

The University of the State of New York

295TH HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Thursday, August 23, 1945 — 8.30 to 11.30 a. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (a) names of schools where you have studied, (b) number of weeks and recitations a week in plane geometry previous to entering summer high school, (c) number of recitations in this subject attended in summer high school of 1945 or number and length in minutes of lessons taken in the summer of 1945 under a tutor licensed in the subject and supervised by the principal of the school you last attended, (d) author of textbook used.

The minimum time requirement is five recitations a week for a school year. The summer school session will be considered the equivalent of one semester's work during the regular session or five recitations a week for half a school year.

For those pupils who have met the time requirement the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1945 or an equivalent program of tutoring approved in advance by the Department is required.

Part II

Answer two questions from part II.

26 Prove that tangents drawn to a circle from an external point are equal. [10]

27 Quadrilateral $ABCD$ is inscribed in a circle. Diagonals AC and BD intersect in E and $BE = CE$. Prove that chord $AB =$ chord CD . [10]

28 Prove that if two triangles have an angle of one equal to an angle of the other and the sides including these angles proportional, the triangles are similar. [10]

Part III

Answer two questions from part III.

29 Equal chords AB and CD of a circle are extended through B and D to meet in P . AD and BC intersect in E and AC is drawn. Angle $P = 18^\circ$ and arc $AC = 60^\circ$. Find arc BD , angle BCD , angle AEC and angle ACB . [3, 2, 2, 3]

30 CD is the altitude on the hypotenuse of right triangle ABC . $AB = 25$ and $AC = 20$. Find segment BD , altitude CD and angle B correct to the nearest degree. [3, 2, 5]

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- 31 A circle and an equilateral triangle each have a perimeter of 132 feet.
- a Find a side of the triangle. [1]
 - b Find the radius of the circle. [Use $\pi = \frac{22}{7}$] [2]
 - c Show that the difference between the area of the circle and the area of the triangle is approximately 549 square feet. [7]

Part IV

Answer one question from part IV.

- 32 a Let ABC be an *acute* triangle with $AB = 12$, $AC = 9$ and altitude CD drawn.
- (1) Express CD in terms of a trigonometric function of angle A . [3]
 - (2) Express K , the area of the triangle, in terms of a trigonometric function of angle A . [2]
- b Let ABC be *any* triangle with $AB = 12$, $AC = 9$ and altitude CD drawn.
- (1) As angle A increases from 0° to 180° , what change takes place in the length of side BC ? [2]
 - (2) As angle A increases from 0° to 90° and then from 90° to 180° , what change takes place in the length of altitude CD ? [3]
- 33 A shore defense gun is located at point A on a stretch of straight coast line. From a port B , 26 miles farther up the coast and out of range of the gun, a boat started out on a straight line course at 20 mph. When it had traveled for 9 minutes, it came within range of the gun and 42 minutes later it passed out of range of the gun.
- a How far had the boat gone when it first came within range of the gun? [1]
 - b For how many miles was the boat within range of the gun? [1]
 - c How far was the boat from the gun at the nearest point? [6]
 - d What was the range of the gun? [2]

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Fill in the following lines:

Name of school.....Name of pupil.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

- 1 In triangle ABC , $AB = 8$, $BC = 10$ and $AC = 14$. Name the largest angle. 1.....
- 2 An exterior angle at the base of an isosceles triangle is 110° . How many degrees are there in the vertex angle of the triangle? 2.....
- 3 A transversal cuts two parallel lines AB and CD at E and F , respectively, and the bisector of angle AEF meets CD in P . If angle $AEF = 64^\circ$, find the number of degrees in angle EPF . 3.....
- 4 In circle O , angle ABC , formed by diameter AB and chord BC , is equal to 60° . If the diameter of the circle is 10, find chord AC . [Answer may be left in radical form.] 4.....
- 5 Find the side of an equilateral triangle whose altitude is $\sqrt{3}$. 5.....
- 6 What is the name of the quadrilateral that is a regular polygon? 6.....
- 7 From point P outside a circle, tangent PD and secant PCA are drawn. $PD = 8$ and the secant is four times its external segment. Find the external segment of the secant. 7.....
- 8 The area of a rhombus is 135 and one diagonal is 18. Find the other diagonal. 8.....
- 9 The bases of a trapezoid are 2.4 inches and 5.6 inches and its altitude is 7.0 inches. Find the number of square inches in the area of the trapezoid. 9.....
- 10 Find the number of degrees in the angle of a sector if its area is 5π and the radius of the circle is 6. 10.....
- 11 Find the sum of the angles of a polygon of 8 sides. 11.....
- 12 If the radius of the circle inscribed in a regular polygon is 6, what is the apothem of the polygon? 12.....
- 13 What is the locus of the midpoints of all chords of a given length in a given circle? 13.....
- 14 The sides of two regular hexagons are in the ratio of 1:3. What is the ratio of the areas of the two hexagons? 14.....
- 15 A doorway is 3 feet wide and 7 feet high. Can a circular table top 8 feet in diameter be taken through the doorway? [Answer *yes* or *no*.] 15.....

Directions (questions 16-23) — If the blank in each statement is replaced by one of the words *always*, *sometimes* or *never*, the resulting statement will be true. Select the word that will correctly complete *each* statement and write this word on the line at the right.

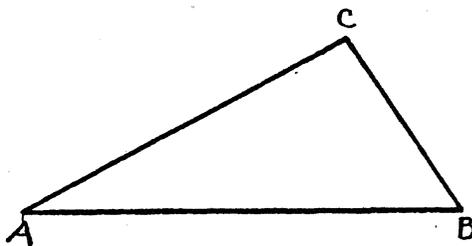
- 16 If the diagonals of a quadrilateral bisect each other, the figure is ... a parallelogram. 16.....
- 17 A median of a triangle ... divides the triangle into two equal triangles. 17.....
- 18 Chord AB of circle O passes through the midpoints of two parallel chords. AB is ... a diameter of the circle. 18.....
- 19 Two externally tangent circles ... have three common tangents. 19.....

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- 20 A parallelogram inscribed in a circle ... has two of its angles acute. 20.....
- 21 A triangle can ... be constructed by using as the three sides the lengths of the diameter and two radii of the same circle. 21.....
- 22 Two rectangles of equal area ... have unequal perimeters. 22.....
- 23 Side AB of triangle ABC is 16 and a line segment cuts the sides AC and BC in D and E , respectively. If DE equals 8, then DE is ... parallel to AB . 23.....

Directions (questions 24–25) — Leave all construction lines on the paper.

24 Find the center of the circle that can be circumscribed about triangle ABC .



25 Draw an obtuse angle on your paper and construct the locus of points within the angle equidistant from its sides.