The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

MATHEMATICS B

Thursday, August 16, 2007 — 8:30 to 11:30 a.m., only

Print Your Name: Steve Sibol

Print Your School's Name: High School For Civil Rights

Print your name and the name of your school in the boxes above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored. Write all your work in pen, except graphs and drawings, which should be done in pencil.

The formulas that you may need to answer some questions in this examination are found on page 23. This sheet is perforated so you may remove it from this booklet.

This examination has four parts, with a total of 34 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice. . .

A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

The use of any communications device is strictly prohibited when taking this examination. If you use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [40]

$3^{-3} - 3^0 + 2^3 = \frac{1}{27} - 1 + 8 = 7\frac{1}{27}$

Use this space for computations.

1 If $f(x) = (x^{-x} - x^0 + 2^x)$, then $f(3)$ is equal to

(1) $8\frac{1}{27}$
(2) $7\frac{1}{27}$
(3) -21
(4) -22

$6i^3 - 15i^2 = 6(-i) - 15(-1) = 15 - 6i$

2 The expression $3i(2i^2 - 5i)$ is equivalent to

(1) $15 - 6i$
(2) $15 - 5i$
(3) $-15 - 5i$
(4) $-1 + 0i$

3 If $\csc \theta = -2$, what is the value of $\sin \theta$?

(1) -2
(2) 2
(3) $-\frac{1}{2}$
(4) $\frac{1}{2}$

$\sin\theta = \frac{1}{\csc\theta} = \frac{1}{-2}$

4 What is 235°, expressed in radian measure?

(1) 235π
(2) $\frac{\pi}{235}$
(3) $\frac{36\pi}{47}$
(4) $\frac{47\pi}{36}$

$235^\circ\left(\frac{2\pi}{360^\circ}\right) = \frac{470\pi}{360}$

5 The flight paths of two Thunderbird jets are plotted on a Cartesian coordinate plane, and the equations of the jets' flight paths are represented by $y = 2^x + 3$ and $y = 0.5^x$. The best approximation of the intersection of the flight paths is

(1) (–1.72,3.3)
(2) (0,1)
(3) (–1.50,2.82)
(4) (–2,–1)

Use a graphing calculator to find the intersection.

Use this space for computations.

$$\frac{\frac{1}{a}-a}{\frac{1}{a}+1} = \frac{\frac{1-a^2}{a}}{\frac{1+a}{a}} = \frac{1-a^2}{a} \div \frac{1+a}{a}$$

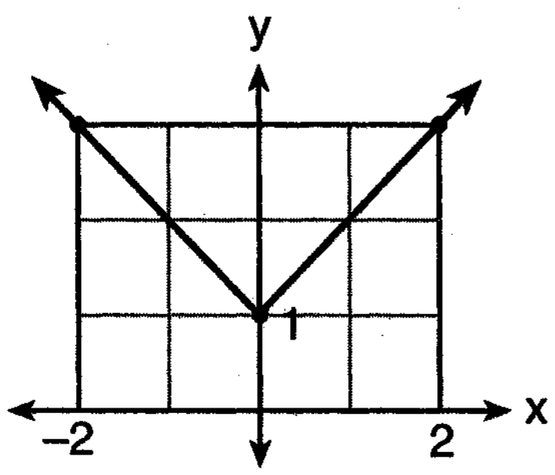
$$\frac{1-a^2}{a} \times \frac{a}{1+a}$$

$$\frac{(1+a)(1-a)}{1+a}$$

6 Which expression is equivalent to the complex fraction $\frac{\frac{1}{a}-a}{\frac{1}{a}+1}$?

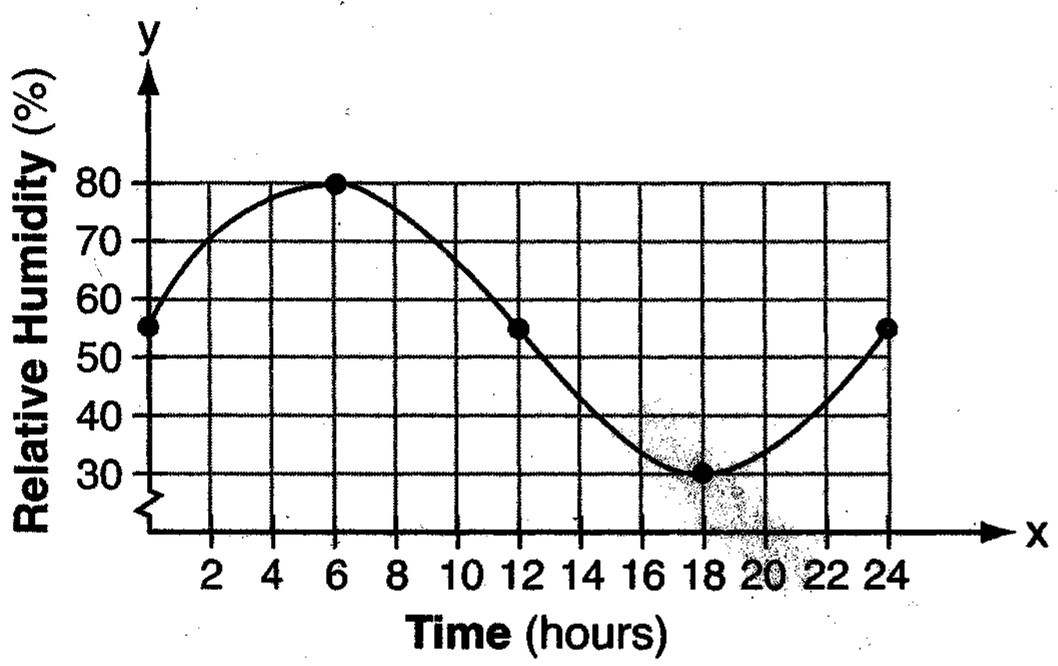
(1) $+1$
(2) -1
(3) $1-a$
(4) $-(1-a)$

7 Which equation represents the function shown in the accompanying graph?



(1) $f(x) = |x| + 1$
(2) $f(x) = |x| - 1$
(3) $f(x) = |x + 1|$
(4) $f(x) = |x - 1|$

8 A meteorologist drew the accompanying graph to show the changes in relative humidity during a 24-hour period in New York City.



What is the range of this set of data?

(1) $0 \le y \le 24$
(2) $0 \le x \le 24$
(3) $30 \le y \le 80$
(4) $30 \le x \le 80$

9 The equation used to determine the time it takes a swinging pendulum to return to its starting point is $T = 2\pi\sqrt{\frac{\ell}{g}}$, where T represents time, in seconds, ℓ represents the length of the pendulum, in feet, and g equals 32 ft/sec^2. How is this equation expressed in logarithmic form?

(1) $\log T = \log 2 + \log \pi + \log \sqrt{\ell - 32}$

(2) $\log T = \log 2 + \log \pi + \frac{1}{2}\log \ell - \frac{1}{2}\log 32$

(3) $\log T = \log 2 + \log \pi + \frac{1}{2}\log \ell - \log 16$

(4) $\log T = 2 + \log \pi + \frac{1}{2}\log \ell - 16$

Use this space for computations.

$\log T = \log 2\pi\sqrt{\frac{\ell}{32}}$

$= \log 2 + \log \pi + \log\sqrt{\frac{\ell}{32}}$

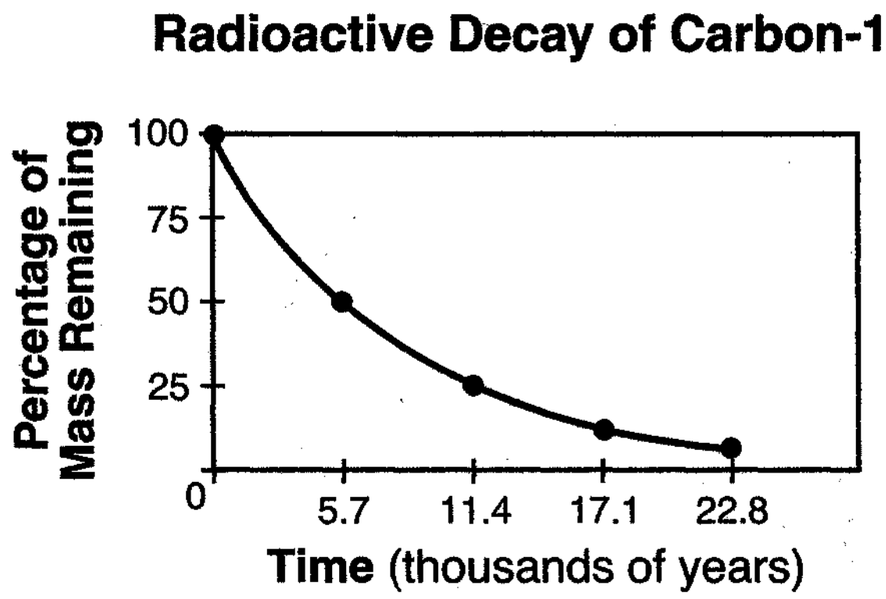
$= \log 2 + \log \pi + \log\left(\frac{\ell}{32}\right)^{1/2}$

$= \log 2 + \log \pi + \frac{1}{2}\log\frac{\ell}{32}$

$= \log 2 + \log \pi + \frac{1}{2}(\log \ell - \log 32)$

$= \log 2 + \log \pi + \frac{1}{2}\log \ell - \frac{1}{2}\log 32$

10 Which type of function could be used to model the data shown in the accompanying graph?



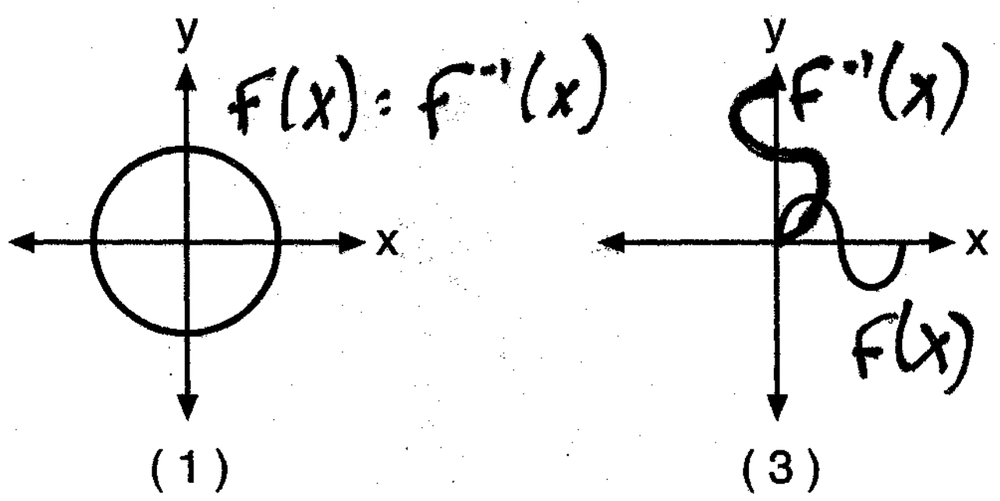
(1) exponential
(2) quadratic
(3) trigonometric
(4) linear

11 Under a dilation with respect to the origin, the image of $P(-15,6)$ is $P'(-5,2)$. What is the constant of dilation?

(1) -4
(2) $\frac{1}{3}$
(3) 3
(4) 10

Use this space for computations.

12 Which graph has an inverse that is a function?



13 What is the solution set of the inequality $x^2 + 4x - 5 < 0$?

(1) $\{x|x < -1 \text{ or } x > 5\}$
(2) $\{x|x < -5 \text{ or } x > 1\}$
(3) $\{x| -1 < x < 5\}$
(4) $\{x| -5 < x < 1\}$

$x^2+4x-5<0$
$(x+5)(x-1)<0$
$x+5<0$ AND $x-1>0$ } NOT
$x<-5$ AND $x>1$ } POSSIBLE
$x+5>0$ AND $x-1<0$
$x>-5$ AND $x<1$

14 The graph of which function is symmetric with respect to the graph of the line $y = x$?

(1) $y = \frac{1}{x}$
(2) $y = x^2$
(3) $y = x^3$
(4) $y = \log x$

15 The coordinates of ΔJRB are $J(1,-2)$, $R(-3,6)$, and $B(4,5)$. What are the coordinates of the vertices of its image after the transformation $T_{2,-1} \circ r_{y\text{-axis}}$?

(1) (3,1), (−1,−7), (6,−6)
(2) (3,−3), (−1,5), (6,4)
(3) (1,−3), (5,5), (−2,4)
(4) (−1,−2), (3,6), (−4,5)

After the reflection on the y-axis:
$J(-1,-2)$, $R(3,6)$, $B(-4,5)$
After the translation $(x+2, y-1)$
$J(1,-3)$, $R(5,5)$, $B(-2,4)$

16 The expression $\frac{2}{1-\sqrt{3}}$ is equivalent to

(1) $1+\sqrt{3}$

(2) $1-\sqrt{3}$

(3) $-1+\sqrt{3}$

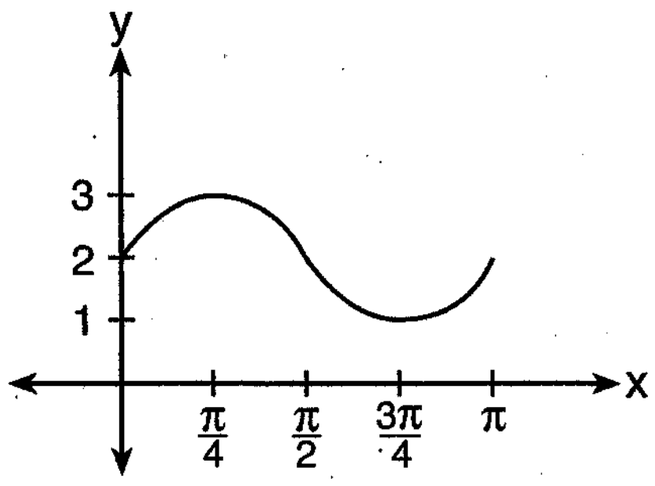
(4) $-1-\sqrt{3}$

Use this space for computations.

$\frac{2}{1-\sqrt{3}} \cdot \frac{1+\sqrt{3}}{1+\sqrt{3}} = \frac{2+2\sqrt{3}}{1-3} = \frac{2+2\sqrt{3}}{-2}$

$= -1-\sqrt{3}$

17 The accompanying graph represents a portion of a sound wave.



Which equation best represents this graph?

(1) $y = 2\sin\frac{1}{2}x$

(2) $y = \sin\frac{1}{2}x + 2$

(3) $y = \sin 2x$

(4) $y = \sin 2x + 2$

The sine wave is translated up by 2, eliminating (1) & (3).

The sine wave has a period of π:

$\frac{2\pi}{a} = \pi$

$a = 2$, eliminating (1) and (2)

18 Which equation has the complex number $4 - 3i$ as a root?

(1) $x^2 + 6x - 25 = 0$

(2) $x^2 - 6x + 25 = 0$

(3) $x^2 + 8x - 25 = 0$

(4) $x^2 - 8x + 25 = 0$

The roots are $4-3i$ and $4+3i$

The sum of the roots $= -\frac{b}{a}$

$4-3i+4+3i = 8$

$a = 1$ so $-b = 8$

$b = -8$

The product of the roots $= \frac{c}{a}$

$(4-3i)(4+3i)$

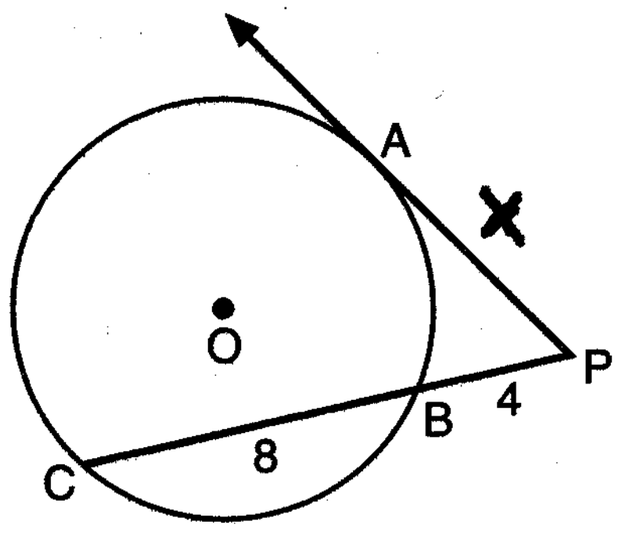
$16 - 9i^2$

$16 - 9(-1) = 25$

$a = 1$ so $c = 25$

Use this space for computations.

19 In the accompanying diagram, $\overrightarrow{PA}$ is tangent to circle O at A, $\overline{PBC}$ is a secant, $PB = 4$, and $BC = 8$.



$x^2 = 4(8+4)$

$x^2 = 48$

$x = \sqrt{48} = \sqrt{16}\sqrt{3} = 4\sqrt{3}$

What is the length of $\overline{PA}$?

(1) $4\sqrt{6}$
(2) $4\sqrt{2}$
(3) $4\sqrt{3}$
(4) 4

20 If $\log_2 a = \log_3 a$, what is the value of a?

(1) 1
(2) 2
(3) 3
(4) 4

$\log_2 a = \log_3 a = x$

$2^x = 3^x$, which is true only if $x = 0$

IF $\log_2 a = 0$

$a = 2^0 = 1$

OR

IF $\log_3 a = 0$

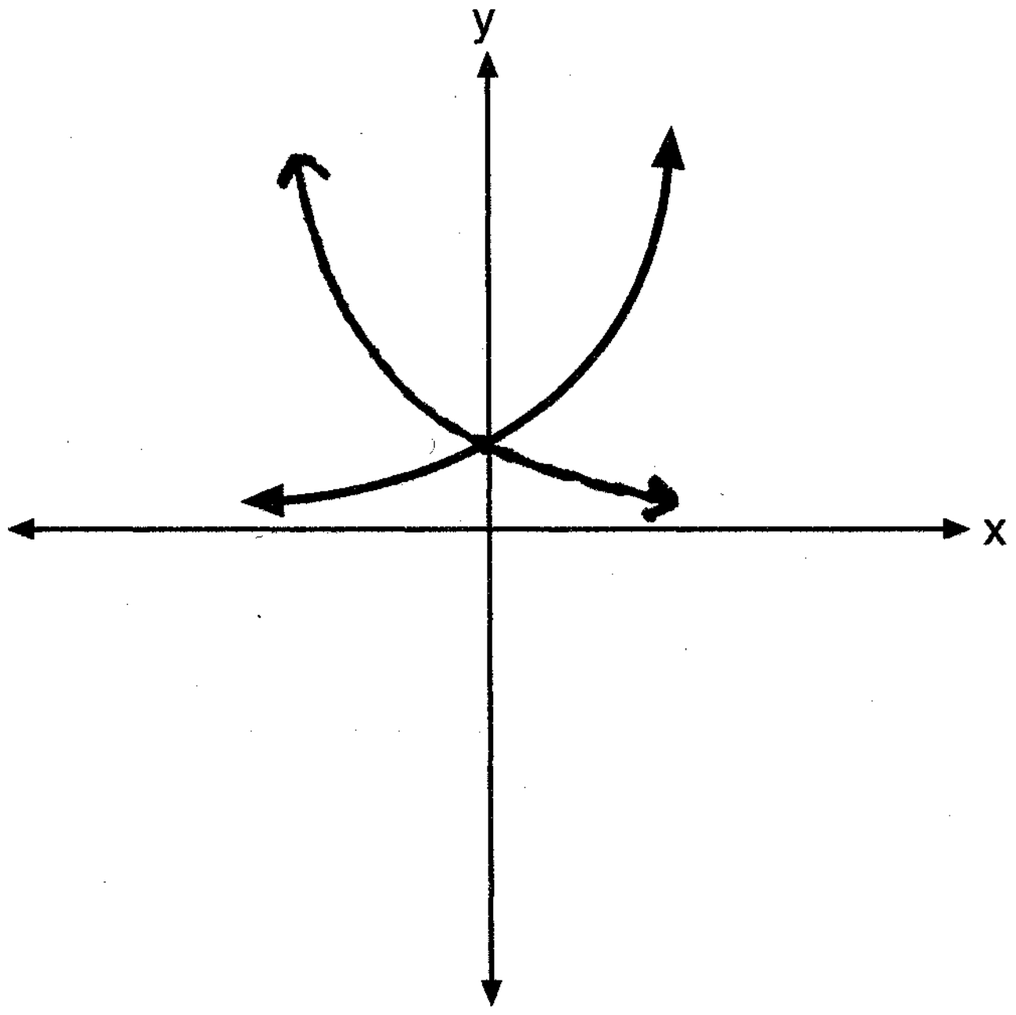
$a = 3^0 = 1$

Part II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

21 The graph of the function $f(x) = a^x$ is shown on the accompanying set of axes. On the same set of axes, sketch the reflection of $f(x)$ in the y-axis.

State the coordinates of the point where the graphs intersect. (0,1)



$F(x) = a^x$

After a reflection in the y-axis

$a^{-x} = \left(\frac{1}{a}\right)^x$

22 Solve for all values of x:

$$\frac{2}{x+1} = \frac{x}{1}$$

$x(x+1) = 2$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2 \quad x = 1$

neither root is extraneous

23 Mr. and Mrs. Doran have a genetic history such that the probability that a child being born to them with a certain trait is $\frac{1}{8}$. If they have four children, what is the probability that *exactly* three of their four children will have that trait?

$${}_4C_3\left(\frac{1}{8}\right)^3\left(\frac{7}{8}\right)^1$$

$$4 \cdot \frac{1}{512} \cdot \frac{7}{8} = \frac{28}{4096} = \frac{7}{1024}$$

24 Classical mathematics uses the term "Golden Ratio" for the ratio $\left(1+\sqrt{5}\right):2$. The Golden Ratio was used by many famous artists to determine the dimensions of their paintings. If the ratio of the length to the width of a painting is $\left(1+\sqrt{5}\right):2$, find the length, in feet, of a painting that has a width of 14 feet. Express your answer in simplest radical form.

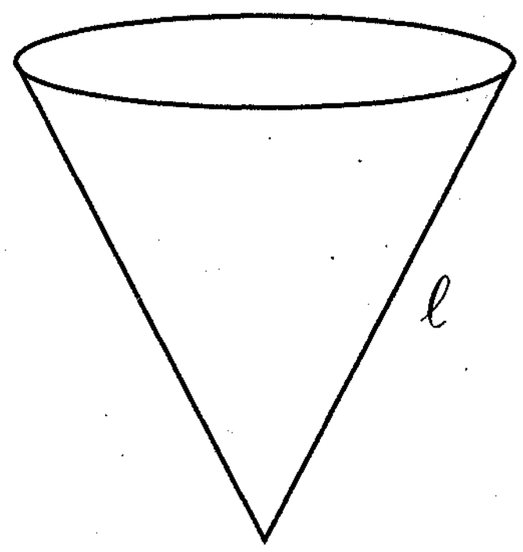
$$\frac{l}{w}\quad\frac{1+\sqrt{5}}{2}=\frac{x}{14}$$

$$\frac{14(1+\sqrt{5})}{2}=x$$

$$7(1+\sqrt{5})=x$$

$$7+7\sqrt{5}=x$$

25 The slant height, ℓ, of the conical water tank shown in the accompanying diagram is $\ell = \sqrt[3]{\frac{8v}{\pi}}$. Solve for v, in terms of ℓ and π.

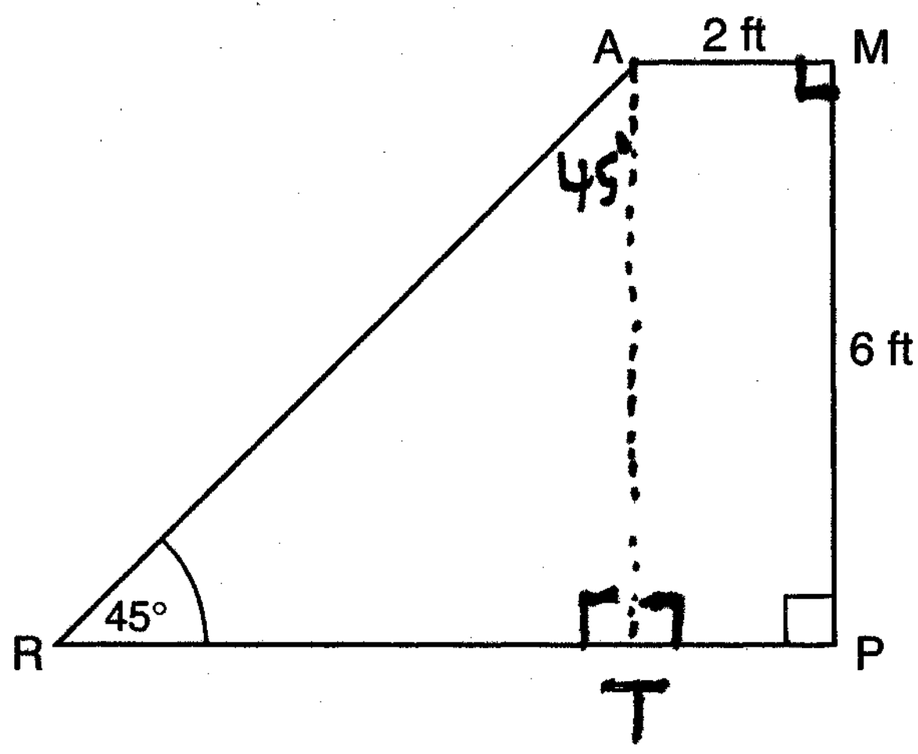


$$\ell = \sqrt[3]{\frac{8v}{\pi}}$$

$$\ell^3 = \frac{8v}{\pi}$$

$$\frac{\pi\ell^3}{8} = v$$

26 The accompanying diagram shows ramp $\overline{RA}$ leading to level platform $\overline{AM}$, forming an angle of 45° with level ground. If platform $\overline{AM}$ measures 2 feet and is 6 feet above the ground, explain why the exact length of ramp $\overline{RA}$ is $6\sqrt{2}$ feet.



Because $\overline{AM}$ is level, $m\angle AMP$ is 90°.

If $\overline{AT}$ is drawn $\perp$ to $\overline{RP}$, measure of $\overline{AT} = 6$.

$m\angle TAR = 45°$

Because $\triangle RAT$ is an isosceles right triangle, the ratio of the hypotenuse to a leg is

$\sqrt{2} : 1$

If a leg is 6, the hypotenuse, $\overline{RA}$, is

$6\sqrt{2}$

Part III

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [24]

27 A rectangular patio measuring 6 meters by 8 meters is to be increased in size to an area measuring 150 square meters. If both the width and the length are to be increased by the same amount, what is the number of meters, to the *nearest tenth*, that the dimensions will be increased?

$(6+x)(8+x) = 150$

$x^2 + 14x + 48 = 150$

$x^2 + 14x - 102 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{-14 \pm \sqrt{14^2 - 4(1)(-102)}}{2(1)}$

$\frac{-14 \pm \sqrt{604}}{2}$

$\frac{-14 - \sqrt{604}}{2}$

negative answer

$\frac{-14 + \sqrt{604}}{2} \approx 5.3$

Check: $(6+5.3)(8+5.3)$

150.29

28 The accompanying table shows the percent of the adult population that married before age 25 in several different years. Using the year as the independent variable, find the linear regression equation. Round the regression coefficients to the *nearest hundredth*.

Using the equation found above, estimate the percent of the adult population in the year 2009 that will marry before age 25, and round to the *nearest tenth of a percent*.

Year (x)	Percent (y)
1971	42.4
1976	37.4
1980	37.1
1984	34.1
1989	32.1
1993	28.8
1997	25.7
2000	25.5

$y = -.58x + 1185.09$

$= -.58(2009) + 1185.09 \approx 19.9\%$

29 Drew's parents invested \$1,500 in an account such that the value of the investment doubles every seven years. The value of the investment, V, is determined by the equation $V = 1500(2)^{\frac{t}{7}}$, where t represents the number of years since the money was deposited. How many years, to the *nearest tenth of a year*, will it take the value of the investment to reach \$1,000,000?

$V = 1500 \cdot 2^{t/7}$

$\frac{1000000}{1500} = \frac{1500 \cdot 2^{t/7}}{1500}$

$\frac{2000}{3} = 2^{t/7}$

$\log \frac{2000}{3} = \log 2^{t/7}$

$\log \frac{2000}{3} = \frac{t}{7} \log 2$

$\frac{7 \log \frac{2000}{3}}{\log 2} = t$

$65.7 = t$

30 Mr. Koziol has 17 students in his high school golf club. Each student played one round of golf. The summarized scores of the students are listed in the accompanying table.

Score	Frequency
70	4
73	3
75	2
80	3
85	1
86	1
90	2
92	1

Find the population standard deviation of this set of students' scores, to the *nearest tenth*. $\sigma_x = 7.5$

How many of the individual students' golf scores fall within one population standard deviation of the mean?

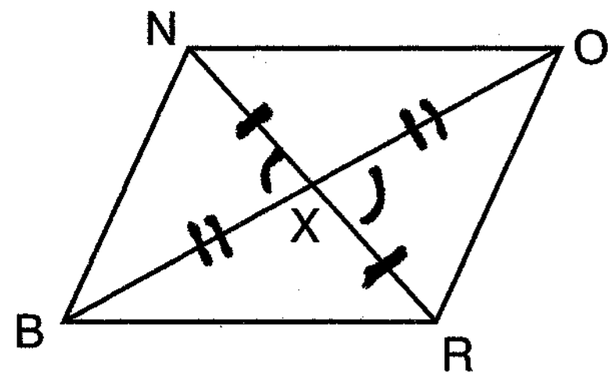
$\bar{x} = 78.4$

78.4 ± 7.5

$70.9 - 85.9$

$3 + 2 + 3 + 1 = 9$ scores

31 The accompanying diagram shows quadrilateral *BRON*, with diagonals $\overline{NR}$ and $\overline{BO}$, which bisect each other at *X*.



Prove: $\Delta BNX \cong \Delta ORX$

STATEMENT	REASON
① Quadrilateral BRON, diagonals $\overline{NR}$ and $\overline{BO}$ bisect each other at X	① Given
② $\angle NXB = \angle RXO$	② Vertical angles are equal
③ $\overline{NX} = \overline{RX}$, $\overline{BX} = \overline{OX}$	③ Bisected lines are divided into equal lines
④ $\Delta BNX \cong \Delta ORX$	④ SAS
Alternatively	
④ $\angle BNR = \angle ORN$ $\angle NBO = \angle ROB$	④ Alternative Interior Angles are equal
⑤ $\Delta BNX \cong \Delta ORX$	⑤ ASA

32 Two circles whose equations are $(x - 3)^2 + (y - 5)^2 = 25$ and $(x - 7)^2 + (y - 5)^2 = 9$ intersect in two points. What is the equation of the line passing through these two points? [The use of the accompanying grid is optional.]

$(x-3)^2+(y-5)^2=25$

$(x-3)^2-25=-(y-5)^2$

$(x-7)^2+(y-5)^2=9$

$(x-7)^2-9=-(y-5)^2$

$(x-3)^2-25=(x-7)^2-9$

$x^2-6x+9-25=x^2-14x+49-9$

SUBSTITUTE

$-6x-16=-14x+40$

$8x=56$

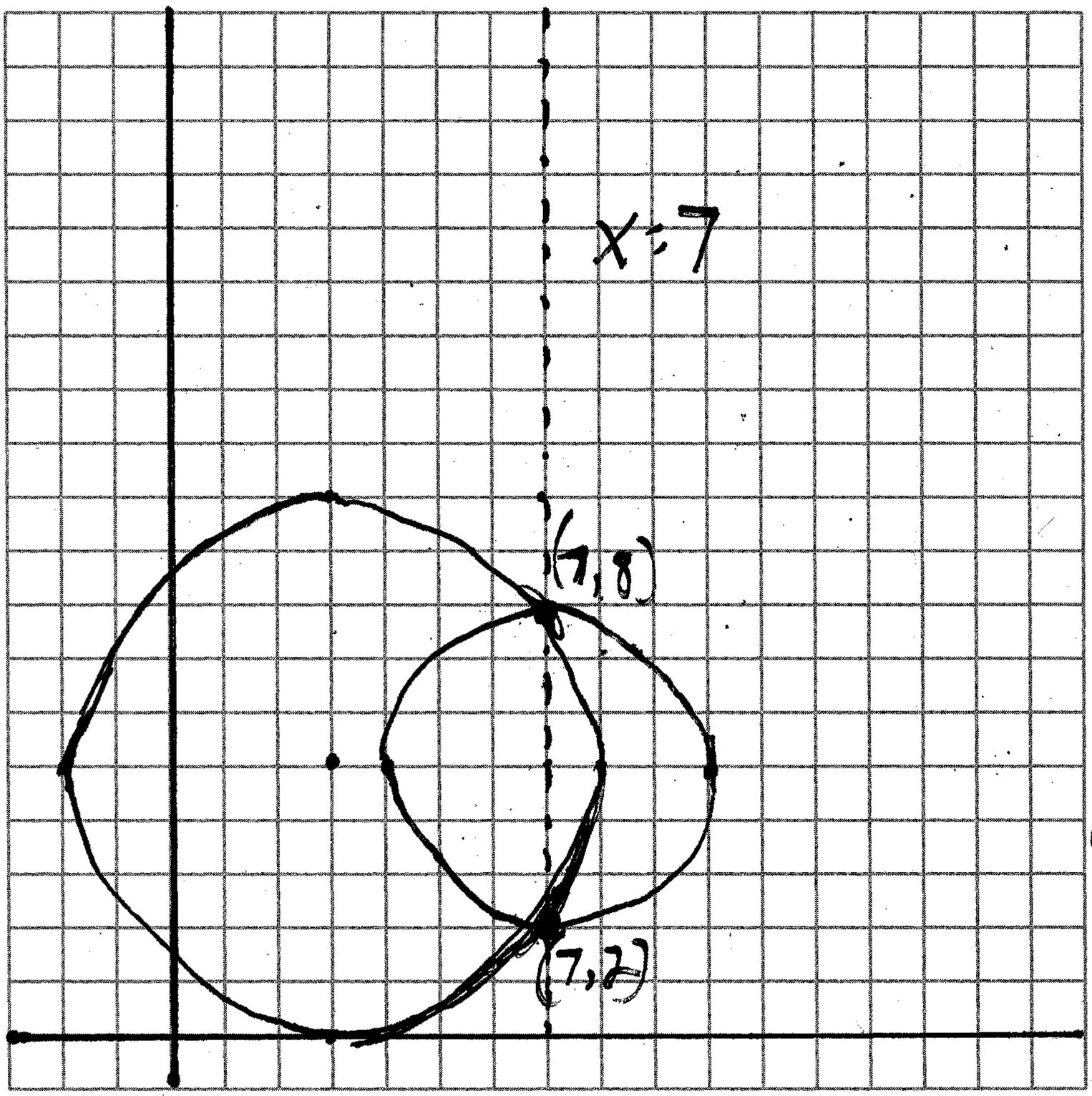
$x=7$

$(7-7)^2+(y-5)^2=9$

$(y-5)^2=9$

$y-5=\pm 3$

$y=8,2$



Points of Intersection

(7,8) (7,2)

$x=7$ passes through both points

Part IV

Answer all questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

33 Express in simplest form:

$$\frac{2x}{x^2-4} \div \frac{4}{x^2-4x+4} + \frac{12}{x^2-4} \bullet \frac{2-x}{3}$$

$$\frac{2x}{(x+2)(x-2)} \times \frac{(x-2)(x-2)}{4_2} + \frac{4\,12}{(x+2)(x-2)} \times \frac{2-x}{3} \cdot -1$$

$$\frac{x(x-2)}{2(x+2)} + \frac{-4}{(x+2)}$$

$$\frac{x^2-2x}{2(x+2)} + \frac{-8}{2(x+2)}$$

$$\frac{x^2-2x-8}{2(x+2)}$$

$$\frac{(x-4)(x+2)}{2(x+2)}$$

$$\frac{x-4}{2}$$

34 A farmer has a triangular field with sides of 240 feet, 300 feet, and 360 feet. He wants to apply fertilizer to the field. If one 40-pound bag of fertilizer covers 6,000 square feet, how many bags must he buy to cover the field?

Use Heron's Formula:

$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$, where a, b, c represent the lengths of the three sides, and s is the semi-perimeter.

$P = 240+300+360 = 900$

$s = 450$

$\text{Area} = \sqrt{450(450-240)(450-300)(450-360)}$

$= \sqrt{1,275,750,000}$ $= 35,718\ Ft^2$

# of bags $\frac{35,718}{6000} =$ 6 bags

OR

Use Law of Cosines

$a^2 = b^2 + c^2 - 2bc\cos A$

$360^2 = 300^2 + 240^2 - 2(300)(240)\cos A$

$-18000 = -144,000\cos A$

$\frac{1}{8} = \cos A$

$\cos^{-1}\left(\frac{1}{8}\right) = A$

$A = 82.8°$

$\text{Area} = \frac{1}{2}bc\sin A$

$= \frac{1}{2}300 \cdot 240 \cdot \sin 82.8°$

$= 35,718\ Ft^2$

The University of the State of New York

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MATHEMATICS B

Thursday, August 16, 2007 — 8:30 to 11:30 a.m., only

ANSWER SHEET

Student ..... Sex: ☐ Male ☐ Female Grade .....

Teacher Steve Sibol ..... School NS For Civil Rights .....

Your answers to Part I should be recorded on this answer sheet.

Part I

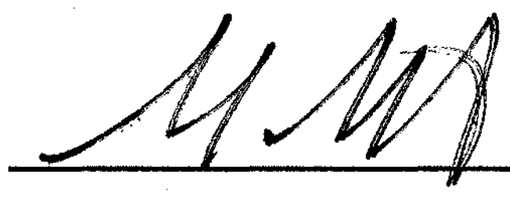
Answer all 20 questions in this part.

1 2	6 3	11 2	16 4
2 1	7 1	12 4	17 4
3 3	8 3	13 4	18 4
4 4	9 2	14 1	19 3
5 1	10 1	15 3	20 1

Your answers for Parts II, III, and IV should be written in the test booklet.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination and that I have neither given nor received assistance in answering any of the questions during the examination.



Signature