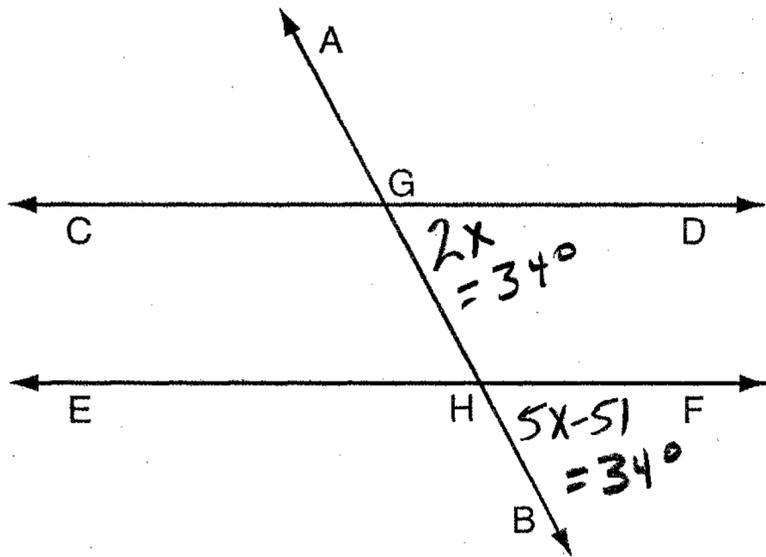
$$\tan 70^\circ(\text{adj}) = 10.39$$

$$\text{adj} = \frac{10.39}{\tan 70^\circ}$$

$$\text{adj} = 3.781650734$$

$$\boxed{\text{adj} = 4}$$

39 In the accompanying diagram, $\overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$, \overleftrightarrow{AB} is a transversal, $m\angle DGH = 2x$, and $m\angle FHB = 5x - 51$. Find the measure, in degrees, of $\angle BHE$.



$$\angle BHE = 180 - (5x - 51)$$

$$2x = 5x - 51$$

$$-2x \quad -2x$$

$$0 = 3x - 51$$

$$+51 \quad +51$$

$$51 = 3x$$

$$17 = x$$

$$\angle BHE = 180 - 34$$

$$\angle BHE = 146^\circ$$