A.APR.B.3: Find Zeros of Polynomials

POLYNOMIALS AND QUADRATICS
A.APR.B.3: Find Zeros of Polynomials

B. Understand the relationship between zeros and factors of polynomials.

3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Overview of Lesson
- activate prior knowledge and review learning objectives (see above)
- explain vocabulary and/or big ideas associated with the lesson
- connect assessment practices with curriculum
- model an assessment problem and solution strategy
- facilitate guided discussion of student activity
- facilitate guided practice of student activity
Selected problem set(s)
- facilitate a summary and share out of student work
Homework – Write the Math Assignment

Vocabulary

Multiplication Property of Zero: The **multiplication property of zero** says that if the product of two numbers or expressions is zero, then one or both of the numbers or expressions must equal zero. More simply, if $x \cdot y = 0$, then either $x = 0$ or $y = 0$, or, x and y both equal zero.

Factor: A **facto**r is:
1) a whole number that is a **divisor** of another number, or
2) an algebraic expression that is a **divisor** of another algebraic expression.

Examples:
- 1, 2, 3, 4, 6, and 12 all divide the number 12,
  so 1, 2, 3, 4, 6, and 12 are all factors of 12.
- $(x-3)$ and $(x+2)$ will divide the trinomial expression $x^2 - x - 6$,
  so $(x-3)$ and $(x+2)$ are both factors of the $x^2 - x - 6$.

Zeros: A **zero** of an equation is a **solution** or **root** of the equation. The words **zero**, **solution**, and **root** all mean the same thing. The zeros of a polynomial expression are found by finding the value of x when the value of y is 0. This done by making and solving an equation with the value of the polynomial expression equal to zero.

Example:
- The **zeros** of the trinomial expression $x^2 + 2x - 24$ can be found by writing and then factoring the equation:

$$x^2 + 2x - 24 = 0$$
$$(x+6)(x-4) = 0$$

After factoring the equation, use the **multiplication property of zero** to find the zeros, as follows:

$$(x+6)(x-4)=0$$

$$\therefore\ x+6=0\ \textit{and / or}\ x-4=0$$

$$\textit{If}\ x+6=0,\ \textit{then}\ x=-6$$

$$\textit{If}\ x-4=0,\ \textit{then}\ x=+4$$

The zeros of the expression $x^2+2x-24=0$ are -6 and +4.

Check: You can check this by substituting both -6 or +4 into the expression, as follows:

Check for -6

$$x^2+2x-24$$

$$(-6)^2+2(-6)-24$$

$$36-12-24$$

$$0$$

Check for +4

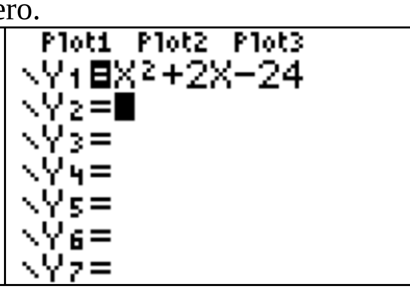
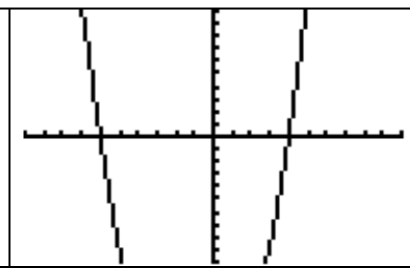
$$x^2+2x-24$$

$$(4)^2+2(4)-24$$

$$16+8-24$$

$$0$$

x-axis intercepts: The zeros of an expression can also be understood as the **x-axis intercepts** of the graph of the equation when $f(x)=0$. This is because the coordinates of the x-axis intercepts, by definition, have y-values equal to zero, and is the same as writing an equation where the expression is equal to zero.

The roots of $x^2+2x-24=0$ are $x=-6$ and $x=4$. These are the x coordinate values of the x-axis intercepts.		

BIG IDEA #1

Starting with Factors and Finding Zeros

Remember that the **factors** of an expression are *related to* the **zeros** of the expression by the **multiplication property of zero**. Thus, if you know the **factors**, it is easy to find the **zeros**.

Example: The factors of an expression are $(2x+2)$, $(x+3)$ and $\left(x-1\right)$.

The zeros are found as follows using the multiplication property of zero:

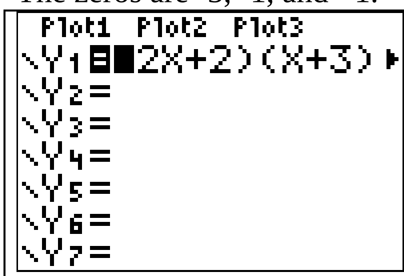
$$(2x+2)(x+3)(x-1)=0$$

$$\therefore\ 2x+2=0\ \textit{and}\ x=-1$$

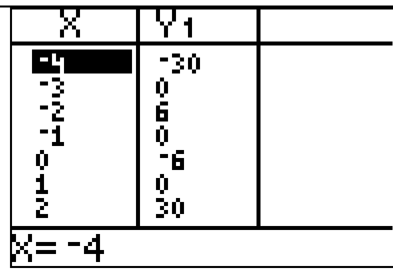
$$\textit{and / or}\ x+3=0\ \textit{and}\ x=-3$$

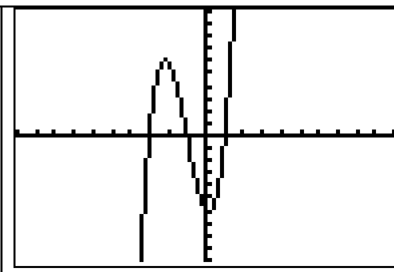
$$\textit{and / or}\ x-1=0\ \textit{and}\ x=1$$

The zeros are -3, -1, and +1.



Be sure to input all factors.





BIG IDEA #2

Starting with Zeros and Finding Factors

If you know the **zeros** of an expression, you can work backwards using the **multiplication property of zero** to find the **factors** of the expression. For example, if you inspect the graph of an equation and find that it has **x-intercepts** at $x=3$ and $x=-2$, you can write:

$$x=3$$

$$\therefore\ (x-3)=0$$

$$\textit{and}$$

$$x=-2$$

$$\therefore\ (x+2)=0$$

The equation of the graph has **factors** of $(x-3)$ and $(x+2)$, so you can write the equation:

$$(x-3)(x+2)=0$$

which simplifies to

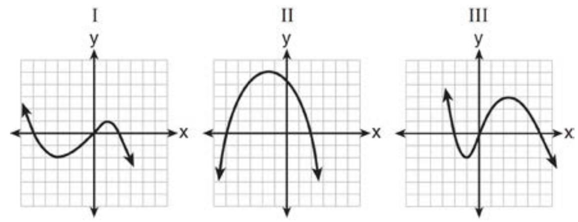
$$x^2+2x-3x-6=f(x)$$

$$x^2-x-6=f(x)$$

With practice, you can probably move back and forth between the **zeros** of an expression and the **factors** of an expression with ease.

REGENTS PROBLEMS TYPICAL OF THIS STANDARD

1. A polynomial function contains the factors x, $x-2$, and $x+5$. Which graph(s) below could represent the graph of this function?



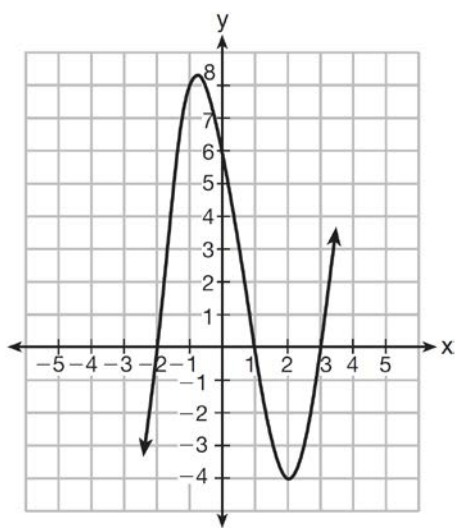
a. I, only
b. II, only
c. I and III
d. I, II, and III

2. Which equation(s) represent the graph below?

I $y=(x+2)(x^2-4x-12)$

II $y=(x-3)(x^2+x-2)$

III $y=(x-1)(x^2-5x-6)$



a. I, only
b. II, only
c. I and II
d. II and III

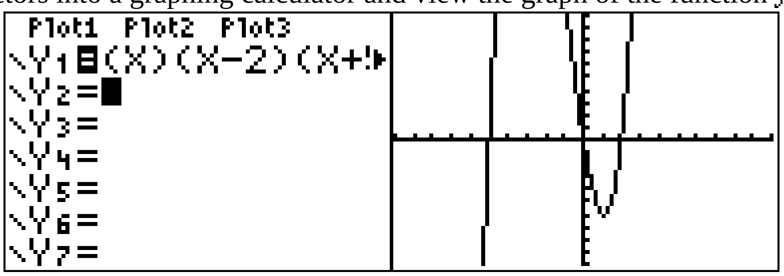
3. The zeros of the function $f(x) = (x + 2)^2 - 25$ are

a. -2 and 5
b. -3 and 7
c. -5 and 2
d. -7 and 3

A.APR.B.3: Find Zeros of Polynomials
Answer Section

1. ANS: A
Stategy 1. Convert the factors to zeros, then find the graph(s) with the corresponding zeros.
STEP 1. Convert the factors to zeros.
A factor of $x-0$ equates to a zero of the polynomial at x=0.
A factor of $x-2$ equates to a zero of the polynomial at x=2.
A factor of $x+5$ equates to a zero of the polynomial at x=-5.
STEP 2. Find the zeros of the graphs.
Graph I has zeros at -5, 0, and 2.
Graph II has zeros at -5 and 2.
Graph III has zeros at -2, 0, and 5.
Answer choice *a* is correct.

Strategy 2: Input the factors into a graphing calculator and view the graph of the function $y=(x)(x-2)(x+5)$.



Note: This graph has the same zeros as graph I, but the end behaviors of the graph are reversed. This graph is a reflection in the x-axis of graph I and the reversal is caused by a change in the sign of the leading coefficient in the expansion of $y=(x)(x-2)(x+5)$. It makes no difference in answering this problem. The zeros are the same and the correct answer choice is answer choice *a*.

PTS: 2 REF: 011524ai NAT: A.APR.B.3 TOP: Zeros of Polynomials

2. ANS: B
Strategy: Factor the trinomials in each equation, then convert the factors into zeros and select the equations that have zeros at -2, 1, and 3.

STEP 1.

I	II	III
$y=(x+2)(x^2-4x-12)$	$y=(x-3)(x^2+x-2)$	$y=(x-1)(x^2-5x-6)$
$y=(x+2)(x-6)(x+2)$	$y=(x-3)(x+2)(x-1)$	$y=(x-1)(x-6)(x+1)$
Zeros at -2, 6, and -2	Zeros at 3, -2, and 1	Zeros at 1, 6, and -1
(Wrong Choice)	(Correct Choice)	(Wrong Choice)

The correct answer choice is *b*.

PTS: 2 REF: 061512ai NAT: A.APR.B.3 TOP: Zeros of Polynomials

3. ANS: D
Strategy: Use root operations to solve $f(x)=(x+2)^2-25$ for $f(x)=0$.

Lesson Plan

$$
\begin{aligned}
f(x) &= (x+2)^2 - 25 \\
0 &= (x+2)^2 - 25 \\
25 &= (x+2)^2 \\
\sqrt{25} &= \sqrt{(x+2)^2} \\
\pm 5 &= x+2 \\
-2 \pm 5 &= x \\
-7 \text{ and } 3 &= x
\end{aligned}
$$

PTS: 2 REF: 081418ai NAT: F.IF.C.8 TOP: Zeros of Polynomials

Homework - Write the Math Assignment

START Write your name, date, topic of lesson, and class on your paper.

NAME: Mohammed Chen
DATE: December 18, 2015
LESSON: Missing Number in the Average
CLASS: Z

PART 1a. Copy **the problem** from the lesson and underline/highlight key words.
PART 1b. State your understanding of **what the problem is asking**.
PART 1c. **Answer** the problem.
PART 1d. Explanation of **strategy** with all work shown.

PART 2a. Create **a new problem** that addresses the same math idea.
PART 2b. State your understanding of **what the new problem is asking**.
PART 2c. **Answer** the new problem.
PART 2d. Explanation of **strategy** used in solving the new problem with all work shown.

Clearly label each of the eight parts.

Grading Rubric

Each homework writing assignment is graded using a four point rubric, as follows:

Part 1. The Original Problem	**Up to 2** points will be awarded for: a) correctly restating the original problem; b) explicitly stating what the original problem is asking; c) answering the original problem correctly; and d) explaining the math.
Part 2. My New Problem	**Up to 2** points will be awarded for: a) creating a new problem similar to the original problem; b) explicitly stating what the new problem is asking; c) answering the new problem correctly; and d) explaining the math.

This assignment/activity is designed to incorporate elements of Polya's four step universal algorithm for problem solving with the idea that writing is thinking. Polya's four steps for solving any problem are:

1. Read and understand the problem.
2. Develop a strategy for solving the problem.
3. Execute the strategy.
4. Check the answer for reasonableness.

EXEMPLAR OF A WRITING THE MATH ASSIGNMENT

Part 1a. **The Problem**

TOP Electronics is a small business with five employees. The mean (average) weekly salary for the five employees is $360. If the weekly salaries of four of the employees are $340, $340, $345, and $425, what is the salary of the fifth employee?

Part 1b. **What is the problem asking?**

Find the salary of the fifth employee.

Part 1c. **Answer**

The salary of the fifth employee is $350 per week.

Part 1d. **Explanation of Strategy**

The arithmetic mean or average can be represented algebraically as:

$$\overline{X} = \frac{x_1 + x_2 + \ldots + x_n}{n}$$

I put information from the problem into the formula. The problem says there are 5 employees, so $n = 5$. The problem also gives the mean (average) salary and the salaries of 4 of the employees. These numbers can be substituted into the formula as follows:

$$360 = \frac{340 + 340 + 345 + 425 + x_5}{5}$$

$$1800 = 340 + 340 + 345 + 425 + x_5$$

$$1800 = 1450 + x_5$$

$$1800 - 1450 = x_5$$

$$350 = x_5$$

Check: $360 = \frac{340 + 340 + 345 + 425 + 350}{5} = \frac{1800}{5} = 360$

Part 2a. **A New Problem**

Joseph took five math exams this grading period and his average score on all of the exams is 88. He remembers that he received test scores of 78, 87, 94, and 96 on four of the examinations, but he has lost one examination and cannot remember what he scored on it. What was Joseph's score on the missing exam?

Part 2b **What is the new problem asking?**

Find Joseph's score on the missing exam.

Part 2c. **Answer to New Problem**

Joseph received a score of 85 on the missing examination.

Part 2d. **Explanation of Strategy**

I substitute information from the problem into the formula for the arithmetic mean, as follows:

$$88 = \frac{78 + 87 + 94 + 96 + x_5}{5}$$

$$440 = 355 + x_5$$

$$85 = x_5$$

$$88 = \frac{78 + 87 + 94 + 96 + 85}{5} = \frac{440}{5} = 88$$

The answer makes sense.