

The University of the State of New York

292D HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Wednesday, August 23, 1944 — 8.30 to 11.30 a. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish this part before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (a) names of schools where you have studied, (b) number of weeks and recitations a week in solid geometry previous to entering summer high school, (c) number of recitations in this subject attended in summer high school of 1944 or number and length in minutes of lessons taken in the summer of 1944 under a tutor licensed in the subject and supervised by the principal of the school you last attended, (d) author of textbook used.

The minimum time requirement is five recitations a week for half a school year. The summer school session will be considered the equivalent of one semester's work during the regular session or five recitations a week for half a school year.

For those pupils who have met the time requirement, the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1944 or an equivalent program of tutoring approved in advance by the Department is required.

Part II

Answer two questions from part II.

21 Prove that if a line is perpendicular to a plane, every plane passed through the line is perpendicular to the given plane. [10]

22 A plane is passed through one diagonal of a parallelogram. Prove that the perpendiculars dropped to that plane from the extremities of the other diagonal are equal. [10]

23 Prove that every section of a circular cone made by a plane parallel to its base is a circle. [10]

*24 Given a plane m and line l perpendicular to m and intersecting it at point P

a What is the locus of points at a given distance d from m ? [3]

b What is the locus formed by a line through P revolving about l as an axis and making an angle θ with l ? [5]

c What is the locus of points that satisfy the conditions in both *a* and *b*? [2]

* This question is based on one of the optional topics in the syllabus.

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Part III

Answer three questions from part III.

- 25 *a* The runway on an air field is to be 60 yards wide and one-half mile long. Find the cost of filling the runway with crushed stone to a depth of 9" if labor and materials cost \$2 per cubic yard. [5]
- b* The sides of a spherical triangle on a sphere whose radius is 9 are 50° , 70° and 80° . Find the area of the polar triangle. [Leave answer in terms of π .] [5]
- 26 A zone of one base on a sphere of radius 13 equals a lune on another sphere, also of radius 13. The radius of the base of the zone is 12. Find, correct to the *nearest degree*, the angle of the lune. [10]
- 27 Two sides of the base of a parallelepiped are 10" and 12" and the angle included between these sides is 54° . Find, correct to the *nearest 10 cubic inches*, the volume of the parallelepiped if its altitude is 15". [10]
- 28 *a* In a right circular cone, s represents the slant height and θ is the angle which the slant height makes with the base. Show that the formula for the volume V of the cone is $V = \frac{1}{3} \pi s^3 \cos^2 \theta \sin \theta$ [5]
- b* By the use of logarithms, find the volume, correct to the *nearest cubic inch*, when $s = 8$ " and $\theta = 63^\circ$ [Use $\pi = 3.14$] [5]

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Fill in the following lines:

Name of school.....Name of pupil.....

Part I

Answer all questions in part I. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

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| 1 How many degrees are there in the angle of a lune whose area contains 50 spherical degrees? | 1..... |
| 2 Find the radius of a sphere whose volume is 288π . | 2..... |
| 3 Find the total area of a cylindrical oil tank that is 30 feet long and has a diameter of 8 feet. [Answer may be left in terms of π .] | 3..... |
| 4 If the slant height of a right circular cone is multiplied by 3, by what number is the lateral area multiplied? | 4..... |
| 5 A pyramid whose altitude is 12" is cut by a plane parallel to the base and 4" from the vertex. Find the volume of the pyramid cut off if the volume of the given pyramid is 27 cubic inches. | 5..... |
| 6 Given two meridians, one 50° west and the other 10° east; how many degrees are there in the portion of the equator included between the two meridians? | 6..... |
| 7 Which of the polygons—equilateral triangles, squares, regular pentagons, regular hexagons— <i>can not</i> be faces of a regular polyhedron? | 7..... |
| 8 A sphere is inscribed in a cylinder whose altitude is 4. Find the area of the sphere. [Answer may be left in terms of π .] | 8..... |
| 9 The base of a pyramid is a right triangle with legs 4" and 5". If the altitude of the pyramid is 6", find the volume. | 9..... |
| 10 The sides of a spherical triangle contain 80° , 100° and 100° . Find the number of inches in the perimeter of the triangle if the radius of the sphere is 9". [Use $\pi = \frac{22}{7}$] | 10..... |

Directions (questions 11–15) — Indicate the correct answer to each question by writing on the line at the right the letter *a*, *b*, *c* or *d*.

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|---|---------|
| 11 Which of the following may be the face angles of a trihedral angle? (a) 40° , 60° , 100° ; (b) 80° , 150° , 120° ; (c) 70° , 80° , 160° ; (d) 120° , 120° , 120° | 11..... |
| 12 <i>A</i> and <i>B</i> are two points on a sphere not the extremities of a diameter. The maximum number of great circles that can be drawn through <i>A</i> and <i>B</i> is (a) 1, (b) 2, (c) 3, (d) infinite. | 12..... |
| 13 The altitude of the Torrid Zone is approximately 3000 miles. If the radius of the earth is assumed to be 4000 miles, the number of million square miles in the zone is most nearly (a) 10, (b) 40, (c) 75, (d) 300 | 13..... |
| 14 If two sides of a spherical triangle contain 100° and 160° , we know the third side must be greater than 60° and less than (a) 100° , (b) 180° , (c) 260° , (d) 280° , and may have any value between these limits. | 14..... |
| 15 If two spheres are tangent to the same line at point <i>P</i> , then we know that (a) they are tangent to each other, (b) they intersect, (c) they coincide, (d) their relative positions are not determined. | 15..... |

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Directions (questions 16–20) — If the blank in each statement is filled by one of the words *always*, *sometimes* or *never*, the resulting statement will be true. Select the word that will correctly complete *each* statement and write the word on the line at the right.

- 16 Two skew lines are ... in the same plane. 16.....
- 17 If a line is parallel to each of two intersecting planes, it is ... parallel to their line of intersection. 17.....
- 18 If the faces of a polyhedron are regular polygons, the polyhedron is ... regular. 18.....
- 19 Two prisms which have unequal bases and unequal altitudes ... have equal volumes. 19.....
- 20 If a plane is parallel to one of two intersecting lines, it is ... parallel to the other. 20.....