223D HIGH SCHOOL EXAMINATION

TRIGONOMETRY

Thursday, June 17, 1920-1.15 to 4.15 p. m., only

Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in trigonometry.

The minimum time requirement for plane trigonometry is two recitations a week for a school year; for plane and spheric trigonometry three recitations a week for a school year.

Students taking this examination may use textbooks and notes prepared previous to the examination, but there must be no communication among students after the examination has begun.

Candidates for plane trigonometry should answer six questions, ineluding three from group I and three from group II.

Candidates for plane and spheric trigonometry should answer six questions, including two from group I, two from group II and two from group III.

A, B and C represent the angles of a triangle ABC; a, b and c represent the respective opposite sides. In a right triangle, C represents the right angle.

Give special attention to neatness and arrangement of work.

Group I

1 a If
$$\tan A = \frac{a}{a+1}$$
 and $\tan B = \frac{1}{2a+1}$
prove that $\tan (A+B)=1$

b Solve the equation $2 \cos^2 A = 1 - \sin A$ for all values of A from 0° to 360° . Check the largest angle found.

2 Prove the identity
$$\frac{\cos 2x}{1+\sin 2x} = \frac{\cot x - 1}{\cot x + 1}$$

3 a Without the use of tables, find the value of sin 15°, leaving the answer in radical form.

b By the use of logarithms find the value of

$$\frac{0.076 \times \sqrt[3]{57.46}}{(2.34)^3}$$

4 If the angle A lies between 180° and 270° and $\tan A = \frac{5}{15}$, (a) find $\sin A$ and $\cos A$, (b) using values found in the answer to (a) find $\sin 2A$ and $\cos \frac{A}{2}$

5 a Show that
$$\frac{\sin 2A - \sin A}{\cos A - \cos 2A} = \cot \frac{3A}{2}$$

b Solve for x: $7^{2x+3} = 43$

Group II

6 In each of the following triangles state the number of solutions and show in full on your paper the reason for your conclusion in each case:

(1)
$$b = 75.3$$
 $a = 49.7$ $A = 40^{\circ}$
(2) $a = 67.4$ $b = 97.6$ $c = 30.2$
(3) $c = 156.3$ $b = 104.8$ $B = 142^{\circ}$
(4) $a = 56.7$ $b = 38.4$ $A = 58^{\circ}$ 20'
(5) $a = 18.0$ $c = 9.0$ $C = 30^{\circ}$

7 From the top of a lighthouse 257 feet above the sea, the angles of depression to two boats, in line with the lighthouse, are observed to be 14° and 32° respectively; find the distance between the two boats.

8 Given a=71.2, b=64.8, c=37; find all the angles of the triangle.

9 The longer diagonal of a parallelogram is 500 feet and the angles it makes with the sides are 46° 36′ and 10° 12′; find the lengths of the sides and the area of the parallelogram.

Group III

10 Solve the right spheric triangle, given

$$a = 36^{\circ} 25' 30''$$

 $b = 85^{\circ} 40'$
 $C = 90^{\circ}$

11 Solve the spheric triangle, given

$$A = 74^{\circ} 40'$$

 $B = 67^{\circ} 30'$
 $C = 49^{\circ} 50'$

12 A triangle on the earth's surface has its vertices respectively at the north pole, zero latitude and zero longitude, and zero latitude and 30° west longitude; considering the earth as a sphere with radius 3956 miles, find the area of this triangle.