

MATHEMATICS B

Thursday, August 13, 2009 — 8:30 to 11:30 a.m., only

Print Your Name:

Steve Sibol

Print Your School's Name:

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Print your name and the name of your school in the boxes above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored. Write all your work in pen, except graphs and drawings, which should be done in pencil.

The formulas that you may need to answer some questions in this examination are found on page 19. This sheet is perforated so you may remove it from this booklet.

This examination has four parts, with a total of 34 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice . . .

A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

The use of any communications device is strictly prohibited when taking this examination. If you use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

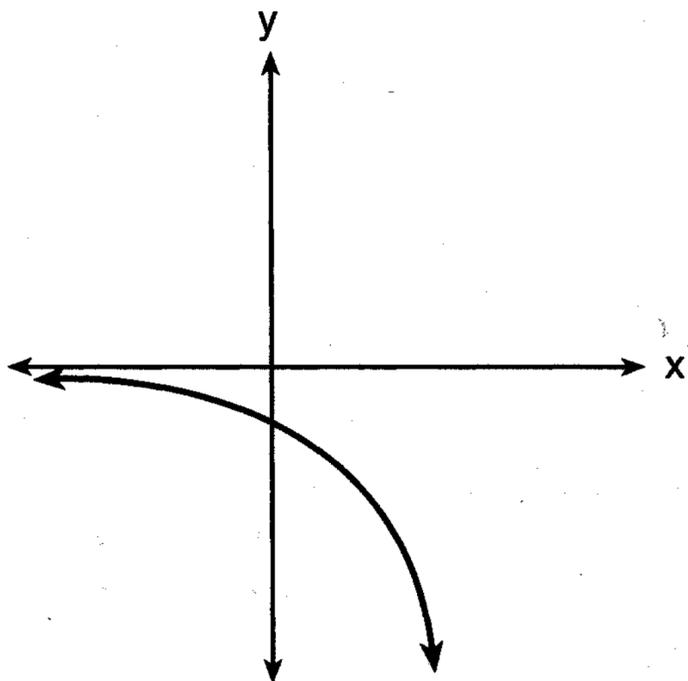
DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [40]

Use this space for computations.

1 Which equation is represented by the accompanying graph?



(1) $y = 2^x$
(2) $y = -2^x$

(3) $y = 2^{-x}$
(4) $y = x^2 - 2$

2 What are the coordinates of the turning point of the parabola whose equation is $y = -x^2 + 4x + 1$?

(1) $(-2, -11)$
(2) $(-2, -3)$

(3) $(2, 5)$
(4) $(2, 13)$

$$x = \frac{-b}{2a} = \frac{-4}{-2} = 2$$

$$y = -2^2 + 4(2) + 1 = 5$$

3 The graph of the equation $y = |\sin x|$ will contain *no* points in Quadrants

(1) I and II
(2) II and III

(3) III and IV
(4) I and IV

Use this space for computations.

4 What is the value of $\sum_{k=0}^2 3(2)^k$?

- (1) 15
- (2) 19

- (3) 21
- (4) 43

$$3 \cdot 2^0 + 3 \cdot 2^1 + 3 \cdot 2^2$$
$$3 + 6 + 12$$
$$21$$

5 Expressed in simplest form, $\frac{\sqrt{-20}}{\sqrt{5}}$ is equivalent to

- (1) $-2i$

- (2) $2i$

- (3) $\sqrt{2i}$

- (4) $\frac{2i}{\sqrt{5}}$

$$\frac{\sqrt{-20}}{\sqrt{5}} = \frac{\sqrt{-4} \sqrt{5}}{\sqrt{5}} = \sqrt{-4} = 2i$$

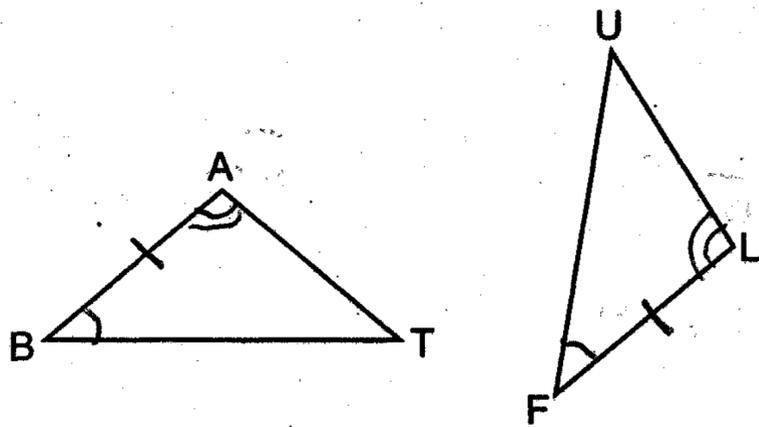
6 On a graph, if point A represents $2 - 3i$ and point B represents $-2 - 5i$, which quadrant contains $3A - 2B$?

- (1) I
- (2) II

- (3) III
- (4) IV

$$3(2 - 3i) - 2(-2 - 5i)$$
$$6 - 9i + 4 + 10i$$
$$10 + i$$

7 In the accompanying diagram of triangles BAT and FLU , $\angle B \cong \angle F$ and $\overline{BA} \cong \overline{FL}$.



Which statement is needed to prove $\triangle BAT \cong \triangle FLU$?

- (1) $\angle A \cong \angle L$ ASA

- (2) $\overline{AT} \cong \overline{LU}$

- (3) $\angle A \cong \angle U$

- (4) $\overline{BA} \parallel \overline{FL}$

Use this space for computations.

8 Which type of transformation is $(x,y) \rightarrow (x+2,y-2)$?

- (1) dilation
(2) reflection
(3) rotation
(4) translation

9 Which functions are positive for angles terminating in Quadrant II?

- (1) sine and cosine
(2) sine and secant
(3) sine and tangent
(4) sine and cosecant

10 What is $\sqrt{\frac{4}{3}} - \sqrt{\frac{3}{4}}$ expressed in simplest form?

- (1) 1
(2) 0
(3) $\frac{\sqrt{3}}{6}$
(4) $2\sqrt{3}$

$$\frac{\sqrt{4}}{\sqrt{3}} - \frac{\sqrt{3}}{\sqrt{4}}$$

$$\frac{2\sqrt{3}}{3} - \frac{\sqrt{3}}{2}$$

$$\frac{4\sqrt{3}}{6} - \frac{3\sqrt{3}}{6}$$

$$\frac{\sqrt{3}}{6}$$

11 Banks use the formula $A = P(1+r)^x$ when they compound interest annually. If P represents the amount of money invested and r represents the rate of interest, which expression represents $\log A$, where A represents the amount of money in the account after x years?

- (1) $x \log P + \log(1+r)$
(2) $\log P + x \log(1+r)$
(3) $\log P + x \log 1+r$
(4) $\log P + \log x + \log(1+r)$

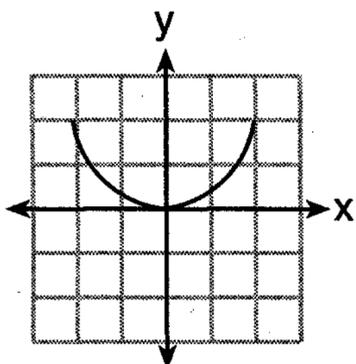
12 If the equation of the axis of symmetry of a parabola is $x = 2$, at which pair of points could the parabola intersect the x -axis?

- (1) (3,0) and (5,0)
(2) (3,0) and (2,0)
(3) (3,0) and (1,0)
(4) (-3,0) and (-1,0)

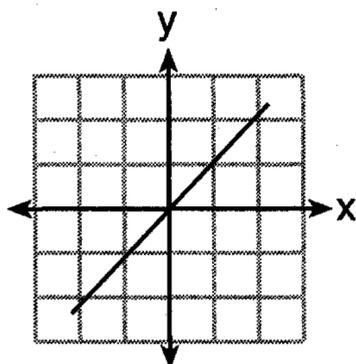
$$\frac{3+1}{2} = 2$$

Use this space for computations.

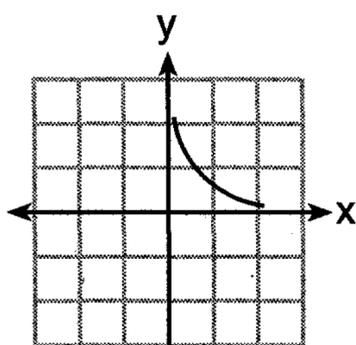
13 Jack is driving from New York to Florida. The number of hours that he drives and the speed at which he drives are inversely proportional. Which graph could be used to describe this situation if one axis represents speed and the other represents hours?



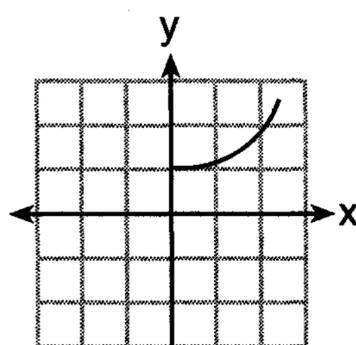
(1)



(3)



(2)



(4)

14 What is the length of the altitude of an equilateral triangle whose side has a length of 8?

(1) 32

(2) $4\sqrt{2}$

(3) $4\sqrt{3}$

(4) 4



$$a^2 + 4^2 = 8^2$$

$$a^2 = 48$$

$$a = \sqrt{48}$$

$$a = \sqrt{16 \cdot 3}$$

$$a = 4\sqrt{3}$$

15 What is the third term in the expansion of $(3x - 2)^5$?

(1) $1,080x^2$

(2) $270x^3$

(3) $540x^3$

(4) $1,080x^3$

$$n = 5$$

$$r - 1 = 2$$

$$a = 3x$$

$$b = -2$$

$${}^n C_{r-1} x^{n-(r-1)} y^{r-1}$$

$$5 C_2 (3x)^3 \cdot (-2)^2$$

$$10 \cdot 27x^3 \cdot 4 = 1080x^3$$

Use this space for computations.

16 If the dilation D_k is an isometry, what must be the value of k ?

- (1) 1
(2) 2
(3) -2
(4) 0

17 If $f(x) = x^2$ and $g(x) = 2x + 1$, which expression is equivalent to $(f \circ g)(x)$?

- (1) $2x^2 + 1$
(2) $2(x + 1)^2$
(3) $4x^2 + 1$
(4) $4x^2 + 4x + 1$

$$\begin{aligned} &(2x+1)^2 \\ &4x^2 + 2x + 2x + 1 \\ &4x^2 + 4x + 1 \end{aligned}$$

18 What is the inverse of the function $y = 2x - 3$?

- (1) $y = \frac{x+3}{2}$
(2) $y = \frac{x}{2} + 3$
(3) $y = -2x + 3$
(4) $y = \frac{1}{2x-3}$

$$\begin{aligned} x &= 2y - 3 \\ x + 3 &= 2y \\ \frac{x+3}{2} &= y \end{aligned}$$

19 If $a > 0$, which function represents the reflection of $y = a^x$ in the y -axis?

- (1) $y = -a^x$
(2) $y = \left(\frac{1}{a}\right)^x$
(3) $y = \left(\frac{1}{a}\right)^{-x}$
(4) $x = a^y$

20 The graph of the equation $2x^2 - 3y^2 = 4$ forms

- (1) a circle
(2) an ellipse
(3) a hyperbola
(4) a parabola

Part II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

21 Evaluate the expression $(x + 3)^{\frac{1}{2}} + (x - 3)^0 + (x + 2)^{-\frac{2}{3}}$ when $x = 6$.

$$(6+3)^{\frac{1}{2}} + (6-3)^0 + (6+2)^{-\frac{2}{3}}$$

$$3 + 1 + \frac{1}{4}$$

$$4\frac{1}{4}$$

22 Solve algebraically for x : $27^x = 9^{x+2}$

$$(3^3)^x = (3^2)^{x+2}$$

$$3^{3x} = 3^{2x+4}$$

$$3x = 2x + 4$$

$$x = 4$$

23 Solve for the negative value of x : $|2x + 5| + 1 = 13$

$$|2x + 5| = 12$$

$$2x + 5 = 12$$

$$2x + 5 = -12$$

$$\frac{2x}{2} = \frac{7}{2}$$

$$\frac{2x}{2} = \frac{-17}{2}$$

$$x = \frac{7}{2}$$

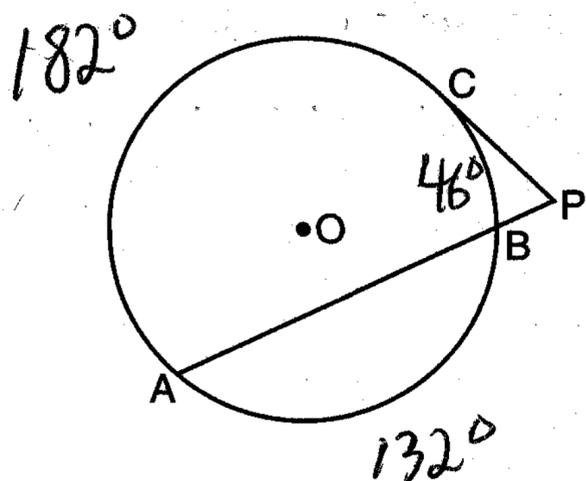
$$x = -\frac{17}{2}$$

24 In physics class, Esther learned that force due to gravity can be determined by using the formula $F = \frac{Gm_1m_2}{r^2}$. Solve for r in terms of F , G , m_1 , and m_2 .

$$\frac{Fr^2}{F} = \frac{Gm_1m_2}{F}$$

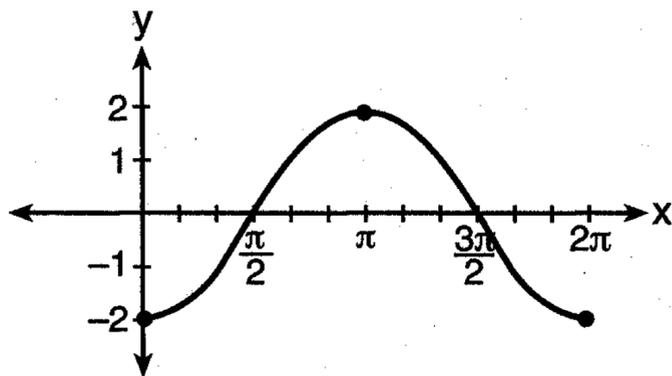
$$r = \sqrt{\frac{Gm_1m_2}{F}}$$

- 25 In the accompanying diagram of circle O , \overline{PC} is a tangent, \overline{PBA} is a secant, $m\widehat{AB} = 132$, and $m\widehat{CB} = 46$. Find $m\angle P$.



$$m\angle P = \frac{182 - 46}{2} = 68$$

- 26 The accompanying graph shows a trigonometric function. State an equation of this function.



$$y = -2\cos x$$

Part III

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [24]

27 Kathy swims laps at the local fitness club. As she times her laps, she finds that each succeeding lap takes a little longer as she gets tired. If the first lap takes her 33 seconds, the second lap takes 38 seconds, the third takes 42 seconds, the fifth takes 50 seconds, and the seventh lap takes 54 seconds, state the power regression equation for this set of data, rounding all coefficients to the *nearest hundredth*.

Using your written regression equation, estimate the number of seconds that it would take Kathy to complete her tenth lap, to the *nearest tenth of a second*.

$$y = 32.35x^{0.26}$$

$$32.35(10)^{0.26} \approx 58.9$$

28 Dave is the manager of a construction supply warehouse and notes that 60% of the items purchased are heating items, 25% are electrical items, and 15% are plumbing items. Find the probability that *at least* three out of the next five items purchased are heating items.

$$n = 5$$

$$r = 3, 4, 5$$

$$p = .6$$

$$q = .4$$

$${}_5C_3 \cdot .6^3 \cdot .4^2 = 0.34560$$

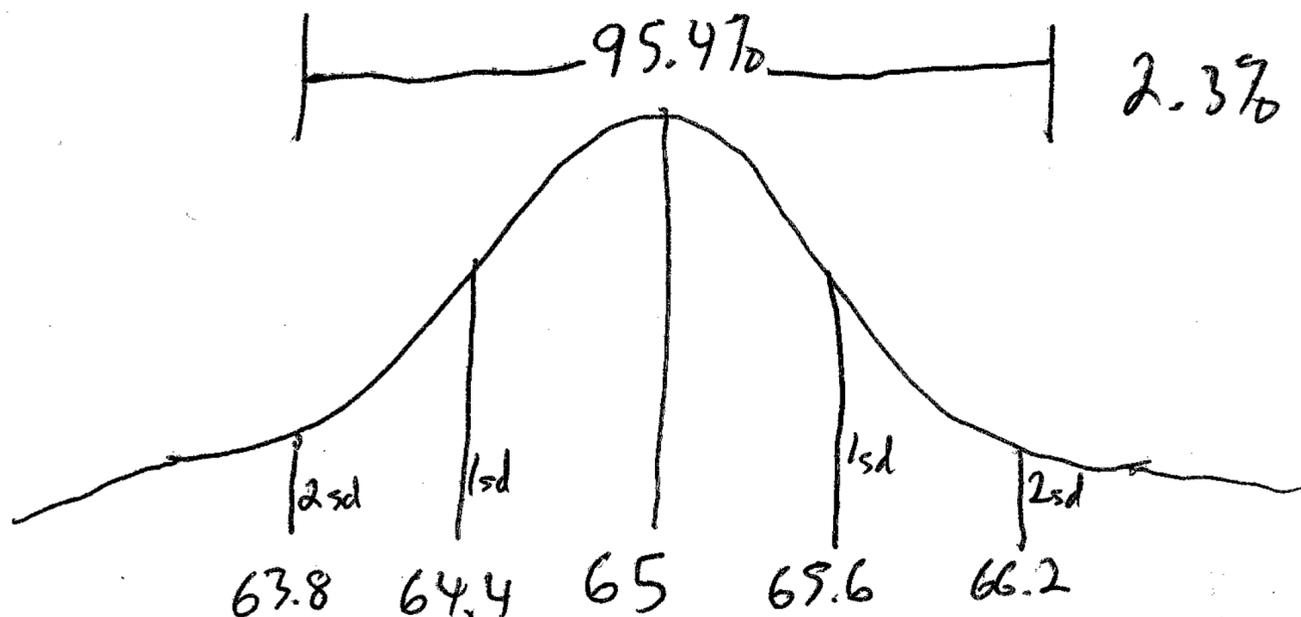
$${}_5C_4 \cdot .6^4 \cdot .4^1 = 0.25920$$

$${}_5C_5 \cdot .6^5 \cdot .4^0 = \frac{0.07776}{0.68256}$$

29 The heights of a sample of female students at Oriskany High School are normally distributed with a mean height of 65 inches and a standard deviation of 0.6 inch.

What percent of this sample is between 63.8 inches and 66.2 inches? 95.4%

Above what height, in inches, would the top 2.3% of this sample population be found? 66.2



30 Express in simplest form: $\frac{\frac{5}{a+b} - \frac{5}{a-b}}{\frac{10}{a^2-b^2}}$

$$\begin{aligned} & \frac{\frac{5(a-b) - 5(a+b)}{(a+b)(a-b)}}{\frac{10}{a^2-b^2}} = \frac{\frac{5a-5b-5a-5b}{\cancel{a^2-b^2}}}{\frac{10}{\cancel{a^2-b^2}}} \\ & = \frac{-10b}{10} \\ & = -b \end{aligned}$$

31 Solve the equation $3x^2 + 5 = 4x$ and express the roots in simplest $a + bi$ form.

$$3x^2 - 4x + 5 = 0$$

$$a = 3$$

$$b = -4$$

$$c = 5$$

$$x = \frac{4 \pm \sqrt{16 - 4(3)(5)}}{2(3)}$$

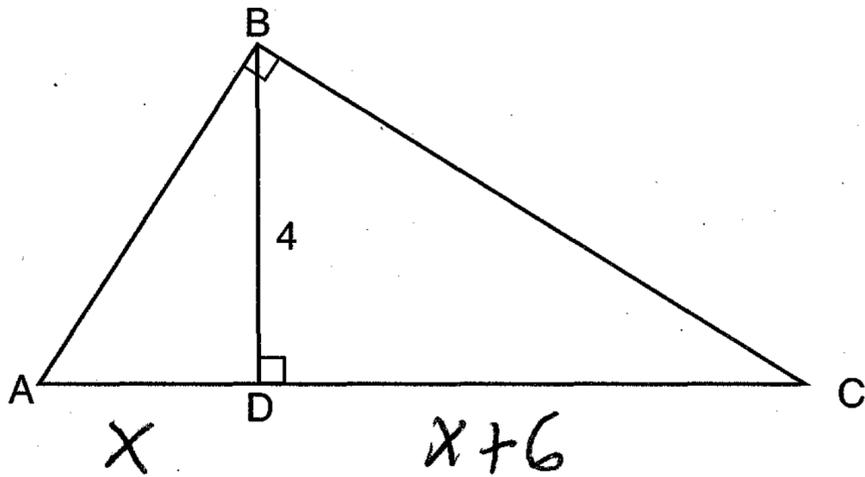
$$x = \frac{4 \pm \sqrt{-44}}{6}$$

$$x = \frac{4 \pm \sqrt{-4} \sqrt{11}}{6}$$

$$x = \frac{4 \pm 2i\sqrt{11}}{6}$$

$$x = \frac{2 \pm i\sqrt{11}}{3}$$

32 The drawing for a right triangular roof truss, represented by $\triangle ABC$, is shown in the accompanying diagram. If $\angle ABC$ is a right angle, altitude $BD = 4$ meters, and \overline{DC} is 6 meters longer than \overline{AD} , find the length of base \overline{AC} , in meters.



$$x(x+6) = 4^2$$

$$x^2 + 6x - 16 = 0$$

$$(x+8)(x-2) = 0$$

$$x = 2$$

$$\overline{AD} = 2$$

$$\overline{DC} = 8$$

$$\overline{AC} = 10$$

Part IV

Answer all questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

33 Given: $T(-1,1)$, $R(3,4)$, $A(7,2)$, and $P(-1,-4)$

Prove: $TRAP$ is a trapezoid.

$TRAP$ is *not* an isosceles trapezoid.

[The use of the grid on the next page is optional.]

To show $TRAP$ is a trapezoid, show that one pair of opposite sides is parallel (have the same slope) and that the other pair of opposite sides is not parallel (have different slope)

$$m_{\overline{TR}} = \frac{1-4}{-1-3} = \frac{-3}{-4} = \frac{3}{4}$$

$$m_{\overline{PA}} = \frac{-4-2}{-1-7} = \frac{-6}{-8} = \frac{3}{4}$$

$$m_{\overline{TP}} = \frac{1-(-4)}{-1-(-1)} = \text{undefined}$$

$$m_{\overline{RA}} = \frac{4-2}{3-7} = \frac{2}{-4} = -\frac{1}{2}$$

To show $TRAP$ is not an isosceles trapezoid, show that the non-parallel opposite sides are not congruent.

$$d_{\overline{TP}} = \sqrt{((-1)-(-1))^2 + (1-(-4))^2} = 5$$

$$d_{\overline{RA}} = \sqrt{(3-7)^2 + (4-2)^2} = \sqrt{20} = 2\sqrt{5}$$

34 Firefighters dug three trenches in the shape of a triangle to prevent a fire from completely destroying a forest. The lengths of the trenches were 250 feet, 312 feet, and 490 feet.

Find, to the nearest degree, the smallest angle formed by the trenches.

Find the area of the plot of land within the trenches, to the nearest square foot.

The smallest angle is opposite the smallest side, so let $a=250$, and Find A .

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$250^2 = 312^2 + 490^2 - 2(312)(490) \cos A$$

$$-274944 = -305760 \cos A$$

$$\frac{274944}{305760} = \cos A$$

$$26 \approx A$$

$$K = \frac{1}{2} ab \sin A$$

$$= \frac{1}{2} (312)(490) \sin 26$$

$$\approx 33,509$$