## HIGH SCHOOL MATHEMATICS: COURSE III—JANUARY 1983 (1)

## Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of  $\pi$  or in radical form.

- 1 Express 144° in radian measure.
- 2 What is the amplitude of the graph of  $y = 3 \cos 2x$ ?

3 Solve for 
$$x$$
:  $\frac{1}{x} + 1 = \frac{3}{2x}$ 

- 4 Given: circle O with  $\overline{QR}$  tangent to the circle at R. If  $m \angle OQR = 20$ , find  $m \angle ROQ$ .
- 5 Find cos (Arc tan 1).
- 6 In a circle, a central angle of 2 radians intercepts an arc of length 12. Find the length of the radius of the circle.
- 7 If  $f(x) = \log(x^2)$ , find f(10).
- 8 Express  $(3 2i)^2$  in the form a + bi.
- 9 In the accompanying diagram of right triangle ABC, AB = 5000 and  $m \angle C = 90$ . If the measure of angle A is  $20^{\circ}$  40', find AC to the nearest unit.



10 Find the value of  $16^{\frac{3}{4}}$ .

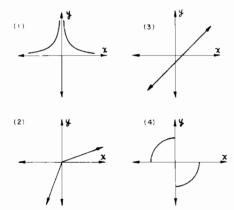
11 Evaluate: 
$$\sum_{n=3}^{5} (2n + 3)$$

- 12 In  $\triangle RST$ , r = 3,  $s = 3\sqrt{2}$ , and  $m \angle R = 30$ . Find  $m \angle S$  if S is an acute angle.
- 13 In circle O, chords  $\overline{AB}$  and  $\overline{CD}$  intersect at E. If AE = 6, BE = 12, and CE = 8, find DE.
- 14 Solve for x:  $\sqrt{2x+3} 5 = 0$
- 15 If  $\cos x = \frac{3}{5}$  and x is a positive acute angle, find  $\tan \frac{1}{2} x$ .
- 16 What value of x satisfies the equation  $\sin x \cos x 2 \cos x = 0$  for  $0^{\circ} < x < 180^{\circ}$ ?
- 17 If  $\sin x = \frac{5}{6}$ , what is the value of  $\cos 2x$ ?
- 18 Team *A* and team *B* play 3 games. If team *A*'s probability of winning a game is  $\frac{2}{5}$ , what is the probability of team *A* winning at least two games?
- 19 Find the value of  $\tan x$  if  $\cos x = \frac{3}{5}$  and  $\sin x < 0$ .
- 20 What is the image of the point (5,-2) under the translation  $T_{2,1}$ ?

Directions (21-35): For each question chosen, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question.

- 21 A dilation is a transformation that does not preserve
  - (1) distance

- (3) orientation
- (2) similarity
- (4) betweenness
- 22 As angle x increases from  $90^{\circ}$  to  $180^{\circ}$ , the value of  $\sin x$ 
  - (1) increases from -1 to 0
  - (2) increases from 0 to 1
  - (3) decreases from 0 to -1
  - (4) decreases from 1 to 0
- 23 Which figure has line symmetry with respect to the line x = 0?



- 24 The expression  $\log \sqrt[3]{\frac{a}{b}}$  is equivalent to
  - $(1) \ \frac{1}{3} \left( \log \ a \ \ \log \ b \right) \qquad (3) \ \frac{1}{3} \left( \frac{\log \ a}{\log \ b} \right)$
  - $(2) \frac{1}{3} \log a + \log b \qquad (4) \ 3(\log a \log b)$

25 In 
$$\triangle ABC$$
, if  $a=4$ ,  $b=9$ , and  $c=8$ , then  $\cos A$  equals

(1) 1 (2) 
$$\frac{43}{48}$$
 (3)  $\frac{161}{144}$  (4)  $\frac{48}{43}$ 

26 If a = 5, b = 7, and  $m \angle A = 30$ , how many distinct triangles can be constructed? (2) 2 $(3) \ 3$ 

(1) 1 (2) 2 (3) 3 (4) 4  
27 A transformation maps (1,3) onto 
$$(-1,-3)$$
. This transformation is equivalent to  
(1) rotation  $R_{90}$  (3) dilation  $D_{-1}$ 

(2) rotation 
$$R_{-90}^{\circ}$$
 (4) translation  $T_{-1,-3}$   
28 If  $a>b$  and  $c<0$ , then which statement is

true?  
(1) 
$$a + c < b + c$$
 (3)  $ac > bc$   
(2)  $a - c < b - c$  (4)  $ac < bc$ 

29 The expression 
$$\sin (\theta + 270^{\circ})$$
 equals
(1)  $\cos \theta$ 
(2)  $2 \cos \theta$ 
(3)  $-\cos \theta$ 
(4)  $-\sin \theta$ 

(4) be tangent to the 
$$x$$
-axis

31 The expression 
$$\frac{5}{4-3i}$$
 is equivalent to

$$(1) \ \frac{20 + 15i}{7} \qquad \qquad (3) \ \frac{20 - 15i}{7}$$

$$(2) \ \frac{4 + 3i}{5} \qquad (4) \ \frac{7i}{5}$$

(1) II and III

32 If 
$$\tan x = -\frac{\sqrt{3}}{3}$$
, in which quadrant(s) may angle x terminate?

(1) II and III

(3) III, only

33 What is the numerical coefficient of the second term of the expansion  $(x - 2y)^4$ ?

$$\begin{array}{cccc}
(1) & -8 & & & & & \\
(2) & 8 & & & & & \\
\end{array}$$

34 On a standardized test, the mean was 75 and the standard deviation was 4.0. Approximately what percentage of the scores would fall within the range 71 to 79?

35 What is the inverse relation of the function whose equation is y = 2x + 3?

$$(1) y = 2x - 3 (3) y = 3x - 2$$

$$(2) \ y = \frac{x - 3}{2} \qquad (4) \ y = x$$

## Part II

Answer four questions from this part. Show all work unless otherwise directed.

- 36 a On the same set of axes, sketch and label the graphs of  $y = 2 \sin x$  and  $y = \cos 2x$  as
  - x varies from 0 to  $2\pi$  radians. [8] b How many values of x in the interval
    - $0 \le x \le 2\pi$  satisfy the equation
      - $2 \sin x = \cos 2x$ ? [2]
  - 37 a Find all values of  $\theta$  in the interval  $0^{\circ} \le \theta \le 360^{\circ}$  which satisfy the equation  $2\cos^3\theta + \cos^2\theta - \cos\theta = 0.$  [6]
    - b For all values of  $\theta$  for which the expressions are defined, prove the following identity:

$$2 \csc^2 \theta = \frac{1}{1 + \cos \theta} + \frac{1}{1 - \cos \theta}$$

- 38 a Solve the equation  $3x^2 + 3 = 5x$  and express its roots in terms of i. [5]
  - b Using logarithms, solve for x to the nearest tenth:

$$3^{2x} = 5$$
 [5]

- 39 The shoe sizes of the ten players on a basket-ball team are as follows: 7, 8, 9½, 10, 10, 10, 10½, 11, 11, 13. Determine the standard deviation of these shoe sizes to the *nearest tenth*. [10]
- 40 *a* In  $\triangle ABC$ , a=11, b=20, and  $m \angle B=42^{\circ}$  30'. Find the measure of angle *A* to the *nearest ten minutes*. [5] *b* Using the result obtained in part *a*, find the area of  $\triangle ABC$  to the *nearest square unit*. [5]
- 41 *a* On graph paper, draw and label  $\triangle ABC$  whose coordinates are A(2,1), B(6,4), and C(8,1). [1] *b* Graph and state the coordinates of  $\triangle A'B'C'$ ,
  - the reflection of  $\triangle ABC$  through the *x*-axis. [3]  $\epsilon$  Graph and state the coordinates of  $\triangle A''B''C''$ .
  - the image of  $\triangle A'B'C'$  after the translation  $T_{-6,-2}$ . [3]
  - d Using the origin as the center of rotation, graph and state the coordinates of  $\triangle A'''B'''C'''$ , the result of rotating  $\triangle A''B'''C'''$  90° clockwise. [3]
- 42 In the accompanying figure, quadrilateral  $\widehat{MNPQ}$  is inscribed in circle O with  $\widehat{ST}$  tangent to the circle at P. Diagonals  $\widehat{PM}$  and  $\widehat{QN}$  intersect at K and  $\widehat{mMN}:\widehat{mNP}:\widehat{mPQ}:\widehat{mQM} = 5:3:8:4.$

Find: 
$$a \quad \widehat{\text{mMN}}$$
 [3]  
 $b \quad \text{m} \angle MQP$  [2]  
 $c \quad \text{m} \angle SPQ$  [3]  
 $d \quad \text{m} \angle PKQ$  [2]

