# The University of the State of New York <br> 326th High School Examination 

## PLANE GEOMETRY

Tuesday, January 24, $1956-9.15 \mathrm{a} . \mathrm{m}$. to 12.15 p.m., only

## Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (a) name of school where you have studied, ( $b$ ) number of weeks and recitations a week in plane geometry, ( $c$ ) author of textbook used. The minimum time requirement is four or five recitations a week for a school year.

## Part II

Answer three questions from this part.
26 Prove: If two sides of a quadrilateral are equal and parallel, the figure is a parallelogram. [10]

27 Prove: An angle formed by two chords intersecting within a circle is measured by one half the sum of the intercepted arcs. [10]

28 In isosceles triangle $A B C, A B=B C$. Points $D$ and $E$ are taken on $A B$ and $B C$ respectively so that $B D=B E . A E$ and $C D$ are drawn and intersect at $H$. Prove that
a $A E=C D \quad[6]$
$b$ triangle $A H C$ is isosceles

29 In the diagram at the right, $A T$ is tangent to the given circle. $C E$ is a chord parallel to $A T$, and $B$ is any point on arc $A E . A C, B C$ and $A B$ are drawn. $A B$ and $C E$ are extended to meet at $D$. Prove that $A C$ is the mean proportional between $A B$ and $A D$. [10]


30 Given two concentric circles whose center is $O$ and whose radii are 4 and 8 , and given straight line $A B$ through $O$.
$a$ Describe fully the locus of points equidistant from the two concentric circles.
$b$ Describe fully the locus of points at a given distance $d$ from $A B$. [4]
$c$ Find the number of points that satisfy the conditions in both part $a$ and part $b$ if
(1) $d=4$
(2) $d=6$
(3) $d=8$

## Plane Geometry

## Part III

## Answer two questions from this part. Show all work.

31 A rectangle and a parallelogram have equa? areas. The base of the rectangle is 16 and its diagonal is 20 . The base of the rectangle and the base of the parallelogram are in the ratio $2: 3$. a Find the altitude of the parallelogram. [8]
3. Indicate the correct completion for the following statement by writing the number (1), (2) or (3) after the letter $b$ on your answer paper: [2]

The perimeter of the rectangle is (1)less than the perimeter of the parallelogram (2)equal to the perimeter of the parallelogram (3)greater than the perimeter of the parallelogram

32 In triangle $A B C$, sides $A B, B C$ and $C A$ are represented by $4 a-1,3 a+3$ and $4 a+8$, respectively. The perimeter of triangle $A B C$ is 54 .
a Find the sides of the triangle.
$b$ Find the area of the triangle.
33 Tangents $P A$ and $P B$ are drawn to a circle irom external point $P$. Angle $A P B=40^{\circ}$ and $P A=25$.
$a$ Find the number of degrees in minor arc.$i B$.
$b$ Find to the nearest tinth the radius of the circle.
$c$ Find to the nearest integer the length of minor arc $A B$. [Use $\pi=\frac{22}{7}$.] [4]
34 In the figure below, $A B C D$ is a trapezoid with bases $A B$ and $D C$ and altitude $A E$. Angle $A D B=90^{\circ}$, angle $A B D=30^{\circ}$ and $A D=6$. Diagonal $A C$ bisects angle $D A B$. For each item listed in column I the numerical value is given in column II. List the numbers $1-5$ on your answer paper and after each number write the letter from column II that indicates the correct numerical value of the corresponding item. [10]


Column I
$1 A B$
2 Angle $A C D$
$3 D C$
$4 A E$
5 Area $A B C D$

Column II
a $30^{\circ}$
b $45^{\circ}$
c $60^{\circ}$
d 3
c $3 \sqrt{3}$
f 6
g $6 \sqrt{3}$
h 12
i $18 \sqrt{3}$
j $27 \sqrt{3}$
k $54 \sqrt{3}$

## Plane Geonetry

## Fill in the following lines:

Name of pupil.
Name of sch

## Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of $\pi$ or in radical form.

1 An angle of $64^{\circ}$ is inscribed in a circle. Find the number of degree. in its intercepted arc.

2 The legs of a right triangle are 8 and 9 . Find the hypotenuse to the nearest integer.

3 In right triangle $A B C, C D$ is the altitude to the hypotenuse $A B$. If $C D=6$ and $A D=3$, find $B D$.

4 The angles of a triangle are in the ratio $1: 4: 5$. Find the number of degrees in the largest angle.

5 In triangle $A B C$, a line parallel to $A B$ cuts $A C$ at $D$ and $B C$ at $E$. If $C D=6, A C=18$ and $B C=27$, find $E C$.

6 Find the area of an equilateral triangle whose side is 4.
7 Find the area of a rhombus whose diagonals are 20 and 29.
8 Find the diagonal of a square whose side is 6 .
9 In triangle $A B C$, angle $C$ is a right angle, angle $A=30^{\circ}$ and $B C=5$. Find $A C$.

10 From a point outside a circle a tangent and a secant are drawn to the circle. The tangent is 8 and the secant is 16 . Find the external segment of the secant.

11 The angle of a sector of a circle is $80^{\circ}$ and the radius of the circle is 9. Find the area of the sector.

12 Find the radius of a circle whose circumference is $16 \pi$.
13 Chords $A B$ and $C D$ of a circle intersect at $E$. If $A E=15, E B=4$ and $D E=10$, find $E C$.

14 Find the number of degrees in an exterior angle of a regular polygon of eight sides.

15 A ladder leans against a house standing on level ground. If the foot of the ladder is 4 feet from the house and the top of the ladder is 11 feet from the ground, find to the nearest degree the acute angle that the ladder makes with the ground.

16 The areas of two similar triangles are in the ratio $4: 9$. Find the ratio of a side of the smaller triangle to the corresponding side of the larger triangle.

## Plane Geometry

Directions (17-19): Indicate the correct completion for each of the following by writing on the line at the right the letter $a, b$ or $c$.

17 Two opposite angles of an isosceles trapezoid are (a) equal (b) complementary
(c) supplementary
17...................

18 Line segment $A B$ is 1 inch long. The number of points 2 inches $\begin{array}{llll}\text { from both } A \text { and } B \text { is } & \text { (a) } 0 & \text { (b) } 1 & \text { (c) } 2\end{array}$

18 $\qquad$
19 A converse of the statement, "Pupils who are G. O. (General Organization) members get a discount at local stores," is
(a) pupils get a discount at local stores if they are G. O. members
(b) pupils who do not get a discount at local stores are not G. O. members
(c) pupils who get a discount at local stores are G. O. members $\qquad$

Directions (20-23): For each of the following, tell whether the statement is always true, sometimes true or never true by writing the word always, sometimes or never on the line at the right.

20 The point of intersection of the perpendicular bisectors of the sides of
a triangle lies on a side of the triangle.

20
21 If the diagonals of a quadrilateral are equal, the quadrilateral is a rectangle.

22 Triangle $A B C$ is isosceles. If $D$ is any point in base $A C$ between $A$ and $C$, then $B D$ is greater than $A B$.

23 Two regular polygons are similar if they have the same number of sides.
$\qquad$
21..................
22..................
23..................

Directions (24-25): Leave all construction lines on your paper.

24 Given triangle $A B C$, construct the median to side $B C$.


25 Construct a tangent to circle $O$ at point $P$.


## FOR TEACHERS ONLY



## INSTRUCTIONS FOR RATING PLANE GEOMETRY

Tuesday, January 24, 1956 - 9.15 a.m. to 12.15 p.m., only
Use only red ink or pencil in rating Regents papers. Do not attempt to correct the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

## Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 17-19, allow credit if the pupil has written the correct answer instead of the letter $a, b$ or $c$.
(1) 128
(13) 6
(2) 12
(14) 45
(3) 12
(15) 70
(4) 90
(16) $2: 3$
(5) 9
(17) $c$
(6) $4 \sqrt{3}$
(18) $c$
(7) 290
(19) $c$
(8) $6 \sqrt{2}$
(9) $5 \sqrt{3}$
(20) sometimes
(10) 4
(21) sometimes
(11) $18 \pi$
(22) never
(12) 8
(23) always

$$
(12) 8
$$

