F – Inequalities, Lesson 3, Modeling Linear Inequalities (r. 2018)

INEQUALITIES

Modeling Linear Inequalities

Common Core Standards	Next Generation Standards
A-CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* PARCC: Tasks are limited to linear, quadratic, or exponential ns with integer exponents.	**AI-A.CED.1** Create equations and inequalities in one variable to **represent a real-world context**. (Shared standard with Algebra II) Notes: • **This is strictly the development of the model (equation/inequality).** • **Limit equations to linear, quadratic, and exponentials of the form $f(x) = a(b)^x$ where $a > 0$ and $b > 0$ ($b \neq 1$).** • **Work with geometric sequences may involve an exponential equation/formula of the form $a_n = ar^{n-1}$, where a is the first term and r is the common ratio.** • **Inequalities are limited to linear inequalities.** • Algebra I tasks do not involve compound inequalities.
A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*	**AI-A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. e.g., Represent inequalities describing nutritional and cost constraints on combinations of different foods.

NOTE: This lesson is related to Expressions and Equations, Lesson 4, Modeling Linear Equations

LEARNING OBJECTIVES

Students will be able to:

1) Model real-world word problems as mathematical inequalities.

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

See key words and their mathematical translations under big ideas.

<u>**BIG IDEAS**</u>

Translating words into mathematical expressions and equations is an important skill.

<u>**General Approach**</u>

The general approach is as follows:

1. Read and understand the entire problem.
2. Underline key words, focusing on variables, operations, and equalities or inequalities.
3. Convert the key words to mathematical notation (consider meaningful variable names other than x and y).
4. Write the final expression or equation.
5. Check the final expression or equation for reasonableness.

<u>**The Solution to a Linear Inequality Can Represent a Part of a Number Line.**</u>

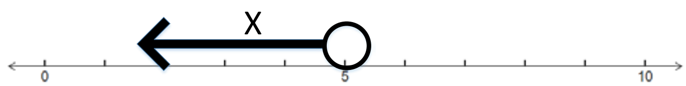
<u>**A linear inequality**</u> describes a part of a number line with either: 1) an upper limit; 2) a lower limit; or 3) both upper and lower limits.

Example - Upper Limit

Let A represent age.

A playground for little kids will not allow children older than four years. If A represents age in years, this can be represented as

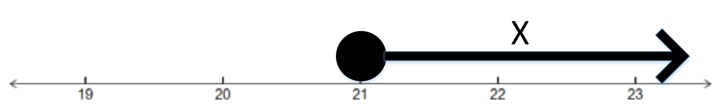
$$A < 5$$



Example - Lower Limit

A state will not allow persons below the age of 21 to drink alcohol. If A represents age in years, the legal drinking age can be represented as

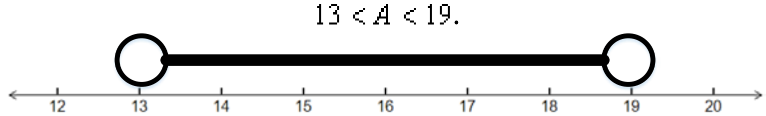
$$A \geq 21$$



Example - Both Upper and Lower Limits

A high school football team limits participation to students from 14 to 18 years old. If A represents age in years, participation on the football team can be represented as

$$13 < A < 19.$$



Key English Words and Their Mathematical Translations

These English Words	Usually Mean	*Examples: English becomes math*
is, are	equals	*the sum of 5 and x is 20 becomes 5 + x = 20*
more than, greater than	inequality $>$	*x is greater than y becomes x > y* *x is more than 5 becomes x > 5* *5 is more than x becomes 5 > x*
greater than or equal to, a minimum of, at least	inequality $\geq$	*x is greater than or equal to y becomes* *the minimum of x is 5 becomes* *x is at least 20 becomes*
less than	inequality $<$	*x is less than y becomes* *x is less than 5 becomes* *5 is less than x becomes*
less than or equal to, a maximum of, not more than	Inequality $\leq$	*X is less than or equal to y becomes* *The maximum of x is 5 becomes* *X is not more than becomes*

Examples of Modeling Specific Types of Inequality Problems

Spending Related Inequalities

Typical Problem in English	Mathematical Translation	Hints and Strategies
Mr. Braun has \$75.00 to spend on pizzas and soda pop for a picnic. Pizzas cost \$9.00 each and the drinks cost \$0.75 each. Five times as many drinks as pizzas are needed. What is the maximum number of pizzas that Mr. Braun can buy?	\$75 is the most that can be spent, so start with the idea that $75 \geq \text{something}$ • Let P represent the # of Pizzas and 9P represent the cost of pizzas. • Let 5P represent the number of drinks and .75(5P) represent the cost of drinks. Write the expression for total costs: $9P + .75(5P)$ Combine the left expression, inequality sign, and right expression into a single inequality. $75 \geq 9P + .75(5P)$ Solve the inequality for P. $75 \geq 9P + .75(5P)$ $75 \geq 9P + 3.75P$ $75 \geq 12.75P$ $\frac{75}{12.75} \geq P$ $5.9 \geq P$	1. Identify the minimum or maximum amount on one side of the inequality. 2. Pay attention to the direction of the inequality and whether the boundary is included or not included in the solution set. 3. Develop the other side of the inequality as an expression.

	It does not make sense to order 5.9 pizzas, and there is not enough money to buy six pizzas, so round down. Mr. Braun has enough money to buy 5 pizzas.	

How Many? Type of Inequalities

Typical Problem in English	Mathematical Translation	Hints and Strategies
There are 461 students and 20 teachers taking buses on a trip to a museum. Each bus can seat a maximum of 52. What is the *least* number of buses needed for the trip?	Write: $\frac{461+20}{52} \geq b$ Solve $\frac{486}{52} \geq b$ $9.25 \geq b$ A fraction/decimal answer does not make sense because you cannot order a part of a bus. Only an integer answer will work. The lowest integer value in the solution set is 10, so 10 buses will be needed for the trip.	Ignore your real life experience with field trips and buses, like how big or small are the students and teachers, or if student attendance will be influenced by how interesting the museum sounds.

Geometry Based Inequalities

Typical Problem in English	Mathematical Translation	Hints and Strategies
The length of a rectangle is 15 and its width is *w*. The perimeter of the rectangle is, *at most*, 50. Write and solve an inequality to find the longest possible width.	The formula for the perimeter of a rectangle is $2l + 2w = P$. Substitute information from the context into this formula and write: $2(15) + 2w \leq 50$ Then, solve for *w*. $2(15) + 2w \leq 50$ $30 + 2w \leq 50$ $2w \leq 20$ $w \leq 10$ The longest possible width is 10 feet.	Use a formula and substitute information from the problem into the formula.

DEVELOPING ESSENTIAL SKILLS

A swimmer plans to swim *at least* 100 laps during a 6-day period. During this period, the swimmer will increase the number of laps completed each day by one lap. What is the *least* number of laps the swimmer must complete on the first day?

Write the left expression and inequality sign as follows:

$100 \geq$

Let d represent the number of laps the swimmer must complete on the 1st day.
Let d+1 represent the number of laps the swimmer must complete on the 2nd day.
Let d+2 represent the number of laps the swimmer must complete on the 3rd day.
Let d+3 represent the number of laps the swimmer must complete on the 4th day.
Let d+4 represent the number of laps the swimmer must complete on the 5th day.
Let d+5 represent the number of laps the swimmer must complete on the 6th day.
Let 6d+15 represent the number of laps the swimmer must complete in total. This is the right expression.

Complete the inequality

$100 \geq 6d + 15$

Solve the inequality

$$100 \geq 6d + 15$$

$$85 \geq 6d$$

$$\frac{85}{6} \geq d$$

$$14.\overline{6} \geq d$$

A swimmer cannot swim a fraction of a lap, so round up to the next integer.
The swimmer must complete 15 laps on the first day.

REGENTS EXAM QUESTIONS (through June 2018)

A.CED.A.1 A.CED.C.3: Modeling Linear Inequalities

151) Connor wants to attend the town carnival. The price of admission to the carnival is \$4.50, and each ride costs an additional 79 cents. If he can spend at most \$16.00 at the carnival, which inequality can be used to solve for *r*, the number of rides Connor can go on, and what is the maximum number of rides he can go on?

1) $0.79 + 4.50r \leq 16.00$; 3 rides
2) $0.79 + 4.50r \leq 16.00$; 4 rides
3) $4.50 + 0.79r \leq 16.00$; 14 rides
4) $4.50 + 0.79r \leq 16.00$; 15 rides

152) Natasha is planning a school celebration and wants to have live music and food for everyone who attends. She has found a band that will charge her \$750 and a caterer who will provide snacks and drinks for \$2.25 per person. If her goal is to keep the average cost per person between \$2.75 and \$3.25, how many people, *p*, must attend?

1) $225 < p < 325$
2) $325 < p < 750$
3) $500 < p < 1000$
4) $750 < p < 1500$

153) The cost of a pack of chewing gum in a vending machine is \$0.75. The cost of a bottle of juice in the same machine is \$1.25. Julia has \$22.00 to spend on chewing gum and bottles of juice for her team and she must buy seven packs of chewing gum. If *b* represents the number of bottles of juice, which inequality represents the maximum number of bottles she can buy?

1) $0.75b + 1.25(7) \geq 22$
2) $0.75b + 1.25(7) \leq 22$
3) $0.75(7) + 1.25b \geq 22$
4) $0.75(7) + 1.25b \leq 22$

154) The acidity in a swimming pool is considered normal if the average of three pH readings, *p*, is defined such that $7.0 < p < 7.8$. If the first two readings are 7.2 and 7.6, which value for the third reading will result in an overall rating of normal?

1) 6.2
2) 7.3
3) 8.6
4) 8.8

155) David has two jobs. He earns $8 per hour babysitting his neighbor's children and he earns $11 per hour working at the coffee shop. Write an inequality to represent the number of hours, *x*, babysitting and the number of hours, *y*, working at the coffee shop that David will need to work to earn a minimum of $200. David worked 15 hours at the coffee shop. Use the inequality to find the number of full hours he must babysit to reach his goal of $200.

156) Joy wants to buy strawberries and raspberries to bring to a party. Strawberries cost $1.60 per pound and raspberries cost $1.75 per pound. If she only has $10 to spend on berries, which inequality represents the situation where she buys *x* pounds of strawberries and *y* pounds of raspberries?

1) $1.60x + 1.75y \le 10$
2) $1.60x + 1.75y \ge 10$
3) $1.75x + 1.60y \le 10$
4) $1.75x + 1.60y \ge 10$

SOLUTIONS

151) ANS: 3
Strategy: Write and solve an inequality that relates total costs to how much money Connor has.

STEP 1. Write the inequality:
The price of admission comes first and is $4.50. Write +4.50
Each ride (r) costs an additional 0.79. Write +0.79*r*
Total costs can be expressed as: $4.50 + 0.79r$
$4.50 + 0.79r$ must be less than or equal to the $16 Connor has.
Write: $4.50 + 0.79r \le 16.00$

STEP 2: Solve the inequality.

Notes	Left Expression	Sign	Right Expression
Given	$4.50 + 0.79r$	$\le$	16.00
Subtract 4.50 from both expressions	$0.79r$	$\le$	11.50
Divide both expressions by 0.79	r	$\le$	$\frac{11.50}{.79}$
Simplify	r	$\le$	14.55696203
Interpret	r	$\le$	14 rides

The correct answer choice is c: $4.50 + 0.79r \le 16.00$; 14 rides

DIMS? Does It Make Sense? Yes. Admissions costs $4.50 and 14 rides cost $14 \times .79 = \$11.06$. After 14 rides, Connor will only have 45 cents left, which is not enough to go on another ride.

$$\$16 - (\$4.50 + \$11.05)$$

$$\$16 - (\$15.55)$$

$$\$0.45$$

PTS: 2 NAT: A.CED.A.1 TOP: Modeling Linear Inequalities

152) ANS: 4
Strategy:

STEP1. Use the definition of average cost.

$$\text{Average Cost} = \frac{\text{total costs}}{\text{number of persons sharing the cost}}$$

Total costs for the band and the caterer are: $\$750 + \$2.25p$

If the average cost is \$3.25, the formula is $\$3.25 = \frac{\$750 + \$2.25p}{p}$

Solve for p

$\$3.25p = \$750 + \$2.25p$

$p = 750$

If the average cost is \$2.75, the formula is $\$2.75 = \frac{\$750 + \$2.25p}{p}$

Solve for p

$\$2.75p = \$750 + \$2.25p$

$.50p = 750$

$p = 1500$

DIMS? Does It Make Sense? Yes. If 750 people attend, the average cost is \$2.25 per person. If 1500 people attend, the average cost is \$3.25 per person. For any number of people between 750 and 1500, the average cost per person will be between \$2.25 and \$3.25.

PTS: 2 NAT: A.CED.A.3 TOP: Modeling Linear Inequalities

153) ANS: 4

Strategy: Examine the answer choices and eliminate wrong answers.

STEP 1. Eliminate answer choices *a* and *c* because both of them have greater than or equal signs. Julia must spend less than she has, not more.

STEP 2. Choose between answer choices *b* and *d*. Answer choice *d* is correct because the term $0.75(7)$ means that Julia must buy 7 packs of chewing gum @ \$0.75 per pack. Answer choice b is incorrect because the term $1.25(7)$ means that Julia will buy 7 bottles of juice.

DIMS? Does It Make Sense? Yes. Answer choice d shows in the first term that Julia will buy 7 packs of gum and the total of the entire expression must be equal to or less than \$22.00.

PTS: 2 NAT: A.CED.A.3 TOP: Modeling Linear Inequalities

154) ANS: 2

Step 1. Recognize that the problem is asking you to identify one pH reading that will result in an average of three readings that is greater than or equal to 7.0 and less than or equal to 7.8.

Step 2. Use algebraic notation to represent the average of three pH readings, then find the answer that gives an average within the required interval.

Step 3.

$$pH_{(average)} = \frac{pH_1 + pH_2 + pH_3}{3}$$

$$pH_{(average)} = \frac{7.2 + 7.6 + pH_3}{3}$$

$$pH_{(average)} = \frac{14.8 + pH_3}{3}$$

Choice a) $pH_{(average)} = \frac{14.8 + pH_3}{3} = \frac{14.8 + 6.2}{3} = \frac{21}{3} = 7.$ This average is not in the required interval, so choice a) is not a correct answer.

Choice b) $pH_{(average)} = \frac{14.8 + pH_3}{3} = \frac{14.8 + 7.3}{3} = \frac{22.1}{3} = 7.3\bar{6} \approx 7.4.$ This average is in the required interval, so choice b) is a correct answer.

Choice c) $pH_{(average)} = \frac{14.8 + pH_3}{3} = \frac{14.8 + 8.6}{3} = \frac{23.4}{3} = 7.8.$ This average is not in the required interval, so choice c) is not a correct answer.

Choice d) $pH_{(average)} = \frac{14.8 + pH_3}{3} = \frac{14.8 + 8.8}{3} = \frac{23.6}{3} = 7.8\bar{6} \approx 7.9.$ This average is not in the required interval, so choice d) is not a correct answer.

Step 4. Does it make sense? Yes.

$$7.0 < p < 7.8$$

$$7.0 < 7.4 < 7.8$$

$$7.0 < \text{choice b} < 7.8$$

PTS: 2 NAT: A.CED.A.1 TOP: Modeling Linear Inequalities

155) ANS:

David must babysit five full hours to reach his goal of $200.

Strategy: Write an inequality to represent David’s income from both jobs, then use it to solve the problem, then interpret the solution.

STEP 1. Write the inequality.

Let x represent the number of hours that David babysits.

Let y represent the number of hours that David works at the coffee shop.

Write: $8x + 11y \geq 200$

STEP 2. Substitute 15 for y and solve for x.

$$8x + 11y \geq 200$$

$$8x + 11(15) \geq 200$$

$$8x + 165 \geq 200$$

$$8x \geq 200 - 165$$

$$8x \geq 35$$

$$x \geq \frac{35}{8}$$

$$x \geq 4.375$$

STEP 3. Interpret the solution.
The problem asks for the number of full hours, so the solution, $x \geq 4.375$, must be rounded up to 5 full hours.

DIMS? Does It Make Sense? Yes. If David works 15 hours at the coffee shop and 5 hours at the library, he will earn more than 200

$$8(5) + 11(15) \geq 200$$

$$40 + 165 \geq 200$$

$$205 \geq 200$$

What does not make sense is why David earns \$8 per hour babysitting and Edith, in the previous problem, only earns \$4 per hour.

PTS: 4 NAT: A.CED.A.3 TOP: Modeling Linear Inequalities

156) ANS: 1

Strategy: There are three terms in each answer choice - sort the information in the problem to write mathematical terms, then eliminate wrong answers.

Given the Words	Write
Strawberries cost \$1.60 per pound.... *x* pounds of strawberries	$1.60x$
raspberries cost \$1.75 per pound.... *y* pounds of raspberries	$1.75y$
she only has \$10 to spend	≤ 10

Combine all three terms and the inequality sign to write: $1.60x + 1.75y \leq 10$

PTS: 2 NAT: A.CED.A.1 TOP: Modeling Linear Inequalities