The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

MATHEMATICS B

Friday, June 20, 2003 — 1:15 to 4:15 p.m., only

Print Your Name: Steven Sibol

Print Your School's Name: HSCR

Print your name and the name of your school in the boxes above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. Any work done on this sheet of scrap graph paper will *not* be scored. All work should be written in pen, except graphs and drawings, which should be done in pencil.

This examination has four parts, with a total of 34 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. The formulas that you may need to answer some questions in this examination are found on page 19.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice. . .

A graphing calculator, a straightedge (ruler), and a compass must be available for your use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [40]

1 For which value of x is $y = \log x$ undefined?

(1) 0
(2) $\frac{1}{10}$
(3) π
(4) 1.483

Use this space for computations.

2 If $\sin \theta > 0$ and $\sec \theta < 0$, in which quadrant does the terminal side of angle θ lie?

(1) I
(2) II
(3) III
(4) IV

$\cos\theta < 0$

If $\sin\theta$ is positive and $\cos\theta$ is negative $(-,+)$, θ ends in Quadrant II

3 What is the value of x in the equation $81^{x+2} = 27^{5x+4}$?

(1) $-\frac{2}{11}$
(2) $-\frac{3}{2}$
(3) $\frac{4}{11}$
(4) $-\frac{4}{11}$

$(3^4)^{x+2} = (3^3)^{5x+4}$

$3^{4x+8} = 3^{15x+12}$

$4x+8 = 15x+12$

$-4x \quad -4x$

$8 = 11x+12$

$-12 \quad -12$

$\frac{-4}{11} = \frac{11x}{11}$

4 The relationship between voltage, E, current, I, and resistance, Z, is given by the equation $E = IZ$. If a circuit has a current $I = 3 + 2i$ and a resistance $Z = 2 - i$, what is the voltage of this circuit?

(1) $8 + i$
(2) $8 + 7i$
(3) $4 + i$
(4) $4 - i$

$E = IZ$

$(3+2i)(2-i)$

$6 - 3i + 4i - 2i^2$

$6 + i - 2(-1)$

$8 + i$

5 Which expression is equivalent to $\frac{4}{3+\sqrt{2}}$?

(1) $\frac{12+4\sqrt{2}}{7}$
(2) $\frac{12+4\sqrt{2}}{11}$
(3) $\frac{12-4\sqrt{2}}{7}$
(4) $\frac{12-4\sqrt{2}}{11}$

$\frac{4}{3+\sqrt{2}} \cdot \frac{3-\sqrt{2}}{3-\sqrt{2}} = \frac{4(3-\sqrt{2})}{9-2} = \frac{12-4\sqrt{2}}{7}$

6 What are the coordinates of point P, the image of point (3,–4) after a reflection in the line $y = x$?

(1) (3,4)
(2) (–3,4)
(3) (4,–3)
(4) (–4,3)

Use this space for computations.

7 The roots of the equation $ax^2 + 4x = -2$ are real, rational, and equal when a has a value of

(1) 1
(2) 2
(3) 3
(4) 4

$ax^2+4x+2=0$

The discriminant must equal 0

$b^2-4ac=0$

$4^2-4(a)(2)=0$

$16-8a=0$

$a=2$

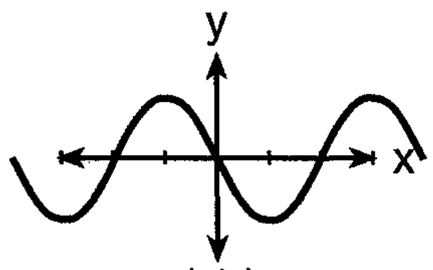
8 Two objects are 2.4×10^{20} centimeters apart. A message from one object travels to the other at a rate of 1.2×10^5 centimeters per second. How many seconds does it take the message to travel from one object to the other?

(1) 1.2×10^{15}
(2) 2.0×10^4
(3) 2.0×10^{15}
(4) 2.88×10^{25}

$t = \frac{\text{distance}}{\text{speed}} = \frac{2.4 \times 10^{20}}{1.2 \times 10^5}$

2×10^{15}

9 If $f(x) = \cos x$, which graph represents $f(x)$ under the composition $r_{y\text{-axis}} \circ r_{x\text{-axis}}$?

(1)


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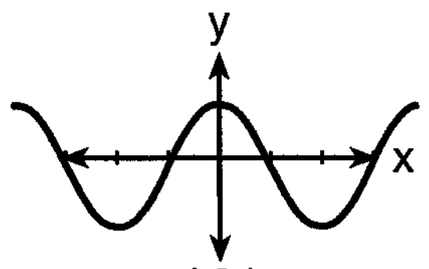

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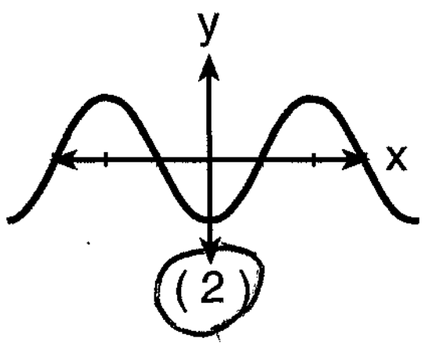

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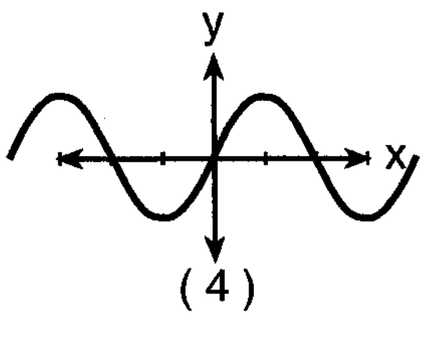
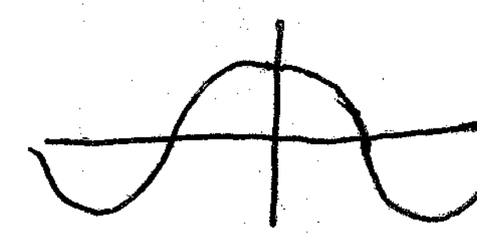
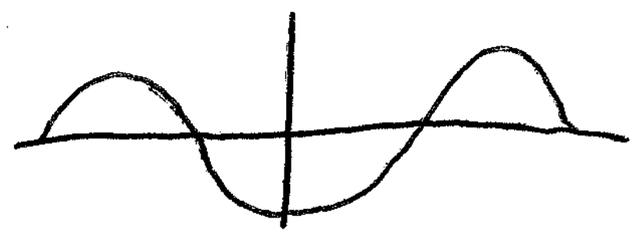


$\cos x$

$r_{x\text{-axis}}$

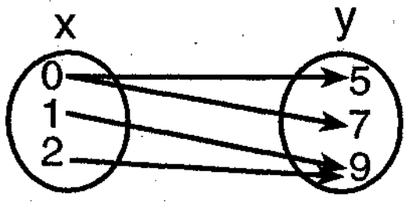


$r_{y\text{-axis}}$

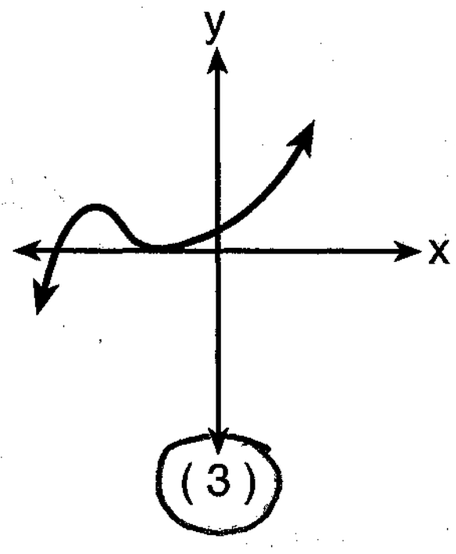


10 Which diagram represents a relation in which each member of the domain corresponds to only one member of its range?

Use this space for computations.

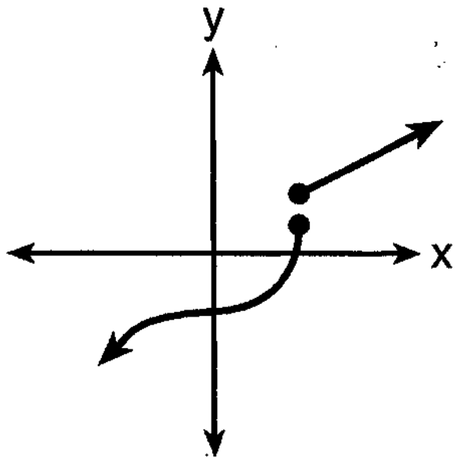


(1)

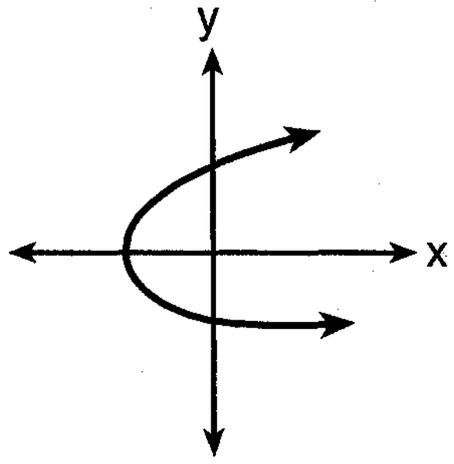


(3)

passed the vertical line test

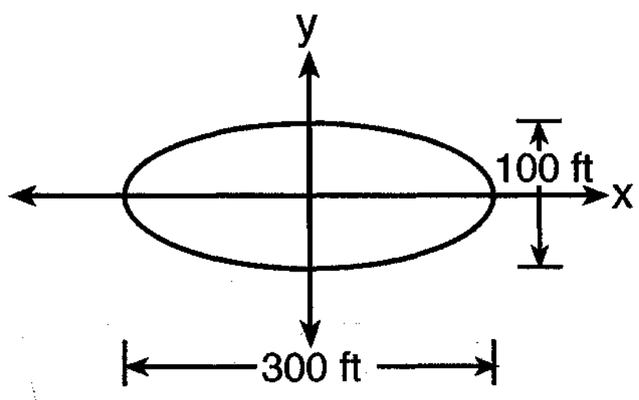


(2)



(4)

11 The accompanying diagram represents the elliptical path of a ride at an amusement park.



Which equation represents this path?

(1) $x^2 + y^2 = 300$ circle

(3) $\frac{x^2}{150^2} + \frac{y^2}{50^2} = 1$ ellipse

(2) $y = x^2 + 100x + 300$ parabola

(4) $\frac{x^2}{150^2} - \frac{y^2}{50^2} = 1$ hyperbola

Use function of the Sum of Two Angles

12 If A and B are positive acute angles, $\sin A = \frac{5}{13}$, and $\cos B = \frac{4}{5}$, what is the value of $\sin (A + B)$?

(1) $\frac{56}{65}$
(2) $\frac{63}{65}$
(3) $\frac{33}{65}$
(4) $-\frac{16}{65}$

$\sin^2 A + \cos^2 A = 1$
$\left(\frac{5}{13}\right)^2 + \cos^2 A = 1$
$\cos A = \frac{12}{13}$

$\sin^2 B + \left(\frac{4}{5}\right)^2 = 1$
$\sin B = \frac{3}{5}$

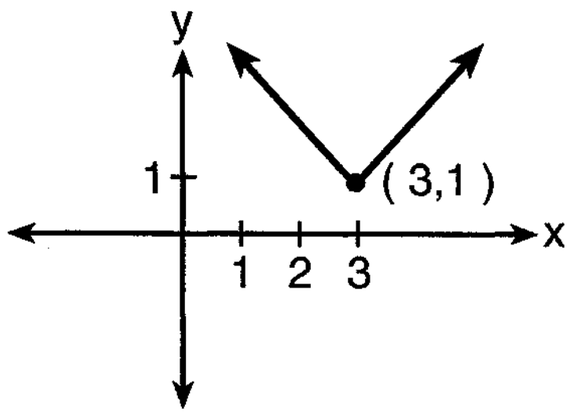
$\sin(A+B) = \sin A \cos B + \cos A \sin B$
$\left(\frac{5}{13}\right)\left(\frac{4}{5}\right) + \left(\frac{12}{13}\right)\left(\frac{3}{5}\right)$
$\frac{20}{65} + \frac{36}{65}$
$\frac{56}{65}$

Use this space for computations.

13 Which transformation is an opposite isometry?

(1) dilation
(2) line reflection
(3) rotation of 90°
(4) translation

14 Which equation is represented by the accompanying graph?



(1) $y = |x| - 3$
(2) $y = (x-3)^2 + 1$
(3) $y = |x+3| - 1$
(4) $y = |x-3| + 1$

15 What is the value of $i^{99} - i^3$?

(1) 1
(2) i^{96}
(3) $-i$
(4) 0

$i^3 - i^3$
0

$\frac{99}{4} = 24$ r3
$i^{99} = i^3$

16 If $\log a = 2$ and $\log b = 3$, what is the numerical value of $\log \frac{\sqrt{a}}{b^3}$?

(1) 8
(2) –8
(3) 25
(4) –25

$\log \sqrt{a} - \log b^3$
$\log a^{1/2} - \log b^3$
$\frac{1}{2}\log a - 3\log b$
$\frac{1}{2}(2) - 3(3)$
$1 - 9 = -8$

17 In simplest form, $\dfrac{\frac{1}{x^2} - \frac{1}{y^2}}{\frac{1}{y} + \frac{1}{x}}$ is equal to

(1) $\frac{x-y}{xy}$ (3) $x-y$

(2) $\frac{y-x}{xy}$ (4) $y-x$

$\frac{y^2-x^2}{x^2y^2} \div \frac{x+y}{xy}$

$\frac{(y-x)(y+x)}{x^2y^2} \times \frac{xy}{x+y} = \frac{y-x}{xy}$

Use this space for computations.

18 What is the solution set of the inequality $|3-2x| \geq 4$?

(1) $\left\{x \middle| \frac{7}{2} \leq x \leq -\frac{1}{2}\right\}$ (3) $\left\{x \middle| x \leq -\frac{1}{2} \text{ or } x \geq \frac{7}{2}\right\}$

(2) $\left\{x \middle| -\frac{1}{2} \leq x \leq \frac{7}{2}\right\}$ (4) $\left\{x \middle| x \leq \frac{7}{2} \text{ or } x \geq -\frac{1}{2}\right\}$

$3-2x \geq 4$ $\quad 3-2x \leq -4$

$-3 \quad -3$ $\quad -3 \quad -3$

$\frac{-2x}{-2} \geq \frac{1}{-2}$ $\quad \frac{-2x}{-2} \leq \frac{-7}{-2}$

$x \leq -\frac{1}{2}$ $\quad x \geq \frac{7}{2}$

19 What value of x in the interval $0° \leq x \leq 180°$ satisfies the equation $\sqrt{3}\tan x + 1 = 0$?

(1) $-30°$ (3) $60°$

(2) $30°$ (4) $150°$

$\sqrt{3}\tan x + 1 = 0$

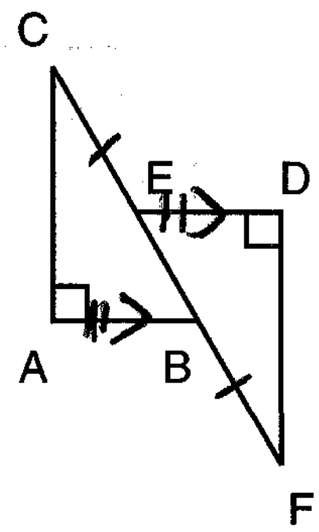
$-1 \quad -1$

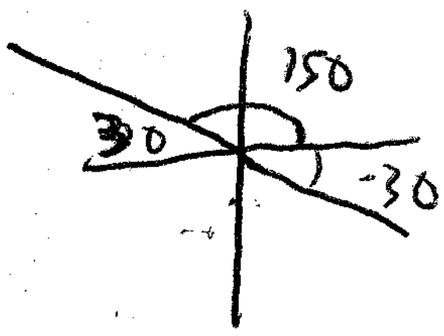
$\frac{\sqrt{3}\tan x}{\sqrt{3}} = \frac{-1}{\sqrt{3}}$

$x = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

$x = -30$

20 In the accompanying diagram, $\overline{CA} \perp \overline{AB}$, $\overline{ED} \perp \overline{DF}$, $\overline{ED} \parallel \overline{AB}$, $\overline{CE} \cong \overline{BF}$, $\overline{AB} \cong \overline{ED}$, and $m\angle CAB = m\angle FDE = 90$.





Which statement would *not* be used to prove $\triangle ABC \cong \triangle DEF$?

(1) SSS $\cong$ SSS (3) AAS $\cong$ AAS

(2) SAS $\cong$ SAS (4) HL $\cong$ HL

SSS cannot be used since no evidence that $\overline{AC} \cong \overline{DF}$

Part II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

21 Vanessa throws a tennis ball in the air. The function $h(t) = -16t^2 + 45t + 7$ represents the distance, in feet, that the ball is from the ground at any time t. At what time, to the *nearest tenth of a second,* is the ball at its maximum height?

$x = \frac{-b}{2a} = \frac{-45}{2(-16)} \approx 1.4$ sec

22 If $f(x) = 2^x - 1$ and $g(x) = x^2 - 1$, determine the value of $(f \circ g)(3)$.

$g(3) = 3^2 - 1 = 8$

$F(8) = 2^8 - 1 = 255$

23 When air is pumped into an automobile tire, the pressure is inversely proportional to the volume. If the pressure is 35 pounds when the volume is 120 cubic inches, what is the pressure, in pounds, when the volume is 140 cubic inches?

$$P_1 V_1 = P_2 V_2$$

$$\frac{35 \cdot 120}{140} = \frac{P_2 \cdot 140}{140}$$

$$30 = P_2$$

24 In a certain school district, the ages of all new teachers hired during the last 5 years are normally distributed. Within this curve, 95.4% of the ages, centered about the mean, are between 24.6 and 37.4 years. Find the mean age and the standard deviation of the data.

Since the ages are normally distributed and centered about the mean, the mean is the midpoint of the two given values.

$$\frac{24.6 + 37.4}{2} = 31$$

The 95.4% figure represent data within 2 standard deviations of the mean.

$$\frac{37.4 - 31}{2} = 3.2$$

25 Express the following rational expression in simplest form:

$$\frac{9 - x^2}{10x^2 - 28x - 6}$$

$$-1\frac{(3-x)(3+x)}{(10x+2)(x-3)} = \frac{-(x+3)}{10x+2}$$

26 Evaluate: $2\sum_{n=1}^{5}(2n-1)$

n	$2n-1$
1	1
2	3
3	5
4	7
5	9

$2 \times 25 = 50$

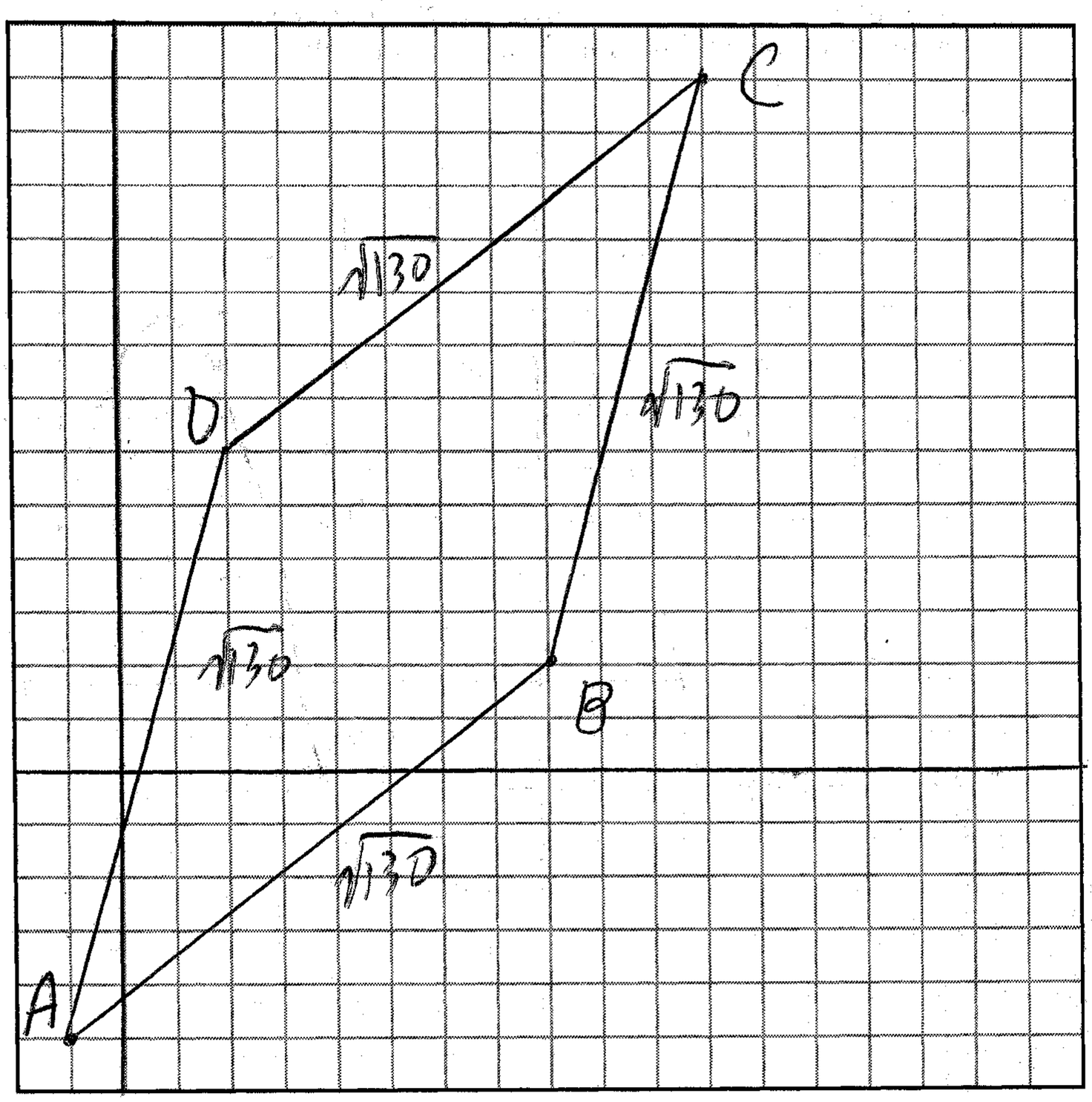
Part III

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [24]

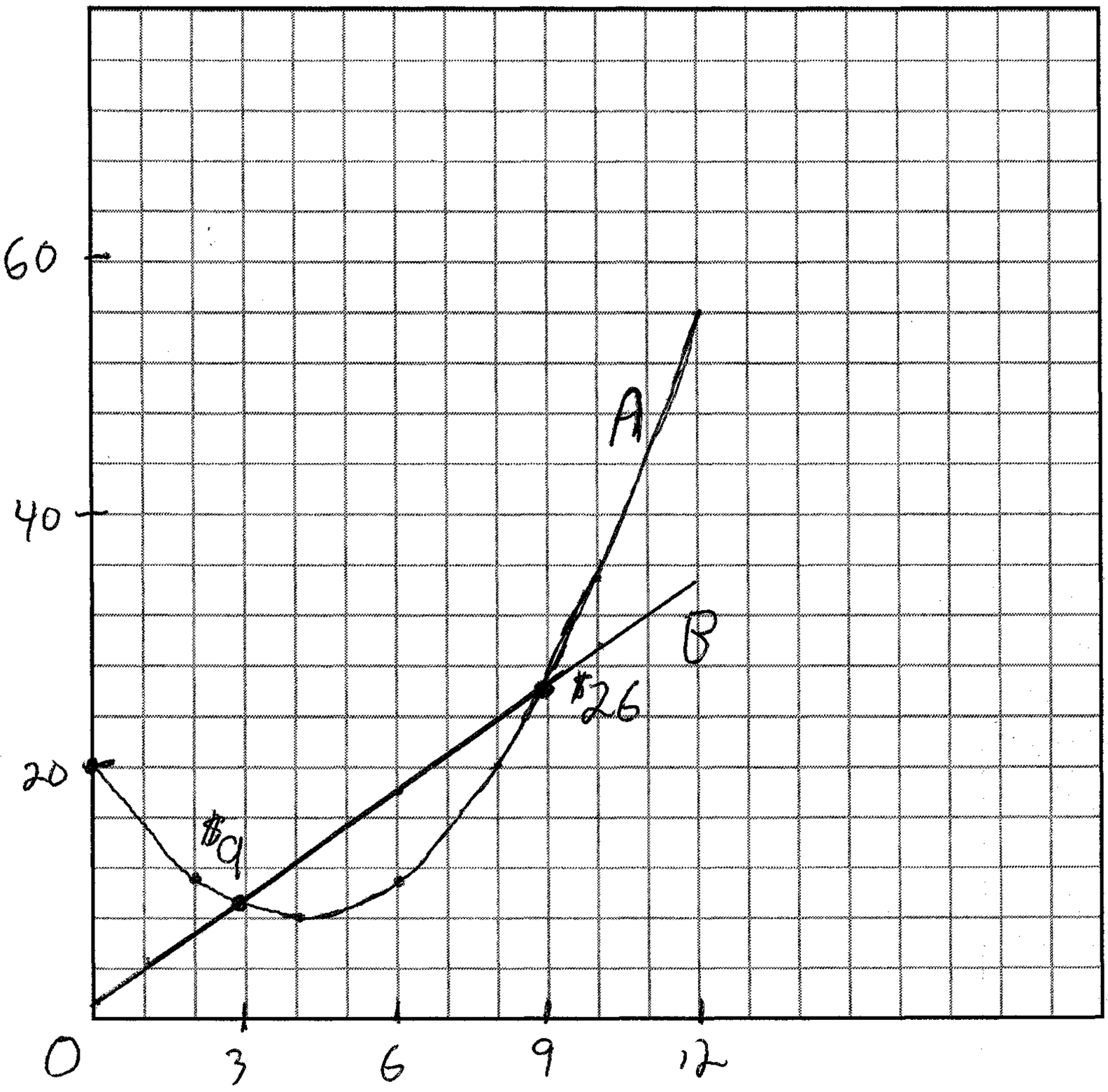
27 The coordinates of quadrilateral $ABCD$ are $A(-1,-5)$, $B(8,2)$, $C(11,13)$, and $D(2,6)$. Using coordinate geometry, prove that quadrilateral $ABCD$ is a rhombus. [The use of the grid on the next page is optional.]

STATEMENT	REASON
(1) Quadrilateral ABCD with coordinates $A(-1,-5)$, $B(8,2)$, $C(11,13)$, and $D(2,6)$.	(1) Given
(2) $d_{\overline{AB}} = \sqrt{(8-(-1))^2+(2-(-5))^2} = \sqrt{130}$ $d_{\overline{BC}} = \sqrt{(11-8)^2+(13-2)^2} = \sqrt{130}$ $d_{\overline{CD}} = \sqrt{(11-2)^2+(13-6)^2} = \sqrt{130}$ $d_{\overline{AD}} = \sqrt{(2-(-1))^2+(6-(-5))^2} = \sqrt{130}$	(2) Distance Formula
(3) ABCD is a rhombus.	(3) A rhombus is a quadrilateral with four equal sides.

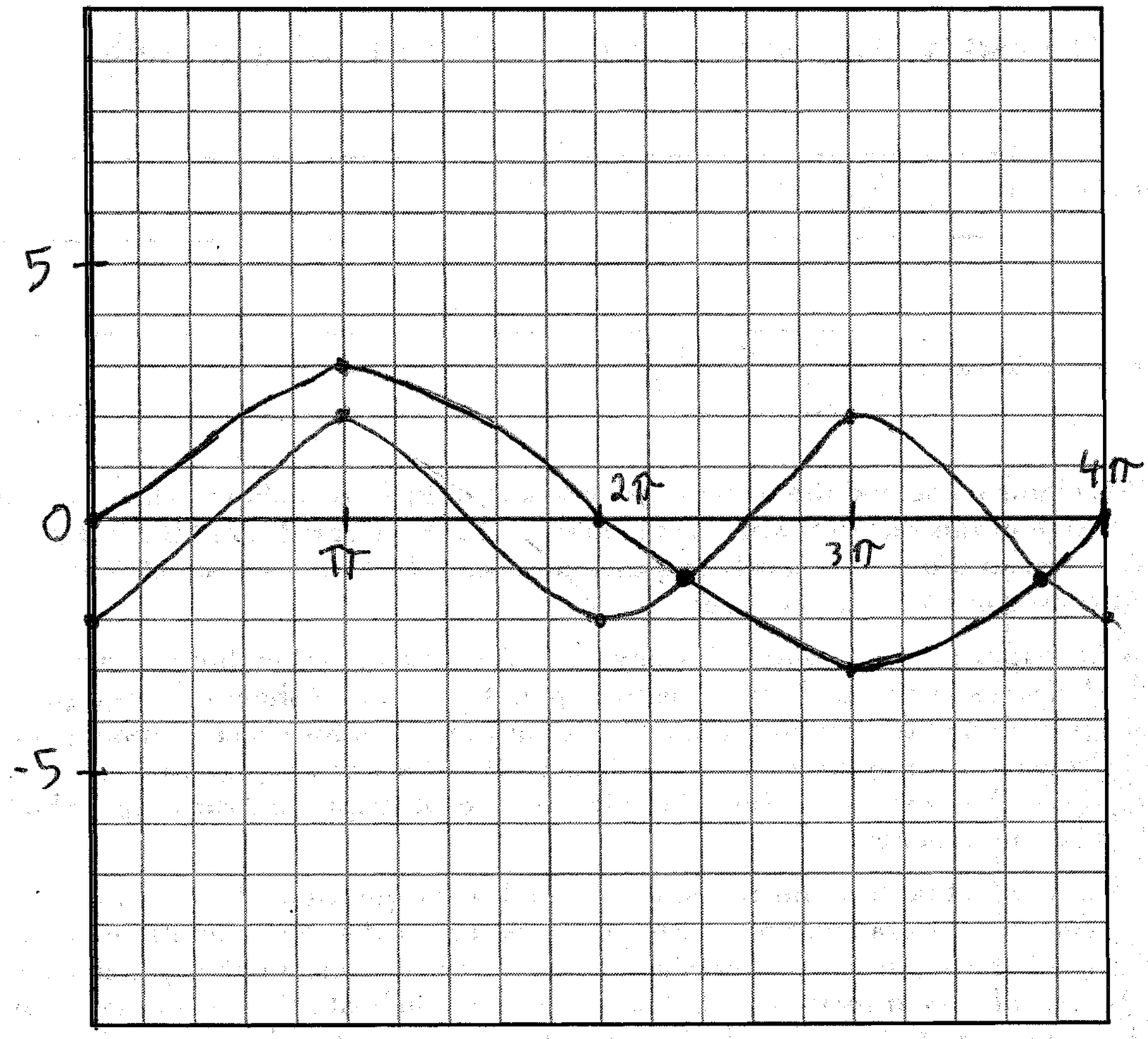
Question 27 continued



28 The price of a stock, $A(x)$, over a 12-month period decreased and then increased according to the equation $A(x) = 0.75x^2 - 6x + 20$, where x equals the number of months. The price of another stock, $B(x)$, increased according to the equation $B(x) = 2.75x + 1.50$ over the same 12-month period. Graph and label both equations on the accompanying grid. State all prices, to the *nearest dollar,* when both stock values were the same.



29 A pair of figure skaters graphed part of their routine on a grid. The male skater's path is represented by the equation $m(x) = 3 \sin \frac{1}{2}x$, and the female skater's path is represented by the equation $f(x) = -2 \cos x$. On the accompanying grid, sketch both paths and state how many times the paths of the skaters intersect between $x = 0$ and $x = 4\pi$.



The paths of the skaters intersect twice.

30 Sean invests \$10,000 at an annual rate of 5% compounded continuously, according to the formula $A = Pe^{rt}$, where A is the amount, P is the principal, $e = 2.718$, r is the rate of interest, and t is time, in years.

Determine, to the *nearest dollar*, the amount of money he will have after 2 years.

$A = Pe^{rt}$

$A = 10{,}000(2.718)^{(.05)(2)}$

$A \approx \$11{,}052$

Determine how many years, to the *nearest year*, it will take for his initial investment to double.

$A = Pe^{rt}$

$\frac{20{,}000}{10{,}000} = \frac{10{,}000(2.718)^{.05t}}{10{,}000}$

$\log 2 = \log 2.718^{.05t}$

$\log 2 = .05t \log 2.718$

$t = \frac{\log 2}{.05 \log 2.718} \approx 14$

31 On any given day, the probability that the entire Watson family eats dinner together is $\frac{2}{5}$. Find the probability that, during any 7-day period, the Watsons eat dinner together *at least* six times.

$n = 7$

$r = 6, 7$

$p = \frac{2}{5}$

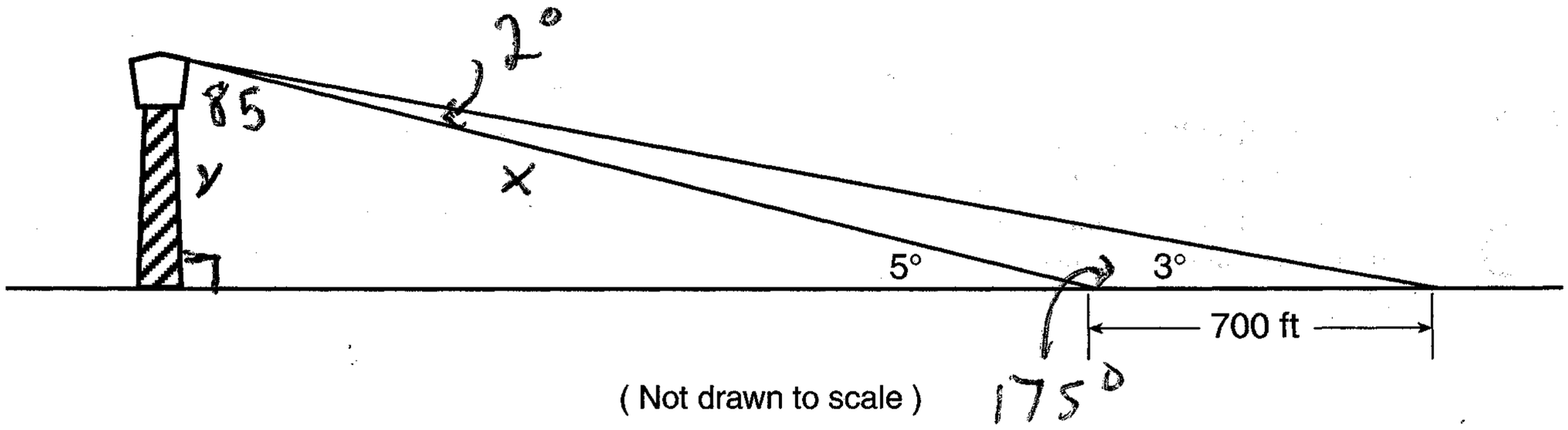
$q = \frac{3}{5}$

$P(6) = {}_7C_6\left(\frac{2}{5}\right)^6\left(\frac{3}{5}\right)^1 = \frac{1344}{78{,}125}$

$P(7) = {}_7C_7\left(\frac{2}{5}\right)^7\left(\frac{3}{5}\right)^0 = \frac{128}{78{,}125}$

$\frac{1472}{78{,}125}$

32 While sailing a boat offshore, Donna sees a lighthouse and calculates that the angle of elevation to the top of the lighthouse is 3°, as shown in the accompanying diagram. When she sails her boat 700 feet closer to the lighthouse, she finds that the angle of elevation is now 5°. How tall, to the *nearest tenth of a foot,* is the lighthouse?



(Not drawn to scale)

$$\frac{x}{\sin 3} = \frac{700}{\sin 2}$$

$$x = \frac{700 \sin 3}{\sin 2}$$

$$x \approx 1049.7$$

$$\sin 5 = \frac{y}{1049.7}$$

$$x \approx 91.5$$

Part IV

Answer all questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

33 A farmer has determined that a crop of strawberries yields a yearly profit of \$1.50 per square yard. If strawberries are planted on a triangular piece of land whose sides are 50 yards, 75 yards, and 100 yards, how much profit, to the *nearest hundred dollars,* would the farmer expect to make from this piece of land during the next harvest?

Use Heron's Formula to find area

$A = \sqrt{s(s-a)(s-b)(s-c)}$ $p = 225$

$s = 112.5$

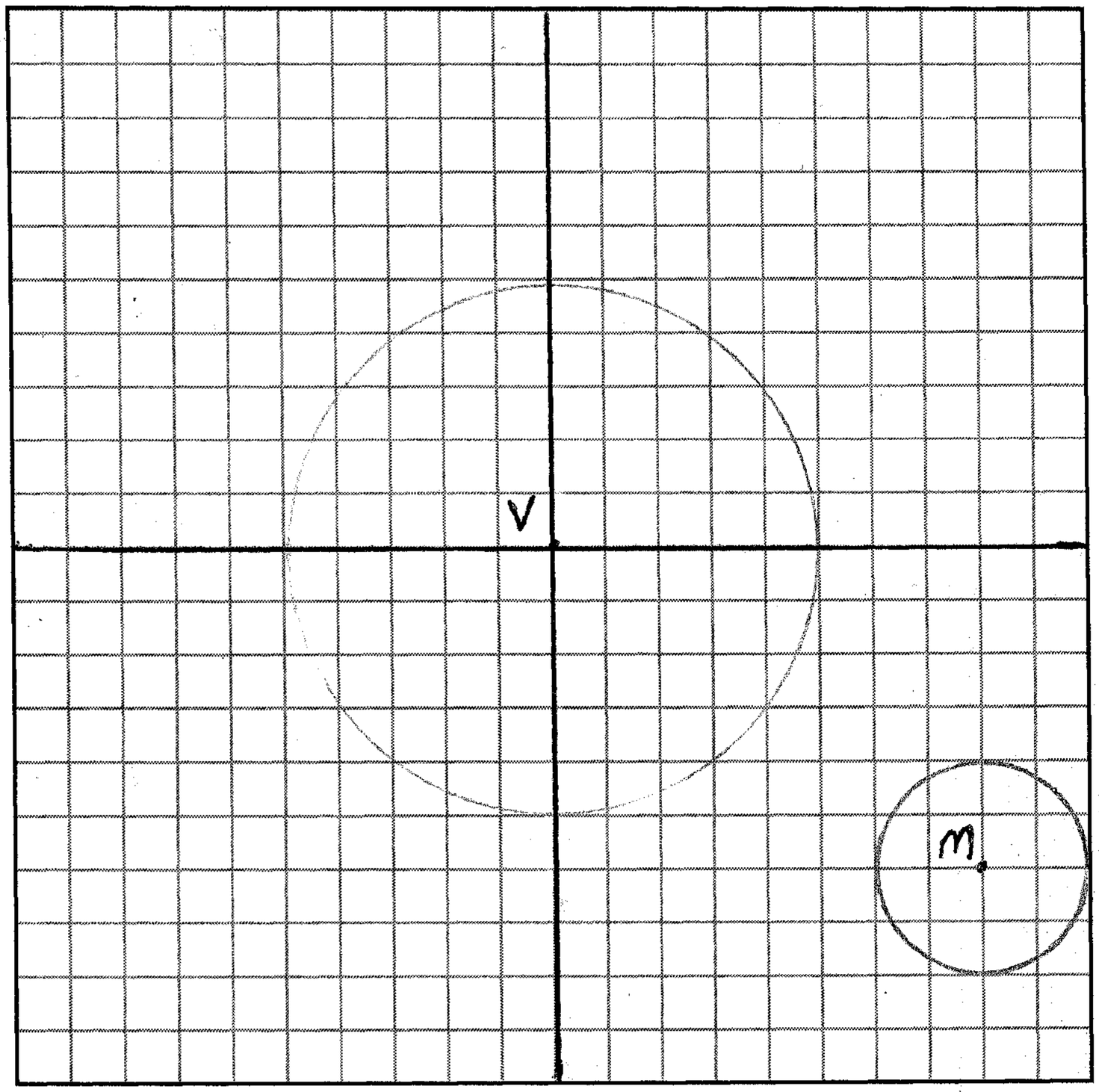
$A = \sqrt{112.5(112.5-50)(112.5-75)(112.5-100)}$

$A \approx 1815$

$Profit = 1815 \times \$1.50 \approx \$2700$

34 For a carnival game, John is painting two circles, V and M, on a square dartboard.

a On the accompanying grid, draw and label circle V, represented by the equation $x^2 + y^2 = 25$, and circle M, represented by the equation $(x - 8)^2 + (y + 6)^2 = 4$.



b A point, (x,y), is randomly selected such that $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$. What is the probability that point (x,y) lies outside both circle V and circle M?

The area of the grid is $20 \times 20 = 400$

The area of the circles is $5^2\pi + 2^2\pi = 29\pi$

$$\frac{400 - 29\pi}{400} \approx 0.77$$