SOLID GEOMETRY

Thursday, January 26, 1956—9.15 a.m. to 12.15 p.m., only

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1. A right section of a prism is a triangle whose sides are 6, 7 and 8. If a lateral edge of the prism is 5, find the lateral area. 1.....

2. A base edge of a regular square pyramid is $2a$ and the altitude is h. Express the slant height in terms of a and h. 2.....

3. The base of a pyramid is an equilateral triangle whose side is 8. If the altitude of the pyramid is 9, find the volume. 3.....

4. The base edges of a frustum of a regular pentagonal pyramid are 3 and 5. If the slant height is 6, find the lateral area. 4.....

5. The lateral area of a right circular cylinder is equal to the area of its base. Express the altitude in terms of the radius (r) of the base. 5.....

6. Find the altitude of a circular cone whose volume is 308 and whose radius is 7. [Use $\pi = 22/7$.] 6.....

7. An element of a right circular cone makes an angle of 72° with the base. If the element is 8, find, to the *nearest tenth*, the radius of the base. 7.....

8. Two face angles of a trihedral angle are 90° and 100°. The third face angle is more than 10 degrees and less than n degrees and may have any value between these limits. Find n. 8.....

9. The base of a right prism is a rectangle whose sides are 2 and 6. If a diagonal of the prism is 11, find the altitude of the prism. 9.....

10. The altitude of a pyramid is 8 inches and its base is 40 square inches. Find the number of square inches in the section made by a plane parallel to the base of the pyramid and 6 inches from its vertex. 10.....

11. A zone is drawn on a sphere whose radius is 15. If the altitude of the zone is 3, find its area. 11.....

12. A lune is drawn on a sphere whose radius is r. If the angle of the lune is 120°, express the area of the lune in terms of r. 12.....

13. A spherical triangle is $\frac{1}{24}$ the area of the sphere on which it is drawn. Find the number of spherical degrees in the area of the triangle. 13.....

Directions (14-16): Indicate the correct completion for *each* of the following by writing on the line at the right the letter *a*, *b* or *c*.

14. The locus of points 2 inches from a sphere whose radius is 5 inches and 1 inch from a plane through the center of the sphere consists of (*a*) 2 circles (*b*) 4 circles (*c*) 6 circles 14.....

15. Plane *M* is parallel to plane *N* if (*a*) *M* passes through two lines parallel to *N* (*b*) *M* and *N* are perpendicular to the same plane (*c*) *M* passes through two intersecting lines, each of which is parallel to *N* 15.....

16. Three of the regular polynedrons have equilateral triangles as faces. The number of faces of these three polyhedrons is (*a*) 4, 6 and 12 (*b*) 4, 8 and 12 (*c*) 4, 8 and 20 16.....

Directions (17-20): For *each* of the following, tell whether the statement is always true, sometimes true or never true by writing the word *always*, *sometimes* or *never* on the line at the right.

17. Circles of a sphere whose planes are parallel have the same poles. 17.....

18. If three sides of one spherical triangle are equal to the three sides of another spherical triangle, the triangles are congruent. 18.....

19. If two lines, one in each face of a dihedral angle, form a right angle, the dihedral angle is a right dihedral angle. 19.....

20. If more than one plane can be passed through one of two non-intersecting lines parallel to the other, the lines are skew. 20.....

Part II

Answer two questions from this part.

21. Prove: If a line is perpendicular to each of two intersecting lines at their point of intersection, it is perpendicular to the plane of the two lines. [10]

22. Prove: If the first of two spherical triangles is the polar triangle of the second, then the second is the polar triangle of the first. [10]

23. The plane passed through two diagonally opposite lateral edges of a parallelepiped intersects the plane passed through the other two lateral edges in line *RS*. If *RS* is perpendicular to the base of the parallelepiped, prove that the parallelepiped is a right parallelepiped. [10]

24. *A*. Given plane *M* perpendicular to and intersecting plane *N*. Describe *fully* the locus of points equidistant from *M* and *N*. [4]

B. Given points *R* and *S*. Describe *fully* the locus of points equidistant from *R* and *S*. [2]

C. Indicate the correct completion for *each* of the following by writing the letter *a*, *b* or *c* after the proper number on your answer paper: [2, 2]

(1) If *R* is in plane *M* and if the line through *R* and *S* is perpen-

dicular to M, the locus of points satisfying *both* conditions given in A and B above consists of (*a*) one line (*b*) two parallel lines (*c*) two perpendicular lines

(2) If both R and S lie on the intersection of M and N, the locus of points satisfying *both* conditions given in A and B above consists of (*a*) one line (*b*) two parallel lines (*c*) two perpendicular lines

Part III

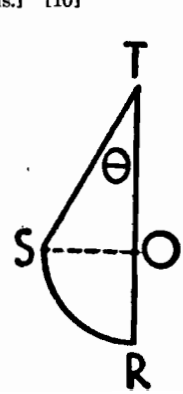
Answer three questions from this part. Show all work.

25. *a.* On a sphere whose radius is 10 inches, an equilateral triangle has an area of $\frac{550}{3}$ square inches. Find one angle of the triangle. [Use $\pi = 22/7$.] [5]

b. Find, to the *nearest tenth of an inch,* the perimeter of the polar triangle. [5]

26. A form into which concrete is to be poured has the shape of a frustum of a regular square pyramid. The lower base edge is 12 feet, the upper base edge is 6 feet and the altitude is 18 feet. If concrete has been poured into the form to a depth of 12 feet, find, to the *nearest cubic yard,* the amount of concrete needed to fill the remaining space in the form.
[$V = 1/3\,h\,(B + B' + \sqrt{BB'})$] [10]

27. A water tank has the form of a right cylinder with a hemisphere at each end. The cylindrical portion has a diameter of 7 feet and an altitude of 8 feet. If the total length of this tank is 15 feet, find its capacity to the *nearest ten gallons.* [Use $\pi = 22/7$ and 1 cubic foot = 7-1/2 gallons.] [10]



28. In the figure at the right, O is the center of the quadrant RS, and angle STR is represented by θ. The length of OR is represented by x.

a. If RST is revolved through 360° about RT as an axis, express in terms of x and θ the area, K, generated by RST. [7]

b. If $x = 5$ and $\theta = 30°$, find K. [3]

TWELFTH YEAR MATHEMATICS

12B

12B (Solid Geometry)

Thursday, January 26, 1956—9:15 a.m. to 12:15 p.m., only

Part III

Directions: The following questions are based upon the optional topics of the twelfth year syllabus. *Either one or both* may be substituted for *any one or two* of the questions on part III of the examination in solid geometry.

29. Given spherical triangle ABC in which angle $C = 90°$, angle $A = 64° 20'$ and side $b = 60°$. Find side c to the *nearest degree.* [10]

30. *a.* Find the coordinates of the mid-point of the line segment whose end points are (1, 2, 7) and (3, —2, 7). [3]

    b. Find the x, y and z intercepts of the plane whose equation is $2x - 3y + 8z = 24$. [1, 1, 1]

    c. Write an equation of the plane parallel to the z-axis and passing through the points (4, 0, 0) and (0, —3, 0). [2]

    d. Write the equation of the plane parallel to the xy-plane and passing through the point (5, —1, 7). [2]

TWELFTH YEAR MATHEMATICS

12B (Solid Geometry)

Wednesday, January 23, 1957—9:15 a.m. to 12:15 p.m., only

Part III

Directions: The following questions are based upon the optional topics of the twelfth year syllabus. *Either one or both* may be substituted for *any one or two* of the questions on part III of the examination in solid geometry.

30. *a.* Write an equation of the plane parallel to the xz-plane and passing through the point (2, 3, —6). [2]

    b. Find the distance between the points (2, 3, —6) and (—5, 6, 1). [3]

    c. Find the coordinates of the midpoint of the line segment joining the points (5, 6, 2) and (—2, 4, —6). [2]

    d. Write an equation of the plane whose x-, y- and z-intercepts are 5, —5 and 4, respectively. [3].

31. Given spherical triangle ABC in which angle $C = 90°$, side $c = 62°$ and side $b = 37°$.

    a. Find side a to the *nearest degree.* [7]

    b. Using the given data, write an equation that could be used to find angle A. [3]