# The University of the State of New York <br> 322nd High School Examination <br> PLANE GEOMETRY 

Tuesday, August 24, $1954-8.30$ to 11.30 a. m., only

## Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (a) names of schools where you have studied, (b) number of weeks and recitations a week in plane geometry previous to entering summer high school, (c) number of recitations in this subject attended in summer high school of 1954 or number and length in minutes of lessons taken in the summer of 1954 under a tutor licensed in the subject and supervised by the principal of the school you last attended, (d) author of textbook used.

The minimum time requirement is four or five recitations a week for a school year. The summer school session will be considered the equivalent of one semester's work during the regular session (four or five recitations a week for half a school year).

For those pupils who have met the time requirement the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1954 or an equivalent program of tutoring approved in advance by the Department is required.

## Part II

Answer three questions from this part.
26 Prove that, if in the same circle, or in equal circles, two chords are equidistant from the center, they are equal. [10]

27 Prove that the area of a triangle is equal to one half the product of its base and its altitude. [10]
28 The bases of trapezoid $A B C D$ are $A B$ and $C D$. Diagonals $A C$ and $B D$ of the trapezoid intersect at $R$. Prove that, if angle $C D R$ equals angle $D C R$, the trapezoid is isosceles. [10]

29 In quadrilateral $A B C D$, side $A D$ is parallel to side $B C$. A line from $A$ intersects $C D$ at $E$ and meets $B C$ extended at $F$. If $A D$ is greater than $D E$, prove that $C F$ is greater than $C E$. [10]

30 In triangle $A B C$ at the right, $D$ is a point on $A B$ and $E$ is a point on $A C$ such that $A D: A C=A E: A B$. If line $D E$ is drawn, prove that $D E \times A C=B C \times A D$. [10]


## Part III

Answer two questions from part III. All work, including computation, should be shown.
31 In hexagon $A B C D E F, A B D E$ is a rectangle and sides $B C, C D, E F$ and $F A$ are equal. Diagonal $A D$ is 13 , angle $D A B$ is $39^{\circ}$ and angle $C B D$ is $35^{\circ}$.
$a$ Find $A B$ to the nearest integer.
$b$ Find $B D$ to the nearest integer.
$c$ Using the answer found in $b$, find to the nearest integer the altitude from $C$ to $B D$. [3]
$d$ Using the answers found in $a, b$ and $c$, find the area of hexagon $A B C D E F$. [3]


32 The bases of an isosceles trapezoid are 26 and 10 , and the lower base angles are each $60^{\circ}$. Find a side of an equilateral triangle that is equal in area to this trapezoid. [10]

33 Diameter $D C$ of a circle is extended through $C$ to point $A$. From $A$ a line is drawn tangent to the circle at $B$. Angle $C A B$ is $32^{\circ}$.
a Find the number of degrees in minor arc $C B$. [3]
$b$ Chord $B E$ is drawn intersecting $D C$ at point $R$. If the ratio of arc $D E$ to arc $E C$ is $2: 3$, find the number of degrees in minor arc $D E$. [2]
$c$ Find angle $D R E$. [2]
$d$ Chord $D E$ is drawn. Find angles $D E R$ and $E D R$.
$e$ Check your answers to this problem by finding the sum of the angles of triangle $D E R$. [1]
34 In parallelogram $A B C D$, angle $A$ is acute and the altitude from $D$ meets $A B$ at $E . E B$ is 5 more than $A E$ and $A D$ is 3 more than twice $A E$.
$a$ If $A E$ is represented by $x$, represent $E B$ and $A D$ in terms of $x$. $[1,1]$
$b$ If the perimeter of parallelogram $A B C D$ is 56 , find $A E, E B$ and $A D$. [2, 1, 1]
$c$ Find $D E$.
[2]
$d$ Find the area of $A B C D$. [2]

## Plane Geometry

## Fill in the following lines:

Name of pupil. Name of school

## Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 Angle $A$ of parallelogram $A B C D$ is $85^{\circ}$. Find the number of degrees in angle $B$.

2 The angles of a triangle are in the ratio $5: 6: 7$. Find the number of degrees in the smallest angle.

3 Find the sum of the interior angles of a polygon of nine sides.
4 A line joining the midpoints of two sides of a triangle is $x$. Express the third side in terms of $x$.

5 Arc $A B$ of a circle is $96^{\circ}$. Find the number of degrees in the acute angle formed by chord $A B$ and the tangent at $B$.

6 A line parallel to $A C$ of triangle $A B C$ intersects $A B$ at $D$ and $B C$ at $E$. If $A D=8, D B=12$ and $B E=9$, find $E C$.

7 A tangent and a secant to a circle from an external point are 8 and 16 respectively. Find the external segment of the secant.

8 Two triangles have equal altitudes and their bases are in the ratio 5:8. Find the ratio of the area of the smaller triangle to the area of the larger.

9 The bases of a trapezoid are 5 and 9 and the altitude is 7. Find the area of the trapezoid.

10 The altitude on the hypotenuse of a right triangle is 6 and one of the segments of the hypotenuse made by the altitude is 2 . Find the other segment of the hypotenuse.

11 Corresponding sides of two similar triangles are 6 and 9. If the area of the larger triangle is 45 , find the area of the smaller.

12 Find a diagonal of a square whose side is 6 . [Answer may be left in radical form.]

13 The area of a regular polygon to the nearest integer is 396 . If the perimeter is 72 , find the apothem.
$14 P$ is a point on diameter $A B$ of a circle such that $A P=4$ and $P B=9$. $R S$ is a chord perpendicular to $A B$ at $P$. Find $R S$.

15 Find the length of an arc of $50^{\circ}$ in a circle whose radius is 18 . [Answer may be left in terms of $\pi$.]

16 Find the area of a rhombus whose diagonals are 7 and 10.
16.

17 The locus of points equally distant from two intersecting straight lines is a (a) straight line (b) pair of straight lines [Answer by writing either $a$ or $b$ on the line at the right.]
17.

18 Two sides of a rectangle are 6 and 8. Find to the nearest degree the angle that the longer side makes with a diagonal.
18.

Directions (19-23): For each of the following, if the statement is always true, write the word true on the line at the right ; if it is not always true, write the word false.

19 If two circles are tangent externally, three common tangents can be drawn to these circles.

19

20 If $B$ is the midpoint of arc $A C$, then chord $A C$ is twice chord $A B$.
20.

21 The sides of an equilateral polygon inscribed in a circle are equidistant from the center.

22 If the bisectors of two consecutive angles of a quadrilateral are perpendicular to each other, then these two consecutive angles are supplementary.

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21
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$\qquad$

22
23 The median drawn to the hypotenuse of a right triangle divides the triangle into two isosceles triangles.
23. $\qquad$

Directions (24-25): Leave all construction lines on the paper.
24 Construct a triangle congruent to triangle $A B C$.


25 Find by construction the point in line $m$ that is equally distant from points $A$ and $B$.

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A
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# INSTRUCTIONS FOR RATING <br> PLANE GEOMETRY 

Tuesday, August 24, 1954 - 8.30 to 11.30 a. m., only
Use only red ink or pencil in rating Regents papers. Do not attempt to correct the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

## Part I

Allow 2 credits for each correct answer; allow no partial credit. For question 17, allow credit if the pupil has written the correct expression instead of the letter $b$.

| $(1)$ | 95 | $(13)$ | 11 |
| :--- | :--- | :--- | :--- |
| $(2)$ | 50 | $(14)$ | 12 |
| $(3)$ | $1260^{\circ}$ | $(15)$ | $5 \pi$ |
| $(4)$ | $2 x$ | $(16)$ | 35 |
| $(5)$ | 48 | $(17)$ | $b$ |
| $(6)$ | 6 | $(18)$ | 37 |
| $(7)$ | 4 | $(19)$ | true |
| $(8)$ | $5: 8$ | $(20)$ | false |
| $(9)$ | 49 | $(21)$ | true |
| $(10)$ | 18 | $(22)$ | true |
| $(11)$ | 20 | $(23)$ | true |
| $(12)$ | $6 \sqrt{2}$ |  |  |

