

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

# MATHEMATICS A

Tuesday, August 13, 2002 — 8:30 to 11:30 a.m., only

Print Your Name:

Imaginary Student

Print Your School's Name:

www.jmap.org

Print your name and the name of your school in the boxes above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. Any work done on this sheet of scrap graph paper will not be scored. All work should be written in pen, except graphs and drawings, which should be done in pencil.

This examination has four parts, with a total of 35 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

**Notice . . .**

A minimum of a scientific calculator, a straightedge (ruler), and a compass must be available for your use while taking this examination.

**DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.**

## Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Record your answers in the spaces provided on the separate answer sheet. [40]

- 1 On a map, 1 centimeter represents 40 kilometers. How many kilometers are represented by 8 centimeters?

(1) 5  
(2) 48

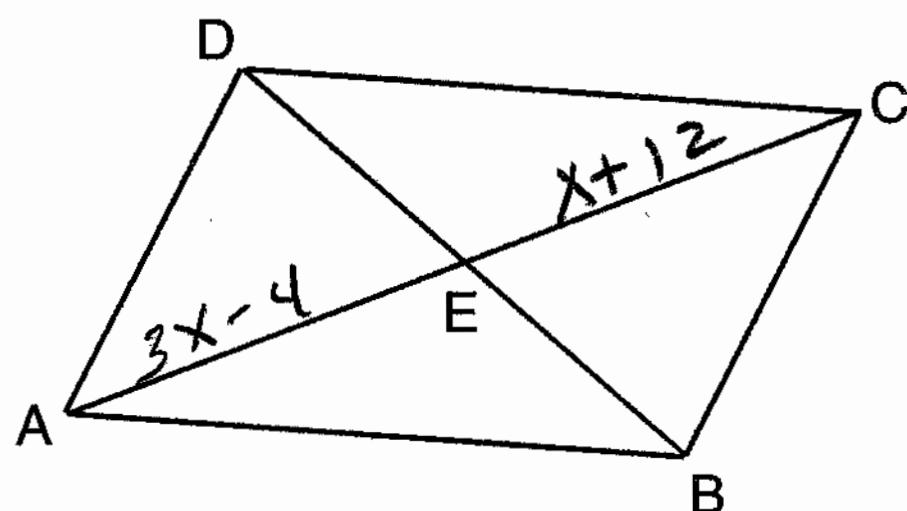
(3) 280  
(4) 320

centimeters  
kilometers

Use this space for computations.

$$\begin{aligned}\frac{1}{40} &= \frac{8}{x} \\ x &= 8(40) \\ x &= 320\end{aligned}$$

- 2 In the accompanying diagram of parallelogram  $ABCD$ , diagonals  $\overline{AC}$  and  $\overline{DB}$  intersect at  $E$ ,  $AE = 3x - 4$ , and  $EC = x + 12$ .



$AE = EC$  {The diagonals of a parallelogram bisect each other.

$$\begin{aligned}3x - 4 &= x + 12 \\ +4 &\quad +4 \\ \hline 3x &= x + 16 \\ -x &\quad -x \\ \hline 2x &= 16 \\ x &= 8\end{aligned}$$

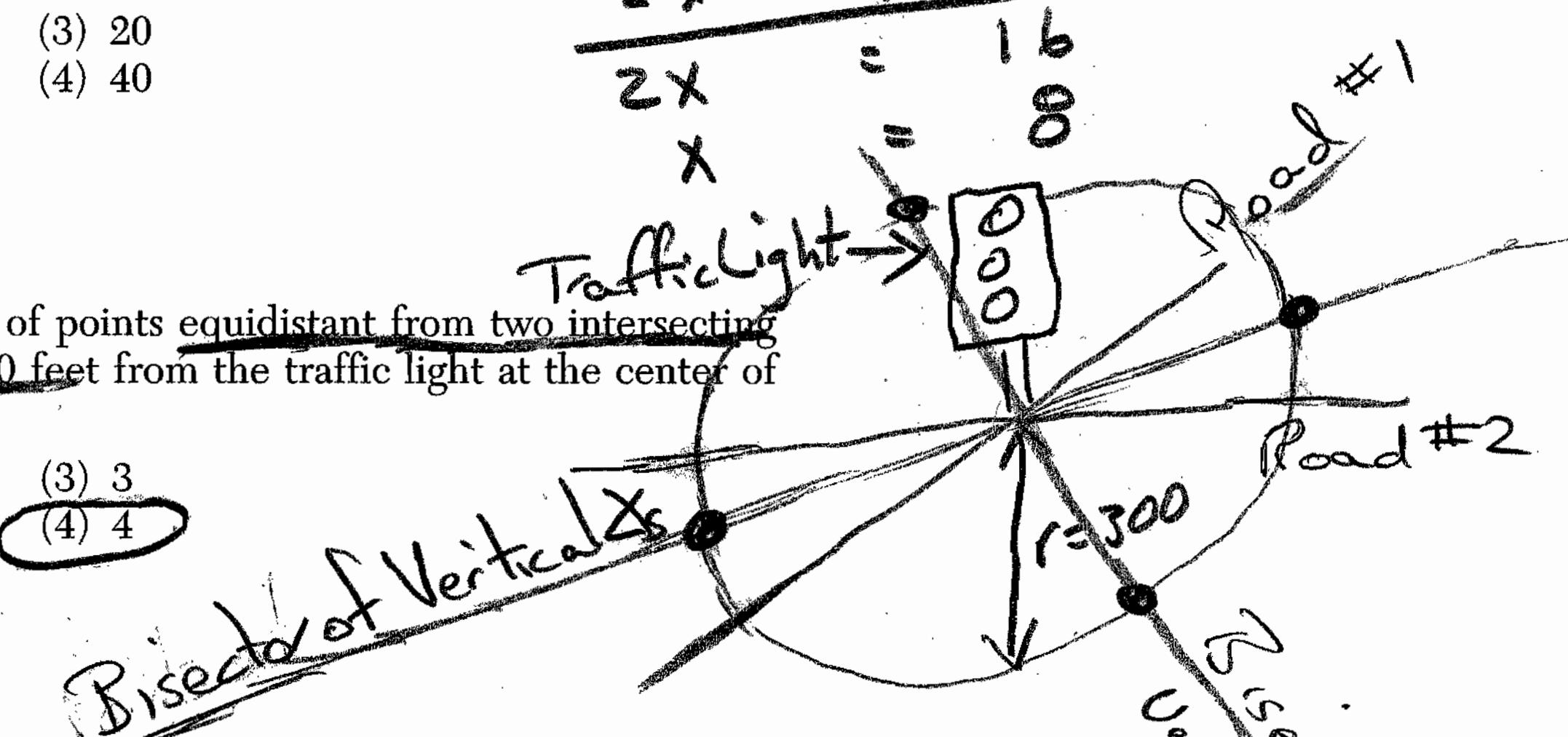
- What is the value of  $x$ ?  
(1) 8  
(2) 16

(3) 20  
(4) 40

- 3 What is the total number of points equidistant from two intersecting straight roads and also 300 feet from the traffic light at the center of the intersection?

(1) 1  
(2) 2

(3) 3  
(4) 4



- 4 Juan has three blue shirts, two green shirts, seven red shirts, five pairs of denim pants, and two pairs of khaki pants. How many different outfits consisting of one shirt and one pair of pants are possible?

(1) 19  
(2) 84

(3) 130  
(4) 420

Shirt Choices      Pants Choices

$$[2] \quad 12 \times 7 = 84$$

$$3+2+7=12$$

$$5+2=7$$

This is true

- 5 Given the statement: "If two lines are cut by a transversal so that the corresponding angles are congruent, then the lines are parallel."

Use this space for computations.

What is true about the statement and its converse?

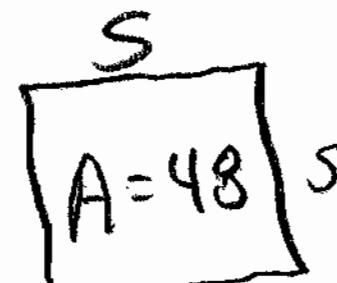
- (1) The statement and its converse are both true.
- (2) The statement and its converse are both false.
- (3) The statement is true, but its converse is false.
- (4) The statement is false, but its converse is true.

Given If 1, then 2  
Converse is If 2, then 1

This is true Converse: If 2 lines are parallel, then a transversal cuts them so that corresponding angles are congruent.

- 6 If the area of a square garden is 48 square feet, what is the length, in feet, of one side of the garden?

- (1)  $12\sqrt{2}$
- (2)  $4\sqrt{3}$
- (3)  $16\sqrt{3}$
- (4)  $4\sqrt{6}$



$$A_{\square} = s^2 \rightarrow \sqrt{48} = \sqrt{s^2} \rightarrow \sqrt{3} \cdot \sqrt{16} = s \rightarrow 4\sqrt{3} = s$$

- 7 The sum of  $\frac{3}{x} + \frac{2}{5}$ ,  $x \neq 0$ , is

- (1)  $\frac{1}{x}$
- (2)  $\frac{2x+15}{5x}$
- (3)  $\frac{5}{x+5}$
- (4)  $\frac{2x+15}{x+5}$

$$\frac{3}{x} + \frac{2}{5} = 0 \rightarrow \frac{3(5) + 2x}{5x} = 0 \rightarrow 15 + 2x = 0 \rightarrow 2x = -15 \rightarrow x = -\frac{15}{2}$$

- 8 The number 0.14114111411114... is

- (1) integral
- (2) rational
- (3) irrational
- (4) whole

All never ending, never repeating decimal s are irrational.

- 9 When  $-2x^2 + 4x + 2$  is subtracted from  $x^2 + 6x - 4$ , the result is

- (1)  $-3x^2 - 2x + 6$
- (2)  $-x^2 + 10x - 2$
- (3)  $2x^2 - 2x - 6$
- (4)  $3x^2 + 2x - 6$

$$\begin{array}{r} x^2 + 6x - 4 \\ - (-2x^2 + 4x + 2) \\ \hline +2x^2 - 4x - 2 \\ \hline 3x^2 + 2x - 6 \end{array}$$

To subtract, change the signs and add

- 10 If  $0.0347$  is written by a scientist in the form  $3.47 \times 10^n$ , the value of  $n$  is

- (1) -2
- (2) 2
- (3) 3
- (4) -3

•0347  
1 2

Move decimal to left  $\Rightarrow$  positive

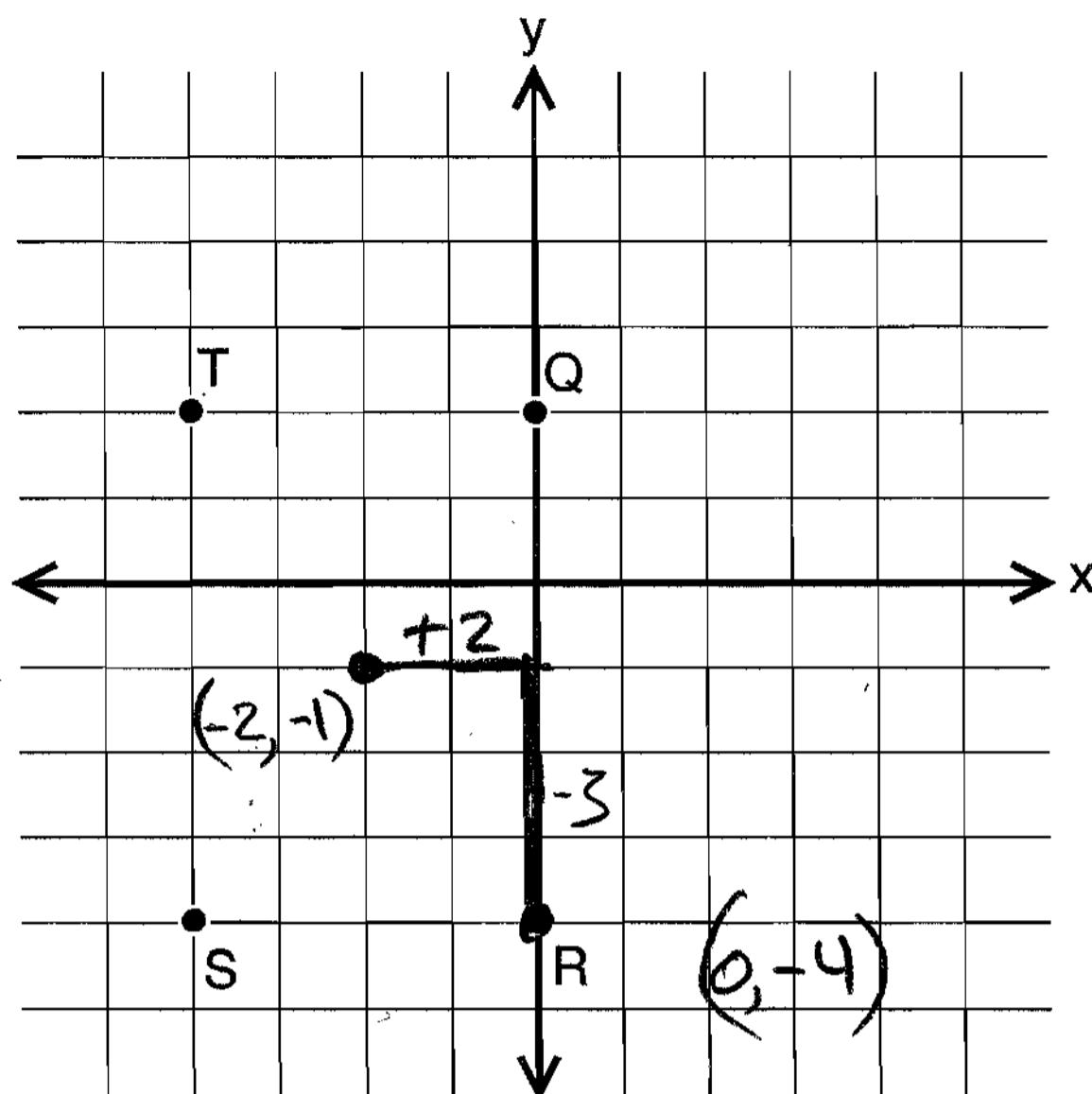
[3]

[OVER]

Move decimal to right  $\Rightarrow$  negative

- 11 If  $x = -2$  and  $y = -1$ , which point on the accompanying set of axes represents the translation  $(x, y) \rightarrow (x + 2, y - 3)$ ?

Use this space for computations.

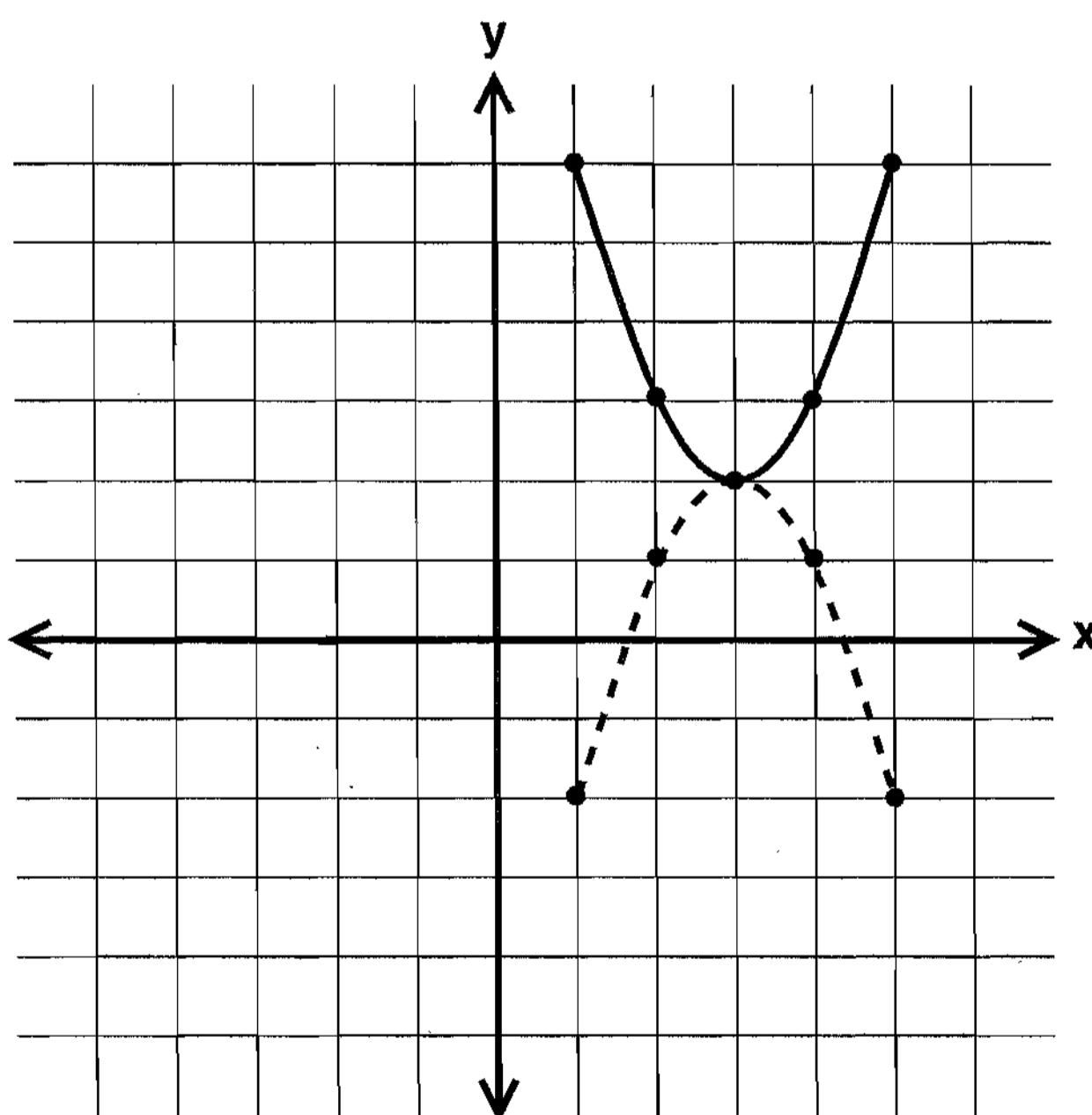


$(x, y)$   
 $(-2, -1)$   
 $(x+2, y-3)$   
 $(-2+2, -1-3)$   
 $(0, -4)$

(1) Q  
 (2) R

(3) S  
 (4) T

- 12 In the accompanying diagram, which transformation changes the solid-line parabola to the dotted-line parabola?



- (1) translation  
 (2) line reflection, only  
 (3) rotation, only  
 (4) line reflection or rotation

13 How many times larger than  $\frac{1}{4}x$  is  $5x$ ?

- (1) 20  
(2) 9

- (3)  $\frac{5}{4}$   
(4)  $\frac{4}{5}$

There are 20  $\Delta$ 's  
in 5  $O$ 's.

Let  $X = O$

Let  $\frac{1}{4}X = \Delta$

Use this space for computations.



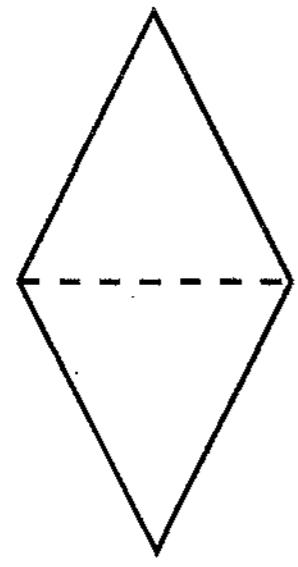
14 If the lengths of two sides of a triangle are 4 and 10, what could be the length of the third side?

→ (1) 6      (3) 14  
→ (2) 8      (4) 16

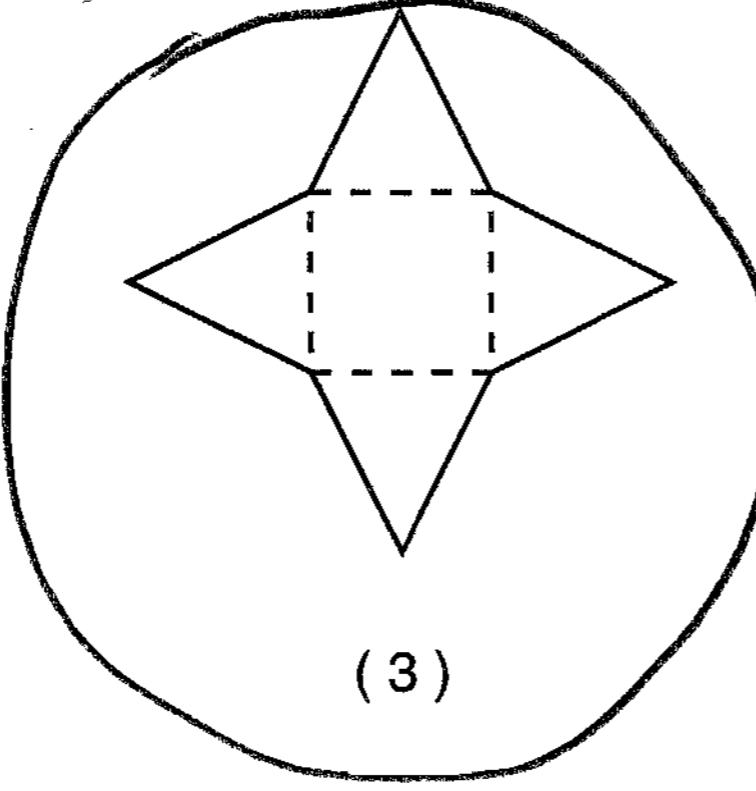
✓  $4+10 > 6$      $4+6 > 10$      $4+10 > 14$   
✓  $4+10 > 8$      $4+10 > 16$

The sum of any 2 sides of a  $\triangle$   
must be greater than the 3rd side.  
 $\{4, 8, 10\}$

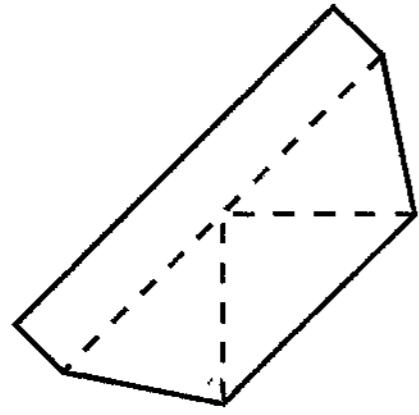
15 Which piece of paper can be folded into a pyramid?



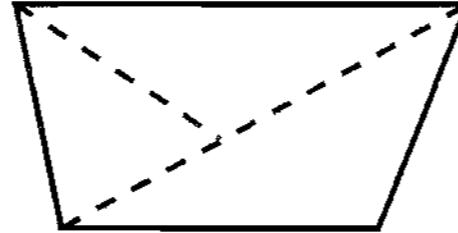
(1)



(3)

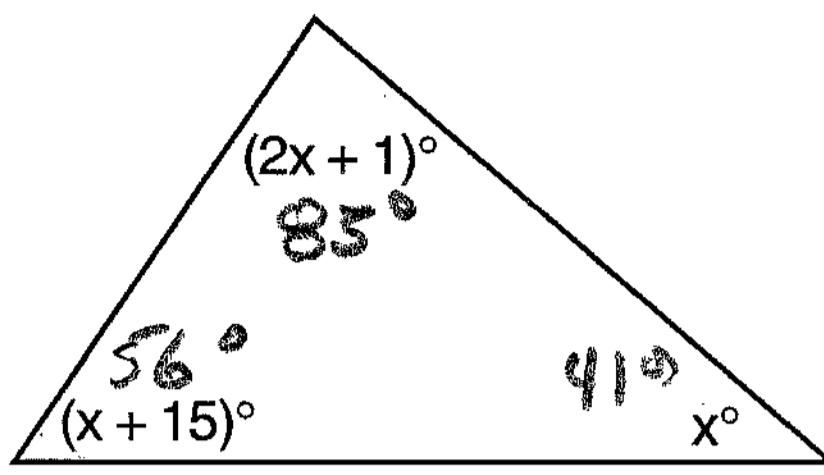


(2)



(4)

16 What is the measure of the largest angle in the accompanying triangle?



- (1) 41  
(2) 46.5

- (3) 56  
(4) 83

$$(2x+1) + (x+15) + (x) = 180^\circ$$

$$\begin{array}{r} 4x + 16 \\ -16 \\ \hline 4x \end{array}$$

$$= 180^\circ$$

$$-16$$

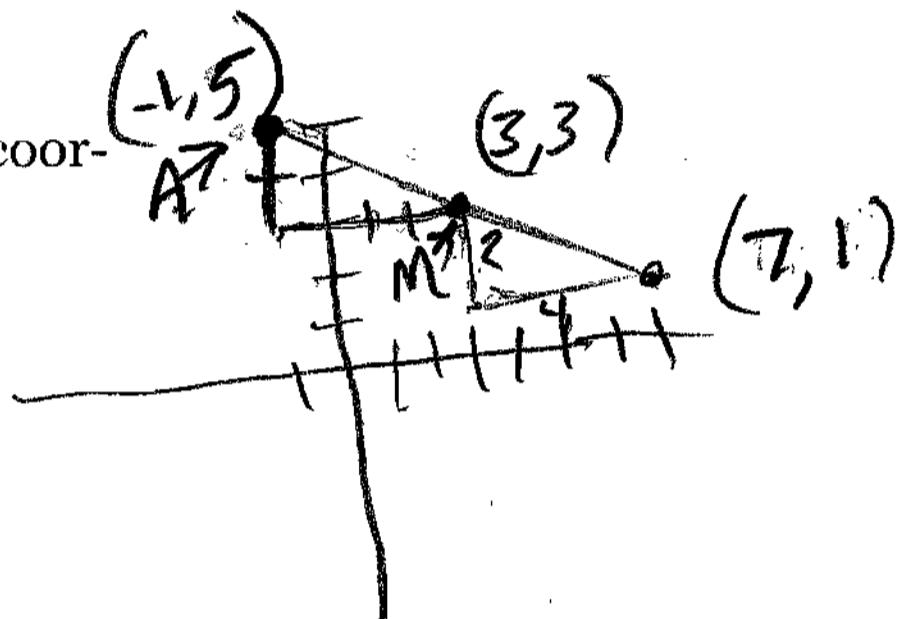
$$= 164$$

$$x = 41$$

17 M is the midpoint of  $\overline{AB}$ . If the coordinates of A are  $(-1, 5)$  and the coordinates of M are  $(3, 3)$ , what are the coordinates of B?

- (1)  $(1, 4)$   
(2)  $(2, 8)$

- (3)  $(7, 1)$   
(4)  $(-5, 7)$



$$mp = \left( \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right) \Rightarrow 3 = \frac{-1+x_2}{2} \Rightarrow 6 = -1+x_2$$

$$7 = x_2$$

18 If  $2m + 2p = 16$ , p equals

- (1)  $8 - m$   
(2)  $16 - m$

- (3)  $16 + 2m$   
(4)  $9m$

$$2m + 2p = 16$$

$$m + p = 8$$

$$\begin{array}{r} m \\ -m \\ \hline p = 8 - m \end{array}$$

19 If  $2x + 5 = -25$  and  $-3m - 6 = 48$ , what is the product of x and m?

- (1) -270  
(2) -33

- (3) 3  
(4) 270

$$2x + 5 = -25$$

$$\begin{array}{r} -5 \\ -5 \\ \hline 2x = -30 \\ x = -15 \end{array}$$

$$-3m - 6 = 48$$

$$+6 +6$$

$$\begin{array}{r} -3m = 54 \\ \hline \end{array}$$

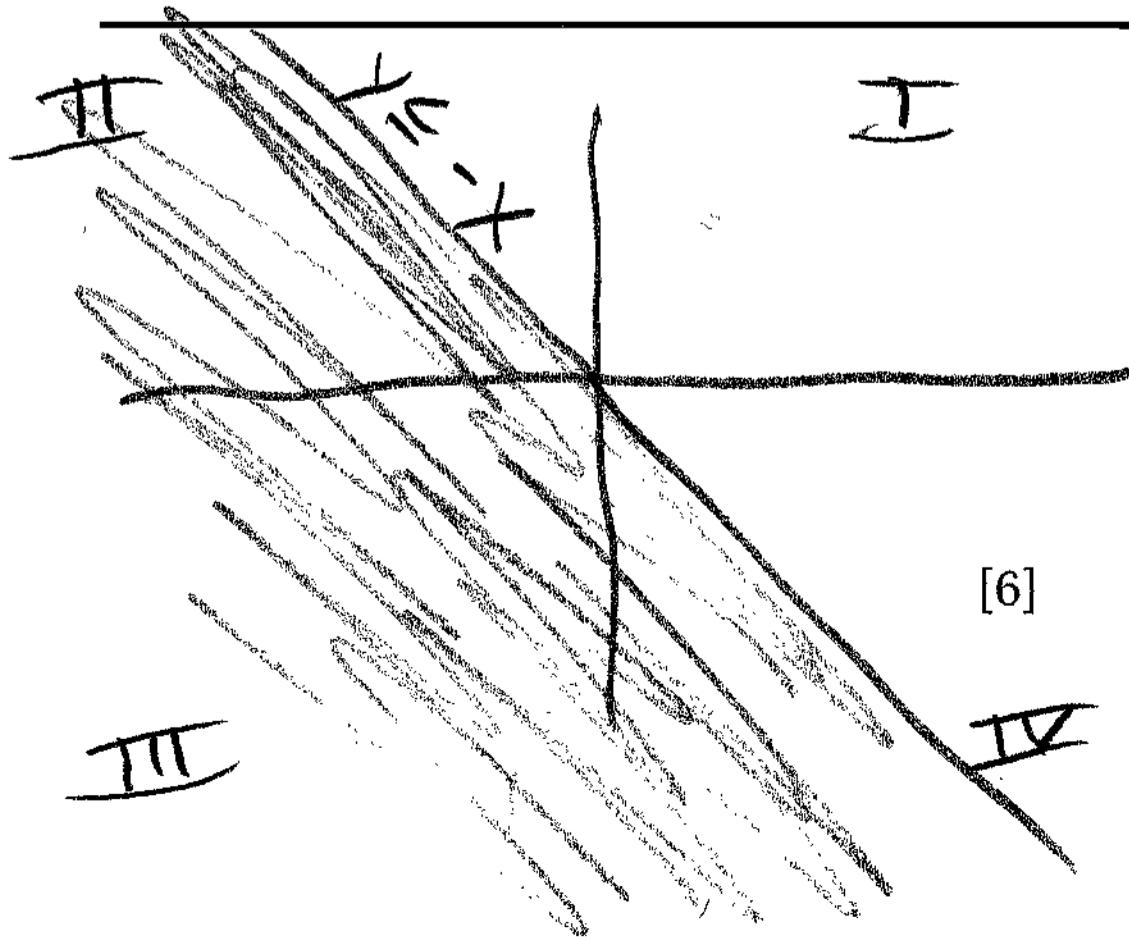
$$D(-3) \quad m = -18$$

$$(-15)(-18) = 270$$

20 In the graph of  $y \leq -x$ , which quadrant is completely shaded?

- (1) I  
(2) II

- (3) III  
(4) IV



## Part II

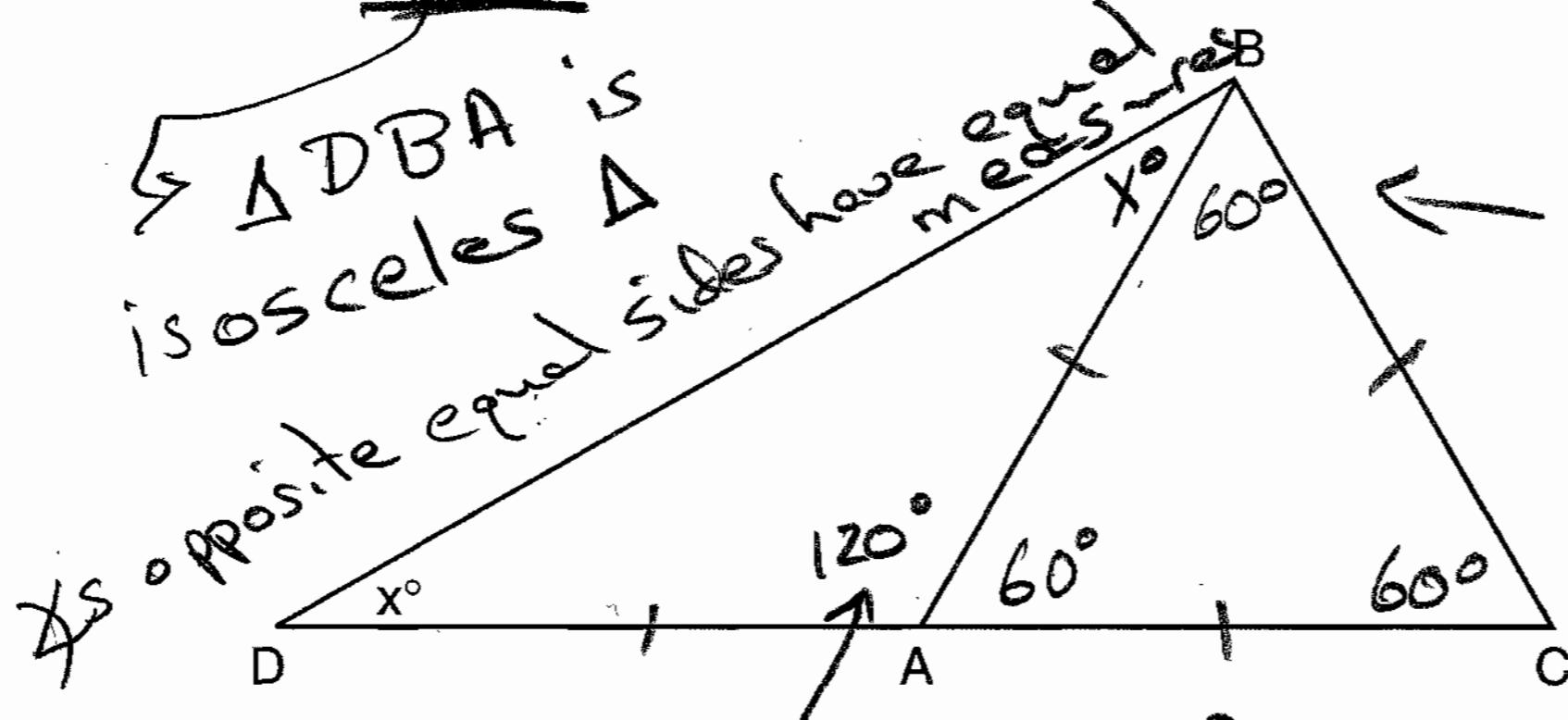
**Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.** [10]

- 21 In the accompanying diagram of  $\triangle BCD$ ,  $\triangle ABC$  is an equilateral triangle and  $AD = AB$ . What is the value of  $x$ , in degrees?

2 3 sides equal

3 & s equal

Each  $\angle$  in an equilateral  $\triangle$  measures  $60^\circ$



This  $\angle$  is  $120^\circ$   
 $180^\circ - 60^\circ = 120^\circ$

$\triangle DBA$

$$2x + 120^\circ = 180^\circ$$

$$\begin{array}{r} -120 \\ \hline \end{array}$$

$$2x = 60^\circ$$

$$\boxed{x = 30^\circ}$$

- 22 In the addition table for a subset of real numbers shown below, which number is the inverse of 3? Explain your answer.

$\oplus$	1	2	3	4
1	2	3	4	1
2	3	4	1	2
3	4	1	2	3
4	1	2	3	4

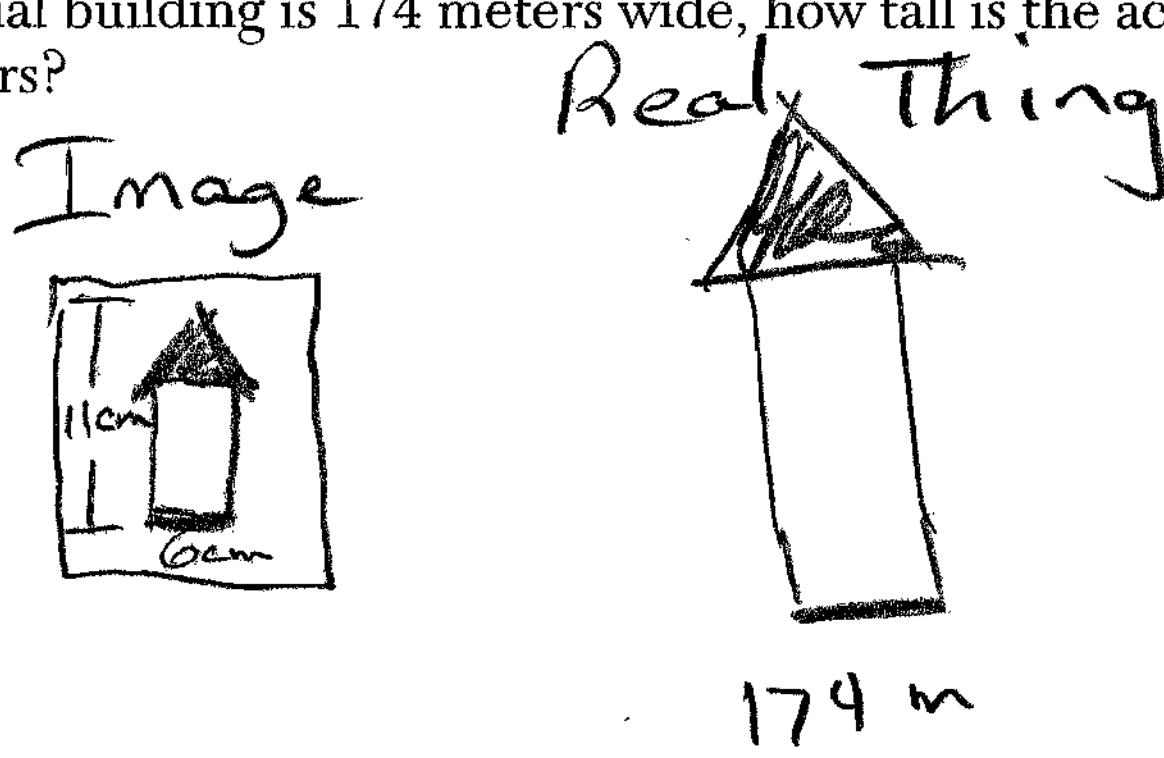
The inverse of a # under a given operation produces the identity element.

The identity element preserves the identity of the other elements under the operation.

In this table, the identity element is 4.  
When 3 operates with 1 in this table, 4 is the result.

Answer

- 23 An image of a building in a photograph is 6 centimeters wide and 11 centimeters tall. If the image is similar to the actual building and the actual building is 174 meters wide, how tall is the actual building, in meters?



$$\frac{\text{Height}}{\text{Width}} \Rightarrow \frac{11}{6} = \frac{x}{174}$$

$$11(174) = 6x$$

$$1914 = 6x \Rightarrow \boxed{319 = x}$$

$$\begin{array}{r} 1790 \\ 174 \\ \hline 1914 \end{array}$$

$$\begin{array}{r} 319 \\ 6 ) 1914 \\ 18 \\ \hline 11 \\ 11 \\ \hline 0 \end{array}$$

- 24 A doughnut shop charges \$0.70 for each doughnut and \$0.30 for a carryout box. Shirley has \$5.00 to spend. At most, how many doughnuts can she buy if she also wants them in one carryout box?

$$\begin{array}{r} \$5.00 \\ - \$0.30 \text{ for carryout box} \\ \hline \$4.70 \text{ for donuts} \end{array}$$

$$70 \overline{)4.70} \\ \quad 420 \\ \quad \quad 50$$

Answer  
She can buy 6  
donuts and one  
carrying box

- 25 In bowling leagues, some players are awarded extra points called their "handicap." The "handicap" in Anthony's league is 80% of the difference between 200 and the bowler's average. Anthony's average is 145. What is Anthony's "handicap"?

$$\text{Handicap} = 80\% (200 - 145)$$

↑ difference between  
200 and Anthony's average

$$\text{Handicap} = .80 (55)$$

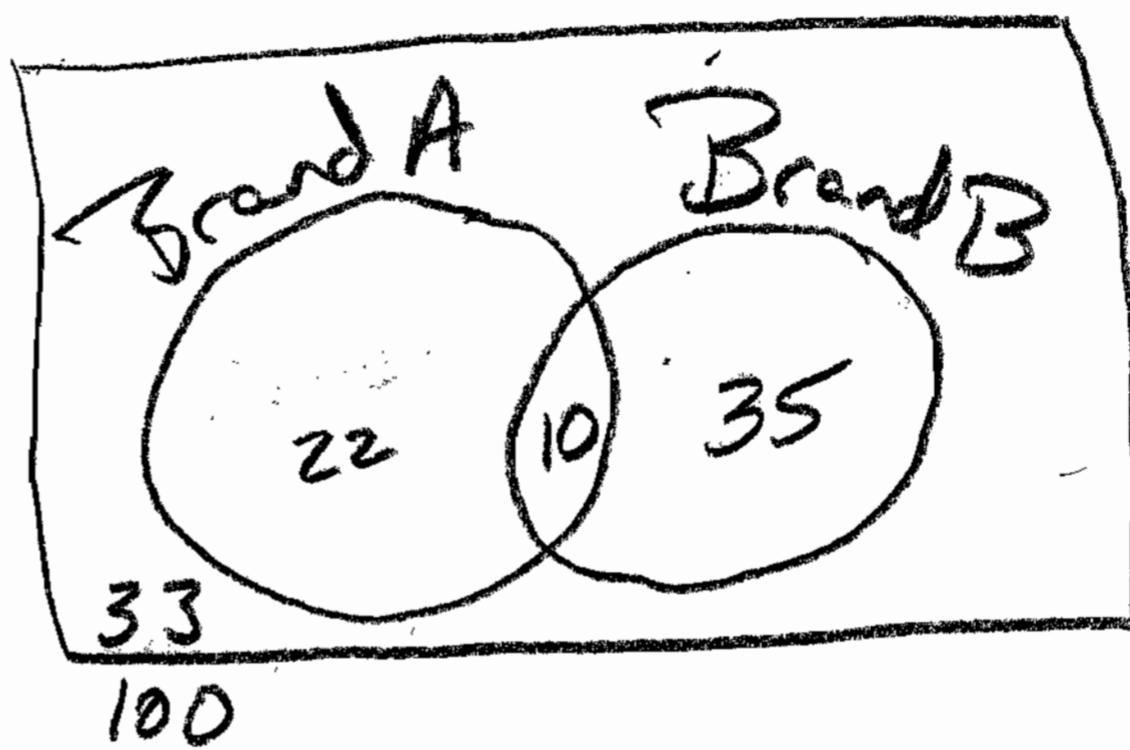
$$\text{Handicap} = \boxed{44}$$

### Part III

Answer all questions in this part. Each correct answer will receive 3 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [15]

- 26 In a telephone survey of 100 households, 32 households purchased Brand A cereal and 45 purchased Brand B cereal. If 10 households purchased both items, how many of the households surveyed did *not* purchase either Brand A or Brand B cereal?

	Brand A	Not Brand A	Total
Brand B	10	35	45
Not Brand B	22	33	55
Total	32	68	100



Answer  $\Rightarrow$  33 households did not purchase either

- 27 Tamika could not remember her scores from five mathematics tests. She did remember that the mean (average) was exactly 80, the median was 81, and the mode was 88. If all her scores were integers with 100 the highest score possible and 0 the lowest score possible, what was the lowest score she could have received on any one test?

Test 1	Test 2	Test 3	Test 4	Test 5
$\square$	$\square$	81	88	88
lowest score				highest score
Part 1	Part 2	Median	Mode	Mode
$x_1 + x_2 + x_3 + x_4 + x_5 = 143$	$x_1 + x_2 + 81 + 88 + 88 = 143$	$x_1 + x_2 + 257 = 143$	$143 - 80 = 63$	

Part 3  
If test 1 and test 2 sum to 143, test 1 score is lowest when test 2 score is highest. The highest test 2 can be is 80 since it must be less than the median.

$$\begin{array}{r} 400 = x_1 + x_2 + 257 \\ -257 \quad -257 \\ \hline 143 = x_1 + x_2 \end{array}$$

Answer: The lowest score was 63

- 28 There are 28 students in a mathematics class. If  $\frac{1}{4}$  of the students are called to the guidance office,  $\frac{1}{3}$  of the remaining students are called to the nurse, and, finally,  $\frac{1}{2}$  of those left go to the library, how many students remain in the classroom?

28

-  $\frac{1}{4}$  who go to guidance  $(\frac{28}{1})(\frac{1}{4}) = \frac{28}{4} = 7$  go

21 left

-  $\frac{1}{3}$  who go to nurse  $(\frac{21}{1})(\frac{1}{3}) = \frac{21}{3} = 7$  go

14 left

-  $\frac{1}{2}$  who go to library  $(\frac{14}{1})(\frac{1}{2}) = \frac{14}{2} = 7$  go

7 left in classroom  $\Rightarrow$  answer

- 29 On a bookshelf, there are five different mystery books and six different biographies. How many different sets of four books can Emilio choose if two of the books must be mystery books and two of the books must be biographies?

# Mystery Books

5

# Biographies

6

# Ways to choose 2  
Order Does Not Matter

$5 C_2$

$$\left( \frac{5 \times 4}{2 \times 1} \right)$$

# Ways to choose 2  
Order does not matter

$6 C_2$

$$\left( \frac{6 \times 5}{2 \times 1} \right) =$$

$$\left( \frac{20}{2} \right)$$

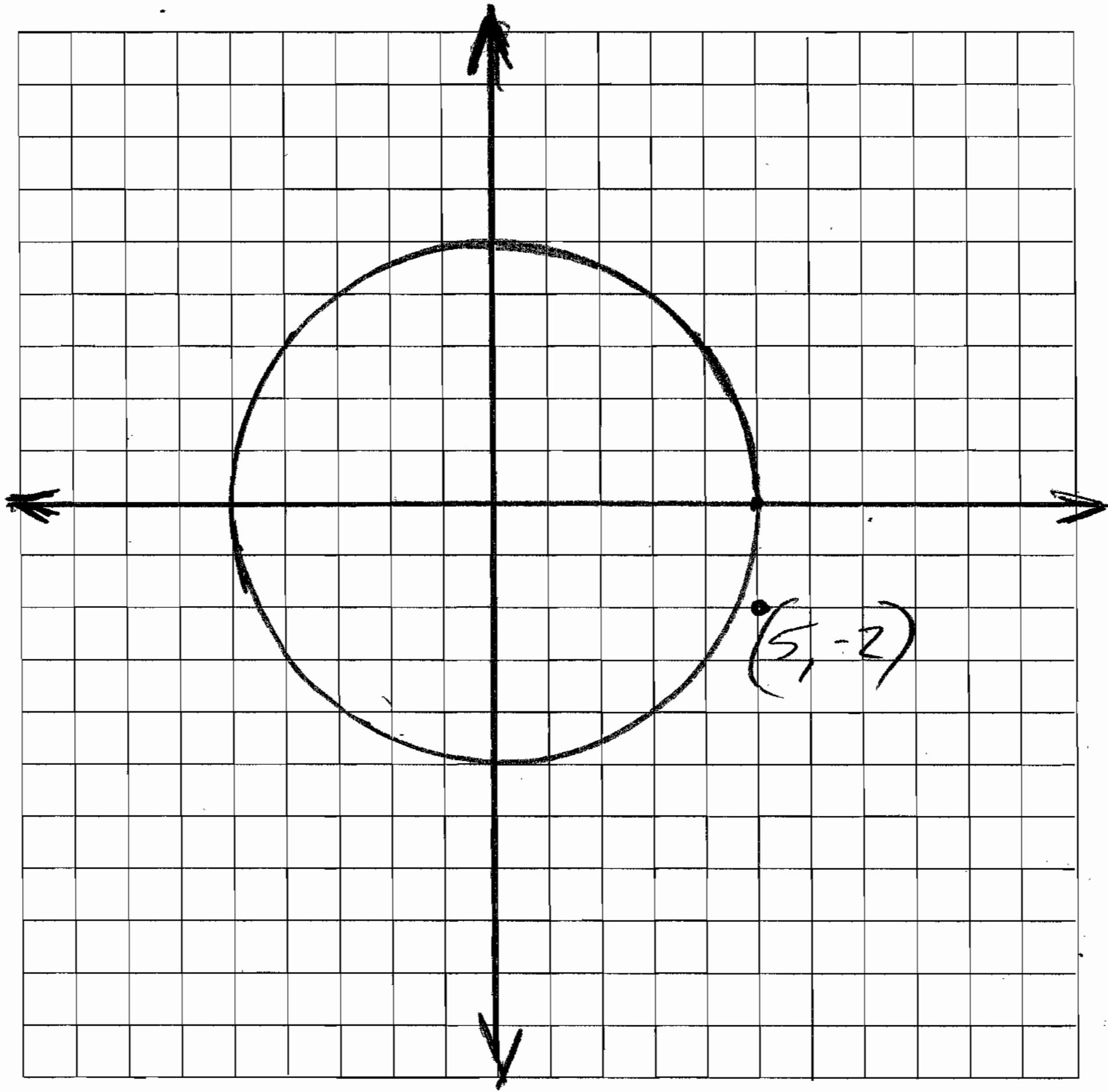
$$(10)$$

$$\left( \frac{30}{2} \right)$$

$$(15)$$

150

- 30 On the accompanying grid, graph a circle whose center is at  $(0,0)$  and whose radius is 5. Determine if the point  $(5,-2)$  lies on the circle.



Answer

$(5, -2)$  does not lie on the circle

Check

Equation of a circle is  $x^2 + y^2 = r^2$

The equation of this circle is  $x^2 + y^2 = 25$

$$(5, -2) \quad (5)^2 + (-2)^2 \stackrel{?}{=} 25 \\ 25 + 4 \neq 25$$

$$29 \neq 25$$

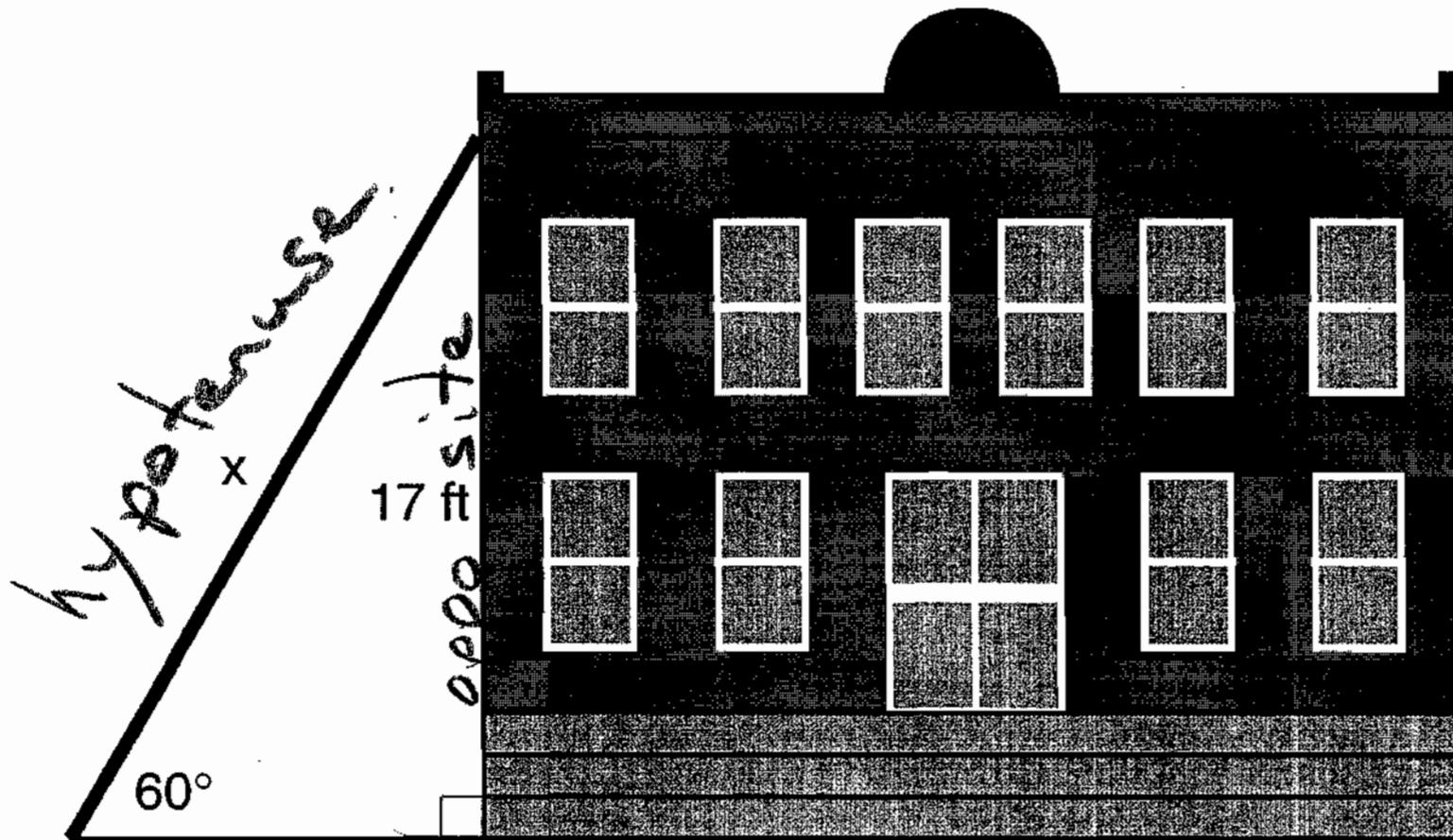
[12]

$\therefore$  The point  $(5, -2)$  is not on the circle

### Part IV

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [20]

- 31 In the accompanying diagram,  $x$  represents the length of a ladder that is leaning against a wall of a building, and  $y$  represents the distance from the foot of the ladder to the base of the wall. The ladder makes a  $60^\circ$  angle with the ground and reaches a point on the wall 17 feet above the ground. Find the number of feet in  $x$  and  $y$ .



Adjacent

$$\begin{aligned} \text{SOH - CAH - TOA} \\ \sin &= \frac{\text{Opp}}{\text{Hyp}} \\ \cos &= \frac{\text{Adj}}{\text{Hyp}} \\ \tan &= \frac{\text{Opp}}{\text{Adj}} \end{aligned}$$

$$\sin 60^\circ = \frac{17}{x}$$

$$\tan 60^\circ = \frac{17}{y}$$

$$\frac{\sin 60^\circ}{1} = \frac{17}{x}$$

$$\frac{\tan 60^\circ}{1} = \frac{17}{y}$$

$$(\sin 60^\circ)x = 17$$

$$\tan 60^\circ(y) = 17$$

Cross  
Multiplication

Set Calculated  
to Degrees

$$x = \frac{17}{\sin 60^\circ}$$

$$x = 19.62990915$$

$$y = \frac{17}{\tan 60^\circ}$$

$$y = 9.814954576$$

$$\text{Check } a^2 + b^2 = c^2$$

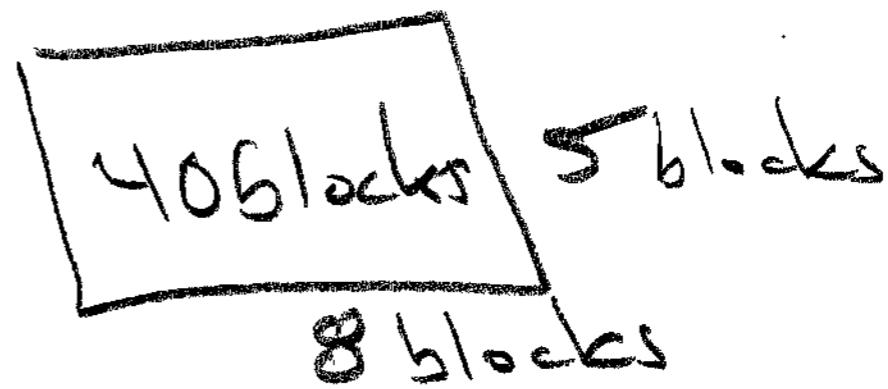
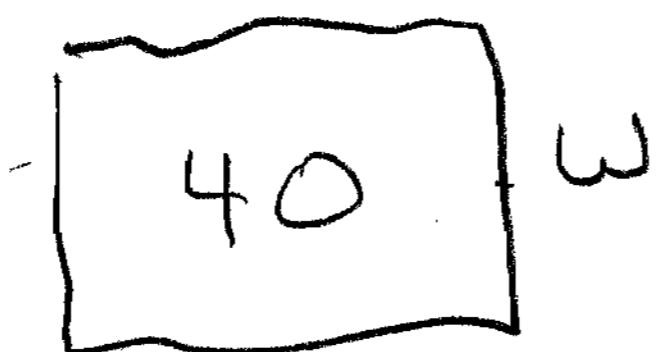
$$(9.81)^2 + (17)^2 \stackrel{?}{=} (19.63)^2$$

$$96.24 + 289 = 385.34$$

$$385.24 = 385.34$$

✓ close enough

- 32 A rectangular park is three blocks longer than it is wide. The area of the park is 40 square blocks. If  $w$  represents the width, write an equation in terms of  $w$  for the area of the park. Find the length and the width of the park.



Answer

$$\begin{aligned} w(w+3) &= 40 \\ w^2 + 3w &= 40 \\ -40 &\quad -40 \\ \hline w^2 + 3w - 40 &= 0 \\ (w+8)(w-5) &= 0 \\ w+8 &= 0 \quad w-5 = 0 \\ w &= -8 \quad w = 5 \end{aligned}$$

The park is  
5 blocks wide  
8 blocks long

Answer

- 33 Tanisha and Rachel had lunch at the mall. Tanisha ordered three slices of pizza and two colas. Rachel ordered two slices of pizza and three colas. Tanisha's bill was \$6.00, and Rachel's bill was \$5.25. What was the price of one slice of pizza? What was the price of one cola?

$$\begin{array}{ll} \text{Tanisha} & 3P + 2C = 6.00 \Rightarrow M(2) \quad 6P + 4C = 12.00 \\ \text{Rachel} & 2P + 3C = 5.25 \Rightarrow M(3) \quad -6P - 9C = -15.75 \\ & \hline & -5C = -3.75 \end{array}$$

$$\begin{aligned} 3P + 2(.75) &= 6.00 \\ 3P + 1.50 &= 6.00 \\ -1.50 &\quad -1.50 \\ \hline 3P &= 4.50 \\ P &= 1.50 \end{aligned}$$

C = .75  
Cola cost 75¢  
Pizza cost \$1.50  
Answer

$$\begin{aligned} 3P + 2C &= 6.00 \\ 3(1.50) + 2(.75) &= 6.00 \\ 4.50 + 1.50 &= 6.00 \\ 6.00 &= 6.00 \checkmark \end{aligned}$$

$$\begin{aligned} 2P + 3C &= 5.25 \\ 2(1.50) + 3(.75) &= 5.25 \\ 3.00 + 2.25 &= 5.25 \\ 5.25 &= 5.25 \checkmark \end{aligned}$$

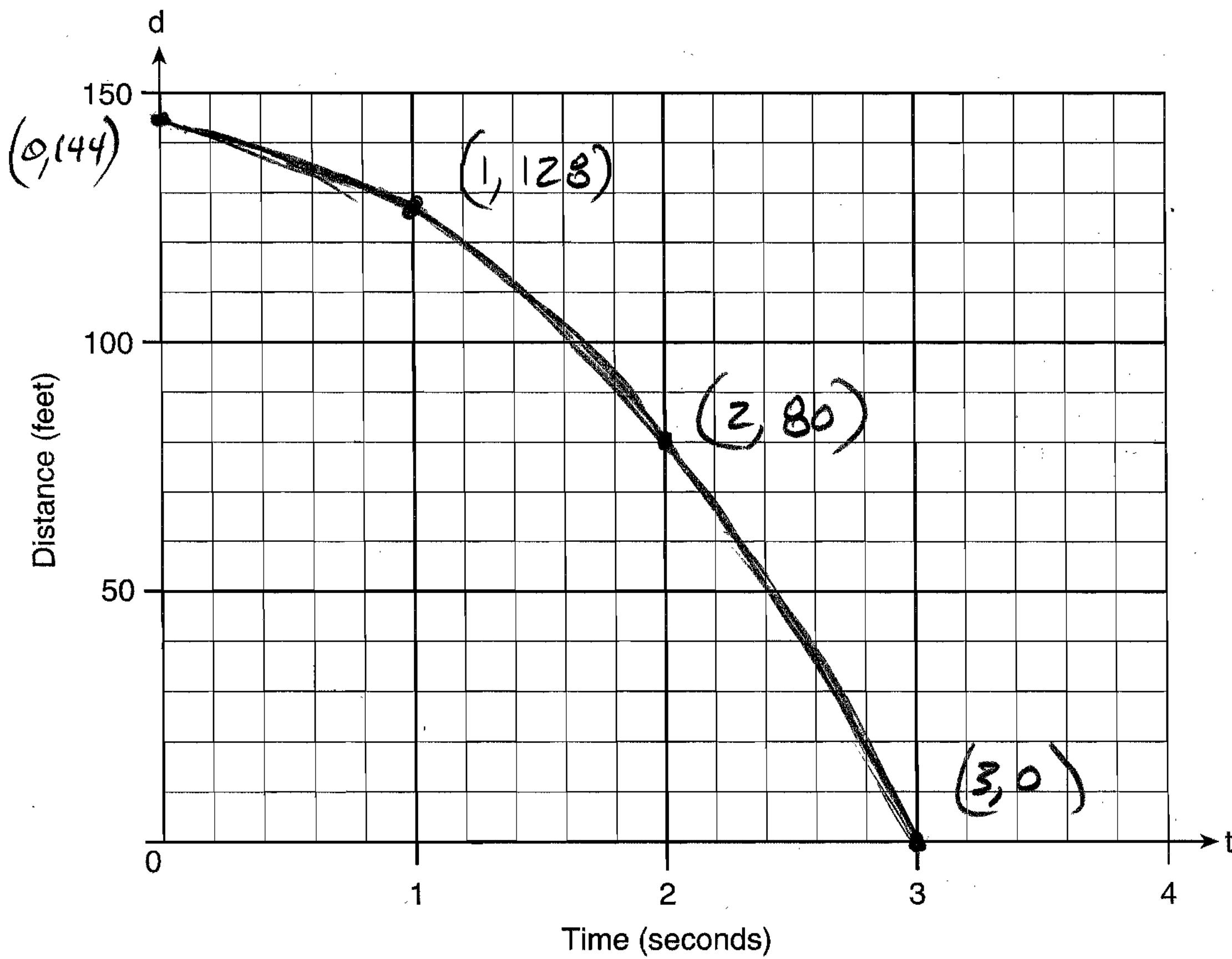
- 34 Greg is in a car at the top of a roller-coaster ride. The distance,  $d$ , of the car from the ground as the car descends is determined by the equation  $d = 144 - 16t^2$ , where  $t$  is the number of seconds it takes the car to travel down to each point on the ride. How many seconds will it take Greg to reach the ground?

For an algebraic solution show your work here.

$$\begin{aligned} d &= 144 - 16t^2 \\ 0 &= 144 - 16t^2 \\ D(6) \quad \frac{0}{16} &= \frac{144}{16} - \frac{16t^2}{16} \\ 0 &= 9 - t^2 \\ +t^2 & \quad +t^2 \\ \hline t^2 &= 9 \\ + &= 3 \end{aligned}$$

*Answer*  
3 Seconds

For a graphic solution show your work here.



- 35 Determine the distance between point  $A(-1, -3)$  and point  $B(5, 5)$ . Write an equation of the perpendicular bisector of  $\overline{AB}$ . [The use of the accompanying grid is optional.]

The line segment  $\overline{AB}$  has slope  $= \frac{8}{6}$   
and midpoint  $(2, 1)$

The perpendicular bisector of  $\overline{AB}$   
passes through its midpoint  $(2, 1)$  and  
has slope  $= (-\frac{6}{8})$ .

$$y = mx + b$$

$$1 = \frac{-6}{8}(2) + b$$

$$1 = -\frac{12}{8} + b$$

$$m = -\frac{6}{8}$$

$$(x, y)$$

$$(2, 1)$$

$$1 + \frac{12}{8} = b$$

$$1 + 1\frac{1}{2} = b$$

$$2\frac{1}{2} = b$$

$$y = \frac{-6}{8}x + b$$

$$\text{Midpoint of } \overline{AB} \text{ is } (2, 1)$$

$$\text{Slope of } \overline{AB} \text{ is } \frac{8}{6} \text{ or } \frac{\text{rise}}{\text{run}}$$

$$a^2 + b^2 = c^2$$

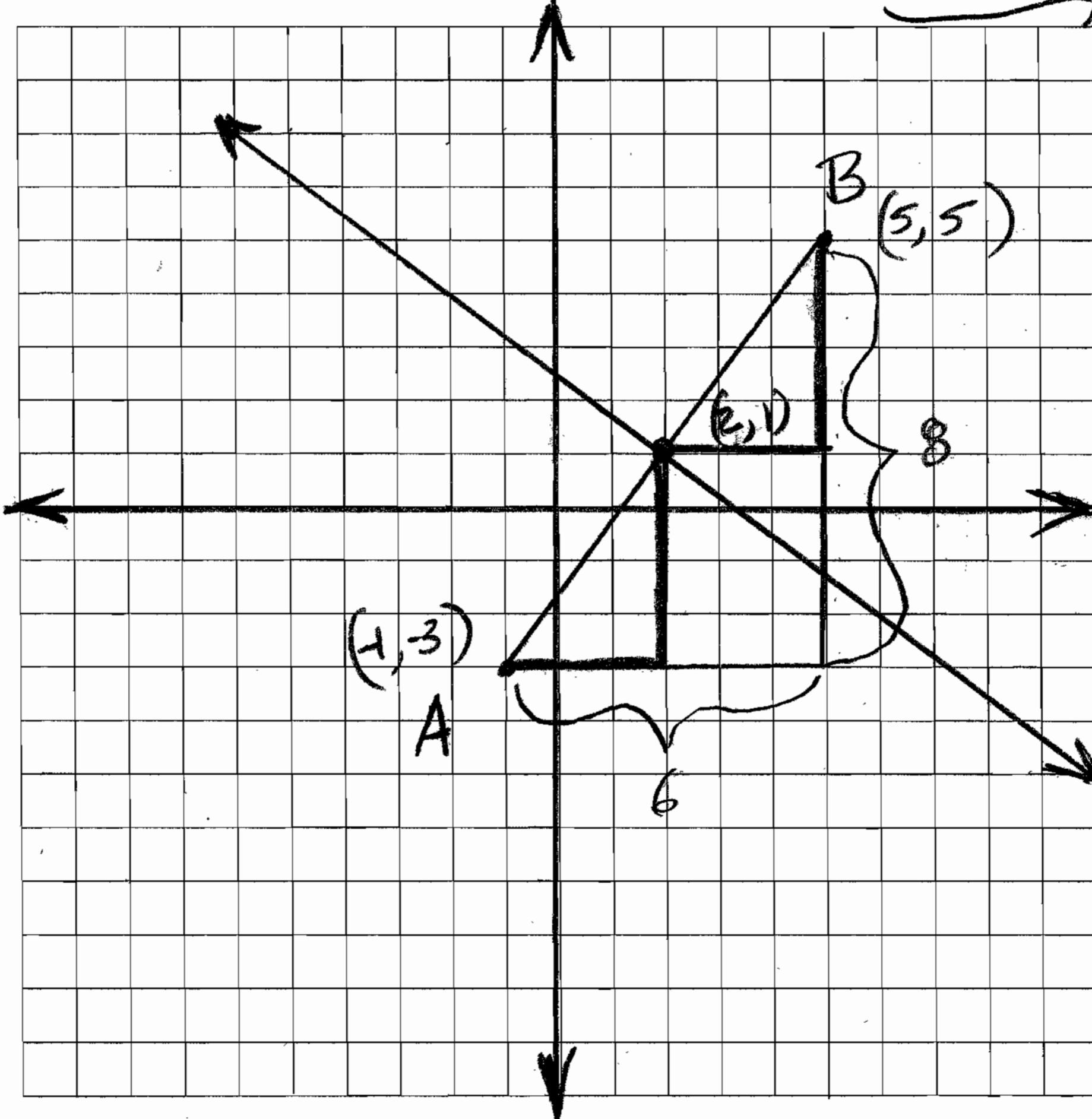
$$8^2 = c^2$$

$$64 = c^2$$

$$64 + b^2 = c^2$$

$$= c^2$$

$$100 = c^2$$



## The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

**MATHEMATICS A****Tuesday, August 13, 2002 — 8:30 to 11:30 a.m., only****ANSWER SHEET**Student Imaginary Student Sex:  Male  Female Grade .....Teacher Mr. Steve School IHS @ PH**Your answers to Part I should be recorded on this answer sheet.****Part I****Answer all 20 questions in this part.**

1	4	6	2	11	2	16	4
2	1	7	2	12	9	17	3
3	4	8	3	13	1	18	1
4	2	9	9	14	2	19	4
5	1	10	1	15	3	20	3

**Your answers for Parts II, III, and IV should be written in the test booklet.****The declaration below should be signed when you have completed the examination.**

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination and that I have neither given nor received assistance in answering any of the questions during the examination.

ASW

Signature