Print your name and the name of your school in the boxes above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. Any work done on this sheet of scrap graph paper will not be scored. Write all your work in pen, except graphs and drawings, which should be done in pencil.

This examination has four parts, with a total of 34 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. The formulas that you may need to answer some questions in this examination are found on page 19.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...
A graphing calculator, a straightedge (ruler), and a compass must be available for your use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.
Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question.  

1 Which graph does not represent a function of x? 

![Graphs](image)

Use this space for computations.

(1) Fails the vertical line test

2 What is the value of x in the equation $\sqrt{5-2x} = 3i$?

(1) 1
(2) 7
(3) -2
(4) 4

3 Which graph represents the solution set of $|2x - 1| < 7$?

![Graphs](image)

Math. B – Aug. ’03
4 The strength of a medication over time is represented by the equation 
\[ y = 200(1.5)^{-x}, \]
where \( x \) represents the number of hours since the medication was taken and \( y \) represents the number of micrograms per millimeter left in the blood. Which graph best represents this relationship?

![Graphs](Image)

Use this space for computations.

5 Written in simplest form, the expression \[ \frac{x^2y^2 - 9}{3 - xy} \] is equivalent to

\[ \boxed{-(3 + xy)} \]

(1) \(-1\)
(2) \(\frac{1}{3 + xy}\)
(3) \(-(3 + xy)\)
(4) \(3 + xy\)

6 Which graph represents data used in a linear regression that produces a correlation coefficient closest to \(-1\)?

![Graphs](Image)
7 Which expression is equal to \( \frac{2 + \sqrt{3}}{2 - \sqrt{3}} \)?

(1) \( \frac{1 - 4\sqrt{3}}{7} \)

(2) \( \frac{7 + 4\sqrt{3}}{7} \)

(3) \( 1 - 4\sqrt{3} \)

(4) \( 7 + 4\sqrt{3} \)

8 Which transformation is not an isometry?

(1) rotation

(2) line reflection

(3) dilation

(4) translation

9 A dog has a 20-foot leash attached to the corner where a garage and a fence meet, as shown in the accompanying diagram. When the dog pulls the leash tight and walks from the fence to the garage, the arc the leash makes is 55.8 feet.

What is the measure of angle \( \theta \) between the garage and the fence, in radians?

(1) 0.36

(2) 2.79

(3) 3.14

(4) 160

\[
\theta = \frac{55.8}{20} = \frac{2.79}{0.36} = 2.79
\]
10 In the accompanying diagram of parallelogram $ABCD$, $DE \parallel BF$.

Triangle $EGC$ can be proved congruent to triangle $FGA$ by

(1) $HL \equiv HL$
(2) $AAA \equiv AAA$
(3) $AAS \equiv AAS$
(4) $SSA \equiv SSA$

Because $BE \equiv BF$, $EC \equiv FA$

$CD \parallel AB$ because opposite sides of a parallelogram are congruent.

11 An architect commissions a contractor to produce a triangular window. The architect describes the window as $\triangle ABC$, where $\angle A = 50^\circ$, $BC = 10$ inches, and $AB = 12$ inches. How many distinct triangles can the contractor construct using these dimensions?

(1) 1
(2) 2
(3) More than 2
(4) 0

$10 = \frac{12}{\sin 50^\circ} \Rightarrow \sin C = \frac{12}{10} \sin 50^\circ$

$C = 67^\circ$ or $113^\circ$

12 The accompanying graph shows the relationship between a person’s weight and the distance that the person must sit from the center of a seesaw to make it balanced.

Which equation best represents this graph?

(1) $y = 12x^2$
(2) $y = -120x$
(3) $y = 2 \log x$
(4) $y = \frac{120}{x}$
13 If \( f \) and \( g \) are two functions defined by \( f(x) = 3x + 5 \) and \( g(x) = x^2 + 1 \), then \( g(f(x)) \) is

1. \( x^2 + 3x + 6 \)
2. \( 9x^2 + 30x + 26 \)
3. \( 3x^2 + 8 \)
4. \( 9x^2 + 26 \)

\[ g(3x+5) = (3x+5)^2 + 1 \]
\[ = 9x^2 + 15x + 15x + 25 + 1 \]
\[ = 9x^2 + 30x + 26 \]

14 What is the product of \( 5 + \sqrt{-36} \) and \( 1 - \sqrt{-49} \), expressed in simplest \( a + bi \) form?

1. \(-37 + 41i\)
2. \(5 - 7i\)
3. \(47 + 41i\)
4. \(47 - 29i\)

\[ (5 + 6i)(1 - 7i) \]
\[ = 5 - 35i + 6i - 42i^2 \]
\[ = 5 - 29i - 42(-1) \]
\[ = 47 - 29i \]

15 The expression \( \frac{2 \cos \theta}{\sin 2\theta} \) is equivalent to

1. \( \csc \theta \)
2. \( \sec \theta \)
3. \( \cot \theta \)
4. \( \sin \theta \)

\[ \frac{2 \cos \theta}{\sin 2\theta} = \frac{2 \cos \theta}{2 \sin \theta \cos \theta} = \frac{1}{\sin \theta} = \csc \theta \]

16 If \( \sin x = \frac{12}{13} \), \( \cos y = \frac{3}{5} \), and \( x \) and \( y \) are acute angles, the value of \( \cos (x - y) \) is

1. \( \frac{21}{65} \)
2. \( \frac{63}{65} \)
3. \( -\frac{14}{65} \)
4. \( -\frac{33}{65} \)

\[ \cos (x - y) = \cos x \cos y + \sin x \sin y \]
\[ = \left( \frac{5}{13} \right) \left( \frac{3}{5} \right) + \left( \frac{12}{13} \right) \left( \frac{4}{5} \right) \]
\[ = \frac{15}{65} + \frac{48}{65} \]
\[ = \frac{63}{65} \]

17 The amount of ketchup dispensed from a machine at Hamburger Palace is normally distributed with a mean of 0.9 ounce and a standard deviation of 0.1 ounce. If the machine is used 500 times, approximately how many times will it be expected to dispense 1 or more ounces of ketchup?

1. \( 5 \)
2. \( 16 \)
3. \( 80 \)
4. \( 100 \)

\[ 0.167 \times 500 \approx 80 \]

Math. B – Aug. '03
18 A commercial artist plans to include an ellipse in a design and wants the length of the horizontal axis to equal 10 and the length of the vertical axis to equal 6. Which equation could represent this ellipse?

- (1) $9x^2 + 25y^2 = 225$
- (2) $9x^2 - 25y^2 = 225$
- (3) $x^2 + y^2 = 100$
- (4) $3y = 20x^2$

19 A function is defined by the equation $y = \frac{1}{2}x - \frac{3}{2}$. Which equation defines the inverse of this function?

- (1) $y = 2x + 3$
- (2) $y = 2x - 3$
- (3) $y = 2x + \frac{3}{2}$
- (4) $y = 2x - \frac{3}{2}$

20 In the equation $ax^2 + 6x - 9 = 0$, imaginary roots will be generated if

- (1) $-1 < a < 1$
- (2) $a < 1$, only
- (3) $a > -1$, only
- (4) $a < -1$

$$b^2 - 4ac < 0$$

$$6^2 - 4a(-9) < 0$$

$$36 + 36a < 0$$

$$36a < -36$$

$$\frac{36}{36} \quad \frac{-36}{36}$$

$$a < -1$$
Part II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

21 The height, \( h \), in feet, a ball will reach when thrown in the air is a function of time, \( t \), in seconds, given by the equation \( h(t) = -16t^2 + 30t + 6 \).
Find, to the nearest tenth, the maximum height, in feet, the ball will reach.

Maximum height occurs at the axis of symmetry,

\[ t = \frac{-b}{2a} = \frac{-30}{2(-16)} = \frac{15}{16} \text{ seconds} \]

\[ h\left(\frac{15}{16}\right) = -16\left(\frac{15}{16}\right)^2 + 30\left(\frac{15}{16}\right) + 6 \]

\[ h \approx 20.1 \text{ seconds} \]

22 Find the value of \((x + 2)^0 + (x + 1)^{-\frac{2}{3}}\) when \( x = 7 \).

\[ (7 + 2)^0 + (7 + 1)^{-\frac{2}{3}} \]

\[ 9^0 + 8^{-\frac{2}{3}} \]

\[ 1 + \frac{1}{8^{-\frac{2}{3}}} \]

\[ 1 + \frac{1}{\sqrt[3]{64}} = 1 + \frac{1}{4} = 1\frac{1}{4} \]
23 Express in simplest form: \[ \frac{x^2 - 16}{4x} \frac{x - 4}{x} \]

\[ = \frac{x^2 - 16}{4} x \frac{x - 4}{x} \]

\[ = \frac{(x + 4)(x - 4)}{4} x \frac{1}{x - 4} \]

\[ = \frac{x + 4}{4} \]

24 The triangular top of a table has two sides of 14 inches and 16 inches, and the angle between the sides is 30°. Find the area of the tabletop, in square inches.

\[ K = \frac{1}{2} ab \sin C \]

\[ = \frac{1}{2} (14)(16) \sin 30 \]

\[ = 56 \]
25 Meteorologists can determine how long a storm lasts by using the function \( t(d) = 0.07d^{3/2} \), where \( d \) is the diameter of the storm, in miles, and \( t \) is the time, in hours. If the storm lasts 4.75 hours, find its diameter, to the nearest tenth of a mile.

\[
\frac{4.75}{0.07} = \sqrt[3]{d^{3/2}}
\]

\[
(4.75)^{3/2} = (d^{3/2})^{2/3}
\]

\[
16.6 \approx d
\]

26 Tom scored 23 points in a basketball game. He attempted 15 field goals and 6 free throws. If each successful field goal is 2 points and each successful free throw is 1 point, is it possible he successfully made all 6 of his free throws? Justify your answer.

No. Tom scored an odd number of total points, since field goals are worth an even number of points, he must have scored an odd number of free throws.

\[
\text{Even} + \text{Odd} = \text{Odd}
\]

\[
2\text{pts} + 1\text{pts} = 23
\]
Part III

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [24]

27 On the accompanying grid, graph and label \( \overline{AB} \), where \( A \) is \((0,5)\) and \( B \) is \((2,0)\). Under the transformation \( r_{y\text{-axis}} \circ r_{y\text{-axis}}(\overline{AB}) \), \( A \) maps to \( A'' \), and \( B \) maps to \( B'' \). Graph and label \( A''B'' \). What single transformation would map \( \overline{AB} \) to \( A''B'' \)?

![Graph of line segment AB and its transformed line segment A'B' with labels A, B, A', and B'.]

Single transformations include \( R_{180} \), \( R_{180} \) or \( R_{(0,0)} \).
28 Express, in simplest $a + bi$ form, the roots of the equation $x^2 + 5 = 4x$.

$$x^2 - 4x + 4 = -5 + 4 \quad \text{Complete the square.}$$

$$(x-2)^2 = -1$$

$$x-2 = \pm \sqrt{-1}$$

$$x = 2 \pm i$$

29 A ship at sea is 70 miles from one radio transmitter and 130 miles from another. The angle between the signals sent to the ship by the transmitters is 117.4°. Find the distance between the two transmitters, to the nearest mile.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = 70^2 + 130^2 - 2(70)(130) \cos 117.4°$$

$$a^2 = 30,175.6$$

$$a \approx 174$$
A student attaches one end of a rope to a wall at a fixed point 3 feet above the ground, as shown in the accompanying diagram, and moves the other end of the rope up and down, producing a wave described by the equation \( y = a \sin bx + c \). The range of the rope's height above the ground is between 1 and 5 feet. The period of the wave is \( 4\pi \).

Write the equation that represents this wave.

\[
\text{Period} = \frac{2\pi}{b}
\]

\[
4\pi = \frac{2\pi}{b}
\]

\[
b = \frac{1}{2}
\]

The sine wave starts at \((0,0)\). To intersect at \((0,3)\), \(c = 3\). Since the minimum and maximum are 1 and 5, \(a = 2\).

\[
y = 2\sin\left(\frac{1}{2}x\right) + 3
\]
The table below shows the results of an experiment that relates the height at which a ball is dropped, \( x \), to the height of its first bounce, \( y \).

<table>
<thead>
<tr>
<th>Drop Height (( x )) (cm)</th>
<th>Bounce Height (( y )) (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>26</td>
</tr>
<tr>
<td>90</td>
<td>23</td>
</tr>
<tr>
<td>80</td>
<td>21</td>
</tr>
<tr>
<td>70</td>
<td>18</td>
</tr>
<tr>
<td>60</td>
<td>16</td>
</tr>
</tbody>
</table>

Find \( \bar{x} \), the mean of the drop heights.
Find \( \bar{y} \), the mean of the bounce heights.
Find the linear regression equation that best fits the data.
Show that \((\bar{x}, \bar{y})\) is a point on the line of regression. [The use of the grid on the next page is optional.]

\[
\bar{x} = 80 \\
\bar{y} = 20.8 \\
y = 0.25x + 0.8 \\
= 0.25(80) + 0.8 \\
= 20.8
\]
Question 31 continued

![Graph showing the relationship between drop height (cm) and bounce height (cm).](image)

- The graph plots the relationship between drop height (cm) on the x-axis and bounce height (cm) on the y-axis.
- A straight line connects the points, indicating a linear relationship.
- The point (80,20.8) is marked on the graph.
A company calculates its profit by finding the difference between revenue and cost. The cost function of producing $x$ hammers is $C(x) = 4x + 170$. If each hammer is sold for $10$, the revenue function for selling $x$ hammers is $R(x) = 10x$.

How many hammers must be sold to make a profit?

How many hammers must be sold to make a profit of $100$?

To make a profit:

\[ 10x > 4x + 170 \]
\[ 6x > 170 \]
\[ x > 28.3 \]
So, $29$ hammers must be sold.

To make a profit of $100$:

\[ 10x - (4x + 170) = 100 \]
\[ 6x - 170 = 100 \]
\[ 6x = 270 \]
\[ x = 45 \]
So, $45$ hammers must be sold.
Part IV

Answer all questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [10]

33 Given circle O with diameter GOAL; secants HUG and HTAM intersect at point H; m\(\overparen{GM}\):m\(\overparen{ML}\):m\(\overparen{LT}\) = 7:3:2; and chord \(\overparen{GU} \equiv \overparen{UT}\).

Find the ratio of \(m\angle UGL\) to \(m\angle H\).

\[
\begin{align*}
180^\circ & \left\{ \begin{array}{l}
m\overparen{GM} \quad \frac{7}{16} \times 180 = 126^\circ \\
m\overparen{ML} \quad \frac{3}{16} \times 180 = 54^\circ \\
m\overparen{LT} \quad \frac{2}{16} \times 180 = 36^\circ \\
\end{array} \right.
\]

\[\overparen{GU} + \overparen{UT} + 36 = 180\]
\[x + x + 36 = 180\]
\[2x = 144\]
\[x = 72\]

\[m\angle UGL = \frac{72 + 36}{2} = 54^\circ\]

\[m\angle H = \frac{126 - 72}{2} = 27\]

ratio is 2:1
34 When Joe bowls, he can get a strike (knock down all the pins) 60% of the time. How many times more likely is it for Joe to bowl at least three strikes out of four tries as it is for him to bowl zero strikes out of four tries? Round your answer to the nearest whole number.

\[ n = 4 \]
\[ r = 3, 4, 0 \]
\[ p = \frac{6}{10} \]
\[ q = \frac{4}{10} \]

\[ P(3) = 4 C_3 \left( \frac{6}{10} \right)^3 \left( \frac{4}{10} \right)^1 = \frac{3456}{10,000} \]

\[ P(4) = 4 C_4 \left( \frac{6}{10} \right)^4 \left( \frac{4}{10} \right)^0 = \frac{1296}{10,000} \]

\[ P(\geq 3) = \frac{4752}{10,000} \]

\[ P(0) = 4 C_0 \left( \frac{6}{10} \right)^0 \left( \frac{4}{10} \right)^4 = \frac{256}{10,000} \]

\[ \frac{\frac{4752}{10,000}}{\frac{256}{10,000}} = 18.5625 \approx 19 \]