

# MATHEMATICS B

Tuesday, June 23, 2009 — 1:15 to 4:15 p.m., only

Print Your Name:

Steve Sibol

Print Your School's Name:

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Print your name and the name of your school in the boxes above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored. Write all your work in pen, except graphs and drawings, which should be done in pencil.

The formulas that you may need to answer some questions in this examination are found on page 23. This sheet is perforated so you may remove it from this booklet.

This examination has four parts, with a total of 34 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice . . .

A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

The use of any communications device is strictly prohibited when taking this examination. If you use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

**DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.**

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [40]

Use this space for computations.

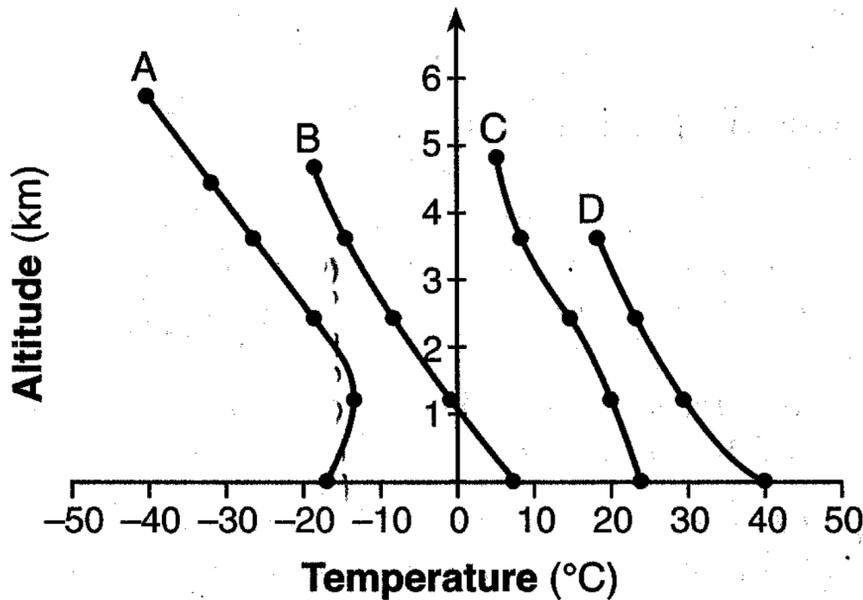
1 The number of degrees equal to  $\frac{5}{9}\pi$  radians is

- (1) 45
- (2) 90

- (3) 100
- (4) 900

$$\frac{5\pi}{9} \cdot \frac{180}{\pi} = 100^\circ$$

2 The accompanying graph shows the curves of best fit for data points comparing temperature to altitude in four different regions, represented by the relations A, B, C, and D.



Which relation is *not* a function?

- (1) A
- (2) B

- (3) C
- (4) D

3 What is the value of  $\sum_{k=1}^3 (2 - k)^2$ ?

- (1) 1
- (2) 2

- (3) 3
- (4) 0

$$\begin{aligned} (2-1)^2 &= 1 \\ (2-2)^2 &= 0 \\ (2-3)^2 &= 1 \\ \hline &2 \end{aligned}$$

Use this space for computations.

4 If  $\sin x = \frac{1}{a}$ ,  $a \neq 0$ , which statement must be true?

- (1)  $\csc x = a$  (3)  $\sec x = a$   
 (2)  $\csc x = -\frac{1}{a}$  (4)  $\sec x = -\frac{1}{a}$

5 The expression  $\frac{5 + \sqrt{7}}{5 - \sqrt{7}}$  is equivalent to

- (1)  $\frac{16 + 5\sqrt{7}}{16}$  (3)  $\frac{16 - 5\sqrt{7}}{16}$   
 (2)  $\frac{16 + 5\sqrt{7}}{9}$  (4)  $\frac{16 - 5\sqrt{7}}{9}$

$$\frac{5 + \sqrt{7}}{5 - \sqrt{7}} \cdot \frac{5 + \sqrt{7}}{5 + \sqrt{7}} = \frac{25 + 10\sqrt{7} + 7}{25 - 7}$$

$$\frac{32 + 10\sqrt{7}}{18}$$

$$\frac{16 + 5\sqrt{7}}{9}$$

6 When the sum of  $-4 + 8i$  and  $2 - 9i$  is graphed, in which quadrant does it lie?

- (1) I (3) III  
 (2) II (4) IV

$$\begin{array}{r} -4 + 8i \\ 2 - 9i \\ \hline -2 - i \end{array}$$

7 What is the solution of the inequality  $|2x - 5| < 1$ ?

- (1)  $x < 3$  (3)  $x > -3$   
 (2)  $2 < x < 3$  (4)  $x \leq 2$  or  $x \geq 3$

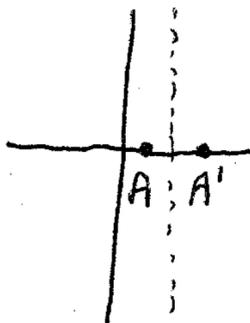
$$\begin{array}{l} 2x - 5 < 1 \\ 2x < 6 \\ x < 3 \end{array} \qquad \begin{array}{l} 2x - 5 > -1 \\ 2x > 4 \\ x > 2 \end{array}$$

8 Point A(1,0) is a point on the graph of the equation  $y = x^2 - 4x + 3$ . When point A is reflected across the axis of symmetry, what are the coordinates of its image, point A'?

- (1) (-1,2) (3) (2,-1)  
 (2) (0,3) (4) (3,0)

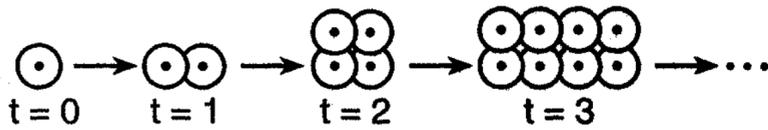
axis of symmetry

$$x = \frac{-b}{2a} = \frac{4}{2(1)} = 2$$



Use this space for computations.

- 9 The accompanying diagram represents the biological process of cell division.



If this process continues, which expression best represents the number of cells at any time,  $t$ ?

- (1)  $t + 2$   
 (2)  $2t$   
 (3)  $t^2$   
 (4)  $2^t$

- 10 The roots of the equation  $x^2 - 5x + 1 = 0$  are

- (1) real, rational, and unequal  
 (2) real, rational, and equal  
 (3) real, irrational, and unequal  
 (4) imaginary

$$b^2 - 4ac$$

$$(-5)^2 - 4(1)(1)$$

$$25 - 4 = 21$$

- 11 Using a drawing program, a computer graphics designer constructs a circle on a coordinate plane on her computer screen. She determines that the equation of the circle's graph is  $(x - 3)^2 + (y + 2)^2 = 36$ . She then dilates the circle with the transformation  $D_3$ . After this transformation, what is the center of the new circle?

- (1)  $(6, -5)$   
 (2)  $(-6, 5)$   
 (3)  $(9, -6)$   
 (4)  $(-9, 6)$

$$(3, -2) \Rightarrow (9, -6)$$

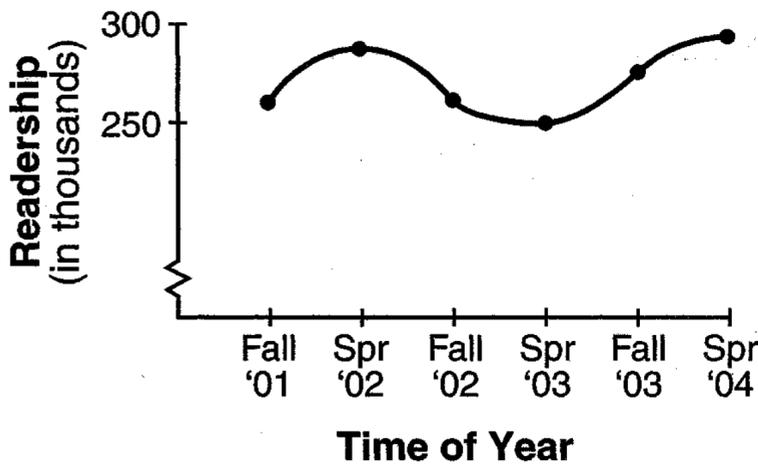
- 12 Which expression is equivalent to  $(\sqrt{a^2 b^{\frac{1}{2}}})^{-1}$ ?

- (1)  $a^{-2} b^{-\frac{1}{2}}$   
 (2)  $-ab^{\frac{1}{4}}$   
 (3)  $-ab^2$   
 (4)  $\frac{1}{ab^{\frac{1}{4}}}$

$$\left( (a^2 b^{\frac{1}{2}})^{\frac{1}{2}} \right)^{-1} = \frac{1}{ab^{\frac{1}{4}}}$$

Use this space for computations.

13 The accompanying graph shows the average daily readership, in thousands, of the newspaper "El Diario La Prensa."



Which type of function best represents this graph?

- (1) exponential  
 (2) logarithmic  
 (3) trigonometric  
 (4) quadratic

14 The expression  $\frac{\sin 2A}{2 \cos A}$  is equivalent to

- (1)  $\cos A$   
 (2)  $\tan A$   
 (3)  $\sin A$   
 (4)  $\frac{1}{2} \sin A$

$$\frac{2 \sin A \cos A}{2 \cos A} = \sin A$$

15 What is the solution set of the equation  $y = 2 + \sqrt{y^2 - 12}$ ?

- (1)  $\{ \}$   
 (2)  $\{2\}$   
 (3)  $\{-4, 4\}$   
 (4)  $\{4\}$

$$y - 2 = \sqrt{y^2 - 12}$$

$$(y - 2)^2 = y^2 - 12$$

$$y^2 - 4y + 4 = y^2 - 12$$

$$-4y = -16$$

$$y = 4$$

16 What is the third term in the expansion of  $(2x - 3)^5$ ?

- (1)  $-1080x^2$   
 (2)  $-720x^3$   
 (3)  $720x^3$   
 (4)  $1080x^3$

$$n = 5$$

$$r - 1 = 2$$

$$a = 2x$$

$$b = -3$$

$${}^5C_2 (2x)^3 (-3)^2$$

$$10 \cdot 8x^3 \cdot 9$$

$$720x^3$$

Use this space for computations.

17 The accompanying table shows the scores on a classroom test.

$x_i$	$f_i$
100	7
90	10
80	4
70	4

What is the population standard deviation for this set of scores?

- (1) 10.2                      (3) 25  
 (2) 10.4                      (4) 88

18 The manager of Stuart Siding Company found that the number of workers used to side a house varies inversely with the number of hours needed to finish the job. If four workers can side the house in 48 hours, how many hours will it take six workers working at the same speed to do the same job?

- (1) 32                              (3) 42  
 (2) 36                              (4) 72

$$w_1 h_1 = w_2 h_2$$

$$48 \cdot 4 = 6h$$

$$192 = 6h$$

$$32 = h$$

19 The expression  $\frac{1 - \frac{x}{x-y}}{\frac{1}{x-y}}$  is equivalent to

- (1)  $1 - x$                        (3)  $y$   
 (2)  $x - y$                        (4)  $-y$

$$\frac{\frac{x-y}{x-y} - \frac{x}{x-y}}{\frac{1}{x-y}} = \frac{\frac{-y}{x-y}}{\frac{1}{x-y}} = \frac{-y}{\cancel{x-y}} \times \frac{\cancel{x-y}}{1} = -y$$

Use this space for  
computations.

20 The Sea Dragon, a pendulum ride at an amusement park, moves from its central position at rest according to the trigonometric function  $P(t) = -10 \sin\left(\frac{\pi}{3}t\right)$ , where  $t$  represents time, in seconds. How many seconds does it take the pendulum to complete one full cycle?

(1) 5  
(2) 6

(3) 3  
(4) 10

period =  $\frac{2\pi}{a}$ ,  $\frac{2\pi}{\frac{\pi}{3}} = 6$

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Part II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

21 If  $f(x) = x^2 + 4$  and  $g(x) = 2x + 3$ , find  $f(g(-2))$ .

$$g(-2) = 2(-2) + 3 = -1$$

$$f(-1) = (-1)^2 + 4 = 5$$

22 In  $\triangle ABC$ ,  $\sin A = 0.6$ ,  $a = 10$ , and  $b = 7$ . Find  $\sin B$ .

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{10}{.6} = \frac{7}{\sin B}$$

$$10 \sin B = 4.2$$

$$\sin B = .42$$

23 Solve algebraically for  $x$ :  $9^{3x} = 3^{3x+1}$

$$(3^2)^{3x} = 3^{3x+1}$$

$$3^{6x} = 3^{3x+1}$$

$$6x = 3x + 1$$

$$3x = 1$$

$$x = \frac{1}{3}$$

$$\log 9^{3x} = \log 3^{3x+1}$$

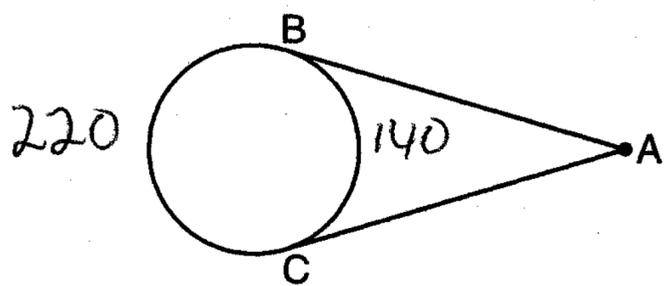
$$\frac{3x \log 9}{\log 3} = \frac{(3x+1)(\log 3)}{\log 3}$$

$$6x = 3x + 1$$

$$3x = 1$$

$$x = \frac{1}{3}$$

- 24 The accompanying diagram shows two lengths of wire attached to a wheel, so that  $\overline{AB}$  and  $\overline{AC}$  are tangent to the wheel. If the major arc  $\widehat{BC}$  has a measure of  $220^\circ$ , find the number of degrees in  $m\angle A$ .



$$\frac{220 - 140}{2} = \frac{80}{2} = 40$$

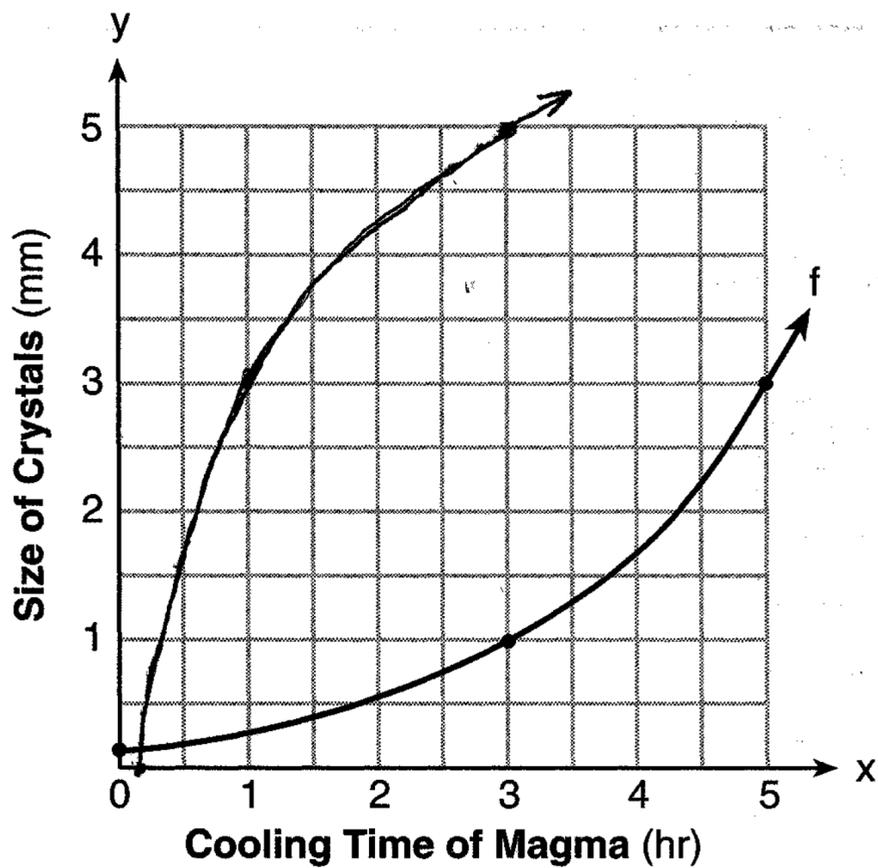
25 Solve for  $x$ :  $\log_8(x+1) = \frac{2}{3}$

$$8^{\frac{2}{3}} = x+1$$

$$4 = x+1$$

$$3 = x$$

26 The accompanying graph shows the relationship between the cooling time of magma and the size of the crystals produced after a volcanic eruption. On the same graph, sketch the inverse of this function.



Part III

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [24]

27 The number of newly reported crime cases in a county in New York State is shown in the accompanying table. Write the linear regression equation that represents this set of data. (Let  $x = 0$  represent 1999.)

Using this equation, find the projected number of new cases for 2009, rounded to the nearest whole number.

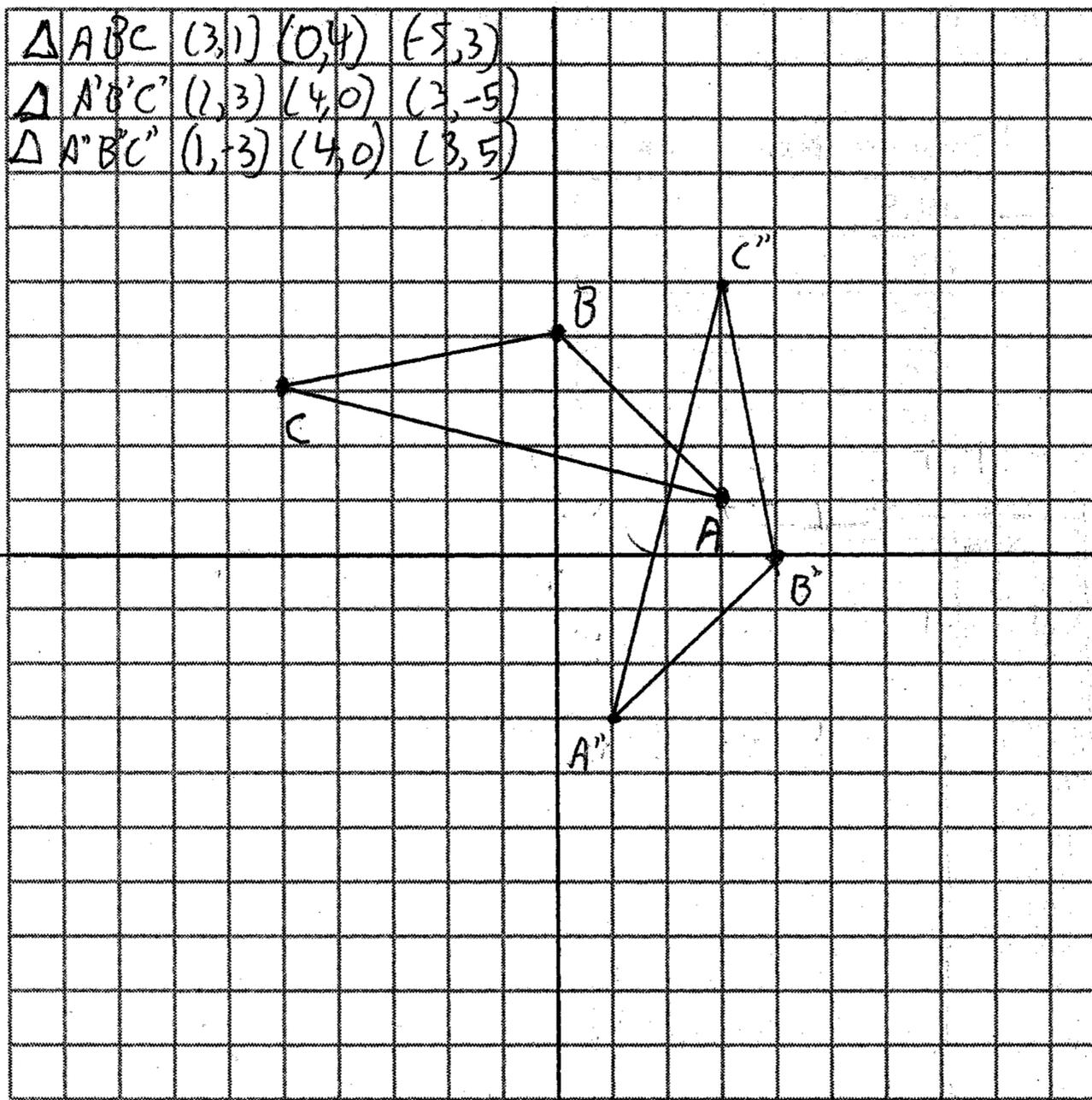
Year ( $x$ )	New Cases ( $y$ )
1999	440
2000	457
2001	369
2002	351

$$y = -35.5x + 457.5$$

$$y = -35.5(10) + 457.5$$

$$y \approx 103$$

28 On the accompanying grid, graph and label  $\triangle ABC$  with vertices  $A(3,1)$ ,  $B(0,4)$ , and  $C(-5,3)$ . On the same grid, graph and label  $\triangle A''B''C''$ , the image of  $\triangle ABC$  after the transformation  $r_{x\text{-axis}} \circ r_{y=x}$ .



29 Express in simplest form:  $\frac{3x}{2x-6} + \frac{9}{6-2x}$

$$\frac{3x}{2x-6} - \frac{9}{2x-6}$$

$$\frac{3x-9}{2x-6}$$

$$\frac{3(x-3)}{2(x-3)}$$

$$\frac{3}{2}$$

30 Dave does *not* tell the truth  $\frac{3}{4}$  of the time. Find the probability that he will tell the truth *at most* twice out of the next five times.

$$n = 5$$

$$r = 0, 1, 2$$

$$p = \frac{1}{4}$$

$$q = \frac{3}{4}$$

$$n C_r p^r q^{n-r}$$

$$r=0 \quad 5 C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^5 = \frac{243}{1024}$$

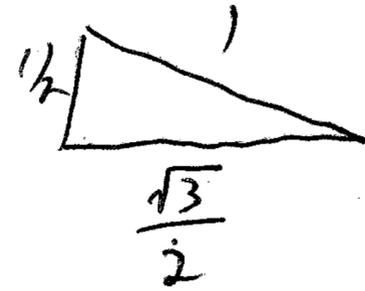
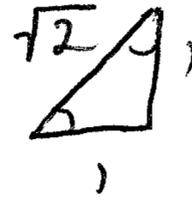
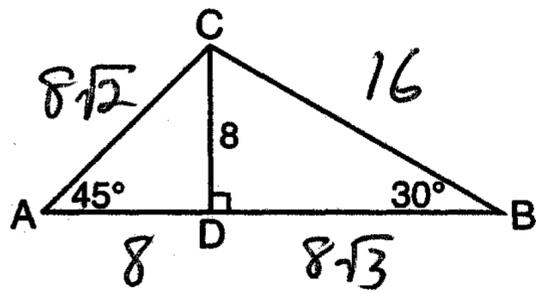
$$r=1 \quad 5 C_1 \left(\frac{1}{4}\right)^1 \left(\frac{3}{4}\right)^4 = \frac{405}{1024}$$

$$r=2 \quad 5 C_2 \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^3 = \frac{270}{1024}$$

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$$\frac{918}{1024}$$

31 In the accompanying diagram,  $\overline{CD}$  is an altitude of  $\triangle ABC$ . If  $CD = 8$ ,  $m\angle A = 45^\circ$ , and  $m\angle B = 30^\circ$ , find the perimeter of  $\triangle ABC$  in simplest radical form.



$$24 + 8\sqrt{2} + 8\sqrt{3}$$

32 Solve the equation  $\cos \theta = 2 + 3 \cos 2\theta$  for all values of  $\theta$ , to the nearest tenth of a degree, in the interval  $0^\circ \leq \theta < 360^\circ$ .

$$\cos \theta = 2 + 3(2 \cos^2 \theta - 1)$$

$$\cos \theta = 2 + 6 \cos^2 \theta - 3$$

$$0 = 6 \cos^2 \theta - \cos \theta - 1$$

$$0 = 6x^2 - x - 1$$

$$0 = (3x+1)(2x-1)$$

$$3x+1=0$$

$$x = -\frac{1}{3}$$

$$2x-1=0$$

$$x = \frac{1}{2}$$

$$\cos \theta = -\frac{1}{3}$$

$$\cos \theta = \frac{1}{2}$$

$$\theta = \cos^{-1}\left(-\frac{1}{3}\right)$$

$$\theta = \cos^{-1}\frac{1}{2}$$

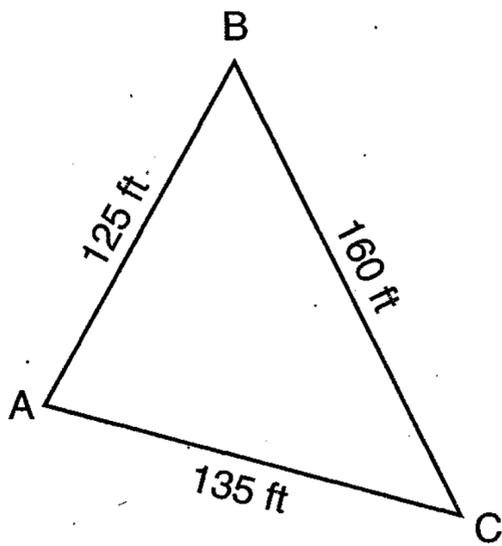
$$\theta = 109.5, 250.5$$

$$\theta = 60^\circ, 300^\circ$$

Part IV

Answer all questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

- 33 The accompanying diagram shows a triangular plot of land located in Moira's garden.



Find the area of the plot of land, and round your answer to the nearest hundred square feet.

$$p = 125 + 135 + 160 = 420$$

$$s = 210$$

Heron's Formula

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

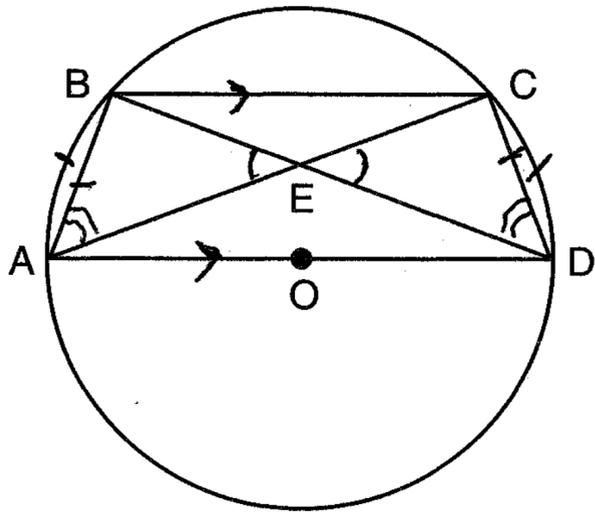
$$= \sqrt{210(210-160)(210-135)(210-125)}$$

$$= \sqrt{210 \cdot 50 \cdot 75 \cdot 85}$$

$$= \sqrt{66937500} \approx 8200$$

34 In the accompanying diagram of circle  $O$ ,  $\overline{AD}$  is a diameter with  $\overline{AD}$  parallel to chord  $\overline{BC}$ , chords  $\overline{AB}$  and  $\overline{CD}$  are drawn, and chords  $\overline{BD}$  and  $\overline{AC}$  intersect at  $E$ .

Prove:  $\overline{BE} \cong \overline{CE}$



STATEMENT	REASON
① Diameter $\overline{AD} \parallel$ chord $\overline{BC}$ Chords $\overline{AB}$ and $\overline{CD}$ Chords $\overline{BD}$ and $\overline{AC}$ intersect at $E$	① Given
② $\widehat{AB} \cong \widehat{DC}$	② Parallel lines intercept congruent arcs
③ $\overline{AB} \cong \overline{DC}$	③ Congruent chords intercept congruent arcs
④ $\angle BEA \cong \angle CED$	④ Vertical angles
⑤ $\angle BAC \cong \angle CDB$	⑤ Inscribed angles intercepting the same arc are congruent.
⑥ $\triangle BAE \cong \triangle CDE$	⑥ AAS
⑦ $\overline{BE} \cong \overline{CE}$	⑦ CPCTC