

The University of the State of New York

328TH HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Tuesday, August 21, 1956— 8:30 to 11:30 a.m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (a) names of schools where you have studied, (b) number of weeks and recitations a week in plane geometry previous to entering summer high school, (c) number of recitations in this subject attended in summer high school of 1956, (d) author of textbook used.

The minimum time requirement is four or five recitations a week for a school year. The summer school session will be considered the equivalent of one semester's work during the regular session (four or five recitations a week for half a school year).

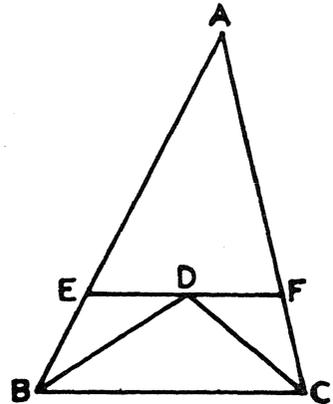
For those pupils who have met the time requirement the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1956 is required.

Part II

Answer three questions from this part.

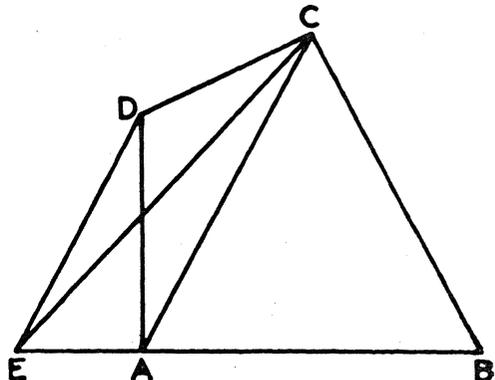
26 Prove: If the opposite sides of a quadrilateral are equal, the figure is a parallelogram. [10]



27 In triangle  $ABC$  at the right,  $DB$  bisects angle  $B$  and  $DC$  bisects angle  $C$ . Through  $D$ , the point of intersection of the two bisectors,  $EF$  is drawn parallel to  $BC$ .

Prove that  $EF = EB + FC$ . [10]

28 Prove: The areas of two similar triangles are to each other as the squares of any two corresponding sides. [10]



29 In the figure at the right,  $AC$  is a diagonal of quadrilateral  $ABCD$ .  $DE$  is drawn through  $D$  parallel to  $AC$  and meets  $BA$  extended at  $E$ .

a Prove that the area of triangle  $ADC$  is equal to the area of triangle  $AEC$ . [7]

b Prove that the area of quadrilateral  $ABCD$  is equal to the area of triangle  $ECB$ . [3]

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30 Triangle  $ABC$  is inscribed in a circle. The bisector of angle  $C$  intersects side  $AB$  at  $F$  and meets arc  $AB$  at  $E$ .

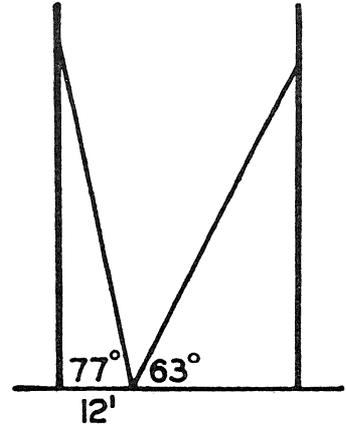
Prove that  $AC \times BC = CF \times CE$ . [10]

Part III

Answer two questions from this part. Show all work unless otherwise directed.

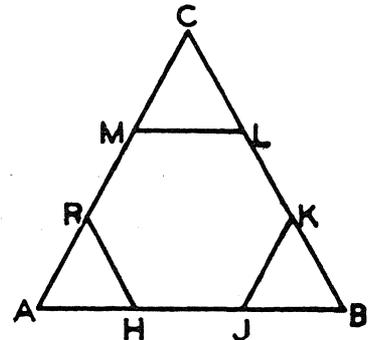
31 Two poles standing on level ground are supported by wires connected to a point on the ground in a straight line between the two poles, as shown in the accompanying diagram. The point is 12 feet from the foot of one pole, and the wire to that pole makes an angle of  $77^\circ$  with the ground. The wire to the other pole makes an angle of  $63^\circ$  with the ground.

- a Find to the *nearest foot* the length of the first wire. [4]
- b If the two wires are equal in length, find to the *nearest foot* the distance between the two poles. [6]



32 In the diagram at the right,  $HJKLMR$  is a regular hexagon and triangle  $ABC$  is equilateral. The area of triangle  $ABC$  is  $9\sqrt{3}$ .

- a Find a side of triangle  $ABC$ . [2]
- b Find a side of the hexagon. [3]
- c Find the area of the hexagon. [Answer may be left in radical form.] [5]



33 Two concentric circles have their center at  $O$ . Radii  $OA$  and  $OB$  of the larger circle meet the smaller circle at  $C$  and  $D$  respectively. If the radii of the circles are 8 and 4 and angle  $AOB = 45^\circ$ , find

- a the length of minor arc  $AB$  in terms of  $\pi$  [3]
- b the ratio of the length of minor arc  $CD$  to the length of minor arc  $AB$  [2]
- c the area of sector  $AOB$  [3]
- d the ratio of the area of sector  $COD$  to the area of sector  $AOB$  [2]

34 List the numbers 1–5 on your answer paper. If the figures described in 1–5 below are always congruent, write *congruent* at the right of the corresponding number. If the figures are not always congruent, determine whether they are similar or equal in area. Then at the right of the corresponding number, write *similar* or *equal in area*, whichever is a correct description of the figures. [10]

- (1) Two squares inscribed in the same circle
- (2) The four triangles formed by the diagonals of a parallelogram
- (3) Two regular polygons having the same number of sides
- (4) Two isosceles triangles having equal vertex angles
- (5) Two trapezoids having equal altitudes, with the sum of the bases of one equal to the sum of the bases of the other

[2]

PLANE GEOMETRY

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of  $\pi$  or in radical form.

- 1 Find the radius of a circle whose circumference is  $16\pi$ . 1.....
- 2 The diagonals  $AC$  and  $BD$  of rectangle  $ABCD$  intersect at  $E$ . If  $AC = 10$ , find  $BE$ . 2.....
- 3 In a given circle, chord  $MN$  is perpendicular to chord  $RS$ . Find the number of degrees in the sum of arcs  $RN$  and  $SM$ . 3.....
- 4 From a point outside a circle a tangent and a secant are drawn to the circle. The secant is 13 and its external segment is 3. Find the tangent. 4.....
- 5 The radius of a circle is 10 inches. How many inches long is a chord which is 6 inches from the center? 5.....
- 6 Find the fourth proportional to 3, 4 and 7 taken in the given order. 6.....
- 7 A square and a parallelogram are equal in area. If the base of the parallelogram is 18 and its altitude is 8, find a side of the square. 7.....
- 8 If one angle of a rhombus is  $60^\circ$  and the shorter diagonal is 12, how long is a side of the rhombus? 8.....
- 9 The median to the hypotenuse of a right triangle is 5. Find the length of the hypotenuse. 9.....
- 10 What is the altitude of an equilateral triangle whose side is 16? 10.....
- 11 Find the number of degrees in an exterior angle of a regular polygon of nine sides. 11.....
- 12 The base of an isosceles triangle is 20 and a base angle is  $42^\circ$ . Find, to the nearest unit, the altitude to the base of the triangle. 12.....
- 13 The altitude to the hypotenuse of a right triangle divides the hypotenuse into two segments 4 inches and 9 inches long, respectively. Find the length of the altitude in inches. 13.....
- 14 In triangle  $ABC$ , angle  $A = 55^\circ$  and angle  $B = 65^\circ$ . Which is the longest side of the triangle? 14.....
- 15 Two exterior angles of a triangle are  $60^\circ$  and  $140^\circ$ . Find the number of degrees in the third exterior angle. 15.....
- 16 Corresponding sides of two similar triangles are in the ratio 2:3. If the perimeter of the smaller triangle is 24, find the perimeter of the larger triangle. 16.....
- 17 In parallelogram  $ABCD$ , angle  $A$  and angle  $B$  are in the ratio 2:7. Find the number of degrees in angle  $A$ . 17.....
- 18 Find the number of points which are a given distance from a given line and also equidistant from two points on the given line. 18.....

PLANE GEOMETRY

*Directions (19–23):* Indicate the correct completion for *each* of the following by writing on the line at the right the letter *a*, *b* or *c*.

19 The minor arc intercepted by two tangents to a circle from a point outside the circle is  $140^\circ$ . The angle formed by the tangents is (a)  $40^\circ$   
(b)  $70^\circ$  (c)  $80^\circ$  19.....

20 The area of a rhombus is 24 and one diagonal is 6. The other diagonal is (a) 2 (b) 4 (c) 8 20.....

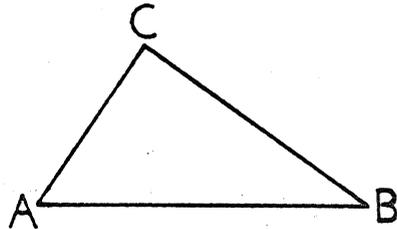
21 Chord  $AB$  of a circle subtends an arc of  $64^\circ$ . The acute angle formed by the chord and a tangent to the circle at  $A$  is (a)  $32^\circ$  (b)  $58^\circ$   
(c)  $64^\circ$  21.....

22 If angle  $A$  is the complement of angle  $B$  and angle  $B$  is the supplement of angle  $C$ , the largest of the three angles is (a) angle  $A$  (b) angle  $B$   
(c) angle  $C$  22.....

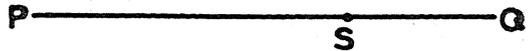
23 Two sides of a triangle are 12 and 15. The third side of the triangle may be (a) 3 (b) 15 (c) 27 23.....

*Directions (24–25):* Leave all construction lines on your paper.

24 In triangle  $ABC$ , construct the median from  $C$  to  $AB$ .



25 Construct a line perpendicular to line  $PQ$  at point  $S$ .



# FOR TEACHERS ONLY

# PG

## INSTRUCTIONS FOR RATING PLANE GEOMETRY

Tuesday, August 21, 1956 — 8:30 to 11:30 a.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

### Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 19–23, allow credit if the pupil has written the correct expression instead of the letter *a*, *b* or *c*.

- |                    |               |
|--------------------|---------------|
| (1) 8              | (19) <i>a</i> |
| (2) 5              | (20) <i>c</i> |
| (3) 180°           | (21) <i>a</i> |
| (4) $\sqrt{39}$    | (22) <i>c</i> |
| (5) 16             | (23) <i>b</i> |
| (6) $9\frac{1}{3}$ |               |
| (7) 12             |               |
| (8) 12             |               |
| (9) 10             |               |
| (10) $8\sqrt{3}$   |               |
| (11) 40°           |               |
| (12) 9             |               |
| (13) 6             |               |
| (14) <i>AC</i>     |               |
| (15) 160°          |               |
| (16) 36            |               |
| (17) 40°           |               |
| (18) two           |               |