

A.APR.A.1: Arithmetic Operations on Polynomials

POLYNOMIALS AND QUADRATICS

A.APR.A.1: Arithmetic Operations on Polynomials

A. Perform arithmetic operations on polynomials.

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials (linear, quadratic).

Overview of Lesson

- activate prior knowledge and review learning objectives (see above)
 - explain vocabulary and/or big ideas associated with the lesson
 - connect assessment practices with curriculum
 - model an assessment problem and solution strategy
 - facilitate guided discussion of student activity
 - facilitate guided practice of student activity
 - facilitate a summary and share out of student work
- Homework – Write the Math Assignment

Vocabulary

- **Polynomial:** A monomial or the sum of two or more monomials whose exponents are positive.

Example: $5a^2 + ba - 3$

- **Monomial:** A polynomial with one term; it is a number, a variable, or the product of a number (the coefficient) and one or more variables

Examples: $-\frac{1}{4}$, x^2 , $4a^2b$, -1.2 , $m^2n^3p^4$

- **Binomial:** An algebraic expression consisting of two terms

Example $(5a + 6)$

- **Trinomial:** A polynomial with exactly three terms.

Example $(a^2 + 2a - 3)$

- **Like Terms:** Like terms must have **exactly the same base and the same exponent**. Their coefficients may be different. Real numbers are like terms.

Example: Given the expression

$$1x^2 + 2y + 3x^2 + 4x + 5x^3 + 6y^2 + 7y + 8x^3 + 9y^2,$$

the following are like terms:

$$1x^2 \text{ and } 3x^2$$

$$2y \text{ and } 7y$$

4x has no other like terms in the expression

$$5x^3 \text{ and } 8x^3$$

$$6y^2 \text{ and } 9y^2$$

$$1x^2 \text{ and } 3x^2 = 4x^2$$

$$2y \text{ and } 7y = 9y$$

4x has no other like terms in the expression = 4x

$$5x^3 \text{ and } 8x^3 = 13x^3$$

$$6y^2 \text{ and } 9y^2 = 15y^2$$

$$1x^2 + 2y + 3x^2 + 4x + 5x^3 + 6y^2 + 7y + 8x^3 + 9y^2 = 4x^2 + 9y + 4x + 13x^3 + 15y^2$$

Adding and Subtracting Polynomials: To add or subtract polynomials, arrange the polynomials one above the other with like terms in the same columns. Then, add or subtract the coefficients of the like terms in each column and write a new expression.

Addition Example

$$\text{Add: } (3r^4 - 9r^3 - 8) + (4r^4 + 8r^3 - 8)$$

$$3r^4 \quad -9r^3 \quad -8$$

$$\underline{4r^4 \quad +8r^3 \quad -8}$$

$$7r^4 \quad -r^3 \quad -16$$

Subtraction Example

$$\text{Subtract: } (3r^4 - 9r^3 - 8) - (4r^4 + 8r^3 - 8)$$

$$3r^4 \quad -9r^3 \quad -8$$

$$\underline{-(4r^4) \quad -(+8r^3) \quad -(-8)}$$

$$-1r^4 \quad -17r^3 \quad +0$$

Multiplying Polynomials: To multiply two polynomials, multiply each term in the first polynomial by each term in the second polynomial, then combine like terms.

Example:

$$\text{Multiply: } (-8r^2 - 9r + 7)(-5r + 1)$$

STEP 1: Multiply the first term in the first polynomial by each term in the second polynomial, as follows:

$$-8r^2(-5r+1)$$

$$-8r^2(-5r)+-8r^2(1)$$

$$\boxed{40r^3-8r^2}$$

STEP 2. Multiply the next term in the first polynomial by each term in the second polynomial, as follows:

$$-9r(-5r+1)$$

$$-9r(-5r)+-9r(1)$$

$$\boxed{45r^2-9r}$$

Lesson Plan

STEP 3. Multiply the next term in the first polynomial by each term in the second polynomial, as follows:

$$7(-5r+1)$$

$$7(-5r)+7(1)$$

$$\boxed{-35r+7}$$

STEP 4. Combine like terms from each step.

$$40r^3 - 8r^2 + 45r^2 - 9r - 35r + 7$$

$$\boxed{40r^3 \quad +37r^2 \quad -44r \quad +7}$$

REGENTS PROBLEMS TYPICAL OF THIS STANDARD

1. If the difference $(3x^2 - 2x + 5) - (x^2 + 3x - 2)$ is multiplied by $\frac{1}{2}x^2$, what is the result, written in standard form?

2. If $A = 3x^2 + 5x - 6$ and $B = -2x^2 - 6x + 7$, then $A - B$ equals
 - a. $-5x^2 - 11x + 13$
 - b. $5x^2 + 11x - 13$
 - c. $-5x^2 - x + 1$
 - d. $5x^2 - x + 1$

3. Subtract $5x^2 + 2x - 11$ from $3x^2 + 8x - 7$. Express the result as a trinomial.

Lesson Plan

4. A company produces x units of a product per month, where $C(x)$ represents the total cost and $R(x)$ represents the total revenue for the month. The functions are modeled by $C(x) = 300x + 250$ and $R(x) = -0.5x^2 + 800x - 100$. The profit is the difference between revenue and cost where $P(x) = R(x) - C(x)$. What is the total profit, $P(x)$, for the month?
- a. $P(x) = -0.5x^2 + 500x - 150$ c. $P(x) = -0.5x^2 - 500x + 350$
b. $P(x) = -0.5x^2 + 500x - 350$ d. $P(x) = -0.5x^2 + 500x + 350$
5. Fred is given a rectangular piece of paper. If the length of Fred's piece of paper is represented by $2x - 6$ and the width is represented by $3x - 5$, then the paper has a total area represented by
- a. $5x - 11$ c. $10x - 22$
b. $6x^2 - 28x + 30$ d. $6x^2 - 6x - 11$
6. Express the product of $2x^2 + 7x - 10$ and $x + 5$ in standard form.

A.APR.A.1: Arithmetic Operations on Polynomials

Answer Section

1. ANS:

$$x^4 - \frac{5}{2}x^3 + \frac{7}{2}x^2$$

Strategy. First, find the difference between $(3x^2 - 2x + 5) - (x^2 + 3x - 2)$, then use the distributive property to multiply the difference by $\frac{1}{2}x^2$. Simplify as necessary.

STEP 1. Find the difference between $(3x^2 - 2x + 5) - (x^2 + 3x - 2)$. To subtract polynomials, change the signs of the subtrahend and add.

Given: $(3x^2 - 2x + 5)$ $-(x^2 + 3x - 2)$	Change the signs and add: $3x^2 - 2x + 5$ $-x^2 - 3x + 2$ $2x^2 - 5x + 7$
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STEP 2. Multiply $2x^2 - 5x + 7$ by $\frac{1}{2}x^2$.

$$\frac{1}{2}x^2(2x^2 - 5x + 7)$$

$$x^4 - \frac{5}{2}x^3 + \frac{7}{2}x^2$$

PTS: 2 REF: 061528ai NAT: A.APR.A.1 TOP: Operations with Polynomials

KEY: multiplication

2. ANS: B

Strategy: To subtract, change the signs of the subtrahend and add.

Given: $3x^2 + 5x - 6$ $-(-2x^2 - 6x + 7)$	Change the signs and add: $3x^2 + 5x - 6$ $+2x^2 + 6x - 7$ $5x^2 + 11x - 13$
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PTS: 2 REF: 061403ai NAT: A.APR.A.1 TOP: Addition and Subtraction of Polynomials

KEY: subtraction

3. ANS:

Strategy: To subtract, change the signs of the subtrahend and add.

Given: $3x^2 + 8x - 7$ $-(5x^2 + 2x - 11)$	Change the signs and add: $3x^2 + 8x - 7$ $-5x^2 - 2x + 11$ $-2x^2 + 6x + 4$
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PTS: 2 REF: 011528ai NAT: A.APR.A.1 TOP: Addition and Subtraction of Polynomials

Lesson Plan

KEY: subtraction

4. ANS: B

Strategy: Substitute $R(x)$ and $C(x)$ into $P(x) = R(x) - C(x)$.

Given: $P(x) = R(x) - C(x)$

$$R(x) = -0.5x^2 + 800x - 100$$

$$C(x) = 300x + 250$$

Therefore: $P(x) = (-0.5x^2 + 800x - 100) - (300x + 250)$

$$P(x) = -0.5x^2 + 800x - 100 - 300x - 250$$

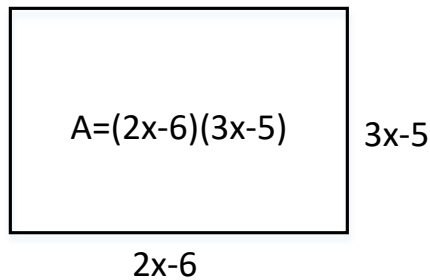
$$P(x) = -0.5x^2 + 500x - 350$$

PTS: 2 REF: 081406ai NAT: A.APR.A.1 TOP: Addition and Subtraction of Polynomials

KEY: subtraction

5. ANS: B

Strategy: Draw a picture and use the area formula for a rectangle: $A = lw$.



$$A = (2x - 6)(3x - 5)$$

$$A = 6x^2 - 10x - 18x + 30$$

$$A = 6x^2 - 28x + 30$$

PTS: 2 REF: 011510ai NAT: A.APR.A.1 TOP: Multiplication of Polynomials

Lesson Plan

6. ANS:

$$2x^3 + 17x^2 + 25x - 50$$

Strategy: Use the distribution property to multiply polynomials, then simplify.

STEP 1. Use the distributive property

$$(2x^2 + 7x - 10)(x + 5)$$

$$2x^3 + 10x^2 + 7x^2 + 35x - 10x - 50$$

$$2x^3 + 17x^2 + 25x - 50$$

STEP 2. Simplify by combining like terms.

$$2x^3 + 10x^2 + 7x^2 + 35x - 10x - 50$$

$$2x^3 + 17x^2 + 25x - 50$$

PTS: 2

REF: 081428ai

NAT: A.APR.A.1

TOP: Multiplication of Polynomials

Homework - Write the Math Assignment

START Write your name, date, topic of lesson, and class on your paper.
 NAME: Mohammed Chen
 DATE: December 18, 2015
 LESSON: Missing Number in the Average
 CLASS: Z

PART 1a. Copy **the problem** from the lesson and underline/highlight key words.
 PART 1b. State your understanding of **what the problem is asking**.
 PART 1c. **Answer** the problem.
 PART 1d. Explanation of **strategy** with all work shown.

PART 2a. Create **a new problem** that addresses the same math idea.
 PART 2b. State your understanding of **what the new problem is asking**.
 PART 2c. **Answer** the new problem.
 PART 2d. Explanation of **strategy** used in solving the new problem with all work shown.

Clearly label each of the eight parts.

Grading Rubric

Each homework writing assignment is graded using a four point rubric, as follows:

Part 1. The Original Problem	Up to 2 points will be awarded for: a) correctly restating the original problem; b) explicitly stating what the original problem is asking; c) answering the original problem correctly; and d) explaining the math.
Part 2. My New Problem	Up to 2 points will be awarded for: a) creating a new problem similar to the original problem; b) explicitly stating what the new problem is asking; c) answering the new problem correctly; and d) explaining the math.

This assignment/activity is designed to incorporate elements of [Polya's four step universal algorithm](#) for problem solving with the idea that writing is thinking. Polya's four steps for solving any problem are:

1. Read and understand the problem.
2. Develop a strategy for solving the problem.
3. Execute the strategy.
4. Check the answer for reasonableness.

EXEMPLAR OF A WRITING THE MATH ASSIGNMENT

Part 1a. The Problem

TOP Electronics is a small business with five employees. The mean (average) weekly salary for the five employees is \$360. If the weekly salaries of four of the employees are \$340, \$340, \$345, and \$425, what is the salary of the fifth employee?

Part 1b. What is the problem asking?

Find the salary of the fifth employee.

Part 1c. Answer

The salary of the fifth employee is \$350 per week.

Part 1d. Explanation of Strategy

The arithmetic mean or average can be represented algebraically as:

$$\bar{X} = \frac{x_1 + x_2 + \dots + x_n}{n}$$

I put information from the problem into the formula. The problem says there are 5 employees, so $n = 5$. The problem also gives the mean (average) salary and the salaries of 4 of the employees. These numbers can be substituted into the formula as follows:

$$360 = \frac{340 + 340 + 345 + 425 + x_5}{5}$$

$$1800 = 340 + 340 + 345 + 425 + x_5$$

$$1800 = 1450 + x_5$$

$$1800 - 1450 = x_5$$

$$350 = x_5$$

$$\text{Check: } 360 = \frac{340 + 340 + 345 + 425 + 350}{5} = \frac{1800}{5} = 360$$

Part 2a. A New Problem

Joseph took five math exams this grading period and his average score on all of the exams is 88. He remembers that he received test scores of 78, 87, 94, and 96 on four of the examinations, but he has lost one examination and cannot remember what he scored on it. What was Joseph's score on the missing exam?

Part 2b. What is the new problem asking?

Find Joseph's score on the missing exam.

Part 2c. Answer to New Problem

Joseph received a score of 85 on the missing examination.

Part 2d. Explanation of Strategy

I substitute information from the problem into the formula for the arithmetic mean, as follows:

$$88 = \frac{78 + 87 + 94 + 96 + x_5}{5}$$

$$440 = 355 + x_5$$

$$85 = x_5$$

$$88 = \frac{78 + 87 + 94 + 96 + 85}{5} = \frac{440}{5} = 88$$

The answer makes sense.